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# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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VOL. XXXVII.

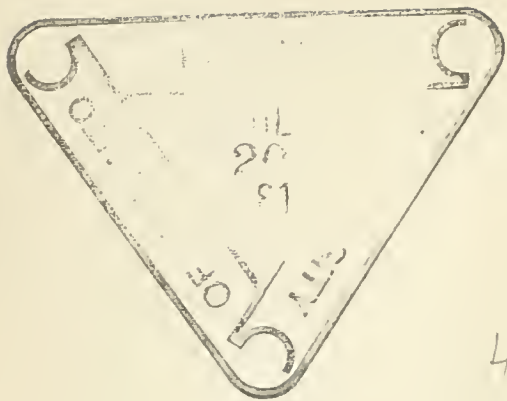
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# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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*Regarding the Mortality Investigation, instituted by the Actuarial Society of America and now in progress. By DAVID PARKS FACKLER, an Ex-President of the Society, Corresponding Member of the Institute of Actuaries.*

IN October 1900, on the morning of the first day of the autumnal meeting of the Actuarial Society of America, Mr. Emory McClintock, Actuary of the Mutual Life Insurance Company of New York, read a paper entitled "The Objects to be attained in Future Investigations of Mortality and Death Loss", in which he stated that probably four-fifths of the publications of companies' general experience possessed only a "casual interest", and that the same amount of effort might much better have been devoted to investigating special classes of business, adding that "it is more important to the future interests of life insurance to learn how fishermen compare with farmers, how physicians compare with clergymen, and the like, than it is to gather all these heterogeneous materials into one grand average in the form of a new life table."

The influence of Mr. McClintock's statements was so much felt that on the second day of the meeting an informal discussion began spontaneously (usually papers are not discussed until the next semi-annual meeting), and the writer had the honour of presenting a resolution voicing the opinion of the society that action should be taken immediately, and naming a Committee of five, Messrs. McClintock, Miller, Van Cise, Weeks, and Wells,

all of whom had previously done more or less in the way of collecting special mortality experience. These gentlemen were requested to consider whether it was desirable and feasible for the Society to collect special mortality experience, and report the result of their deliberations at the next regular meeting of the Society, May 1901.

During the following six months the Committee held many sessions, attended—nearly always—by every member, and on 16 May 1901, reported unanimously in favour of investigating certain classes of risks, of which they submitted a schedule. The Committee's recommendations were adopted unanimously, and they were requested to take charge of the work. In accordance with this request, the Committee immediately sent all the companies the blank forms upon which to report the data, with full directions as to the mode of reporting. Somewhat later Mr. John K. Gore was added to the Committee by vote of the Council, and put in charge of the practical work.

The "Schedule of Classes", with its subsidiary "Table of Weights and Heights", and the "Instructions to Clerks" are given below, with the headings of the Policy Slips on which the companies were to make their returns:—

## SCHEDULE OF CLASSES—MALE LIVES.

### INSURED ON NEW APPLICATIONS.

*Excluding risks taken before 1870 and after 1899, also all taken outside of United States or Canada, and all industrial or sub-standard risks.*

#### POLICY.

1. Amount \$20,000 or over on same application.
2. Amount applied for reduced by Company.
3. Granted on plan not applied for.

#### RACE.

4. Born in Germany.
5. Born in Ireland.
6. Born in Sweden or Norway.
7. Coloured.

#### OCCUPATION.

8. Officer in army. Exclude issued after 1896.
9. " " navy.
10. Marshal, sheriff, police, constable, &c.
11. In city fire department.
12. Physician.
13. Exposed in occupation to electricity.
14. In sawmill.
15. Steel grinder.

16. Pudler, foundry or rolling mill worker, &c., not machinist.
17. Glass worker.
18. Potter.
19. House painter.
20. Printer, not publisher.
21. Tailor.
22. Butcher or meat dealer.
23. Labourer.
24. Contractor.
25. Theatrical occupation.
26. Travelling salesman.
27. Cattle dealer or drover.
28. Hotel keeper, not attending bar.
29. Wine or liquor seller, abstainer.
30.     "         "         "     non-abstainer.
31. Brewer or employee.
32. Distiller or employee.
33. Railway passenger conductor.
34.     "         "     trainmen.     Exclude prior to 1890.
35.     "     mail clerk.
36.     "     express messenger.
37.     "     locomotive engineer.
38.     "         "     fireman.
39. Officer of ocean steam vessel.
40.     "     sailing vessel on ocean or great lakes.
41. Pilot.
42. Seaman or fisherman.

#### PERSONAL RECORD.

43. Has intermittent or irregular pulse.
44. Has pulse below 60.
45. Has taken cure for, or has reformed from, intemperance.
46. Has had asthma.
47.     "     inflammatory rheumatism once.
48.     "         "         "     oftener than once.
49.     "     gout.
50.     "     syphilis.
51.     "     otorrhœa, or running from ear.
52.     "     hepatic colic, or gall stone.
53.     "     renal colic, calculus, or gravel.
54.     "     inflammation of bowels, peritonitis, or appendicitis.
55.     "     blood spitting.
56.     "     hip-joint disease.
57.     "     dyspepsia, with weight A, B or C.
58.     "         "         "     D.

#### WEIGHT AND FAMILY RECORD COMBINED.

59. Weight A : neither parent noted as dead below 70.
60.     "     A : one parent, or both, dead below 70.
61.     "     A : girth of abdomen greater than chest expanded.
62.     "     B : neither parent noted as dead below 70.

63. Weight B: one parent, or both, dead below 70.  
 64. „ B: girth of abdomen greater than chest expanded.  
 65. „ C: parent dead below 70 of consumption, phthisis  
     or tuberculosis.  
 66. „ C: parent dead below 70 of kidney or Bright's disease.  
 67. „ C: „ „ heart disease.  
 68. „ C: „ „ apoplexy or paralysis.  
 69. „ C: both parents dead below 60.  
 70. „ C: both parents reached 75.  
 71. „ D: parent dead below 70 of any lung disease.  
 72. „ D: not included under 58 or 71.

*Height alone.*

73. Height above 6 feet 3 inches.  
 74. „ below 5 feet.

*Family Record alone.*

75. Parent, brother or sister died of cancer.  
 76. „ „ „ has been insane.

CLASSES ACCORDING TO RESIDENCE, BY COUNTIES.

*Last ten years only.*

*Exclude all insured before 1890.*

- |                     |                       |
|---------------------|-----------------------|
| 77. Ala. Mobile.    | 88. Miss. Warren.     |
| 78. „ Montgomery.   | 89. N.C. New Hanover. |
| 79. Ark. Jefferson. | 90. N.M. Santa Fe.    |
| 80. „ Phillips.     | 91. S.C. Charleston.  |
| 81. Col. Arapahoe.  | 92. Tenn. Shelby.     |
| 82. Fla. Duval.     | 93. Tex. Bexar.       |
| 83. „ Monroe.       | 94. „ Dallas.         |
| 84. Ga. Chatham.    | 95. „ Galveston.      |
| 85. Ky. McCracken.  | 96. „ Harris.         |
| 86. La. Orleans.    | 97. „ Travis.         |
| 87. Miss. Adams.    | 98. Va. Norfolk.      |

SUBSIDIARY TABLE OF WEIGHTS.

AGES 15—24.

Height.	D.	C.	B.	A.
5 ft. 0 in.	Under 102	102—144	145—156	Over 156
5 „ 1 „	„ 104	104—146	147—159	„ 159
5 „ 2 „	„ 105	105—149	150—161	„ 161
5 „ 3 „	„ 108	108—152	153—165	„ 165
5 „ 4 „	„ 111	111—157	158—170	„ 170
5 „ 5 „	„ 114	114—161	162—174	„ 174
5 „ 6 „	„ 117	117—166	167—179	„ 179
5 „ 7 „	„ 121	121—170	171—185	„ 185
5 „ 8 „	„ 124	124—175	176—190	„ 190
5 „ 9 „	„ 128	128—180	181—195	„ 195



Height.	D.	C.	B.	A.
5 ft. 10 in.	Under 131	131—185	186—200	Over 200
5 „ 11 „	„ 135	135—191	192—207	„ 207
6 „ 0 „	„ 140	140—198	199—215	„ 215
6 „ 1 „	„ 145	145—204	205—221	„ 221
6 „ 2 „	„ 150	150—211	212—229	„ 229
6 „ 3 „	„ 154	154—217	218—235	„ 235

## AGES 25—29.

Height.	D.	C.	B.	A.
5 ft. 0 in.	Under 106	106—150	151—163	Over 163
5 „ 1 „	„ 107	107—151	152—164	„ 164
5 „ 2 „	„ 109	109—154	155—166	„ 166
5 „ 3 „	„ 111	111—157	158—170	„ 170
5 „ 4 „	„ 115	115—162	163—176	„ 176
5 „ 5 „	„ 117	117—166	167—179	„ 179
5 „ 6 „	„ 121	121—170	171—185	„ 185
5 „ 7 „	„ 125	125—176	177—191	„ 191
5 „ 8 „	„ 128	128—181	182—196	„ 196
5 „ 9 „	„ 132	132—186	187—202	„ 202
5 „ 10 „	„ 135	135—191	192—207	„ 207
5 „ 11 „	„ 139	139—197	198—213	„ 213
6 „ 0 „	„ 145	145—204	205—221	„ 221
6 „ 1 „	„ 150	150—212	213—230	„ 230
6 „ 2 „	„ 156	156—221	222—239	„ 239
6 „ 3 „	„ 162	162—228	229—247	„ 247

## AGES 30—39.

Height.	D.	C.	B.	A.
5 ft. 0 in.	Under 110	110—155	156—168	Over 168
5 „ 1 „	„ 111	111—156	157—169	„ 169
5 „ 2 „	„ 112	112—158	159—172	„ 172
5 „ 3 „	„ 115	115—162	163—176	„ 176
5 „ 4 „	„ 118	118—167	168—181	„ 181
5 „ 5 „	„ 121	121—170	171—185	„ 185
5 „ 6 „	„ 124	124—175	176—190	„ 190
5 „ 7 „	„ 128	128—181	182—196	„ 196
5 „ 8 „	„ 132	132—187	188—202	„ 202
5 „ 9 „	„ 136	136—193	194—209	„ 209
5 „ 10 „	„ 141	141—199	200—215	„ 215
5 „ 11 „	„ 145	145—205	206—222	„ 222
6 „ 0 „	„ 150	150—212	213—230	„ 230
6 „ 1 „	„ 156	156—220	221—238	„ 238
6 „ 2 „	„ 162	162—228	229—247	„ 247
6 „ 3 „	„ 168	168—237	238—257	„ 257

## AGES 40 AND OVER.

Height.	D.	C.	B.	A.
5 ft. 0 in.	Under 114	114—161	162—174	Over 174
5 „ 1 „	„ 115	115—163	164—176	„ 176
5 „ 2 „	„ 117	117—165	166—179	„ 179
5 „ 3 „	„ 119	119—169	170—183	„ 183



Height.	D.	C.	B.	A.
5 ft. 4 in.	Under 123	123—173	174—188	Over 188
5 „ 5 „	„ 126	126—177	178—192	„ 192
5 „ 6 „	„ 129	129—182	183—197	„ 197
5 „ 7 „	„ 133	133—188	189—204	„ 204
5 „ 8 „	„ 137	137—194	195—210	„ 210
5 „ 9 „	„ 142	142—200	201—216	„ 216
5 „ 10 „	„ 146	146—206	207—223	„ 223
5 „ 11 „	„ 150	150—212	213—230	„ 230
6 „ 0 „	„ 155	155—218	219—237	„ 237
6 „ 1 „	„ 160	160—226	227—244	„ 244
6 „ 2 „	„ 165	165—233	234—253	„ 253
6 „ 3 „	„ 171	171—242	243—262	„ 262

### INSTRUCTIONS TO CLERKS PREPARING THE NECESSARY MATERIAL.

The Actuarial Society provides policy slips in quantities as called for by the companies.

For writing the policy slips, clerks working together should have before them the original applications\*, beginning with the first policy dated in 1870, and also the charts entitled Schedule of Classes and Table of Weights. One of them will write in ink the policy slips. The applications should be gone over by another clerk in numerical order, and any application temporarily removed should be noted, and subsequently looked up and attended to. The following classes of applications should not be reported:—

Policies marked off as not taken.

Policies terminated, otherwise than by death, in the same calendar year as dated.

Policies on female lives.

Policies issued in exchange without new applications.

Policies taken outside of United States or Canada.

Sub-standard policies and industrial policies.

The phrase “sub-standard policies” refers to those in which a greater premium has been charged for a given insurance, or a smaller insurance granted for a given premium, because the risk has been found below the usual medical standard. It does not refer to policies on which extra premiums have been charged for occupation or locality.

Each application which is not passed over for one of the reasons just mentioned, should be scrutinized in connection with the schedule of classes, to see whether or not it falls within any one or more of the classes named in the schedule. If not, no attention will be paid to it. If it is found to belong to one or more of these classes, the clerk engaged in writing the policy slip will enter (1) the policy number; (2) the class number or numbers; and (3) the age at issue. (If any company’s records are so kept that it is more convenient to enter the age subsequently, that course should be adopted). Each such policy will occupy one line on the slip, except that joint life policies will

\* Or proposals for insurance (D.P.F.).

occupy one for each life. *Policy numbers not represented in any "class" should not be written on the policy slips at all.* The class numbers may be written in any order. One slip must not contain policies dated in different years, the calendar year of date being noted in the space provided for it at the top of the slip. Policies dated back out of order to the preceding year should therefore be noted on some slip pertaining to that year. When more than one policy has been written on the same application, the successive numbers for the same calendar year of date should be noted in successive lines and connected by a bracket, the class numbers and age being noted only on one line opposite such bracket. Another policy or policies, on same application but dated in a different year, must be disregarded. It is not positively necessary that the policy numbers should invariably be in numerical order, such order being desirable for convenience only.

If the custom of the company is to rate ages by next birthday, special attention should be paid to the age, so as to have it appear on the Policy Slip according to the birthday nearest the date of issue.\*

Fractions of a pound, in the weight of an applicant, are to be disregarded.

The policy slips when written, need to be taken to some register which indicates the date of actual total termination of the risk under the original policy bearing the original number. *This does not mean the date when the policy is marked off.* No attention is to be paid to the exact day or month of termination, but only to the calendar year in which it takes place, unless the termination is by death.

If the policy has terminated before 1901 otherwise than by death, the calendar year of date is to be subtracted from that of termination, and the difference is to be regarded as the number of years during which the policy has been in force, and this number is to be entered on the policy slip in the column headed duration.

If the policy has been terminated, otherwise than by death, in the same calendar year in which it has been dated, all entries relating to it must be crossed out from the policy slip. That is to say, if the duration as found by the foregoing rule is 0, it is not to be so marked in the column headed duration, but the whole line is to be marked out.

If the termination has been by death, examination must be made to determine whether or not death has occurred before or after the anniversary of the date of the policy. If death has occurred before the anniversary of the policy in the calendar year of death, the year of date is to be subtracted from the year of death, and the difference is to be regarded as the duration, and so entered in the policy slip, together with a cross mark or X in the column headed D, which stands for death.

If the death has occurred after the anniversary of the date of the policy, the duration must be increased by one year. For example, if a policy dated on 1 July 1870, has terminated by death on 15 August 1880, the number of years to be entered in the column headed duration is not 10, but 11. The reason for this is that the death

\* Which is the almost universal rule in America.—D.P.F.





being made to exclude policies issued upon the same life, even in the same office, except when issued upon the same application (or proposal), thus if a man obtained five policies of \$1,000 each in the same company at the same time, it would be regarded as one insurance, but if he applied year after year for additional insurance, each case would be considered as a separate risk, and, if death occurred, there would be as many "deaths" as policies.

The amount insured is not noted, except where \$20,000 or more is issued at the same time and in the same company, regardless of whether several policies are taken or whether all is under one policy.

As the volume of life insurance in America prior to 1870 was so small that no satisfactory results could be obtained by minute classification of risks insured before that date, it was resolved to omit all cases prior to 1870.

It will be seen that there are :—

- 3 classes based on facts connected with the amount or the form of the policy.
- 4 classes according to race.
- 35 classes according to occupation.
- 16 classes based on personal record.
- 14 classes based on weight (allowing for age and height) along with family record.
- 2 classes based on height alone.
- 2 classes based on certain points of family record only.
- 22 classes based on residence.

There are thus 98 classes in all, but it is estimated that about 48 per-cent of the policies actually put in force will not fall under any of these special classifications. The remaining 52 per-cent to be tabulated will, it is thought, number about 2,260,000. Sometimes one policy falls in as many as five or six classes, though rarely in more than one, and it is believed that on an average about 130 cards will be required for every hundred cases, as a separate card is used for each class under which a policy falls ; in this way it is expected that the cards will number about 2,940,000, or say three millions.

The preparation of the data was begun by the companies in May 1901, and will be concluded shortly after the middle of 1902. Meantime the preparation of the cards is going forward, and everything is progressing so satisfactorily that it is confidently

expected that the Committee will be able to report at the Annual Meeting of the Society in May 1903.

Through the courtesy of the Prudential Insurance Company of America, the tabulating will be done at the office of that Company. The preparation and sorting of the cards will be accomplished by machinery.

Mr. John K. Gore, the accomplished Actuary of the American Prudential, in charge of the compilation of the statistics, has kindly prepared the following sketch of the *modus operandi*.

The card to be used is shown in Diagram 1; it is four and one-half inches long and two and three-quarters inches wide. Each card is to be perforated, and the perforations will represent the required data for one policy in one of the designated classes. The first two columns are for the class, the next two for age at entry, and the two following for duration. The letter D is to be punched if death has occurred while the policy was under observation, otherwise the letter L is to be punched. Diagram 2 shows a perforated card recording the following data, a policy in class 46 ("has had asthma"), age at entry 35, duration 12, the person assured alive when observation ceased. The perforation not accounted for in the above explanation is used simply to hold the card steady in the process of sorting explained later.

DIAGRAM 1.

*Specimen of the Card for use in recording the Data.*

CLASS		AGE		DURATION		
00	0	00	0	00	0	L
10	1	10	1	10	1	D
20	2	20	2	20	2	
30	3	30	3	30	3	
40	4	40	4		4	
50	5	50	5	●	5	
60	6	60	6		6	
70	7	70	7		7	
80	8	80	8		8	
90	9	90	9		9	



DIAGRAM 3.  
*First View of the Machine for Perforating the Cards.*







DIAGRAM 4.  
*Second View of the Machine for Perforating the Cards.*

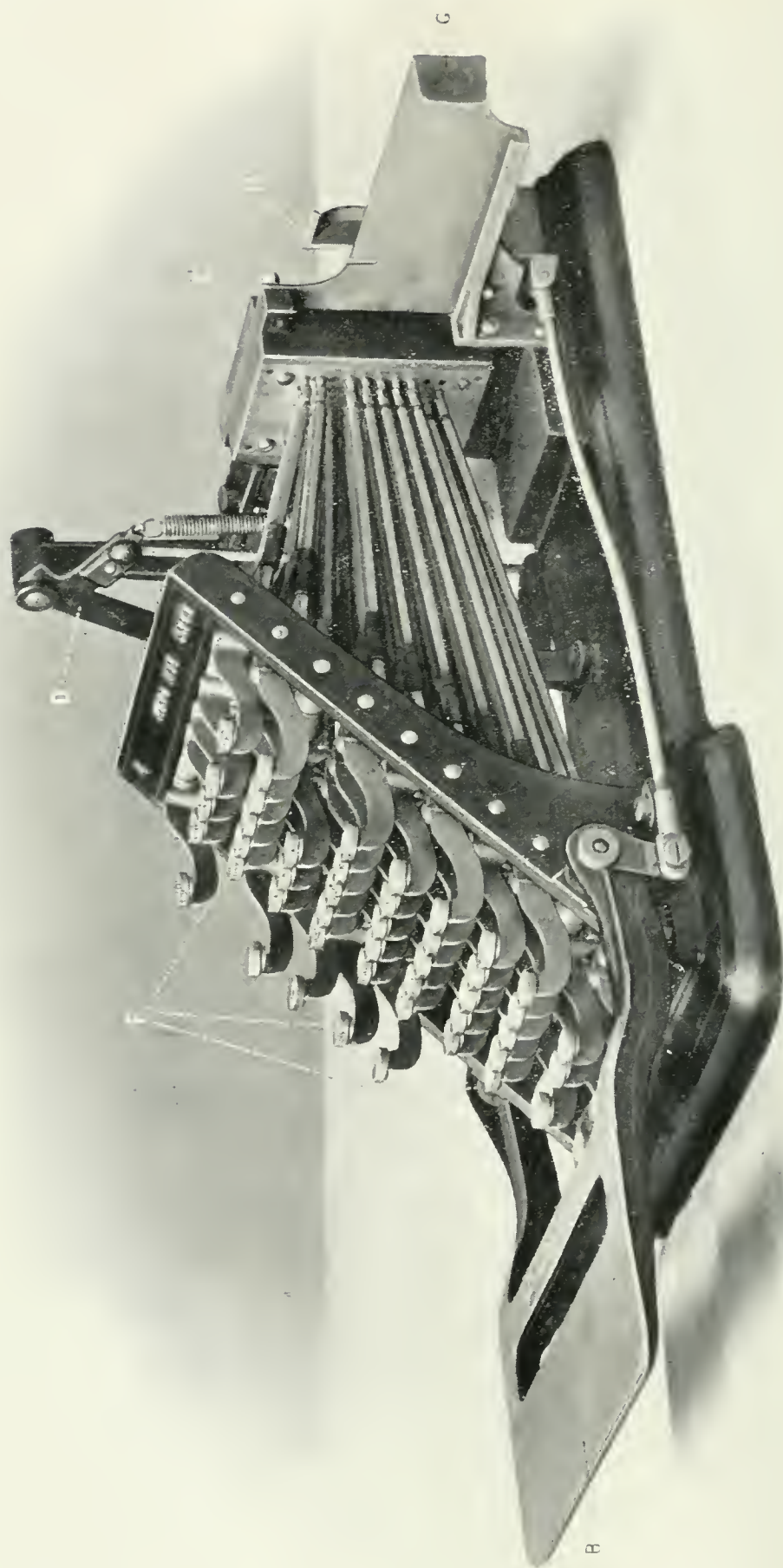


DIAGRAM 2.

*Card perforated for a Policy in Class 46, Age at Entry 35,  
Duration 12, Alive at the Close of the Observations.*

CLASS		AGE		DURATION		
00	0	00	0	00	0	●
10	1	10	1	●	1	D
20	2	20	2	20	●	
30	3	●	3	30	3	
●	4	40	4		4	
50	5	50	●	●	5	
60	●	60	6		6	
70	7	70	7		7	
80	8	80	8		8	
90	9	90	9		9	

THE ACTUARIAL SOCIETY  
OF AMERICA.

The machines which perforate the cards are not finished at this writing, but the illustrations (Diagrams 3 and 4) of perforators now in use by the American Prudential will answer our purpose. Seventy-five to one hundred cards are placed in the box, *a*, being held against the front of the box by a coil spring in the rear, and by depressing the plate, *b*, the lever, *c*, and bell crank, *d*, push the front card of the pack to the right and in front of the punches at *e*. By means of the keys at *f*, the card in front of the punches is properly perforated by the operator, the data being read from a policy slip. By again depressing the plate, *b*, a second card is pushed to the right from the box, that card in turn pushing out the first card to *g* at the right. The stop, *h*, moves toward the operator as the plate, *b*, is depressed, and serves the double purpose of throwing the first card forward face down, so as to preserve the original order, and of placing the second card at the exact position desired in front of the punches. In a working day of six and one-half hours, a clerk of average ability can punch from four to five thousand cards.

[*Diagrams 3 and 4.*]

The next operation is that of sorting or separating the cards, of which there will be about three millions, into classes. The cards of each class are to be separated according to age at issue, and those of each age according to duration. The separating machine is shown in Diagram 5; it is about six feet high. The machine is operated from below by a quarter horse-power motor, *a*, and connected with an operating shaft, *b*. *c*, *c'*, *c''*, *c'''*, *c''''* are circular tiers of boxes, open at the top, and supporting the cards to be placed in them by means of movable lips below them, instead of by a solid bottom. Each of the four tiers is independent of the others. There are ten boxes in a tier, and each box will hold about two hundred cards, so that about eight thousand cards may be put into the machine at a time.

[Diagram 5.]

Below each tier of boxes is a revolving disc, *d*, *d'*, *d''*, *d'''*, *d''''* on which are arranged at equal distances ten detachable plates, of the same size as the cards, each plate having one hundred and ten holes corresponding in position to all the perforations that may occur in the various kinds of cards sorted by the machine. For example, the American Prudential in sorting weekly the issue cards for its Industrial Department, uses about ninety of these holes. In the investigation of the Actuarial Society, less than sixty holes will be used. Each plate is so arranged as to be exactly under a box when the disc is not revolving.

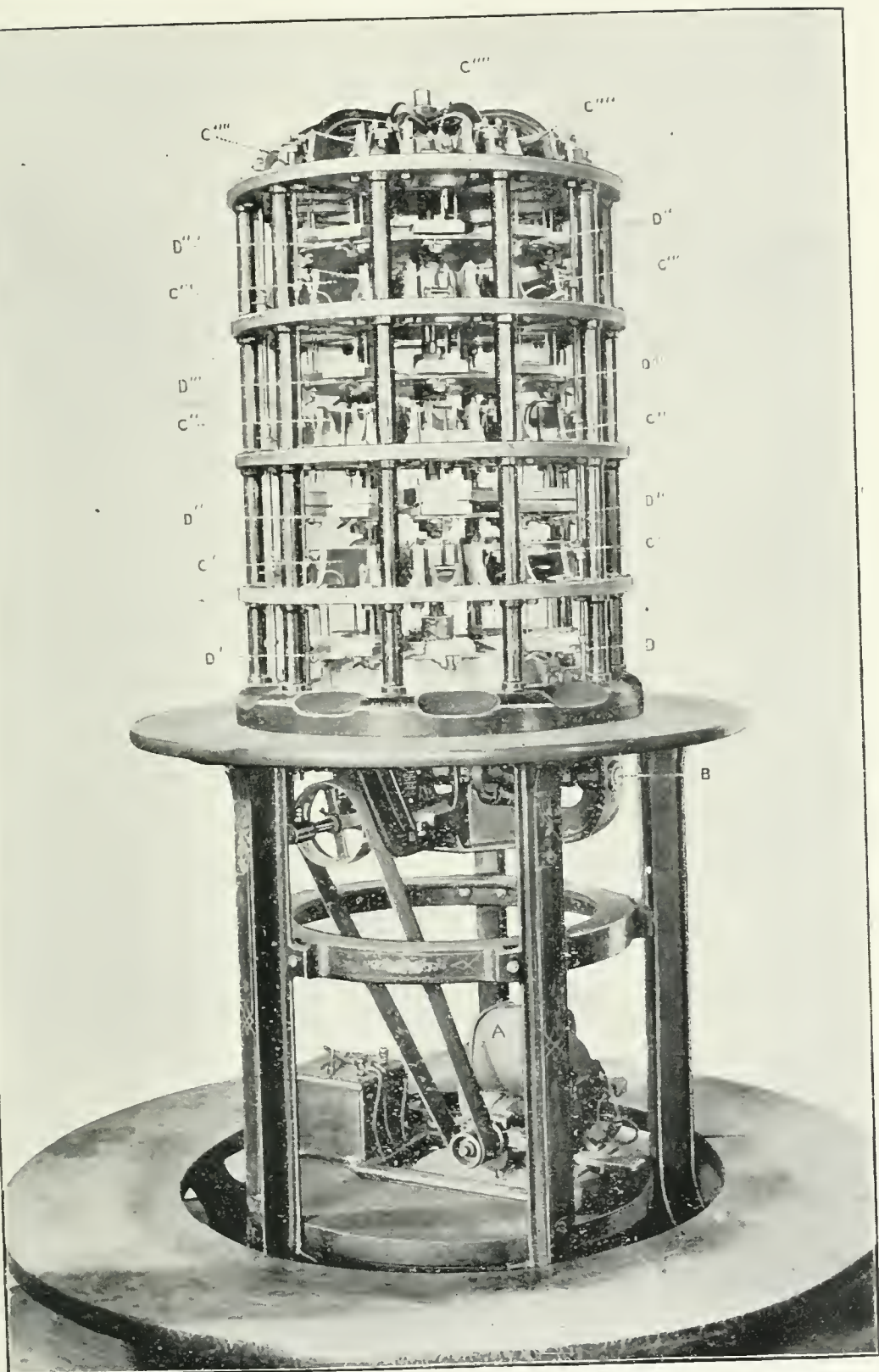
The cards used in the Actuarial Society investigation will first be sorted according to the numbers of the classes into tens of class, and then into units of class. The method of sorting is as follows :

Two upright steel pins are inserted in the proper holes of each plate. One of these pins will remain in the same position throughout the entire operation. This pin will always be under the perforation shown at the centre of the card in Diagram 1. Its purpose in connection with the other pin, whose position will change with each successive step of the sorting, is to hold the cards in position as they fall from the boxes above. In sorting according to tens of class, the second pin will be placed so as to come under the first column of the cards, as shown in Diagram 1. On each disc the second pin of one plate, when the disc is not revolving, will be under the figures 00 of the first column, and this plate will receive all cards of classes one to nine. The second disc of another plate will be under the figures 10 of the



DIAGRAM 5.

*Machine for Sorting the Cards.*





same column, and will receive all the cards belonging to classes ten to nineteen. Another will receive all cards of classes twenty to twenty-nine, and so on.

Forty boxes of the separator having been filled with unsorted cards and the pins set as explained above, the machine is started by a lever which does not appear in the illustration. Each of the revolving discs with its empty plates revolves one-tenth of the circumference, each plate coming to rest beneath a box. The discs with their plates then move automatically upward, until the tops of the pins support the packs of cards above. At the same time the lips which have been supporting the cards move aside automatically, and the bottom card or cards having a perforation in the column of tens of class corresponding to the pin beneath drop upon the plate. The lips then return to their original position under the cards and the discs descend, to repeat the operation, each plate seeking to find bottom cards corresponding to its pins. In a few minutes the boxes are empty, and all the cards sorted as to tens of class. Each plate may be quickly detached, and its cards removed when they reach the top of the pins or when the boxes are empty. In the illustration, the boxes of tier *c''* have been partly emptied, the cards appearing sorted on the plates of disc *d''* below.

By the process just described, the three million cards will be divided into ten groups, with an average of three hundred thousand cards in each group.

The pins will then be transferred to the next row of holes to the right, and each three hundred thousand cards will be sorted in the same manner for units of class into ten groups, each group containing about thirty thousand cards on the average. Continuing the operation, the cards will be separated successively into tens of age, units of age, tens of years of duration, units of years of duration, and finally into "living" and "dead."

As only one sorting machine is available, the entire work of separating the cards must be done by a single clerk, but so rapidly does the machine do its work, that the three million cards will be sorted within four months. This is at the rate of twenty-five thousand cards per day completely sorted by one clerk.

After the sorting is completed the cards will be counted, and the results tabulated upon the sheets supplied by the Committee in charge.

The perforating machines, as well as the sorting machine, are



the inventions of Mr. John K. Gore, Actuary of the Prudential Insurance Company of America. They have been in use in the office of the Prudential for about seven years.

The cards to be used in connection with the Actuarial Society investigation will be destroyed after the investigation is completed.

In the ordinary statistical work of the Prudential, each card, representing an Industrial policy, is numbered accordingly. When these cards have been sorted and counted, and the results tabulated, it is necessary to rearrange them in numerical order. This also is done by the sorting machine.

A special machine, invented by Mr. J. H. Reinhardt, of Newark, N.J., according to suggestions from Mr. Gore, receives the cardboard from rolls, prints the cards, numbers them, perforates them according to their number, as explained later, cuts the card from the cardboard (including the cutting off of a corner of the card, as shown in Diagram 6), and throws the card face down in a box so as to preserve the numerical order. Diagram 6 shows a card printed, numbered, and perforated according to number. This card is ready to be punched by a perforating machine to record the data required by the Prudential in its industrial business.

DIAGRAM 6.

*Specimen of Card used by the Prudential Assurance Company of America.*

Form 1293.

15506030

DIST.

AGE

AMT.

PREM

9 9

8 8

7 7

6 6

5 5

4 4

3 3

2 2

1 1

0 0

PREM.

3 45

5 50

6

10 Ot

15 RV

20 RX

25 SV

30 SX

35

40

AGE

00 0

10 1

20 2

30 3

40 4

50 5

60 6

70 7

8

9

KIND

AM

AF

IM

IF

W

C

●

Sp

Ex

UX

9 9

8 8

7 7

6 ●

5 5

4 4

3 3

2 2

1 1

● 0

Year of Issue, 1901. Year of Exit,

Rev. 190 NO. OF CLAIM,

Amount Paid,



*Machine for Printing, Numbering, and Cutting Cards.*

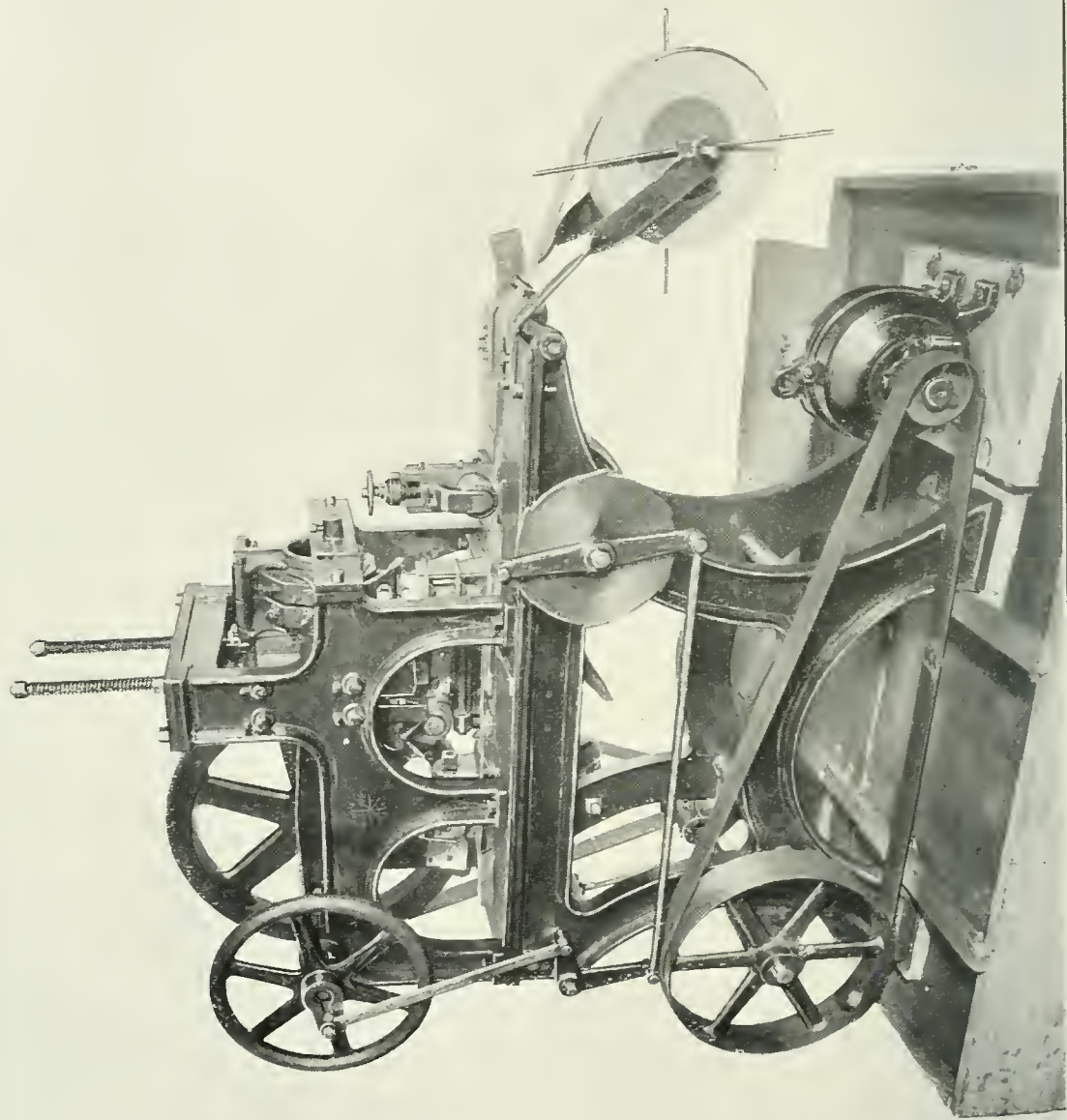


Diagram 7 shows the combination printing, numbering, perforating, and cutting machine.

The cardboard enters the machine at the extreme right, and at the extreme left the cards are dropped in numerical order into a box made to receive them.

[*Diagram 7.*]

When the cards of the Prudential are to be sorted in numerical order, the pins of the sorting machine are set for the first vertical file of figures in the column at the extreme left in Diagram 6, representing ten-thousands of number. It will be seen that the perforation in this file corresponds to the numeral, 0, in the ten-thousand's place of the printed number. After this sorting the cards are further separated according to thousands. Here again the perforation in the second vertical line of figures in the column in the left of Diagram 6 will be seen to correspond to the numeral 6, in the thousands place of the printed number. The operation is repeated for hundreds, and may also be for tens, using the holes in the right hand column. In practice, however, the cards are sorted by hand according to tens as well as units of number.

*An Enquiry into the Methods and General Principles to be adopted in calculating the Rates of Contribution, or Percentage Deductions from Salary, in the case of Superannuation Funds and Pension Funds generally. By JAMES MCGOWAN, Actuary to the Cape of Good Hope Government, and Fellow of the Institute of Actuaries.*

[Read before the Institute, 30 December 1901.]

**Preliminary Observations.** A STRIKING indication of the little interest taken by the actuarial profession generally in the subject of Pension Funds is shown by the fact that in the many published volumes of the *Journal of the Institute*, hardly any information is to be found with reference to such Funds.

This lack of interest in Pension Funds was noticed some time ago by Mr. Ryan (page 470, vol. xxxi, of the *Journal*), who remarked that "it was a very curious thing that the important question of the valuation of Widows' Funds had on no occasion



“ been treated by contributors to the proceedings of the Institute.  
 “ In Dr. Sprague’s 20-volume Index there was absolutely no  
 “ reference to the question of Widows’ Funds. . . . He might  
 “ add that the Index to the *Journal* contained no entry relating  
 “ to superannuation schemes, which was again a matter of great  
 “ public importance.”

It certainly does seem that Superannuation and Pension Funds are a branch of actuarial work which has been much neglected, and it is hoped that the present paper will induce some members of the Institute better qualified than myself to impart some of the information which they possess with regard to these Funds.

In making these remarks, I do not mean to overlook Mr. Huie’s well-known work on the “Valuation of Widows’ Funds”, or Mr. Archibald Hewat’s very valuable book on “Widows’ and Pension Funds.” On pages 18–23 of his book, Mr. Hewat makes some very interesting observations with regard to Pension Funds and superannuation schemes, which are well deserving careful study.

The term “superannuation” has been defined as  
 Superannuation. “the state of being superannuated, or too old for office or business, or of being disqualified by old age.”

In its simplest form a system of superannuation is merely an arrangement to secure deferred life annuities, commencing at a given age, the consideration being premiums payable for a limited number of years while the contributors are drawing salary. If, in each case, the amount of the annuity or superannuation allowance could be fixed upon beforehand, quite independently of the salary, and if it could be arranged for the periodical premium during the term of years, to be a fixed amount, also without reference to the salary, then, given the amount of the required superannuation allowance, obviously the amount of premium could readily be deduced. Or, put it in the other way: given the amount of the premium intended to be paid, the superannuation allowance could readily be ascertained; the age at entry, and the age at which superannuation is to commence being, of course, known.

Average  
Salary.

But, in practice, the superannuation allowance is hardly ever fixed upon beforehand. Neither is the contribution a fixed amount. The superannuation allowance is generally reckoned as a percentage of the average salary. It may be that the average salary is ascertained by taking into account the whole period of service, or by taking into account a limited

number of years prior to the date of superannuation, but in any case the average salary can only be ascertained at the time the superannuation allowance is to commence; that is to say, the exact benefit in the way of pension is only known when the premiums for such benefit have all been paid up. This obviously complicates the calculations. Further, the contribution is usually a percentage on the salary, and thus follows the salary through all its changes, the percentage, however, remaining fixed throughout.

In arriving at the superannuation allowance, it has been held that to deduce the average salary by taking into consideration the whole period of service is more scientific than to derive such average by basing it upon the salary for the last few years only. This may be so, but to the contributor it must be more satisfactory to make the pension a percentage of the salary which he has been receiving at or about the time when he retires.

Seeing, however, that the contribution is generally a fixed percentage on the salary, and follows the salary in all its changes, it is obvious that there is something to be said for deriving the average salary by taking into account all the amounts on which the percentage deductions have been made, that is, by taking into account the whole term of service.

Methods of  
deriving  
Average  
Salary.

There are four principal methods of deriving the average salary, namely, by basing such average on—

- (1) The salary over the whole term of service.
- (2) „ for the last 10 years of service.
- (3) „ „ „ 7 „ „
- (4) „ „ „ 3 „ „

In the Civil Service of the United Kingdom (4) is the rule, that is, the last three years are taken, but a Royal Commission, reporting in the year 1888, suggested (2) as a rule, that is, that the last ten years should be taken.

Formula for  
Average  
Salary.

To produce a formula for the average salary, I assume that  $b_x$  is the initial salary;  $\kappa_1, \kappa_2, \kappa_3 \dots$  the various increments. Taking particular cases, it is found in

practice that there may be decrements as well as increments in the salary, but for practical purposes it may be assumed that  $\kappa_1, \kappa_2, \kappa_3 \dots$  are all increments, and therefore positive quantities.

It may be further assumed that the changes in salary take place once a year, so that if the total service be  $n$  years, the total amount of salary received during these  $n$  years will be—

$$b_x + (b_x + \kappa_1) + (b_x + \kappa_1 + \kappa_2) + \dots \\ + (b_x + \kappa_1 + \kappa_2 \dots \kappa_{n-1})$$

or 
$$nb_x + (n-1)\kappa_1 + (n-2)\kappa_2 \dots \kappa_{n-1}$$

Accordingly, the average salary over the whole term of service will be—

$$\frac{nb_x + (n-1)\kappa_1 + (n-2)\kappa_2 \dots \kappa_{n-1}}{n} \\ = \text{Last year's salary} - \frac{(\kappa_1 + 2\kappa_2 \dots (n-1)\kappa_{n-1})}{n}$$

If it be assumed that the increments are all of the same amount and  $= \kappa$ , then the formula becomes—

$$b_x + \frac{(n-1)}{2}\kappa = \text{average salary.}$$

Suppose that the average be taken over the last 10 years of service, and assume as before a uniform increment of  $\kappa$ , then—

$$b_x + n\kappa - \frac{55}{10}\kappa = \text{average salary taken over last 10 years,}$$

$n$  being equal to or greater than 10.

Further, if the average be taken over the last 7 years, there being, as before, a uniform increment of  $\kappa$ , then—

$$b_x + (n-4)\kappa = \text{average salary taken over last 7 years,}$$

$n$  being equal to or greater than 7.

Also, if the average be taken over the last 3 years, there being, as before, a uniform increment of  $\kappa$ , then—

$$b_x + (n-2)\kappa = \text{average salary taken over last 3 years.}$$

As a rule, there must be at least 10 years' service to count for pension.

Collecting these formulæ, the average salary is represented as follows—

- (1)  $b_x + \frac{(n-1)}{2}\kappa$  [over whole term]
- (2)  $b_x + n\kappa - \frac{55}{10}\kappa$  [over last 10 years]
- (3)  $b_x + (n-4)\kappa$  [over last 7 years]
- (4)  $b_x + (n-2)\kappa$  [over last 3 years]



Thus, it is seen that—

(4) exceeds (3) by  $2\kappa$

(4) „ (2) „  $\frac{35}{10}\kappa$

(4) „ (1) „  $\frac{(n-3)}{2}\kappa$

Also that—

(3) exceeds (2) by  $\frac{15}{10}\kappa$

(3) „ (1) „  $\frac{(n-7)}{2}\kappa$

While—

(2) exceeds (1) by  $\frac{(n-10)}{2}\kappa$

If it be now assumed that  $b_x$ , the initial salary, is £100, and that £4 is the annual increment, the following are the comparative results (40 years' service)—

Average salary :

Whole term = £178.

Last 10 years = £238.

„ 7 „ = £244.

„ 3 „ = £252.

It will be noticed that the average salary calculated over the whole term is considerably smaller than that calculated over the last 3 years. It is therefore of the utmost importance, in comparing one Pension Fund with another, to note the methods adopted in deriving the average salary upon which the pension is to depend.

As I have already stated, the exact amount of the pension is, as a rule, only known when the premiums have all been paid up. We are thus compelled to make some estimate beforehand of what the pension will amount to, and this leads to the practical question: How are the probable increments of salary to be estimated?

In the case where a Fund has been in existence for a considerable number of years, and where an amount of data has been collected showing the past experience in regard to the increments of salary, it may not unreasonably be assumed that the past results, or something like them, will be repeated in

Estimate as to  
Increments of  
Salary.

the future. Any actual experience, however, would undoubtedly show great variations in the salaries of men of the same age. For example, if we assume that 500 persons, all aged 20, join a fund, each in receipt of a salary of £100 per annum, actual experience would no doubt show that after the expiration of, say 10 years, the survivors would not all be receiving the same salary. There would be undoubtedly considerable differences in the amounts. Some might be receiving £300 a year, others perhaps only £150, although all might be of the same age, and might have the same number of years' service. It is obvious that it would be impossible to predict what the salary age by age would be in any particular case. All that can be done is to ascertain from past experience what *average salary* has been paid at each year of age. We have to deal with the "average" man, and not the individual.

It cannot, however, be pretended that this "average salary" method leads to altogether satisfactory results. Looking at the members of a Fund *individually*, it may be said that hardly anyone pays his proper contribution. On the average it is no doubt all right, but individually it is not.

As to the best method of working out the average salaries, there may be some difference of opinion. As a rule the results to the nearest pound would be quite sufficient. In the case of some Funds I gather that the "numbers exposed to risk" were first obtained, then the total salaries were taken out, and the results were shown =  $\frac{\text{total salaries}}{\text{No. exposed to risk}}$  for each year of age.

It is doubtful, however, if some simpler method would not suffice. As a matter of fact, the proper method to be adopted would depend very much on the particular circumstances connected with the Fund under observation. For instance, if a member retires or dies, would he be immediately replaced? Then there might be promotion, either with or without an increase of salary. Various other points would no doubt suggest themselves in an investigation, but bearing in mind, as already said, that the results are only taken out to pounds, a simple method giving a good approximation seems all that is necessary.

I have now dealt with the methods of determining the average salary for the purpose of arriving at the amount of pension, and I have also referred to the method of deriving the probable future salaries.

The Fraction  
of Salary  
secured as  
Pension.

There is much diversity amongst Pension Funds with regard to the rule for determining the fraction of the salary secured by each year's service. The Civil Service of the United Kingdom gives one-sixtieth in respect of each year's service; some railway Funds give one-fiftieth; others a still larger proportion. There, in fact, seems to be a great variety of opinion on the subject. First of all there are various methods of determining the average salary on which to base the pension (*i.e.*, the average may be over the whole period of service, or over the last 10, 7, 3, or other number of years); then there are various fractions of such average salary for determining the amount of pension in respect of each year's service (*i.e.*, such as one-sixtieth, one-fiftieth, &c.).

The  
Contribution.

Turning now our attention from the pension to the contribution to secure the pension, it has to be noticed that it is the usual plan to make the contribution a percentage on salary. Moreover, the percentage must be an easily-workable percentage. A deduction, for example, of 5 per-cent would be readily calculated in making out the pay sheets, whereas a deduction of £4. 18s. 11d. would not be workable. The question may generally be put in this way: Given a certain percentage on salary which it is intended to pay, find what scale of pensions can be granted? Perhaps the simplest plan would be to fix the contribution as so many pence per pound, discarding the term percentage. Thus we might have the contribution, 9d., 10d., 11d., 1s., 1s. 2d., 1s. 4d. per pound, which, like income tax, could be easily calculated.

The general idea, however, seems to be to fix upon a simple percentage, such as 4 or 5 per-cent.

Dr. Sprague  
and Mr. Sutton. It may here be mentioned that some interesting evidence *re* Pension Funds was given by Dr. Sprague and the late Mr. Sutton before a Select Committee of the House of Commons on the School Board for London (Superannuation) Bill, in the year 1891. The Committee published a report by Dr. Sprague, dated 1 November 1884, in which he stated (page 114):

“*Deductions from Salaries.*—The point upon which my opinion appears to be especially desired is the amount of the deductions that should be made from the salaries in order to provide for the above-mentioned pensions. I think that no more satisfactory plan can be devised than that of making the deduction a uniform percentage upon all salaries, both large and small ”



This is a definite expression of opinion in favor of the percentage deduction from salaries.

On the other hand, there is to be found in the same report (page 134) the following excellently expressed statement of the case against the percentage deduction system, drawn up by Mr. Sutton—

“It may be said that, with few exceptions, Superannuation Funds as generally constituted are radically wrong in principle when looked at from the actuarial point of view. Instead of resting content with the introduction of as few assumptions as possible, they are made to involve not only assumptions as to the rates of mortality to be experienced among the members, and as to the rate of interest to be earned by the accumulated funds—these may be fairly said to be indispensable—but they are also made to depend upon such capricious elements as the rates of secession of members (that is of members leaving active service otherwise than by death or retirement), and the rates of salary the members will receive, and on which the nature and amount of their contributions to the Fund will depend, as well as the amount of the pension they will receive. It thus follows, that in bringing into the question, rates of secession and rates of salary, matters which cannot be prognosticated with any certainty for any length of time, classes of members get lumped together whose real circumstances and conditions in respect of these matters are as different as possible. In the particular case before the Committee this would certainly seem to apply. Thus, for instance, it appears to have been considered that a large proportion of teachers with comparatively small salaries would secede in the early years of membership, whereas it may be, on the other hand, safely inferred that members of the official staff as distinguished from the teachers’ staff would rarely secede. Again, by fixing a uniform percentage deduction from salaries, the further assumption is implied, contrary to the facts, that all members of the Fund will have their salaries uniformly and proportionately increased throughout membership, with the result that classes of members whose salary only rises to a small extent, or is practically stationary, will be called upon to pay heavier contributions to meet the shortcomings of those whose salaries have throughout membership steadily if not heavily increased. Put in another way, this is exactly one of the financial blunders which has been more or less the cause of the downfall of countless friendly societies.”

Mr. Sutton’s views appear to have carried weight with the Select Committee, as in the Teachers’ Superannuation Act passed a year or two ago (presumably as a result of the labours of the Select Committee) the principle of a percentage deduction was not adopted.

Some  
Percentage  
Rates of  
deduction.

I now proceed to deduce some percentage rates of deduction for a simple deferred annuity at age 60 and 65, without return of contribution.

In practice, the rate of interest assumed will be 3,  $3\frac{1}{2}$ , or 4 per-cent, and to simplify the calculations, I assume that the average salary increases at the rate of £3, £4, or £5 per annum, until the superannuation age is reached. In the case of several Pension Funds whose experience has been worked up, I have ascertained that the probable increases to salary will range from £3 to £5 per annum.

Table of  
Mortality.

As to the Table of Mortality to be employed, it will probably be found that the English Life Table No. 3 will be the most suitable; or, better still, a Table constructed from the actual experience of the Fund in question, or of some other Fund on similar lines.

The pensions may be payable monthly, quarterly, or annually—probably monthly. The contributions also would probably be payable monthly. In finding the annuities to value the contributions, it might be thought desirable to adopt the continuous method, but the results would be complicated, and to simplify the formulas, I assume the contributions to be payable at the end of the year, which is an assumption on the safe side. In the case of the deferred annuity, .5 is added.

As regards the age, it is necessary to premise that by age  $x$  is meant the exact age  $x$ . This assumption is required, otherwise complications arise. For instance, if a person of the actual age  $19\frac{1}{2}$  years, be assumed to be age 20, and is supposed to retire at age 60, the term might be assumed to be 40 years, while it would actually be  $40\frac{1}{2}$  years. It will therefore avoid fractions of years if we assume the given age to be the exact age in every case, and not an age “next birthday.” Moreover, by assuming the premiums to be payable at the end of each year, the interval between the date of the last payment of contribution, and the payment of the pension cannot exceed one year.

Let  $x$  = exact age at entry.

$x + n$  = pension age.

P = amount of pension.

Then present value of the pension will be—

$$P \times \frac{[N_{x+n} + \frac{1}{2}D_{x+n}]}{D_x}$$

Also, if  $\varpi$  = percentage,  $b_x$  the initial salary, and  $\kappa$  the annual increase, then

$\varpi$  = percentage on salary

$$= \frac{100 \times P [N_{x+n} + \frac{1}{2}D_{x+n}]}{(N_x - N_{x+n})b_x + \kappa [S_{x+1} - S_{x+n} - (n-1)N_{x+n}]}$$



(As being most convenient, I have taken the H<sup>M</sup> Table of Mortality throughout this paper.)

The following are some results from this formula :

Initial salary £100 (all ages at entry).

(“ Average ” salary over whole term of service

$$=b_x+\frac{n-1}{2}\kappa)$$

TABLE NO. 1.

H<sup>M</sup> Mortality.—Pension Age 60.

		Age at Entry	Service in Years	Average Salary	Pension £ <sup>6</sup> th for each Year of Service	PERCENTAGE ON SALARY		
						Interest 3 %	Interest 3½ %	Interest 4 %
Salary Increases £3 per Annum	}	20	40	£ 158·5	£ 105·6	7·384	6·374	5·495
		30	30	143·5	71·75	9·055	8·057	7·168
		40	20	128·5	42·83	11·193	10·250	9·391
Salary Increases £4 per Annum	}	20	40	178·0	118·6	7·536	6·521	5·636
		30	30	158·0	79·0	9·189	8·190	7·298
		40	20	138·0	46·0	11·293	10·351	9·492
Salary Increases £5 per Annum	}	20	40	197·5	131·6	7·664	6·645	5·754
		30	30	172·5	86·25	9·304	8·304	7·411
		40	20	147·5	49·16	11·379	10·439	9·581

To show the effect of taking the “ average salary ” for the last 3 years, instead of for the whole term of service, the foregoing table is re-calculated, and the following are the results :

Initial salary £100 (all ages at entry).

(“ Average ” salary taken over *last three years* of service

$$=b_x+(n-2)\kappa)$$

TABLE NO. 2.  
H<sup>M</sup> Mortality.—Pension Age 60.

		Age at Entry	Service in Years	Average Salary	Pension Path for each Year of Service	PERCENTAGE ON SALARY		
						Interest 3 %	Interest 3½ %	Interest 4 %
Salary Increases £3 per Annum	}	20	40	£ 214	£ 142·6	9·969	8·606	7·419
		30	30	184	92·0	10·610	10·331	9·191
		40	20	154	51·3	13·414	12·284	11·254
Salary Increases £4 per Annum	}	20	40	252	168·0	10·679	9·240	7·936
		30	30	212	106·0	12·322	10·983	9·787
		40	20	172	57·3	14·071	12·898	11·827
Salary Increases £5 per Annum	}	20	40	290	193·3	11·251	9·755	8·447
		30	30	240	120·0	12·942	11·551	10·309
		40	20	190	63·3	14·656	13·445	12·340

Comparing this table with the preceding one, it is seen that if the method of deducing the average salary by taking the last three years instead of the whole term of service be adopted, then, other things being equal, the change of method from the whole term to the last three years means an additional deduction by way of contribution, ranging from 2 to 3 per-cent on salary.

The fraction representing the amount of pension for each year of service may, of course, be raised. In Tables Nos. 1 and 2 it has been assumed that the fraction is *one-sixtieth*. In the following Table No. 3 the pension is taken as proceeding by *fiftieths*. The results are derived from Table No. 1 by adding one-fifth.

Initial salary £100 (all ages at entry).

(“Average” salary over whole term of service

$$=b_x+\frac{n-1}{2}\kappa)$$

TABLE NO. 3.

H<sup>M</sup> Mortality.—Pension Age 60.

	Age at Entry	Service in Years	Average Salary	Pension $\frac{1}{60}$ th for each Year of Service	PERCENTAGE ON SALARY		
					Interest 3 %	Interest 3½ %	Interest 4 %
Salary Increases £3 per Annum	20	40	£ 158·5	£ 126·8	8·861	7·649	6·594
	30	30	143·5	86·1	10·866	9·669	8·601
	40	20	128·5	51·4	13·432	12·300	11·269
Salary Increases £4 per Annum	20	40	178·0	142·4	9·043	7·825	6·763
	30	30	158·0	94·8	11·027	9·828	8·748
	40	20	138·0	55·2	13·551	12·421	11·390
Salary Increases £5 per Annum	20	40	197·5	158·0	9·197	7·974	6·905
	30	30	172·5	103·5	11·165	9·965	8·893
	40	20	147·5	59·0	13·655	12·527	11·497

When pensions are given according to the one-sixtieth scale, and where the salary is taken on the average for the last three years, it is usual to introduce a maximum of  $\frac{40}{60}$ ths. But in Pension Funds where the average salary is taken over the whole term of service, a fraction larger than one-sixtieth is taken, and sometimes no maximum is fixed.

Thus, to compare three typical Pension Funds, we might have—

*Pension Fund X.*—Pension proceeding by *sixtieths* of average salary (average for last 3 years) with maximum of two-thirds of average salary.

*Pension Fund Y.*—Pension proceeding by *fiftieths* of average salary (average for last 7 years) with maximum of two-thirds of average salary.

*Pension Fund Z.*—Pension proceeding by *forty-fifths* of average salary (average for whole term of service) with no fixed maximum.

In calculating the pensions under each of these Funds, we have three variable quantities to consider, namely:

- (1) The initial salary ( $b_x$ ).
- (2) The term of years ( $n$ ).
- (3) The annual increment ( $\kappa$ ).

Comparing, say, Fund X with Fund Z, it may be ascertained under what conditions an equality in the amounts of pension can be produced. Assuming the same term of years, and also the same annual increments, then—

$$\{b_x + (n-2)\kappa\} \frac{n}{60} = \left\{b'_x + \frac{n-1}{2}\kappa\right\} \times \frac{n}{45}$$

where  $n$  is not to exceed 40.  $b_x$  being initial salary Fund X,  $b'_x$  initial salary Fund Y.

This reduces to  $b'_x = \frac{3b_x + (n-4)\kappa}{4}$ , so that if the initial salary under Fund Z be taken as  $\frac{3b_x + (n-4)\kappa}{4}$ , the same pensions will be produced as under Fund X.

Thus, to take a numerical example, if  $b_x$  (Fund X) be 100,  $n=40$ , and  $\kappa=4$ , then  $b'_x$  (Fund Z) will be = 111 to produce the same amounts of pension.

Similarly, if we make  $\kappa$ , the increment, the variable, it appears that—

$$\kappa' = \frac{(3n-6)\kappa - b}{2n-2}$$

where  $\kappa$  = increment Fund X, and  $\kappa'$  = increment Fund Z.

Taking a numerical example, if  $\kappa$  under Fund X be = 4, and  $b=100$ ,  $n=40$ ,

$$\kappa' = \frac{178}{39} = 4.564$$

Thus far it has been assumed that the simple deferred annuity (or pension) has to be provided for by a contribution *not* returnable in case of death before pension age is reached.

In actual practice, however, a variety of additional benefits, beyond the mere deferred annuity, are introduced. Amongst these are the following:

- (I) As regards the pension, if at the time the pensioner dies the total amount contributed by him should exceed the amount drawn by him as pension, then the amount of the excess is to be paid to his representatives.

(II) If a contributor should die before being superannuated, his contributions are to be paid to his representatives.

(III) If a contributor resigns or withdraws at any time before being superannuated, he is entitled to withdraw all he has paid in, but without interest.

Leaving, for the present, the benefit (III), which involves the question of secessions, and looking at the effect of combining the simple deferred annuity with benefits (I) and (II), it may be remarked that Mr. Manly, in his valuable paper in the volume of the *Proceedings of the Actuarial Congress of 1898*, has thrown considerable light on problems of this kind. In his Problem No. V (page 865), Mr. Manly gives the formula for—

“The annual premium, payable for  $n$  years for an annuity of 1 on the life of  $(x)$ , to commence at the age of  $(x+n)$ , with the condition that if death occurs before the annuity commences, all the premiums paid shall be returned, and if death occurs after the annuity commences, but before the annuity payments amount to the total premiums paid, the balance shall be returned.”

Here, instead of Mr. Manly's fixed premium, we have an increasing premium, and a rather complicated formula is the result.

Dividing the sum of the contributions by the annuity we get the term of the decreasing assurance. If the result of the division be an integer plus a fraction, say  $=\left(m + \frac{1}{y}\right)$ , then the term will be  $(m+1)$ .

The present value of the contributions is as before, namely :

$$= \frac{\frac{\omega}{100} [b_x(N_x - N_{x+n}) + \kappa(S_{x+1} - S_{x+n} - (n-1)N_{x+n})]}{D_x}$$

The benefits are—

(a) The deferred annuity or pension.

(I) The return of the excess of amount contributed over the amount drawn as pension.

(II) The return of contributions in the event of death before superannuation.

The present value of (a) is—

$$= \frac{P_x[N_{x+n} + \frac{1}{2}D_{x+n}]}{D_x}$$



In the case of (I), let  $\Sigma c$  be the sum of the contributions, then  $\frac{\Sigma c}{P} = m + \frac{1}{y}$ , so that  $\Sigma c = mP + \frac{P}{y}$ , and the value of this benefit will be—

$$\begin{aligned} & \frac{\left(mP + \frac{P}{y}\right)C_{x+n} + \left[(m-1)P + \frac{P}{y}\right]C_{x+n+1}}{D_x} \\ & + \frac{\left[(m-2)P + \frac{P}{y}\right]C_{x+n+2} + \dots}{D_x} \\ & + \frac{\left[(m-m)P + \frac{P}{y}\right]C_{x+n+m}}{D_x} \cdot (\text{in all } m+1 \text{ terms}) \\ & = \left\{ \begin{aligned} & \frac{mP[C_{x+n} + C_{x+n+1} \dots C_{x+n+m}]}{D_x} \\ & - \frac{P(C_{x+n+1} + 2C_{x+n+2} \dots mC_{x+n+m})}{D_x} \\ & + \frac{\frac{P}{y}(C_{x+n} + C_{x+n+1} \dots C_{x+n+m})}{D_x} \end{aligned} \right. \\ & = \left\{ \begin{aligned} & \frac{mP(M_{x+n} - M_{x+n+m+1})}{D_x} \\ & - \frac{P(R_{x+n+1} - R_{x+n+m+1} - mM_{x+n+m+1})}{D_x} \\ & + \frac{\frac{P}{y}(M_{x+n} - M_{x+n+m+1})}{D_x} \end{aligned} \right. \end{aligned}$$

Thus the present value of (I)

$$= \left\{ \begin{aligned} & \frac{mPM_{x+n} - PR_{x+n+1} + PR_{x+n+m+1}}{D_x} \\ & + \frac{\frac{P}{y}(M_{x+n} - M_{x+n+m+1})}{D_x} \end{aligned} \right.$$

Next, to determine the present value of (II). It may be noted that the contributions are assumed to be paid at the end of each year. The last contribution would not be assured, as it would fall due on the day on which the pension is assumed to be entered upon.

Thus,  $b_x$  being the initial salary, and  $\kappa$  the annual increment, the value of this benefit will be

$$\left. \begin{aligned} & \frac{b_x [C_{x+1} + 2C_{x+2} + 3C_{x+3} \dots (n-1) C_{x+n-1}]}{D_x} \\ & + \frac{\kappa [C_{x+2} + 2C_{x+3} \dots (n-2) C_{x+n-1}]}{D_x} \\ & + \frac{\kappa [C_{x+3} + 2C_{x+4} \dots (n-3) C_{x+n-1}]}{D_x} \\ & \dots \dots \dots \\ & + \frac{\kappa C_{x+n-1}}{D_x} \end{aligned} \right\} \times \frac{\omega}{100}$$

$$= \left\{ \begin{aligned} & \frac{b_x [R_{x+1} - R_{x+n} - (n-1) M_{x+n}]}{D_x} \\ & + \frac{\kappa [R_{x+2} - R_{x+n} - (n-2) M_{x+n}]}{D_x} \\ & + \frac{\kappa [R_{x+3} - R_{x+n} - (n-3) M_{x+n}]}{D_x} \\ & \dots \dots \dots \\ & + \frac{\kappa (R_{x+n-1} - R_{x+n} - M_{x+n})}{D_x} \end{aligned} \right\} \times \frac{\omega}{100}$$

Thus, the present value of Benefit (II) is

$$= \left\{ \begin{aligned} & \frac{\frac{\omega}{100} b_x [R_{x+1} - R_{x+n} - (n-1) M_{x+n}]}{D_x} \\ & + \frac{\frac{\omega}{100} \kappa \left[ \sum_{x+2}^{x+n-1} R - (n-2) R_{x+n} - \frac{(n-1)(n-2)}{2} M_{x+n} \right]}{D_x} \end{aligned} \right.$$

Collecting the present values of the benefits (a), (I), and (II), the following is the result:—

$$\frac{P(N_{x+n} + \frac{1}{2}D_{x+n}) + mPM_{x+n} - PR_{x+n+1} + PR_{x+n+m+1} + \frac{P}{y}(M_{x+n} + M_{x+n+m+1}) + \frac{\varpi}{100}b_x[R_{x+1} - R_{x+n} - (n-1)M_{x+n}] + \frac{\varpi}{100}\kappa\left[\sum_{x+2}^{x+n-1} R - (n-2)R_{x+n} - \frac{(n-1)(n-2)}{2}M_{x+n}\right]}{D_x}$$

Noting that  $\Sigma c$  (or sum of contributions)  $= mP + \frac{P}{y}$ ; also that  $\Sigma c = \frac{\varpi}{100}\left(nb_x + \frac{n(n-1)}{2}\kappa\right)$ ; and equating the value of the benefits to the value of the contributions, there results  $\varpi =$  percentage deduction from salary

$$= \frac{100 \times \left[ P(N_{x+n} + \frac{1}{2}D_{x+n} - R_{x+n+1} + R_{x+n+m+1}) - \frac{P}{y}M_{x+n+m+1} \right]}{b_x(N_x - N_{x+n}) + \kappa[S_{x+1} - S_{x+n} - (n-1)N_{x+n}] - \left[ nb_x + \frac{n(n-1)}{2}\kappa \right] M_{x+n} - b_x[R_{x+1} - R_{x+n} - (n-1)M_{x+n}] - \kappa \left[ \sum_{x+2}^{x+n-1} R - (n-2)R_{x+n} - \frac{(n-1)(n-2)}{2}M_{x+n} \right]}$$

Applying this formula, and assuming  $n=40$ ,  $x=20$ ,  $b_x=100$ ,  $\kappa=5$ , and  $P=\frac{2}{3}$  of average salary over whole term, the following are the contributions:—

- (1) The total contributions  $= \left[ nb_x + \frac{n(n-1)}{12}\kappa \right] \times \frac{\varpi}{100} = 7,900 \times \frac{\varpi}{100}$ .
- (2) The pension  $= 131\cdot\dot{6}$ .
- (3) The term of the decreasing assurance is found by dividing  $7,900 \times \frac{\varpi}{100}$  by  $131\cdot\dot{6}$ . If there be a fraction in the result, the term will be the integral part plus 1.

As a trial, I assume  $\varpi$  to be 10, so that  $\frac{7,900}{131\cdot6} \frac{\varpi}{100}$  will be  $\frac{790}{131\cdot6} = 6$  years (exactly).

*To find the Numerator.*

$  \begin{array}{r}  + \\  N_{60} = 102,272\cdot4 \\  \frac{1}{2}D_{60} = 4\cdot995\cdot75 \\  R_{67} = 43,275\cdot11 \\  \hline  150,543\cdot26 \\  77,406\cdot64 \\  \hline  73,136\cdot62 \\  100 \times 73,136\cdot62 \times 131\cdot6 \\  \hline  = 962,965,496 = \text{Numerator.}  \end{array}  $	$  \begin{array}{r}  - \\  R_{61} = 77,406\cdot64 \\  \frac{P}{y}M_{67} = 0 \\  \hline  77,406\cdot64  \end{array}  $
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*To find the Denominator.*

$  \begin{array}{r}  + \\  N_{20} = 1,174,345 \\  - N_{60} = 102,272\cdot4 \\  \hline  100 \times 1,072,072\cdot6 = 107,207,260 \\  \\  S_{21} = 20,241,319 \\  - S_{60} = 853,723\cdot8 \\  \hline  19,387,595\cdot2 \\  - 39 N_{60} = 3,988,623\cdot6 \\  \hline  5 \times 15,398,971\cdot6 = 76,994,858 \\  \\  \hline  184,202,118 \\  86,483,163 \\  \hline  97,718,955 = \text{Denominator.}  \end{array}  $	$  \begin{array}{r}  - \\  7,900 + M_{60} = 53,101,350 \\  R_{21} = 550,587\cdot5 \\  - R_{60} = 84,128\cdot32 \\  \hline  466,459\cdot18 \\  - 39 M_{60} = 262,145\cdot91 \\  \hline  100 \times 204,313\cdot27 = 20,431,327 \\  \\  \sum_{22}^{59} R = 10,767,745\cdot56 \\  38 R_{60} = 3,196,876\cdot16 \\  \hline  7,570,869\cdot40 \\  - 741 M_{60} = 4,980,772\cdot29 \\  \hline  5 \times 2,590,097\cdot11 = 12,950,486 \\  \hline  86,483,163  \end{array}  $
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Thus  $\varpi = \frac{962,965,496}{97,718,955} = 9\cdot854$ .

This shows that  $\varpi$  is a little under 10 per-cent.

Making another trial, it will be found that 9·896 is the true percentage.

As will be seen on reference to Table No. 1, the deferred annuity by itself would require a percentage deduction of 7·664, so that the additional benefits (I) and (II) necessitate a further deduction from salary of over 2 per-cent.

It is now necessary, in order to get the value of the benefit (III), to look into the question of secessions.

Undoubtedly the secessions are a very uncertain element in any calculations, but there is no getting over the fact that, although an uncertain and, from an actuarial point of view, an undesirable element, they cannot be ignored in the case of Pension Funds generally. The influence of secessions amongst friendly societies and amongst many Pension Funds is most marked.

In the case of friendly societies, the rule of course is that a member seceding forfeits everything. In some Pension Funds the same rule prevails ; in others, however, the contributions without interest are returned on secession.

To get some idea of the benefit (III) to which I have already referred, it is necessary to construct a table of secessions. As I have used the  $H^M$  Table throughout this paper, I thought it might be useful to see what results would be obtained by working up the “discontinuances” as shown on pages 128–129 of the Institute of Actuaries’ Mortality Experience,  $H^M$  Table.

The following are the calculations :

Taking the numbers “lived”, and deducting one-half of the entrants and one-half of the deaths, gave the *exposed to risk of discontinuance*.

In this way, that is, by taking the ratios,

$$\frac{\text{Discontinued}}{\text{Exposed to risk of Discontinuance}}$$

the following table was formed :



TABLE NO. 4.

*H<sup>M</sup> Rates of Discontinuance.*

Age	Rate of Discontinuance	Age	Rate of Discontinuance	Age	Rate of Discontinuance
10	·03896	40	·02871	70	·00838
11	·05163	41	·02684	71	·01140
12	·05743	42	·02478	72	·00994
13	·05068	43	·02524	73	·01043
14	·03898	44	·02318	74	·00911
15	·04946	45	·02316	75	·00815
16	·04671	46	·02314	76	·00933
17	·06614	47	·02243	77	·00798
18	·06018	48	·02163	78	·00763
19	·07290	49	·02074	79	·00765
20	·09125	50	·02095	80	·01182
21	·11914	51	·02061	81	·01517
22	·07762	52	·01906	82	·00358
23	·07386	53	·01951	83	·01904
24	·06627	54	·02043	84	...
25	·06494	55	·01782	85	·01313
26	·05826	56	·01850	86	·00607
27	·05284	57	·01735	87	·03433
28	·05103	58	·01778	88	·01266
29	·04645	59	·01736	89	·01905
30	·04461	60	·01575	90	·02564
31	·04109	61	·01389	91	...
32	·03764	62	·01417	92	...
33	·03680	63	·01391	93	...
34	·03320	64	·01270	94	...
35	·03363	65	·01348	95	·02857
36	·03021	66	·01186	96	...
37	·03009	67	·01298		
38	·02896	68	·01303		
39	·02945	69	·00950		

Comparing the  $H^M$  rates of discontinuance with the Foresters' and Oddfellows' rates of secession, we have the following (see page 86 of Mr. Neison's Foresters' Experience) :

TABLE NO. 5.

Ages	A.O.F., 1871-1875	Oddfellows, 1866-1870	$H^M$
20-	45·93	26·16	42·81
25-	32·29	23·26	27·35
30-	21·88	16·63	19·33
35-	14·66	11·77	15·23
40-	8·78	7·79	12·88
45-	5·04	4·79	11·11

Taking a few separate ages, the following further comparative table is interesting :

TABLE NO. 6.  
*Rates of Secession per-cent.*

Age	H <sup>M</sup>	Foresters	Oddfellows	Actual Experience of a Pension Fund
20	9·125	9·928	4·903	6·97
25	6·494	7·536	5·189	5·55
30	4·461	5·072	3·835	4·27
35	3·363	3·441	2·678	3·58

It will be noticed that the H<sup>M</sup> runs pretty close to the Foresters' Experience.

No doubt objections will be raised to the construction and use of a table of secessions based on the H<sup>M</sup> discontinuances. There can, however, be no harm in seeing what the results are like, and I have accordingly prepared tables which show the H<sup>M</sup> mortality with secessions.

The following is a comparison of the annuity-values (with secessions) :

Age	H <sup>M</sup> <i>a'</i> <sub><i>x</i></sub>	Foresters (complete)	Oddfellows <i>a'</i> <sub><i>x</i></sub>
15	9·916	...	...
20	9·720	9·814	12·336
25	11·900	12·062	13·304
30	12·923	13·783	14·415

To apply these tables, I take first the case of the simple deferred annuity, without return, at age 60. The age 20 at entry, term 40 years, increase of salary £5 per annum, interest 3 per-cent. (See Table No. 1, where the percentage is shown as 7·664.)

To find the percentage, allowing for secessions. There will, of course, be no secessions when the annuity is actually entered upon, and the formula will be modified as follows :

$$100 \times P \left[ \frac{N_{x+n}}{D_{x+n}} + \frac{1}{2} \right] \times D_{x+n}^1$$
$$\frac{\hspace{10em}}{(N_x^1 - N_{x+n}^1) b_x + \kappa (S_{x+1}^1 - S_{x+n}^1 - (n-1) N_{x+n}^1)} = 4\cdot150 \text{ percentage}^*$$

\* D<sub>*x*</sub><sup>1</sup>, N<sub>*x*</sub><sup>1</sup>, and S<sub>*x*</sub><sup>1</sup>, are based on a table of survivors constructed on the H<sup>M</sup> rates of mortality and secession.

Thus the secessions reduce the percentage from 7.664 to 4.150.

Returning to the problem where the benefits are—

(a) The deferred annuity or pension.

(I) The return of the excess of the amount contributed over the amount drawn as pension.

(II) The return of the contributions in the event of death before superannuation.

(III) The return of the contributions in the event of secession.

Bearing in mind that there will be no secessions after the annuity has been entered upon, the numerator will be

$$\left(100 \times [P(N_{x+n} + \frac{1}{2}D_{x+n} - R_{x+n+1} + R_{x+n+m+1})] - \frac{P}{y} M_{x+n+m+1}\right) \frac{D_{x+n}^1}{D_{x+n}}.$$

As a trial, I assume  $\varpi$  to be 7 (seven) per-cent. The term of the deferred assurance therefore = 4.2.

$$\text{Thus} \quad \varpi = \frac{135,288,700}{22,365,400} = 6.04.$$

Thus  $\varpi$  lies between 6 and 7 per-cent.

It is not, of course, suggested that these tables should be used in the valuation of a Pension Fund: the Fund's own experience is, no doubt, the proper guide.

In bringing to a conclusion this paper, which I am afraid has not gone very deeply into the subject, I may draw attention to a very difficult matter, which has occasionally presented itself of late years, and which is likely to arise frequently in future. This may be called the "proper apportionment" of the cost of a pension where two or more Funds have to share the cost. Take the case of an officer in the service of the Imperial Government who is transferred to the service of the Government of a British Colony. On retirement, the pension would be calculated upon the whole period of continuous service under the two Governments, and the Imperial Government has laid it down as a rule, that, in respect of the period during which the officer was in its service, it will contribute to the pension at the usual rate of one-sixtieth, taking the average salary *as at the date when the officer is taken over* by the other Government. This rule has the merit of

simplicity in working out the results, but it is not exactly equitable.

The cases, however, are not likely to be restricted to two Governments. For instance, an officer transferred from the service of the Imperial Government to the Cape Government may afterwards be transferred to, say, the Rhodesian Government; then, perhaps, to the Government of one of the new Colonies (the Transvaal or Orange River Colony). Each Government would pay its share according to the rule laid down by the Imperial Government. It is much easier, however, to find fault with the rule than to find a proper substitute for it.

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### DISCUSSION.

The PRESIDENT (Mr. C. D. Higham), in inviting discussion, reminded the members that the paper was written and sent to England before the publication of Mr. Manly's very excellent—he might almost say monumental—essay. In its original state it was consequently not altogether in a condition to be submitted to the Institute, and, on the author being communicated with, he requested that any necessary alterations should be made. The members must therefore remember that, by the exigencies of the case, the author had not had an opportunity of seeing the paper as it appeared that evening.

Mr. C. E. THOMAS, in opening the discussion on the paper, said the subject of Pension Funds, in addition to being a vastly complex and interesting one, bulked so largely in the public view, in one form or another, that any paper which threw further light upon the matter was sure of a hearty welcome from the Institute. Mr. Manly's paper was described by more than one speaker as a text-book upon the subject, and it certainly seemed to cover the ground so completely as to leave very little further to be said. Mr. McGowan, however, had succeeded in presenting some of the points involved in a different light, and in giving some interesting and useful tabular comparisons. His general line of thought was theoretical rather than practical. A problem investigated by Mr. Manly first under simple conditions, and later under the conditions of actual practice, was treated in the paper in a somewhat different manner, and elaborated into a very complicated formula on the basis of a scale of salaries increasing by constant first differences. Finally there was the element of discontinuance brought into account, and previous formulas modified and worked out on a hypothetical table allowing for withdrawals. The first point likely to provoke discussion was as to the most equitable and convenient method of assessing the future salaries for the purpose of fixing the pension benefits. The question at once arose, under what circumstances was the scale of salaries required? Was it for the purpose of valuing an existing Fund, or fixing the



benefits for a new one? In the first place they had to deal with facts and existing rules, irrespective of their justice or injustice to individuals. If a scale of salaries could be obtained which represented approximately the rate of increase likely to be experienced, it could be fearlessly used in valuing the assets and liabilities. As Mr. Manly pointed out, by reducing everything to a standard of 1 at age  $x$ , and operating with the multipliers on the individual salaries, it was not a uniform rate of salary that was assumed but a uniform rate of increase, which was quite another thing. It would be noticed, on reference to the interesting comparison made in Table 1, that an error even in the actual rate of increase would not have an overwhelming effect upon the results. The author showed that an addition of 67 per-cent to the initial rate of increase caused a rise in the contribution of from .2 per-cent to .3 per-cent only. That sank into insignificance compared with the effect caused by a variation of  $\frac{1}{2}$  per-cent in the rate of interest employed, which caused a difference of .8 per-cent to 1.1 per-cent in the contribution; and, unless the rate of interest was guaranteed by the employers, it would probably be admitted that a decline of  $\frac{1}{2}$  per-cent in that factor was quite as likely as a revolutionary change in the rate of increase of salaries. To consider the second case, that of starting a Fund, the important thing to know was, whether, apart from this question, the Fund was being formed upon equitable lines. If it was, then it would seem that, providing the entrants to the Fund were in an equal position at the beginning of their career, with the same chances of promotion in each case, no great injustice was worked by the assumption of a uniform scale of salaries to assess the benefits. It would not be fair to include, say, the messengers of a bank in the same scheme as the official staff, or the porters of a railway company with the clerical staff, because the chances of promotion as between the two classes were obviously unequal. On page 31 a very complicated-looking formula was given, to which reference had already been made. In connection with that formula he wished to notice a point, trivial in itself but for the fact that special attention was given to it in the paper, namely, the term of the insurance in the last year. To take a practical example, suppose the total contributions to amount to £200 and the pension to be £30 per annum; an allowance had to be made for a decreasing assurance for six years with an outstanding fraction. The author said that that fraction must be insured for the whole of the seventh year. That, no doubt, would be correct if one was dealing with the strict annuity of theory, namely, one payable yearly at the end of the year and not apportionable, but that was hardly ever the annuity of practice, which was to all intents and purposes continuous. The author had given an interesting example showing the effect on the contribution required when allowance was made for an assumed rate of withdrawal. In the face of his frank admissions on the subject, criticism would be out of place as to the particular lapse rate used, that of the  $H^M$  Table. He (the speaker) had personally been taught to consider the  $H^M$  withdrawals as being entirely unsuitable for use unless considered as a function of the age and duration together, or of the duration alone. It was



somewhat of a surprise to him to find the  $H^M$  experience agreeing so closely with the Foresters' experience, which agreement, however, was not maintained when compared with the Oddfellows. It would be noticed that the comparison was carried only to age 45, and by that time the  $H^M$  rate was considerably in excess of the Foresters' rate. It might be expected that that divergence would be accentuated at the higher ages where the  $H^M$  figures were still considerable. It was there that the principal danger of the  $H^M$  rate came in, because the figures were over-estimated just at that part of the table where they would have most weight when applied to a Pension Fund, namely, after considerable terms of service and contribution. Apart from that it seemed a little dangerous to argue from the similarity, that the same results would be experienced in the case of a Pension Fund, because the governing circumstances of a Friendly Society were entirely different. The fact that in one case membership was voluntary, and in the other generally compulsory, was quite sufficient to cause wide divergences in that respect. The author certainly strengthened his position by his exhibition of the actual experience of a Pension Fund. The rates shown were different in character from most that one had met with previously, which had tended to run down much more rapidly from the younger ages to the older, but there again it probably turned upon the method of treating those retiring from service through ill-health. Generally speaking, the latter class began to multiply somewhere near the point where the withdrawals proper sank into insignificance. If, therefore, the invalided members were written off the books, either by receiving a cash payment in lieu of pension or a return of their contributions like the withdrawals, the result would be one fairly steady rate, instead of two, one decreasing and the other increasing. Comparing the results obtained with those where no withdrawals were allowed for, it would be found that the necessary contributions were reduced at age 20 from 7.664 to 4.150. That seemed a very large reduction, but it must be borne in mind that no benefits whatever were allowed for on withdrawal; although in practice, whatever happened to the *early* withdrawals it would be impossible for any Fund to ignore the claims of those leaving after a long term of membership, whether they were called withdrawals, retirements, or anything else. In the concluding problem he thought a little more explanation was wanted of the requirements and the formula. As he understood it, the benefits were the same as in the problem on page 28, plus the return of the contributions on withdrawal (Benefit III), the whole to be calculated on a table allowing for secessions as well as deaths. They were led to infer that the only alteration in the numerator was to defer the benefit by the combined instead of the ordinary table, and in the denominator to substitute the modified D, N and S columns for the normal functions. One could only suppose, therefore, that the C, M and R columns were based on the total decrements at each age, and not on the deaths alone, so as to include the withdrawal benefits. That was the most convenient course, where (as in the present instance) the benefits were the same on withdrawal as on death. Where, however, there was any difference in the treatment

of the two classes, special commutation columns must be formed for each class. It was impossible to follow the author's calculations in the absence of the tables, but the decrease in the rate of contribution now obtained seemed to him surprisingly large. The figures were not given exactly, but approximately they seemed to be in the first case slightly under 10 and in the second a trifle over 6. The reduction (about 40 per-cent) was nearly as great as in the previous problem (46 per-cent), where no benefits were given on withdrawal. Bearing in mind that the whole of the contributions were to be returned on withdrawal, in the present problem the only source of profit was the interest on the said contributions which remained in the Fund. Assuming the facts to be as stated, the reduction, if caused by that element alone, was rather startling, and emphasized in a remarkable manner the danger of over-estimating the withdrawals.

Mr. T. G. ACKLAND agreed that it was somewhat unfortunate that the author had not had an opportunity of fully and carefully considering the classical paper written by Mr. Manly upon the same subject. At the same time, he was sure the Institute was much indebted to Mr. McGowan for bringing the subject again forward, because every contribution to their proceedings tended to throw further light upon a somewhat difficult subject, and they welcomed a paper coming from a member in the Colonies, and from one with whom they had been familiar in earlier days in London. He agreed with Mr. Thomas's remark that it is very much better, in dealing with such problems, to compute commutation columns, introducing not only the elements of mortality and interest but also the element of the variable salary at each age. The author had evidently been very much hampered by his different method of approaching the subject. He had had to make assumptions, some of them of a rather unsatisfactory nature, as to the increase in the salary, and to proceed on the lines of an increase by constant first differences. Formulas of a complicated character were also deduced, which compared unfavourably with those given in Mr. Manly's paper. The introduction of the salary was a comparatively simple matter, and once that was done almost the whole of the difficulties connected with the problem were manageable. Like Mr. Thomas, he had also been very much struck with the effect of introducing a secession rate, and he ventured to think that the author would find that there was some doubt as to his assumptions and results in this respect. When one found so material a fall in the computed contribution, from about 10 per-cent to 6 per-cent on the introduction of the element of secession, some further consideration of the matter seemed to be required. It was especially difficult for him to believe that such a result could be confirmed, because he had always contended that the effect of the secessions on the computed contributions was probably comparatively trifling. The author had referred to some discussion in connection with the School Board Fund, and to remarks made by Dr. Sprague and Mr. Sutton some years ago on the question of a percentage deduction from the salary. He ventured to think that Mr. McGowan had entirely misunderstood Mr. Sutton's remarks. If he read those remarks rightly—and he



remembered reading them with their context on a previous occasion—Mr. Sutton was not contending that a percentage on the salary was in itself an improper form of contribution, or one unsuitable in the particular case. What he did contend, as he understood it, was that, when people were being taken in classes, for example, to take the instance referred to by Mr. Thomas, as messengers in banks and the official or managerial staff—it was not right to charge a uniform percentage for those two classes. The scale of salary was different for the several classes, and the percentage must properly be computed for each class separately. There was nothing in the paragraph quoted by the author which indicated that Mr. Sutton objected to the percentage deduction in itself, but only to the fact that it was not separately assessed upon the salary of the class. The author had referred to the English Life Table No. 3 as a suitable table of mortality to be employed in connection with Pension Funds. In his opinion that no doubt was generally so, but he thought it would be found that, when the pension age was arrived at, it was hardly suitable to employ such a table. A table would have to be used which brought in a lower rate of mortality, or it would have to be adjusted in some arbitrary way to give effect to the higher vitality of the pensioners. He entirely agreed with Mr. Thomas that the author was not adopting the best course in ignoring the fraction of the period of the term of insurance. Assumptions such as those made in the paper might not introduce a material error, but it was very simple to allow for the fraction in the calculation, and once the scale of salary was introduced into the table the calculations might as well be completed thoroughly without introducing unnecessary assumptions.

Mr. S. J. H. W. ALLIN said that, while joining with the other speakers in thanking the author, he was rather surprised that, in a paper dealing with Pension Funds, he raised no protest against the methods in general use for fixing the superannuation allowance. To his mind there was no particular charm or advantage in the pension being made a percentage of the average salary or salary for the last few years. On the other hand, as Mr. Sutton pointed out, by adopting that method not only do some members of the Fund pay too largely towards the benefits of others, but, what was more important to them as actuaries, an additional assumption was introduced which needed most careful handling. He wished to put forward the following suggestion as a means of avoiding an otherwise necessary assumption. A table could be calculated showing at each age the benefit and pension which 1 per annum would secure. Then, to find the pension of any particular member, that table had only to be entered at his present age and the resultant figure multiplied with the percentage of his salary. He was assuming the usual method of fixing the contribution as a fixed percentage of salary to hold. On his salary being increased, the table was again entered at his increased age and multiplied by the percentage of his increase in salary and the result added to the pension he was previously entitled to. Of course, a record would have to be kept. That would entail very little additional trouble, and would probably, if made on cards, be of great use in the valuations of the Fund. He wished to make an additional

suggestion, namely, whether it would not be simpler to substitute for Benefit (I) on page 28 the guarantee of a limited number of payments of the annuity.

Mr. H. W. MANLY said the author seemed to have studied all the works on the subject, not only the published works by different authors who had treated the subject of Widows' Pension Funds, but also blue-books and actuarial reports upon several large Funds, which had been printed and circulated for private use. The author must, therefore, be given credit for having read thoroughly everything that had previously been written upon the subject. Like many others, he had been groping about for light, and had discovered a little. He had not attacked the problem in anything like the way that he (Mr. Manly) did, and he did not think the way in which Mr. McGowan had approached the subject was likely to give him any really satisfactory results. For instance, the problem with which the author had started, namely, that of finding the percentage of salary at different ages to provide a certain pension based upon the average salary, was not the problem which had to be solved. The problem which nearly always presented itself was: Given a contribution of a percentage of salary, what percentage of average salary could be given as a pension? Individual cases had not to be taken, but the whole class. Thus, supposing a whole body of men elected to contribute a certain percentage of their salary, what percentage of average salary could be given to all as a pension? That problem necessarily involved a consideration of the increase in the salaries. As he pointed out in his paper, after all the problem was not so much the actual salary as the annual average ratio of increase in the salary. He would illustrate that by taking the author's Table No. 1. Mr. McGowan had there given an illustration at age 20, based on a salary of £100, increasing by constant differences of £3, £4, or £5 per annum. At age 30 he also took the same figures—£100, increasing by a constant addition of (say) £5. That was not right. If the salary at age 20 was £100, and increased by £5 a year up to the age of 65, at age 30 the salary became £150 increasing by a constant addition of £5, which was a different thing to £100 increasing by £5. The percentage of increase grew less. At 40 £100 of salary was not increased by £5 a year, but it was £200 of salary increasing by £5 a year. Therefore the ratio of increase was less as the ages increased. He wished to make some remarks about discontinuances, although Mr. Thomas had covered the ground so fully that he had left very little for him to say. He held it to be a first principle that in all such investigations as these, like must be always compared with like. Secessions in life assurance companies were purely voluntary; but secessions in Pension Funds, with which they had to deal, were not voluntary. If a man left the service it was compulsory that he left the Fund, and he could not cease to contribute to the Fund so long as he remained in the service. So that the two things were totally dissimilar, and it would be a pure accident if the results even approached each other, which they did not. The author had compared his figures with the Foresters and the Oddfellows Societies, where secessions also were purely voluntary. It would be absolutely misleading to base



calculations for a Pension Fund on tables constructed on purely voluntary secessions. Mr. Allin made a suggestion that the pension might be ascertained by investing the man's salary in a deferred annuity, any increase which he received to be applied to the purchase of a further deferred annuity at his then age. There was a large railway Fund which was carried out in that way, and it gave the lower-grade men, whose salaries never increased much, a very decent pension. But the higher officials, whose salaries did not amount to a very large figure until they got into middle life, so that the amount of the deferred pension which their increased contribution would purchase began to be very small, did not like it, and wanted to know whether it was not possible to alter the system so that they could have the same proportion of pension as the lower-grade men. Mr. King, Mr. R. P. Hardy, and himself were consulted in the matter, and they set their faces strongly against any alteration. They said that the system as existing was the most scientific and the best that had ever been devised, and that the higher-paid officials had sufficient salary to provide elsewhere for their pensions. Of course the officials did not see it, and he did not know whether the rules had been altered since. Perhaps a few years hence they might be asked to re-value upon a totally different basis. As he pointed out in his paper, one was seldom consulted a second time without there being some alteration in the rules, which meant a very material alteration in the value of the benefits. In nearly all the schemes, the men tried to get all their money back somehow, as well as a pension; and he had before him now a case where they wanted their money back with compound interest and to have a good pension as well.

Mr. GEORGE KING said that, in view of the circumstances which the President had so clearly set forth in his opening sentences, he did not propose to criticize the paper. He felt that it was a creditable effort in a very difficult field, and it was well worth the attention of students as showing how problems might be worked out. At the same time, he must frankly say that his experience led him to feel that, from a practical point of view, the attack must be made in quite a different manner. The principal question which came before the actuary was the valuation of Funds, and nominally that was the title of Mr. Manly's paper, Mr. McGowan's paper being on the calculation of rates. But Mr. Manly had devoted a very considerable and important section of his paper to the preliminary enquiry which Mr. McGowan had tackled in a different way. The starting of a Fund was difficult. There was no recorded experience, and one could only judge from the experience of similar institutions, and he did not think it was possible to calculate with any accuracy either the contributions which should be paid for given benefits, or the benefits that could be granted for given contributions. It was essential to see that the rules were elastic, so that in course of time adjustments might be made to prevent the Fund from getting into difficulties; or, better still, to have a wealthy and generous employer, who would put right any errors that might unavoidably occur. Great caution must be exercised by actuaries when they advised with regard to the starting of Funds. One very remarkable case came before him



a few years ago. A great financial institution, employing many hundreds of clerks, started a Fund about 25 years ago. They consulted an eminent actuary, one of the President's predecessors in the chair, who, however, was no longer with them. He gave a report, which he (Mr. King) had not seen, but which, he understood, stated what percentage of salaries would be required for certain benefits. He assumed that the Fund was to be started by new entrants only. The Institution took a different view, and started the Fund with all its then employed. The result was that it started with a deficit amounting to well over six figures sterling. That showed how necessary it was to enquire into all the circumstances before advising definitely upon the starting of a Fund. He did not think it was the fault of the actuary, who simply gave answers to the questions placed before him. But those who consulted him had not understood the great difference between the present employed and those who would join later on. In order to show what happened he would mention that one of the officials, the general manager of the concern, who was drawing a salary of £3,000 a year, paid for three years 5 per-cent of his salary into the Fund, and then retired with a pension of £2,000 a year, and at the time of the valuation he had drawn that pension for 17 years. On several occasions he (Mr. King) had been asked to start such Funds. Wherever possible he had followed the lines advocated by Mr. Allin. With that system it was quite easy to adjust the rates of pension for each £ of purchase-money, so as to allow for other benefits, and, barring the discontent of some of the highly-paid officials, the scheme worked very well. There were two ways of carrying it into effect. They might either take the payment made each year by the employee as a single premium to buy a certain amount of pension, or they might assume an annual premium—it did not matter which. In one case the percentage of salary was taken as an annual premium to buy a pension, and in the other case each payment was taken as a single premium. The question of average salary was a very important one, and he entirely agreed with the method which had been set forth by Mr. Manly, of dealing with it by taking the experience of the Fund, without reference to arbitrary assumptions as to increase. But there was one point which he did not think had been definitely referred to before, namely, that from the very nature of the case, the experience of the Fund was not always to be taken without adjustment. The great railway companies were good examples. They had grown rapidly. In the early days they had their general manager, their chief engineers, and so on, receiving fairly large salaries, but only had a comparatively small staff of the lower-grade men. When the system developed they did not require more general managers and secretaries, and in the same proportion did not require highly-paid officials, but took on only lower-grade men. The result was that if the past experience of the Fund over a series of years was taken, the average salaries at the higher ages were exaggerated, because the average age of the lower-grade men was comparatively low. Therefore the average pension which would have to be paid was exaggerated, as were also the contributions which would be paid by the staff generally. In dealing with a Fund of that kind it was necessary to enquire

further into its progression, and properly adjust the table of average salaries upon which the calculations were to be based. The next great point, the rates of mortality, was not so simple as Mr. McGowan seemed to assume, because several rates of mortality had to be considered. First, there was the rate of mortality among the active members, which was remarkably light, because those who got into bad health withdrew in one way or another. Then there was the rate of mortality among the members superannuated, which was practically normal, and, thirdly, the rate of mortality among those who drew what had been called early pensions—pensions prior to the pension age—and that rate of mortality was enormously heavy. Then, with regard to the rates of secession, it was not sufficient to take merely the experience of the Fund. If the experience of the Fund for a long series of years were taken, the rate of secession would be exaggerated. It was necessary to take the experience in periods. From his analysis of the Funds which had come before him, the rate of secession was diminishing, which fact was lost sight of unless the rates were taken in sections. If the rate of the last five years was taken, it was necessary to adjust it for the probable continued diminution in future, which was very important, seeing the great effect secessions had in the valuations of the Fund. Unfortunately it was necessary to bring them into account, although it would be much better to devise a means of avoiding doing so; but, seeing that the contributions depended upon membership, and that there were benefits payable on secession, one could not avoid bringing in the rate of secession. He merely wished to mention that one could not take the most recent experience of the Fund as an infallible guide; the progress of the rates must be continually watched. As regards the benefits, in such Funds one dealt only with averages. One result, unfortunately, was that the lower-grade officials, whose salaries practically did not increase, had in one sense to pay for the pensions of the higher-grade officials who were drawing five or ten times as much income, and who received very much larger pensions. That might seem to be very hard, but there was one consideration which took away the apparent hardship, namely, that in almost all cases the employer supplemented the contributions of the employed by possibly an equal percentage of the salary, and the employer was quite entitled to apply his share in any direction he pleased. He was entitled to give a large salary to a good man in an important post, and he was also entitled to say that, for the good of the whole service, his (the employer's) proportion of contribution should be largely applied to that man's pension, so long as the benefits of the lower-grade officials were not less than would have been provided by their own contributions alone. While there was no real hardship, at the same time it caused a great deal of grumbling among the lower ranks, which it was well to avoid if possible. Some Funds limited the pensions of the higher-grade officials to a fixed amount—a highest possible pension—and if the directors wished to give more they had to provide it from other sources, and not out of the Pension Fund. It was wonderful how little saving was thus made upon the general finances of the Fund, because the pensions of the higher-grade officials, although they seemed very large, were almost a negligible quantity in looking at the Fund as a great whole.



Mr. T. E. YOUNG said that in his limited experience of such Funds he had found that in practice they generally broke down or involved considerable and increasing trouble and perplexity, so that their elaborate statistical and mathematical structure, as not infrequently happened in other cases, simply formed an important exhibition of mere theoretical treatment. Not merely were they presented with the difficulty, in practical usage, of securing effective representation upon the Board of Control of the interested members, but they were also confronted with the specific difficulties arising from the admission into the service with which these Funds were associated of experts, at somewhat advanced ages, which tended to dislocate all their assumptions; and in the case of banks, for example, they possessed the additional trouble produced by the absorption of other banks or by the amalgamation between banks, with the importation of differences of ages and of official positions affecting the calculations of the Fund. In such instances he invariably advised that the bank or other institution should itself simply and directly guarantee a certain proportion of the average salary for whatever number of years might be decided on, as a pension, without the intervention of a specific Fund; and, with a view to avoiding a serious burden upon future shareholders when the pension ages were closely and numerously approaching, that the bank should set aside an adequate fund each year out of profits for the purpose, with its accumulations, of equalizing the general strain upon resources. Happily, however, even with that suggestion, the services of the actuary could not entirely be dispensed with, since it would be necessary from time to time to determine approximately, from an examination of the statistics of the staff, whether these accumulated annual sums were sufficient to provide the contingent obligations. With regard to the inclusion of the element of secession, where a separate Fund was established, he would add that if that element be involved it would, on the whole, distinctly produce the effect of diminishing the future rate of secession in consequence of the greater strength and attractiveness which the Fund would be able to display by reason of its introduction, and would thus tend gradually to defeat and countervail the effect which originally it was assumed would be produced.

The PRESIDENT, on behalf of the members, proposed a hearty vote of thanks to Mr. McGowan for his paper.

The resolution was carried by acclamation.

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*A Few Notes upon Certain Methods of allotting Surplus. By*  
THOMAS EMLEY YOUNG, B.A., *Past-President of the*  
*Institute of Actuaries.*

1. WITH a view to preparing the way, in some humble degree, to a generally satisfactory method of distribution hereafter, I beg leave to submit a few brief observations upon the merits and demerits of certain plans already in operation. Unhappily, I

cannot at present, in the existing state of our enquiries, present any constructive notions capable of being fused into a symmetric system.

2. The primary defect of all modes—and the remark possesses an application to organizations generally—is the absence of *Elasticity* in the original structure, so that subsequent modifications, indicated by Experience, are incompetent of introduction in reasonable harmony with the fundamental scheme itself, and thus, when their practical recognition becomes imperative, tend to assume the form of mechanical accretions instead of organic developments. Hence, the plan must either be completely re-fashioned, from time to time, or exhibit the character of incongruity and partial inequity.

3. With respect to the employment in any scheme of the office premium or the loading as one of the measures of apportionment, a comment may at once be submitted. I deal only with the *constant* for expenses. If this be  $x$  per-cent upon the sum assured, a Policy for £50,000 is charged 500 times the annual contribution levied upon an Assurance for £100 only. It is true that experience has proved that smaller Policies entail a greater proportionate trouble and cost than Policies for larger amounts, so that it is usually easier and cheaper to deal with one Assurance for a considerable sum than with several Policies of minor amounts. The reasons for this difference will be recognized by all Actuaries. Hence equity, on this ground alone, would require that a Policy for £50,000 should not be burdened with so seriously disparate a contribution towards expenses. On the other hand, a part of the loading is intended to provide for fluctuations of mortality-experience, and these are likely to prove more considerable and perplexing in the case of the larger Policies. And again, by the disappearance of these higher assurances, the Company is subjected to a more serious strain in supplementing the loss of premium income thereby incurred. But, viewing the whole of the circumstances from all sides, it is unjustifiable to require, in the instance I have cited as an illustration, that the contribution from a single Policy for £50,000 towards the current (and largely fixed) expenditure of a Company should be 500 times as great as that demanded from a single Policy for £100. I draw attention to this anomaly at the present moment simply for the purpose of emphasizing the contention (justly urged by Mr. Lidstone in vol. xxxii) that the element of loading should be assigned a prominent place in the allotment of

Surplus in order that compensation may thus be afforded for the disproportionate assessment which I have pointed out.

4. Dr. Sprague's method of Distribution is described in vol. xiv, page 396.

Its defects may be thus summarized :—

- (1) The principal (and not impartial) care is directed to the interests of Policies effected prior to the current Quinquennium, while the more recent Policies are relegated to a general and indiscriminated treatment.
- (2) The Policies possessing Reserves at the commencement of the Quinquennium are thus absolutely benefited (beyond contingencies), seeing that the primary source of surplus consists of interest, and all adverse fluctuations of mortality, for example, even though they may have occurred upon the older Assurances, solely or mainly, are thrown principally upon the fresh Assurances, while the favoured Policies again participate, proportionately with the more recent ones, in the lighter Mortality of Selection.
- (3) The basis of allotment in respect of the newer Policies consisting exclusively of the office premiums, it is sufficient to indicate the injustice of this arrangement, from the point of view of equity, by referring to my comment in section 3.
- (4) It will at once be noticed that, as time proceeds, the older Policies, with their increasing Reserves—and the Reserves for their Bonuses which, on account of this protected treatment, are likely to be accepted chiefly in the Reversionary form—will more and more completely absorb the major portion of the surplus. And this effect is intensified when we consider the vast augmentation of Endowment Assurances, especially those of shorter terms, and the considerable Reserves required to be retained.
- (5) When the general Reserves of the Company are increased by the adoption of a higher rate of mortality or a reduced rate of interest, the requisite amounts are derived from the *general* resources of the Company, including the contributions of the new Policies; hence, the older Policies obtain their



enhanced protection partly from the payments of fresh entrants, and then receive the surplus-interest upon the augmented Reserves thus created. An extending range of injustice is continuously produced.

- (6) Regard, again, the expenses incurred in procuring new business. I cannot agree that these should be chiefly applied in diminution of the share of surplus of the Policies thus secured; the cost of excessive business extension is not demanded by the public, but expresses a voluntary act and policy on the part of the Companies themselves, presumably and ostensibly in the interests of *all* their assured. The effective cause is the competition of the offices alone; the unsagacious interpretation of the significance and real effects of an abnormal acquisition of new business; with the involved and serious fallacy that the index of prosperity is to be discovered there. Success in this direction will usually mean, in the long run, a deviation from the path which leads to true success. Hence, for example, the suggestion sometimes submitted that the first year's premium should be excluded in the allotment of surplus is unjust.

And since surplus-interest constitutes the principal element of Profit, the older Policyholders are, by Dr. Sprague's method (where the total business charges deplete that portion of the Profit-fund in which the fresh Assurances alone participate), inequitably benefited at the expense of the younger members, who, by extravagant expenditure and unwise competition, are compelled to assure. Why should they be denuded of a larger share of Profit simply because the Company, for some reason or another, has decided on excessive expenditure in order to augment its income to an extravagant and needless degree?

- (7) It requires no capacity of prediction to forecast that Dr. Sprague's plan is doomed to failure as the public become more cognizant, from results, of the unjustifiable extent to which the Policies of longer duration are accorded a distinctive and unmerited preference.

The amount of new business must, therefore, necessarily diminish, and, in order to maintain it, the Company, adopting Dr. Sprague's method, must either (i) reconstruct the system of Division, and thus demonstrate its original inefficiency and inadaptability, or (ii) still further augment its expenditure to the detriment of its assured.

- (8) With respect to the history of the suggestion that surplus-interest should first be appropriated to older Policies, I find that Dr. Sprague's plan was published in October 1868; the date of its actual practical introduction I am not acquainted with: the Contribution-scheme of Mr. Sheppard Homans (of which this method is an application in a particular direction) is described in vol. xi for October 1863, page 121; but the notion appears to be originally traceable to Mr. W. P. Pattison, who, in the Messenger Prize Essay (vol. ix for July 1861, page 341) distinctly and precisely expressed the view (page 345) that surplus-interest should primarily be apportioned to former Reserves.

5. Mr. T. G. C. Browne's method is described in vol. xxxii, page 194, and incorporates the decided improvement of averting the main incidence of mortality-fluctuations from the newly-effected Policies. It forms an ingenious and interesting experiment, but appears to present, I sincerely regret to add, the following grave defects:

- (1) It was adopted when the Company valued at 3 per-cent, and its inelasticity of application will, from the nature of the case, always prevent the Office from basing its Reserves *directly* upon a specific rate of interest lower than 3 per-cent. For if the Company proceeded direct to  $2\frac{1}{2}$  per-cent, for example, the constitution of Group I (the loading) and of Group II (the surplus-interest) would be fatally affected, and the integrity of the system would consequently be violated.

The loading would be seriously reduced if a  $2\frac{1}{2}$  per-cent pure premium were directly introduced; and the surplus-interest on the augmented  $2\frac{1}{2}$  per-cent Reserves would be very appreciably increased,

so that the practical symmetry, so to speak, at present existing between Groups I and II, on the 3 per-cent basis, would be hopelessly destroyed.

- (2) The Company accordingly can only adopt a  $2\frac{1}{2}$  per-cent Reserve by valuing at 3 per-cent and including a supplementary fund embodying the difference of values between the two rates. This obviously introduces an anomaly, for the public might justly comment upon the Company pursuing the cumbrous method of a supplemental Reserve instead of valuing directly and professedly upon the basis of the diminished rate of interest.
- (3) In Group II, the surplus-interest allotted is the difference between the net rate actually realized and 3 per-cent; but by the supplemental addition to the Reserves the real Valuation-rate is constructed at  $2\frac{1}{2}$  per-cent, so that an inequality is involved,—looking to the foundation of the system,—since the Reserves obtain one-half per-cent less than the actual condition warrants, and this half is included in Group III for division between the remaining groups. Hence, if Group II were framed on the basis of the real Valuation-rate, not only would its amount be increased, but its participation also in the contents of Group III would assume an enhanced ratio, so that a twofold advantage would ensue.

In this Miscellaneous Group, Group I also shares on its 3 per-cent basis for  $\pi$ ; although, if the plan were rigorously carried out, Group I would be reduced by the increased pure premium, and would therefore also obtain a diminished portion of the Profits incorporated in Group III; *i.e.*, Group I would be doubly diminished.

By the introduction of the real Valuation-rate accordingly both the primary Groups would be affected in a twofold fashion, and the scheme be irretrievably ruined in its integrity.

It is obvious also that the ratio of participation in the results of Group III (on the basis of the results of Groups I and II) is necessarily of an arbitrary character, and is thus opposed to a consistent scheme of equity, particularly where the

professed object is to secure a division of Profits founded upon the several sources from which they have been derived.

- (4) Moreover the loading (diminished by the expenditure) has also earned surplus-interest during the Valuation-period; but this element is comprised in Group III for distribution between Groups I and II, so that Group II, which in no degree has contributed to the formation of this portion of the total Profits, improperly secures a share.
- (5) Mr. Browne adopts a truer course than Dr. Sprague in employing as a ratio the Reserves created at the *termination* of the Quinquennium. This method gives practical effect to the fact that surplus-interest has been produced by *all* the Policies during the period, and seems to me to form the appropriate measure of allotment of the Profit realized upon the non-participating business if a discrimination of sources is to be maintained. The difficulty, however, ought to be noted that this course erroneously assumes that the augmented Reserves for previous Bonuses have earned surplus-interest throughout the *complete* Valuation-term.
- (6) The plan, therefore, while exceedingly ingenious, unhappily does not seem destined to become the supreme method of assessment.
- (7) I omit the consideration of certain minor points which invite attention and criticism, for it has been sufficiently proved that inadaptability to changing circumstances of administration is inherent in the conception of the scheme.
- (8) I desire unaffectedly to add that the submission of these comments is accompanied by a sincere sense of respect and courtesy to Dr. Sprague and Mr. Browne.

6. I might finally refer to the very just contention of Mr. G. F. Hardy, in connection with discriminative plans, that the Assurance premiums should be relieved of that portion of the expenses of management which appertains to the Investments. The fees of the members of the Finance Committee, and a share of the office (and legal) costs—which can be approximately ascertained with reasonable exactitude—should be assessed upon the Interest Revenue.



7. Of the compound Reversionary system, it need only be mentioned at this point that, inasmuch as its equitable retention depends upon a certain excess in the net rate of interest realized beyond the Valuation-rate, its validity is not permanent, but diminishes as the market-rate definitely decreases.

8. I at once admit that no method, so far as my consideration extends, seems capable of being devised which can be confidently anticipated to harmonize, without some solution of continuity, with the possible variations of business conditions in the future. Every plan must obviously be based upon the best obtainable compromise between theory and experience; and the view underlying the criticisms upon which I have ventured is the imperative necessity of such adequate elasticity in the principles of the scheme selected as shall present the most pliant capacity of meeting changes without a fundamental violation of the structure of the scheme itself.

9. I may possibly, on a future occasion, submit more extended observations, and especially I may hazard a tentative prediction of the views which a future Actuary, surveying the varied history of past attempts, may endeavour to embody in a revised scheme. I would, at the moment, suggest that he may gradually be compelled to regard the *office* premium itself (more equitably adjusted) as the prime factor in his mode of distribution, and may remodel his treatment of Bonuses—the form of option permitted to the public (with the resulting difficulties involved), and the question of the allotment of Bonus upon Bonus—in a radically modified manner. He will probably more and more be induced to consider his Company as a homogeneous Commonwealth, and treat, as an unhappy and inefficient adjunct in such a community, the apparently illusive attempt to introduce discriminative distinctions, and the endeavour to divide, with impracticable consistency of scrupulous equity, the resulting profit according to an analysis of its several sources.

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#### POSTSCRIPT.

Since preparing these Notes, my attention has been called by a friend to Mr. George King's remarks in the discussion which followed Mr. Lidstone's Paper (*vide* p. 108 *et seq.* of vol. xxxii). I am consequently confirmed in my criticism of Dr. Sprague's method by the concurrent conclusions of so competent and practical a mind.

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*On Life Insurance Legislation in Denmark.* By PROFESSOR T. N. THIELE, Ph.D. Copenhagen, Corresponding Member of the Institute of Actuaries.

LIFE insurance in Denmark has certain peculiarities, which may be interesting perhaps also to foreign actuaries.

In 1871 the Danish Government thought to satisfy all the life insurance necessities of the country by establishing by law, and with the full guarantee of the State, one single institution, "Statsanstalten for Livsforsikring." By this all legislation with respect to private life insurance was supposed to be rendered superfluous.

This course was not only immediately severely criticized, but in the next year two private Danish companies were established, the proprietary life insurance company "Hafnia", and the mutual "Danmark", in order, through competition with the State institution, to expose the errors of the latter, and, as far as possible, remedy them.

The struggle was a prolonged one, and some 20 years later the State institution was compelled to inaugurate essential reforms, which not only entitled the insured to bonus, in addition to the guarantee from the State, but even trenched upon the funds which had been accumulated before participation was introduced, in order thereby to augment the future bonuses. Yet by this it only managed to effect insurances at a proportionately low agency commission, but not at all to supplant its competitors.

On the other hand, the two private companies did not succeed in their attempts at moving the Government to pass any law for the regulation of private life insurance. In the face of the companies the Government ventured to maintain its doctrine: those who want to be safe can go to the "Statsanstalt"; those wise in their own conceit may take care of themselves.

But now, of late years, the complete absence of legislation has been followed by its natural consequences. Almost simultaneously some 10 new life insurance companies have been started in Denmark.

There are no political motives for this movement, and only in a few cases any deeper cause. It was enough that manifestly it was possible to set up a competition, and that there was no law to limit the same. Then astute insurance brokers discovered

that it paid better to start new companies than to supply the old ones with business. No regard was paid to the evident impossibility of supplying as many companies at once with an efficient staff. Only in extremely few cases has an actuary taken part in the establishment of the new companies. The want of actuaries was not felt till later on, and though we do not in Denmark any more than elsewhere on the Continent require any knowledge of the law on the part of our actuaries, and though several scientifically educated mathematicians were appointed actuaries, the want is unsupplied. Nay, as lately as the year 1900 a company, in reply to an official inquiry, reported that their computations were made by an apothecary, who was clever in computation, even with rather large numbers.

By these events the Government has been compelled to recognize the necessity of legislation for private life insurance. In the winter of 1899-1900 the Home Secretary brought in a Bill dealing with this question; the Bill, however, meeting with general opposition, and the want of such a law being felt also in Norway and Sweden, a Scandinavian International Commission drafted another Bill, which is now being discussed in our life insurance circles.

The Commission has had no easy task. On a former occasion I declared that, in my opinion, it was impossible to accomplish it. None of the three countries has as yet so numerous and powerful a profession of actuaries, as a system of publicity, such as the English, implies. And as to the other system, with Government concession and control, it is well known that it is difficult to steer between the gulf of superficiality on the one side and the rocky wall of a destructive rigorism on the other.

The Commission, however, has worked well, and its Bill might, with a few amendments and much good-will from all sides, really be used. But in all three countries a considerable resistance seems to grow up against it, particularly perhaps in Denmark. After a preliminary meeting pamphlets have been published against the Bill, both by Danes and Swedes, and its adversaries summoned a remarkable life insurance Congress in Stockholm (26-28 August 1901). This Congress is remarkable, because it does not form a continuation of the long series of earlier meetings of delegates from the older Scandinavian life insurance companies. The latter were invited to the Congress, certainly, but only after they had agreed to hold no meeting in 1901, and

but few of them came. It was the representatives of the newly-established companies and of the mutual companies who met. In Sweden the two categories are nearly identical, and consider themselves prejudiced by the law. The fact that no tangible results were produced by the Congress has only aggravated the estrangement between the old and the new Swedish Life Offices, and within the last few months one of the new Swedish mutual societies, the "Vanadis", has been forced to amalgamate with another, the "Valand."

In Denmark other groups also have joined the adversaries of the Bill.

First, there are almost all the newly-established companies, also the proprietary companies. They assert that the Bill will make it impossible in future to start new companies, and certainly, it cannot be denied that it will compel young companies to go to work with greater circumspection and prudence than have been displayed by the companies which have been started in our legislationless time. And as the cost of acquiring new business, by this sudden augmentation of the intensity of competition, has become greater than was calculated, it may be very difficult for these new companies to re-model themselves according to the requirements of the Bill. But in so small a country, where foreign companies are actively represented, perhaps, moreover, there will be no use for a dozen Danish companies, most of which are rather uninteresting copies of a very few prototypes.

Another less numerous but more interesting group of Danish adversaries of the Bill is called to arms by the fact that the Bill suffers the Danish State institution to remain open as a competitor of the private companies, and in such a way that it is not to be subject to the same legal provisions as its private rivals. When the Minister of Finance of the State has the chief administration of the Government institution, the Home Secretary of the same State can scarcely be an impartial administrator of a very real control over its private competitors. More particularly, this group fears that the provision, which has been inserted in the Bill in accordance with Swedish regulations, enacting that a Government Officer is to control any change in the securities in which the premium reserve of the company is invested, will almost necessarily call forth pressure and encroachment on the part of the Government. In Sweden, which has no State institution, and which lacks certain legal protective measures which



exist in Denmark, these Government Officers may be useful. It is not easy to devise International laws, when such a national curiosity as a State institution for life insurance is maintained, and particularly when such an institution is found only in one of the countries.

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*Some Notes on the Net Premium Method of Valuation, as affected by recent tendencies and developments. By SAMUEL GEORGE WARNER, Fellow of the Institute of Actuaries, and Actuary and Secretary of the Law Union and Crown Insurance Company.*

[Read before the Institute, 24 February 1902.]

IT may well seem an act of no small temerity to attempt a fresh paper on a subject so often and so ably treated in essays read before this Institute as that of the net premium method of valuation. I am fully conscious of this, and the more so that I cannot profess to be able to add anything appreciable to what has already been said, either expressly or by implication, upon the matter. It will be admitted, however, that the practical importance of the subject, its vital bearing upon the conduct of the business of life assurance, can scarcely be exaggerated. Mr. J. Hill Williams, commenting upon Mr. A. H. Bailey's paper on "The Pure Premium Method of Valuation", in March 1878 (*J.I.A.*, xxi., 126), said "The subject of this paper is one upon which we can never have too much writing or too much discussion, for it is the very life of our science, and upon the views which we take of this most important question depends the safety of many of the greatest financial institutions in the country"; and with the spirit of such remarks we shall probably all agree. That being so, I think I am right in saying further that we have had no express discussion of this topic at our Institute for several years past. It has, doubtless, on many occasions been referred to—as a thing so intimately intertwined with our daily practice could hardly help being—in connection with other matters primarily before us; but it has not been the actual and avowed theme for consideration. Meanwhile, the march of events has not ceased, and several things have happened which are of interest in their bearing on the subject. In these circumstances perhaps there may be room for, and some use in, an

attempt at something like a review of the position ; which, without pretension to novelty, will endeavour to bring together into convenient compass from various sources certain considerations which may help us to “take our bearings” on the question up to date. Such a review must contain much that is familiar, perhaps little or nothing that is not ; but if it serves no other useful end there seems at least the prospect that, by re-ventilating the subject, it may elicit a suggestive and instructive discussion. Should it succeed in doing so, its purpose is attained.

To summarize briefly, then, the situation which we have to consider, we may say it is that of a method of valuation based on scientific principles, which has long held the field, served as a standard, and become, in the public mind, practically synonymous with financial soundness ; but which has also been subject to serious criticism, based on divers practical considerations. This criticism, as we shall see, is not suggested by one aspect of the subject only, but operates in varying directions. We must try to appraise its value, measure the importance given to it by the present conditions and tendencies of our business, and ascertain how far it may be held to have effectively shaken the prestige and power of the net premium method of valuation.

For what we may describe as the classical defence of the net premium method, we cannot do better than go to such a source as Mr. H. W. Manly’s Messenger Prize Paper on policy-values, read before the Institute in March 1868. There we find it demonstrated (*J.I.A.*, xiv, pp. 252—253) that if a Company computes its premiums at a given rate of interest, by a given table of mortality, and with a fixed loading for expenses ; and if, further, it experiences exactly its assumed mortality, earns exactly its assumed interest, and spends exactly its loadings, then the sum standing to the credit of any policy, effected at age  $x$  for the whole of life at annual premiums and in force for  $n$  years, will be

$$1 - \frac{1 + a_{x+n}}{1 + a_x}$$

“ This then *must* be the *true value* ; for, all the conditions “ assumed at the outset remaining the same, it is impossible for “ the Office to have either more or less than this amount to the “ credit of each policy.”

To the assumptions involved in this demonstration we shall have to return. Meanwhile let us consider the first serious criticism directed against the method.



It is easy to trace the history of this, and to appreciate its causes. In his memorable paper "On the Proper Method of Estimating the Liability of a Life Insurance Company under its Policies", read in April 1870, Dr. Sprague refers to a pamphlet by Dr. Zillmer, President of the German Life Insurance Institute, published at Stettin in 1863, and entitled "Beiträge zur Theorie der Prämien-Reserve bei Lebens-Versicherungs-Anstalten." Although it is now nearly forty years since that work appeared, I venture to think it is still well worth reading, for the lucidity of its reasoning as well as for its picturesque and frank description of a revolution in life assurance methods which is familiar to us all, and with the consequences and developments of which we have to deal daily.

That revolution was in the mode of paying for the introduction of new business. After describing the great and gratifying increase in the volume of new life assurances effected in Germany which the few years before the publication of his treatise had witnessed, Dr. Zillmer proceeds to explain that this must not be wholly set down to the growing appreciation throughout the community of the importance of life assurance to the individual, the family, and the State. The result has also been brought about by the assiduous labour of the Insurance Companies and their agents. Brought about by the latter, moreover, because "a mode has been devised of granting, on the completion of an assurance, a higher reward to the agent (who had hitherto only taken a uniform commission, throughout the duration of the policy, on the annual premium, and thus secured for himself after years of labour an advantage of but nominal value); thus securing as agents active business men who may find it possible to give up their former business and devote their entire energy to the procuration of new policies."

What had taken place, in fact, was the introduction of a system of commuted commission, and this system, bringing in its train a great and sudden accession of new business, had a serious effect upon reserves computed strictly according to the net premium method. "The cost of acquiring a new policy, providing for the first year's mortality, and setting up, under any Table, the necessary reserve at the end of the first year, must and will in such circumstances exceed the premium received for the policy. The deficit so arising will naturally increase with the influx of new business; and, provided that influx be sufficiently great, the deficit may attain such dimensions

“ that even a Company of considerable wealth may be brought  
“ into a position of embarrassment. Here even a non-expert  
“ must see that a contradiction is involved. On the one hand,  
“ an abundant accession of new business is desired ; on the other  
“ hand we are thereby brought into financial trouble—and that  
“ (as will be presently shown) without excessive payment of  
“ commission.” Arising out of these conditions the author sees  
the possibility of a grave public danger. If a Company adopting  
what Dr. Zillmer describes at the date when he writes as “ the  
new custom”, of paying a commuted agency commission of  
1 per-cent on the sum assured, be expected to make for such  
business in its first year a strict net premium reserve, and be  
held at fault if unable to do so, the result will be “ to shake the  
“ confidence of the public in Life Assurance Companies ; and to  
“ injure precisely those Societies which publish the details of  
“ their business with the greatest candour, and make no secret of  
“ their commission payments.”

Here then is a dilemma, the cause of which is obvious.  
The net premium, like the office premium, has been treated as a  
fixed uniform annual payment throughout the currency of the  
assurance. Now, however, a very important part of the  
expenditure which the loading is imposed to cover—namely,  
the agency commission—is to be treated in a new way. Instead  
of running concurrently with the premium, as hitherto, it is to  
be commuted for a lump sum paid on the completion of the  
policy and the receipt of the first premium. Clearly it will not  
be possible, out of the first year’s uniform loading, to provide  
this capitalized payment. In these circumstances, Dr. Zillmer  
proposes to carry into the net premium the variation set up as  
regards the gross premium. “ As a rule”, he says, “ the net  
“ premium is so fixed that it shall remain constant throughout  
“ the duration of the assurance, as does the office premium ; but  
“ clearly there exists no reason why other distributions of its  
“ incidence should not be arranged, provided only the  
“ computation be properly made and the Company’s obligations  
“ fully covered. There exist, indeed, assurances at increasing or  
“ decreasing premiums, where naturally not only the office but  
“ the net premiums change at prescribed periods during the  
“ currency of the assurance.” Proceeding on this principle, the  
suggestion is that a new series of net premiums be employed, of  
which all after the first year shall be uniform, while that for the



Let  $\pi'_x$  be the new net premium assumed

$$\text{Then } \pi'_x \cdot a_x + \pi'_x \eta = \pi_x (1 + a_x)$$

$$\therefore \pi'_x - \pi'_x \eta = \frac{\pi_x (1 + a_x) - \pi'_x \eta}{a_x} - \pi'_x$$

$$= \frac{(\pi_x - \pi'_x \eta)(1 + a_x)}{a_x}$$

$$= \frac{\pi_x (1 + a_x) - v q_x}{a_x} - \pi'_x \eta$$

$$= \frac{1 - d(1 + a_x) - v + v p_x}{a_x} - \pi'_x \eta$$

$$= \frac{v p_x - d a_x}{a_x} - \pi'_x \eta$$

$$= \frac{1}{1 + a_{x+1}} - d - \pi'_x \eta$$

$$= \pi_{x+1} - \pi'_x \eta$$

$$\therefore \pi'_x = \pi_{x+1}$$



first year shall be less than each of the others by 1 per-cent on the sum assured; the present value of the whole, at the date of commencement of the assurance, being the same as the like value at the like date of the ordinary uniform net premium throughout the currency of the policy. It is further pointed out that on the same principle, if necessary, larger initial payments of a commutative character may be dealt with. The limits within which the principle may be applied are discussed, and are shown to be determined by the rule that the portion of the first year's premium retained in hand must not fall short of the net premium required to cover that year's risk. This, it is further shown, limits the maximum "initial cost" which may, according to the method advocated, be spread as an annuity over subsequent years, to

$$\frac{(\pi_x - \pi_{x+1}^1)(1 + a_x)}{a_x}$$

(where  $\pi$  represents the ordinary net premium), an expression which is further shown to be reducible to a result given verbally in the paper, but which may be symbolically represented by

$$\pi_{x+1} - \pi_{x+1}^1$$

a form which is not only convenient because of its simplicity, but also valuable for exhibiting to us, as it were in pictorial fashion, that the effect of cutting down the amount kept in hand for the first year to the net term premium for that period is to make the increased net premium, taken credit for throughout the remainder of the policy's currency, identical with the ordinary net premium for the age one year higher than that at which the assurance was effected.

Having thus before us Dr. Zillmer's position, let us proceed to the paper to which I have already referred, read to the Institute by Dr. Sprague in April 1870. It is one of the most cogent and luminous essays to be found in the volumes of our *Journal*, and its text is doubtless so familiar that a comparatively brief reference here will be sufficient. It covers much ground, and all the representative varieties of criticism on the net premium method we shall find referred to, or indicated, in its pages. I think, however, it may be best dealt with at this stage, and in connection with the question of initial expenses; as the chief force of its attack was directed against the mode in which this

item was treated by a strict adherence to net premium valuation, and its suggestions on this head have probably proved its most practical and enduring result.

Dr. Sprague, then, very strongly maintains that a net premium valuation does not take proper account of the relative incidence of expenditure in the first year, and during the subsequent years of assurance. "The supposition is tacitly made that the expenses chargeable to a policy are equally spread over its existence. But this is very far indeed from the truth, as the expenses incurred in the first year of the policy's existence far exceed those of subsequent years." After quoting Dr. Zillmer's suggested method of dealing with commuted commission, he leads up to the following well-known conclusion:—

"If the expenditure in obtaining new business has been such that the current claims and the expenses have together absorbed the whole of the first year's premiums, then a simple method of valuation, which in my opinion is perfectly justifiable, will be to reserve for policies in their first year only sufficient to meet the unexpired current risk, and to consider that all other policies have been effected at the next higher age. The value will thus be

$$A_{x+n} - (1 + a_{x+n})\pi_{x+1} = (1 + a_{x+n})(\pi_{x+n} - \pi_{x+1})."$$

Two points remain to be noticed. The argument is of wider applicability than Dr. Zillmer's, inasmuch as it is not based on a commuted commission, but on the general consideration of the cost of procuring new business—to which, it is contended, "by far the greatest part of the expenses of every Life Insurance Company are chargeable—in particular, all the advertising, inspection of agencies, and medical fees, and a considerable proportion of the postage, directors' fees, salaries, and office expenses." On the other hand, the suggested method of relief is not put forward as necessarily for universal use. It is admitted that the net premium method may be expected to lead to satisfactory results "when, from the magnitude of the business transacted or other causes, the total expense of conducting the business, irrespective of commission, amounts only to a small percentage of the premiums."

A good deal has happened since 1870. I think I am right in saying that then the system of initial commission had not, or not to any substantial extent, been adopted in Great Britain. Now, while the ratio to premium income of the more general

expenses enumerated by Dr. Sprague shows some tendency to diminish, we are all working under a commission system which leaves Dr. Zillmer's commuted one per-cent a thing to be envied; being fortunate if we can limit our outgo in this respect to an initial payment at that rate with an appreciable renewal allowance in addition. In these circumstances it is interesting to find that Dr. Sprague has again, and much more recently, dealt with the subject, in a paper read before the first International Actuarial Congress, held at Brussels in 1895. I have no doubt that this essay also is well and widely known, but, perhaps, as it forms no part of our own proceedings, it may be permissible to give here rather a fuller notice of it than of the paper of 1870.

The question proposed for the day's discussion was: "In valuing life policies, should any allowance be made on account of the expense of obtaining new business?" and the paper was entitled "Explanation of a method of valuation which takes account of this expense." After some preliminary explanations of the net premium method, a concrete instance of its working is shown by the construction of a table. The number of survivors at age 40 by the  $H^M$  table, 82,284, is taken as a starting point—and it is supposed that this number of persons, aged 40, effect whole-life policies of £100 each. The first table follows the fortunes of these policies for 10 years, on the suppositions that net 3 per-cent premiums only are paid, and no expenses incurred; that the exact  $H^M$  mortality is experienced; and exactly 3 per-cent interest realized. The final column of the table gives the sum in hand for each existent policy at the end of each year, which is of course the  $H^M$  3 per-cent reserve. The problem of the incidence of initial expenses is then stated—and a second table constructed to show, upon certain assumptions, the effect of this element on reserves. "From an examination", says the author, "of the published accounts of the British Offices, I arrive at the conclusion that the average expenditure of prosperous and well-managed offices may be taken as 80 per-cent of the new premiums, and  $7\frac{1}{2}$  per-cent of the renewal premiums." In the second table, therefore, these rates of expenditure are assumed; and an office instead of a net premium is taken—£3. 5s. per-cent being the premium assumed, as a probable average with profit rate at age 40. The reserves shown by this table are of course very different from the first set. In the first year there is, instead of the  $H^M$  reserve of 1.653, an actual deficit of .365—and the reserves for the succeeding years are less than the  $H^M$



reserves up to and including the sixth year, after which they exceed them. The rate of interest assumed in this table is, as in the first, 3 per-cent, and the deaths are again exactly those of the  $H^M$  table. Dr. Sprague proceeds, however, to point out that the two latter assumptions are less favourable than the facts, and that here may be found an offset, in some degree, to the effect of the expenses. Expressing the opinion that "most offices are still able to realize an average rate of interest of 4 per-cent", he constructs yet a third table, differing from the second in assuming four instead of 3 per-cent interest and in substituting for the ordinary  $H^M$  mortality that shown by his own Select Mortality Tables. The reserves so obtained are, of course, intermediate between the other two sets during the early years. They fall below the  $H^M$  reserves for the first three years, exceeding them thereafter; and the reserve at the end of the first year is  $\cdot 127$ . Taking this last table as the nearest practicable approximation to the real conditions, Dr. Sprague proposes to fix, for the valuation of the policy throughout its currency after the first year, a modified net premium; and in order to ascertain the sum,  $K$ , to be deducted from the office premium so as to obtain the modified net premium, he says "we give  $K$  such a value as will make the reserve at the end of the first year equal to the sum in hand." Accordingly, from the equation

$$100A_{41} - (1 + a_{41})(3\cdot 250 - K) = \cdot 127$$

we find  $K$  to be  $\cdot 5755$ , and the modified net premium therefore  $2\cdot 6745$ ; which would be substituted for the  $H^M$  net premium of  $2\cdot 5891$  in all subsequent valuations of the policy throughout its currency. It had previously been pointed out that, in order to avoid negative values, the modified net premium must not be allowed to exceed the  $H^M$  net 3 per-cent premium for age 41, which is  $2\cdot 6816$ . As the value found falls within this limit, that essential condition is satisfied; as, of course, was sure to happen with any positive quantity on the right-hand side of the equation.

Recalling our opening recital of the three assumptions involved in the theoretical defence of the net premium method, as regards mortality, interest, and expenditure, we see that what has practically happened in the foregoing investigation is, that all three assumptions have been found to vary from fact, and that an attempt has been made to substitute for them hypotheses more in accord with experience. It is especially to be noticed



that the introduction of 4 per-cent interest, and select mortality, into the final table, is not with the view of advocating a 4 per-cent valuation, or one which shall employ select tables. The valuation contemplated throughout the paper is an  $H^M$  3 per-cent valuation in everything except the net premium; for the purpose of modifying which, and for that purpose only, the other elements are introduced; while the modification itself is simply a rearrangement of magnitude as between the first and the subsequent years, the capital values of the uniform and the modified series of net premiums, at the date of commencement of the assurance, being equal.

The discussion upon the paper was interesting and significant. Mr. Sheppard Homans warmly approved the suggestion in principle, but expressed preference for a formula which should proceed at once to the maximum limit for the modified net premium, by making it identical with the ordinary net premium for one year older—this being in fact Dr. Sprague's own suggestion in his paper of 1870. Mr. Homans seemed prepared to contemplate as desirable a general application of this method to all classes of policy—including short period and endowment assurances. The prevailing tone of the speakers, however, was on the whole to deprecate a change, which, however logically justifiable, might have the practical effect of tending to encourage initial expenditure. Out of seven speeches reported in the "Documents" of the Congress, which form its official public record, four convey this view with more or less emphasis. No question is raised as to the theoretical defensibility of the method, or the legitimacy of its application to young Companies; but it is interesting to note the tone of caution, one had almost said of alarm, which runs through the utterances of representative actuaries of France, Belgium, Germany, and Canada at the prospect of any modification in reserve bases which might, however unjustifiably or ignorantly, be conceivably wrested into an excuse for giving a looser rein to initial expenditure. Dr. Sprague himself, in the few words with which he accompanied his paper, said that the question dealt with therein was, practically, "of very great importance to young and weak Companies; the latter would appear insolvent if the ordinary net premium method of valuation were applied to them, while they would be found solvent were the method described in my paper adopted."

Having now before us what we may perhaps describe as the

attack on the net premium method from the standpoint of initial cost, we see, in the first place, that its strength lies in the change which has occurred in the relative incidence of expenditure. That is a matter of history. The device of paying a substantial commission upon the introduction of a policy and abolishing or limiting the renewal allowance has had far-reaching results. The case as put by Dr. Zillmer is a very strong one. There the question is one of a commuted allowance, which extinguishes future liability. At the same time, however, by the large accession of new business which it brings, it involves immediate payments so heavy that they upset the average, and make it difficult, perhaps impossible, to show a satisfactory valuation balance-sheet by a method which assumes in effect that the expenses are distributed equally over all the premiums, including the first. In such circumstances the argument seems irresistible that the net premium for the first year, which bears the brunt of the commuted commission, shall be consequently diminished; and credit taken throughout the future for a slightly higher net premium, which the office premiums—relieved of an important part of their customary burden of expense—should be able to provide.

Dr. Sprague's contentions cover a wider ground. He applies the same method to other expenditure than that upon commission, pointing out that a large proportion of all office organization, with its consequent cost, is made necessary because of the efforts to obtain new business. Here we are no longer considering an initial payment which has become greater by making subsequent payments either disappear or grow less. The general outlay now referred to, if it grow greater, does so as a positive addition to expenditure and brings no compensating future reduction in its train. Now it seems to me that it is essentially on the future reduction, rather than the immediate increase, that the justification of the suggested modes of dealing with the reserve depends. With all deference, therefore, to the great authority and experience of Dr. Sprague, I feel more difficulty in following his extension of the argument to cover general expenses of management than in appreciating its force as applied to the altered system of paying commission.

In any case, however, I think we shall admit that these reasonings have been largely, perhaps mainly, instrumental in achieving one important result. They have shown that a net premium valuation is not a necessary test of solvency. The

adjustments of reserve which they advocate are suggested, primarily at least, as exceptional measures to meet exceptional difficulties. A new Company, which must necessarily face an initiatory period of heavy expense, and expense that, once incurred, will not have to be repeated; an old and prosperous Company (as in Dr. Zillmer's instances) suddenly obtaining an unprecedentedly large accession of new business which involves a high initial cost; or a Company which has been so unfortunate as to render necessary a strict examination of its ability to meet its engagements; each of these might conceivably be unable, for a time, to show adequate reserves upon a strict net premium basis, and yet be demonstrably solvent. If the first year's premiums on the whole-life business cover the first year's risk and the expenses, that year may, if circumstances absolutely demand it, be for valuation purposes thereafter disregarded, and the policies treated as if effected at the opening of the next.

I describe the method in this way because it at once marks the limit up to which it may be safely applied, and furnishes a simple and general rule for its application. It is true that Dr. Sprague, in his Congress paper of 1895, suggests, as we have seen, a modification, based on certain assumptions as to the proportion of new to renewal expenses, the rate of interest realized, and the mortality experienced. As these assumptions, however, must at best be of a temporary kind, and as the method suggested would involve a separate investigation for each age at entry, I think that it was clearly used for illustrative purposes only, and that any discussion of the merits of the proposed change may best consider it carried to its limit as indicated above.

There is, however, one consideration which, I cannot help thinking, is important to the question, and likely to become more so; the great relative increase, in our new business, of endowment assurances. To mention this is, of course, to utter a commonplace; we all know how rapid this increase is, and have, in other connections, considered it often enough. As bearing upon the present subject, it may suffice if I quote, from Mr. Monilaws' "Surplus Funds", and the summary extracted by him from the Valuation Returns of "British Offices transacting 'Ordinary' Business", the figures for 1895 (the year of Dr. Sprague's Congress paper) and 1900:—



	SUMS ASSURED	
	1895	1900
	£	£
Whole Term of Life (including limited payments) with and without profits . . . .	433,950,131	469,983,338
Endowment Assurances, with and without profits . . . .	68,982,280	132,802,633

Here we see in whole-life assurances an increase of £36,000,000, about 8·3 per-cent, and in endowment assurances an increase of £64,000,000, nearly double the other intrinsically, and about 94 per-cent. This is a development so vigorous and so constant that it bids fair to change the whole character of our business. To such policies, of course, the suggested method is only applicable with such modifications as would considerably complicate the formulas. The mode of operation, too, would probably have to be varied according to the term of the assurance. If that were comparatively short it would not be possible to sacrifice much of the pure endowment section of the premium of the first year, lest the resulting burden on the remaining years should too seriously reduce the margin of loading. It might, indeed, be held that in no case should that part of the premium be touched, but that it should be preserved intact along with the provision for the first year's mortality risk. This would reduce the relief obtainable by the system to something so small as to be of little value. In any case, the extension of the suggested principle to endowment assurances involves difficulties and complications which would substantially diminish its effectiveness; and the phenomenally rapid increase of this element in our new business may consequently before long make the question of adopting the method less important.

On general grounds, moreover, and quite apart from the theoretical aspect of the matter, we shall probably all be inclined to approve an emphatic limitation of the process to the exceptional circumstances it is professedly designed to meet, and to regard it as a temporary expedient only. A well-established Company, stable and prosperous, transacting year by year a steady business, at an average rate of expenditure which exhibits no undue fluctuations, should deprecate, for practical reasons, the permanent diminution of reserves to provide for initial outlay on



new business. The inducements to that outlay are already so strong, and modern competitive conditions present them so forcibly, that anything which acts as a check upon their influence must be salutary. Such a check is undoubtedly provided by the recognition of an obligation to reserve for new policies on the same principle as for others, and to treat the special cost attaching to their acquirement as an item to be merged in the general expenses of the Company, which will be recouped for such a proportion of it as the new premiums themselves will not, on such a basis, bear, by the lighter cost of the business in subsequent years. Thus, in the end, the equilibrium will be restored, and justice practically done as between the older and the newer members. After all, true as it is that the greater part of the expenditure of a life office is undertaken for the purpose of extending its business, this is so vital a feature of its existence that its policyholders can hardly expect to be exempt from providing for it, within reasonable limits. They joined the Company as a "going concern", and share the advantages which presumably accrue to it from a steady annual accession of new entrants. Their interests would undoubtedly be prejudiced if that annual accession were too dearly bought, but a valuable safeguard against such a danger is provided by the very principle, of making reserves for old and new policies on a uniform basis, which has been under discussion.

Considerations such as these, however, which are politic rather than scientific, lie somewhat outside the scope of the problem before us. They might apply to any method of valuation. What seems to me to be important is that the attack on the net premium method, as regards its general applicability, must be admitted to have been successful. It is difficult to see any way of escape from the conclusion that in certain important practical contingencies such a valuation will fail, and must be abandoned; for abandoned it undoubtedly is if the suggested modifications are made. The study of the argument and its results indicate to us that the assumptions which underlie the net premium system may get out of accord with fact, its value being thereby seriously impaired, and in some circumstances destroyed.

These conclusions are, I think, brought home to us still more forcibly if we turn to another aspect of the subject, and consider a line of criticism which tends in a different direction. It may, perhaps, best be introduced by a reference to Mr. A. H. Bailey's paper of July 1878, already mentioned. There great stress is

laid by the author on the very important question of the rate of interest to be assumed in a valuation. It is pointed out that this is commonly assumed to be the rate at which the office premiums were calculated; but against that practice an earnest protest is made, and it is urged that the only satisfactory way in which to determine the net (or, as Mr. Bailey prefers to call it, the "pure") premium is "to form an opinion—and to act upon it when formed—as to what rates of mortality and interest will prevail in future." The question of mortality we will leave for the present and consider that of interest. The great difficulty of the task here depicted by Mr. Bailey, and the wide differences of opinion which will arise in an attempt at its accomplishment, are interestingly illustrated by the consideration, in the light of subsequent events, of some of the estimates made in the paper and the resulting discussion. Mr. Bailey, as the result of his own study of the problem, declared for 4 per-cent as the proper rate to employ, and with this view the late Mr. Baden expressed emphatic agreement. Other speakers thought  $3\frac{1}{2}$  per-cent a safer rate—but the whole tone of the discussion, so far as it referred to the subject, was fairly sanguine as to the maintenance of the rate of interest, for at all events as long a future as actuarial forecasts then required, at a rate not lower than  $3\frac{1}{2}$  per-cent. We all know, of course, the elements which combine to bring complexity and uncertainty into estimates of this kind; the general undoubted tendency to reduction in the rate, the temporary fluctuations, the problem how far any immediate movement is passing or permanent, and the speculations on future economic tendencies which no one can foresee. Anyhow, it must, we fear, be confessed that the history of the period since the paper was read has hardly justified the optimists. We have had a steady and substantial downfall in the average rate realized on the assets of British life offices. This, some twelve or thirteen years ago, had the important effect of suggesting to some Companies the prudence of making valuations at a rate lower than the 3 per-cent which had till then been considered a standard minimum. The tendency has grown, and out of the 231 millions sterling shown in the Board of Trade returns for 1900 as the funds of British offices transacting "ordinary" business, about 27 millions—more than 10 per-cent—represent reserves made on a  $2\frac{1}{2}$  per-cent basis.

From this development arises a very important consideration with reference to the net premium method. If that system is

strictly applied, a lowering of the valuation rate means, of course, an automatic increase in the net premium valued. The office premium, however, remaining constant, we have to ask what is the effect, upon the loading, of this change. In the case of many non-participating assurances it is not difficult to see that the margin will be seriously narrowed—may indeed sometimes all but disappear. In such cases the Company may be strong enough to make a special deduction from the net premium so as to secure that every office premium is diminished for valuation purposes by not less than a fixed percentage. Upon this two observations at once suggest themselves: in the first place, such a step is no part of a net premium valuation, and any Company omitting it might state with perfect truth that a net premium valuation was made; in the second place, if anything of the kind be done, however prudent as a precaution and gratifying as an exhibition of strength, it is an abandonment of the net premium method. I have instanced the case of non-profit premiums, because there the possibility of a depleted loading is most strikingly apparent; but there is surely no logical ground for limiting the comment to this branch of the business; because the additional loading which is put upon the with-profit premium exists for the purpose of providing profit. It is ear-marked to that special end, which is the reason of its existence, and therefore, if a non-profit premium show too small a loading for expenses, the with-profit premium for the same assurance at the same age is in like case.

It may be said in reply to such criticisms that, after all, the net result of a valuation is the thing of chief importance; and that, as the reduction in the rate of interest must increase the reserve held by the Company against its liabilities, it is evident, on commonsense grounds, that greater strength and greater safety are secured. This is of course true, but the answer is that the increase of strength is not so great as the assumption underlying the form of the change demands. If it be the deliberate opinion of the Company that financial conditions have so altered as to make  $2\frac{1}{2}$  per-cent interest take the place in its calculations which 3 per-cent has hitherto occupied, a net  $2\frac{1}{2}$  per-cent valuation has reduced its provision for future expenses. If, on the other hand, the view is that a sufficient provision is made for the financial conditions of the future by such an additional reserve as a net  $2\frac{1}{2}$  per-cent valuation gives, then it would surely be better to assume an intermediate rate of interest and leave the loadings unaltered—valuing the old net premiums at the new rate. That,



at all events, would have the merit of showing what had actually been done, instead of throwing a cloud over the results. What are the assumptions involved in the net  $2\frac{1}{2}$  per-cent valuation? Firstly, that it is necessary to hold in hand such a sum as, improved at only  $2\frac{1}{2}$  per-cent compound interest, will suffice to meet the Company's engagements at maturity. Secondly, one of two things; that the future premiums contracted for are increased, which has not happened and cannot happen; or that the former provision for future expenses may be diminished, which is probably not intended. If the first assumption is too stringent, it were better so to relax it as to be able to deal with the future premiums by a method which would avoid either of the others.

We are thus brought to the heart of the difficulty, and find our criticisms of the net premium method centre themselves in the charge that it withdraws our observation from the actual premium we are entitled to receive, and fixes it upon something else, which is not that premium, and which only on certain rigid and definite assumptions bears the relation to it that the method implies. These assumptions may in course of time cease, even approximately, to represent the facts. The system, however, has from its nature no room to make allowance for such changes, and may thus eventually produce an altogether arbitrary and artificial result. No weightier words on this subject have been spoken than those of Dr. Sprague in his Congress paper of 1895: "the premium received by the office is not the net premium but the office premium." Obvious as this seems, its importance in the present connection cannot be too clearly recognized; and in proportion as we accustom ourselves to look at valuation questions from this standpoint we shall, I venture to think, discern the real issues more clearly and be better able to adapt our methods to the conditions of the time.

If we go back for a moment to the theoretic or what I ventured to call the classical defence of the net premium method, quoted at the beginning of this paper, we see at once how natural and indeed inevitable it is that such questions as we have been considering should arise. The office premium is there regarded as a scientific construction on the basis of precise assumptions as to mortality, interest and expenditure; and upon the exact verification of those assumptions its exact adequacy depends. The premiums with which we work, however, are by no means always traceable to such definite sources. As Mr. Bailey pointed out, in the paper to which I have already referred, they



have frequently been fixed by no more scientific agent than competition. In any case, the assumptions which, whether definitely employed in their construction or not, approximately represented them when framed, are subject to continual change. But the premium does not change; it remains the property of the office under its contract; and I confess inability to see why this fact should not be allowed its full weight in valuation as elsewhere; or why, when changes of condition occur in the other factors of that process, we should, in deference to a theory or a formula, import them into this one, where, in reality, they have no place.

We have seen how this occurs, on the net premium system, in connection with the rate of interest. Let us look for a moment at its operation as regards mortality. Here, the brevity of our reference must not be taken as an inadequate recognition of the importance of the subject. The reason is rather to be sought in the somewhat exceptional position in which we are just now placed. What will undoubtedly prove the most valuable, as it has been the most extensive, of all investigations of the mortality among assured lives, has just been completed. A large body of valuable results has been made public, but the processes of graduation, and analysis to show the effect of selection, are yet in progress. As the mass of material available will admit of the fullest and most accurate examination of selection results we have ever been able to make, the value and importance of that work will be very great; and, doubtless, during the next few years, able and eager students will be occupied in revising by its light many aspects of our science. Its effect on valuation questions must be of the first importance; and it would, therefore, be inappropriate now to deal with this branch of our enquiry otherwise than in the most general terms. I would just say then that here we have another striking illustration of the anomalous effect which might follow a rigid adherence to the net premium method.

Of all our assumptions, probably those respecting mortality must be the most continuously subject to change, as successive examinations of experience are made. It would, moreover, be shutting our eyes to fact, and adopting a policy which would be generally and justly condemned as retrograde were we not to adopt into our practice, so soon as they become available, the best and latest mortality statistics; giving to them the place hitherto accorded to those which they will

have rendered obsolete. Now it is probable that the new experience, while showing an unquestionably improved mortality, will also, from the incidence of that mortality, bring out, for the majority of ages and durations, higher reserves—accompanied by lower net premiums. Here the net premium method of valuation would demand, as regards future premiums, an operation the reverse of that which has been noticed in connection with a reduction of interest; and it would practically be assumed, in opposition to fact, that the premiums still to be received by an office under its contracts had been lessened. I can only say that to me there seems no reason, in fact or common sense, why such a modification should be made, or why the new mortality experience should not be used, as the truest available standard, without arbitrarily extending the assumption of change to something which we know to be unchanged—the premiums receivable.

The fact that these criticisms of the net premium method of valuation operate in divergent, and to some extent in contrary directions suggests the practical question whether it might not put in an effective claim to maintenance as a well-known and convenient standard, which probably on the whole attains its end with sufficient accuracy. This recalls a valuable and interesting paper by Mr. Sorley, read before the Actuarial Society of Edinburgh in January 1878, and printed in our *Journal* (*J.I.A.*, *xxi*, p. 192), suggested by Dr. Sprague's investigations, and those of Mr. King and Mr. Berridge, of the effect of selection as shown by the  $H^M$  experience; and also by Dr. Sprague's arguments, to which we have been referring to-night, with regard to the effect of expenditure upon reserves. The essay of Mr. Sorley sets one element against the other, with a view to showing that they are mutually counteractive, so that the result of allowing for both is, at the end of five years, identical with an  $H^M$  net premium reserve. As the rates of expenditure assumed are only 50 per-cent of new premiums and 7 per-cent of renewals, and as the selection statistics will soon in the light of the new experience be superseded, the precise figures of the investigation are hardly applicable now. The argument, however, is important as educing the fact, which is worthy of full consideration, that some of the chief defects in the system tend to check each other, and the plea for the net premium method, with which the author sums up his case, is a powerful one.

“The opinion”, he says, “has been expressed, that the net premium method is one which must inevitably disappear before the advance of true actuarial science. Circumstances, however, seem to favour its continuance; not on the ground of any algebraical demonstration of its correctness—for the question is not a mathematical one—nor that it gives the highest reserves, or is in any other respect extreme, but rather as a compromise or *via media*, where selection, expenses, occupation, residence, and other influences—some affecting the result in one way and some in another—are all alike ignored, and whose defence takes its stand on the broad principle of averages, which lies at the foundation of all actuarial questions.”

The words of moderation are generally wise words, and a philosophical plea like this carries much weight. It reminds one, at any rate, carefully to acknowledge the great service which the net premium method of valuation has done, both as a convenient, easily applied, well understood comparative measure of the standing of Companies, and as a standard which Companies, not quite strong enough for its requirements, have striven after as a “counsel of perfection”, always with salutary and beneficent result to themselves.

At the same time I must personally confess that, in view of all that has happened since those words were written, I am inclined to think that the possibilities of its divergence from fact are becoming too wide and too numerous for us to be able to look forward much longer to finding in the net premium valuation the “guide, philosopher and friend” which it has been to our predecessors.

It is possible that full analysis of its excesses and deficiencies might show something like a balance of errors, but this would be a coincidence, and we should not be justified in expecting it as a permanent condition. We are only safe as we keep in touch with actualities; and this the method in question does not seem to do, especially as regards the liberties it takes with the premium of practice, the office premium, the asset actually receivable.

A defence based on practical expediency implies that the method will only be of permanent value so long as the variations of its assumptions from fact keep within certain manageable limits. For some time that might fairly be said to be the case, but now those variations seem to present possibilities so large and so complex as to demand more definite recognition.



Expenditure, interest and mortality are the three elements vital to an estimate of a Life Assurance Company's liabilities. Considering each of them in turn, we have seen how the changes to which they are subject fail to find adequate reflection in the rigidity of a net premium valuation. Expenditure has altered the ratio of its incidence, so that the constant net premium assumed by the method is not really received. This difficulty, however, though acute in special cases, is of narrower effect than that which arises with regard to interest and mortality. The question suggested by these factors is one of greater importance. Changing financial and social conditions alter the data on which our calculations rest. Money grows cheaper, and men live longer. For these things we must, in our estimates of liability, make allowance; but net premium valuation, in the act of doing so, introduces at the same time, and automatically, corresponding variations into the premium assumed to be receivable throughout the remainder of the contract. As that premium is a fixed quantity, such assumptions are unsound. The argument is general, and not concerned with any special views as to the right rate of interest to assume at a given time, or with any particular mortality investigation. It only postulates that these forces must, in the course of years, be subject to change. The fact that their changes operate on our results in contrary directions does not warrant the assumption that they will neutralize each other.

Mr. G. H. Ryan, in the exhaustive and able paper on the whole subject of valuations which he contributed to the last Actuarial Congress, expresses the opinion that the net premium method is destined to supersession, but that we are hardly competent yet to say what shall take its place, except that it will probably be a method giving greater prominence to the actual premiums paid. This is clearly the point at which the chief weakness of the method is apparent. It seems as though the future will lie with some form of gross premium valuation, which shall make an adequate reserve for expenses and strictly exclude negative values.

If there is one thing which it is desirable in such a connection to state clearly and to emphasize, it is that no criticism here offered of a system of valuation so widely and justly esteemed, and so generally associated with the idea of stability, contemplates for a moment anything in the nature of a weakening of reserves. The desire and impulse of the writer, as probably of



us all, would rather be to maintain them at such a point as shall permit no possible doubt of their adequacy. This, however, we shall surely best be able to do the more clearly we see our facts; and the more explicitly our valuation processes show, with the minimum of technicality, and the maximum of intelligibility to the non-expert, exactly what we have done, and precisely where we stand.

I end as I began, very sensible of the inadequacy of this attempt to deal with a great subject, and of the paucity of original matter my paper contains. I would only, also in conclusion, repeat the hope that it may elicit many expressions of opinion of greater and more permanent value than any of its own.

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#### DISCUSSION.

Mr. B. A. BERRY said that at the outset he wished to congratulate the author on the extreme lucidity with which he had brought into a very small space all the most important arguments that could be advanced for and against the net-premium method of valuation. No doubt there were many present who would defend the system, and as his remarks would be very few he would confine them to the drawbacks to its use that suggested themselves. At the commencement of the paper the author spoke of the postponement for a year of the normal reserves, but in view of it being only a temporary expedient, designed to help perhaps an extravagant or a weak office over a period of difficulty, he would not further allude to it. The next thing which struck him was that a strict adherence to the system would allow no special reserve for a limited payment or single premium policy after the premium period had elapsed, and the method had to be stretched to include those necessary reserves. The inherent weakness of the system was, that if there were two offices, A with a high scale of premiums and a low rate of expenditure, and B with a low scale of premiums and a high rate of expenditure, the system, strictly carried out, would make the same reserve for the two offices. In the same way, a single office, perhaps from an amalgamation or from some other reason, might have contracts running at two different rates of premium, and yet the net reserve be the same in the two cases. No doubt the reason thirty or forty years ago for the general adoption of the method was the knowledge that negative values had been introduced into valuation balance sheets under the hands of unpractical computers with, he was afraid, painful results. At page 71 the author had, to his thinking, stated very clearly the objections to the universal use of the method. No doubt some forty years ago, when the system made very general headway, it was justifiable, because at that time the net premiums employed left a margin which adequately represented the necessary reserves for expenses and future profits; but now that  $2\frac{1}{2}$  per-cent

valuations were coming into vogue that margin had to a very great extent run off. If the two parts of the contract were looked at as a debtor and creditor account, the liability to pay the sum assured on the one side was over-valued, and, on the other side, a premium was valued which did not leave any margin for expenses, so that reliance had to be placed on the excess of interest in the valuation of the liability to provide the future expenses and profits, the result, as the author had stated, being thus thrown under a cloud. It was said that the system gave substantially accurate results and was safe; but it seemed to him if that was all that could be said in its favour it could not be considered scientific; and where would insurance companies be in a few years' time if 2 per-cent net premium valuations came into vogue? Another thing said in its favour was that the man in the street could understand it; but he did not think actuaries should necessarily follow any particular course because the man in the street understood it. Were they not, then, in the author's words, approaching the time when the variations of the assumptions were no longer within manageable limits? If so, he agreed with the statement that the future would lie with some form of gross premium valuation which would make an adequate reserve for future expenses, and strictly exclude negative values.

Mr. H. W. MANLY thought the Institute was indebted to the author for bringing the subject of the net premium method of valuation again forward for discussion. As had been well said, actuaries could not have too much discussion upon the fundamental principles of their institutions, seeing that the questions of solvency and profits were involved in the valuation of the liabilities. Therefore it was extremely useful to discuss this subject from time to time, and if one saw reason to alter opinions which were previously formed, to boldly say so. He was sorry the author had not so boldly expressed his opinions as one would have liked; in fact, he had rather left the members to infer what his views were upon the subject, than expressed them with that courage which one would expect to see from such an eminent member of the Institute. The author, at the beginning of his paper, had referred to his (Mr. Manly's) explanation of the net premium method in the Messenger Prize Essay which he wrote nearly thirty-five years ago. At that time he was a young man, and as a young man he was inclined to take a strong view of the scientific side of questions, simply because at that age he had not the wisdom which came by experience. He wished to call to the recollection of the members of the Institute that at that time (1867) the theoretical principle upon which the net premium valuation was based was a real and living creed, and necessarily so, because the Albert, a large office which had been grossly mismanaged with regard to its expenditure, had failed, and the European was just on the point of failure. So that it was necessary for the managers of other offices to show that they, at any rate, were not in the same disastrous position as those two large companies. The net premium creed had served its purpose exceedingly well, for, having been established as the soundest principle upon which the valuations could be made, all the offices began to put

themselves on that basis. He thought they might very largely attribute the soundness of the offices at the present day to the net premium method, which was then insisted upon as the only safe principle upon which valuations should be made. Like many other creeds which were real and living in their day, the significance of this creed gradually died away, and it was now felt that perhaps the time had come when it might be superseded by some other and better principle. Distinctly the times had altered. In 1867 and 1868 insurance companies had, as the author had reminded them, a very comfortable rate of commission of 10 per-cent upon the first premium and 5 per-cent on renewals; and it was considered, amongst the better class of offices, to be exceedingly sinful to depart from those rates. The introduction of the heavy procuration commission into this country came shortly afterwards by the advent of the American offices—he believed the first American office was established in this country somewhere about 1868—and from that time one had to date the large expenditure on the first premium for the acquisition of the business, and a smaller commission, if such smaller commission was ever allowed, on renewals. Originally the attempt was made to abolish the commission on renewals altogether, but it resulted in so much lapsing of policies that it was found necessary, in order to keep the business on the books, to give some small commission on the renewals. Then, of course, the net premium method of valuation gave rather curious results; it made the actuary reserve something which did not exist. Still, the offices grew so strong that they were prepared to provide for that reserve upon the new policies out of the funds which had been accumulated from the old business. He had said, if one had changed one's opinions one ought to say so. His opinions had certainly changed since he wrote the Messenger Prize Essay thirty-five years ago. He stated his matured opinions in a letter which he addressed to the *Insurance Record* in February 1901, when a discussion was proceeding in that paper upon the proper reserves for discounted bonus policies. He was then induced to express his view of the proper way of making valuations, by which the discounted bonus premiums themselves would be taken into consideration in the valuations in the same way as the full office premiums on the ordinary policies ought to be. With the permission of the members he would read extracts from the letter, as they expressed his latest views upon the question.

“In a letter published in your issue of 25 January, Dr. Sprague says, ‘There is a good deal of truth in Mr. Manly’s statement that, if discounted bonus policies are valued at a lower rate than  $3\frac{1}{2}$  per-cent we make a reserve larger than we can possibly have in hand from the premiums; but the same remark generally applies to all the new policies when a net premium reserve is made for them, and this really constitutes a very strong argument against the use of the net premium method of valuation.’ With this remark I quite agree.

“The doctrine that the net premium method of valuation is the only true and infallible one has been preached so long, that in many



minds it has become an article of faith, and it is held to be rank heresy to say anything against it. In order to maintain this doctrine we have been driven into many absurdities; not the least of these being an  $H^M$  and  $H^{M(5)} 2\frac{1}{2}$  per-cent net premium valuation. Now the ostensible reason given for the use of such a basis, is the desire to maintain a permanent rate of reversionary or compound reversionary bonus, and the proper method to carry out that desire would be to make a Valuation Balance Sheet at the highest safe rate of interest, thus:—

*Account I.*

LIABILITIES.	ASSETS.
1. Present value of sums assured and bonuses.	4. Assurance fund.
2. Reserve for future expenses.	5. Present value of future premiums.
3. Present value of a quinquennial bonus of X per-cent per annum for the past five years and for the future existence of the policies.	

Where such a value is assigned to X as will make both sides of the account balance. X would then be the rate of bonus to be declared.

“Now it is asserted that if the bonus is to be a constant reversionary addition, the rate X can be found approximately by making a net premium valuation at  $\frac{1}{2}$  per-cent under the highest safe rate; and if the bonus is to be a compound reversionary bonus, X can be found approximately by making a net premium valuation at 1 per-cent under the safe rate. By adopting these plans, we outwardly maintain the doctrine of the net premium valuation, while in theory we abandon it. Hence to keep to the true and only faith, we add the formulas:—If  $3\frac{1}{2}$  per-cent is a safe rate, then to maintain a constant reversionary bonus you must value at 3 per-cent, and to maintain a permanent compound reversionary bonus you must value at  $2\frac{1}{2}$  per-cent.

“What happens when a portion of X, say B, has been anticipated by making an equivalent reduction in the premium? We must either add to the asset side of the account ‘Present value of a quinquennial bonus of B per-cent per annum for the past five years and for the future existence of the policies,’ or substitute (X—B) for X on the liability side; valuing, of course, only the reduced premium charged, thus:—

*Account II.*

LIABILITIES.	ASSETS.
1. Present value of sums assured and bonuses.	4. Assurance fund.
2. Reserve for future expenses.	5. Present value of future premiums payable.
3. Present value of a quinquennial bonus of X—B per-cent per annum for the past five years and for the future existence of the policies.	

“If  $B=X$  and the reduced premium is equal to the net premium at the safe rate (say  $3\frac{1}{2}$  per-cent) used in the valuation with an



addition for expenses, the reserve would be a net premium valuation. If the reduced premium is  $3\frac{1}{2}$  per-cent net, with no loading for expenses, then the expenses must be provided by the interest earned over  $3\frac{1}{2}$  per-cent, or the account would show a deficiency.

"Let us now consider the real object and significance of a  $2\frac{1}{2}$  per-cent net premium valuation. We nominally reserve more for a recently effected policy than we have in hand, and therefore, apparently, take something from previously accumulated funds and so deprive the older policyholders of their earnings; but in reality we are only adopting that method of valuation to arrive at the additional reserve which should be made, in respect of the older policies, above the  $3\frac{1}{2}$  per-cent net premium valuation reserve. It is not scientific, and is, in fact, absurd; but the doctrine of the infallibility of the net premium method of valuation demands it."

Thus, his opinion was that the premiums receivable by the office, whether the full annual premiums or the discounted bonus premiums, should be taken credit for, and a proper reserve made for the expenses. That would be a test of solvency. The surplus should then be applied to provide present and future bonuses. We thus come back, though by another way, to the principle of the net premium method, namely, that proper provision should be made for future expenses and profits. That had always been the crux with regard to breaking away from the net premium valuation. If it was once admitted that something less than the loading might be reserved, one did not know where the innovation would stop; because if one could value  $P_{x+1}$ , why not  $P_{x+2}$ , and no doubt good reasons could be found for going to  $P_{x+3}$ , or  $P_{x+4}$ . The question of solvency and the question of a bonus distribution were the two main questions to be considered in a valuation. If one broke away from the net premium method without making proper provision for future expenses and profits, one might bring into account for present distributions as bonus profits which were going to be earned in the future, and that, he maintained, would be wrong. If one made a gross premium valuation, with a proper reserve for future expenses, and thus ascertained the profits of to-day and the profits of the future, and distributed them over the whole period of the policies as he proposed, then he saw no harm whatever; in fact, he saw every reason why such a method should be adopted.

Mr. J. CHISHOLM gave expression to his unhesitating adherence to the net premium system of valuation. It might be old-fashioned to say so, but he had a very strong opinion of the danger of departing from it. In speaking of the net premium method of valuation, however, he thought they had misunderstood what was originally meant by such a term. The term was used in contradistinction to the gross premium system. The net premium system advocated was simply meant to ear-mark the difference between the premium valued and set aside to provide for the risk and the premium actually received. Previously there had been no ear-marking of what had to be set aside for expenses and future profits. He did not recognize any system of net premium valuation which did not take account of the actual premium received, and did not consider

the portion of the actual premium that was reserved, *i.e.*, the difference between the net premium valued and the gross premium. His reason for adhering to the net premium system of valuation was so simple that it seemed almost a *reductio ad absurdum*. He recognized, in the first place, the very great ability shown by the author in dealing with all the various aspects of the subject. Everybody was conscious of the dangers and difficulties which surrounded them in the practical application of valuations to the circumstances, but what did the author tell them? The rate of interest was going down, and the rate of expenses was continually going up—that was to say, there was a less ability to make future profits than there had been in the past, and with that the departure from the net premium system would have the effect of setting aside a less reserve for future expenses, which were going to be larger. It did not seem to be right, if expenses were going to be greater in the future, to make less instead of more provision for them. If additional expense was incurred in getting business there was only one cure for it, namely, to increase the premiums chargeable. There was no justification for a  $2\frac{1}{2}$  per-cent valuation if the premium which was necessary for such a valuation went above the actual premium being received. That was a proof that the rates of premium wanted raising. Mr. Warner, in his paper, alluded to the fact that the limit of expense was the first premium. He could not see that any such limit existed in principle. It was true enough that they could not spend more than the first premium received in the first year, but the next thing that would happen would be that they would be asked to distribute the procuration expenses to agents and others over two years instead of one year, taking the amount out of the first two years' premium, after providing for the cost of insurance. How was provision to be made for that, supposing it became the rule of practice? There would be the same justification for altering the system of valuation to suit the expenditure, as when the first year's premium only was spent. It would only be necessary to take credit in a valuation for the net premium at an age two years older. But, after going to two, why not go to three or four, as Mr. Manly had said. Of course, nobody believed for a moment that the net premium system was applicable in all circumstances and in all cases. It did not apply to a young or a weak company; it applied to the ordinary, steady-going body of companies who transacted the bulk of the business in this country. In those circumstances he thought the net premium system of valuation as he had spoken of it—*i.e.*, taking account also of the gross premium payable, so as to show what was being reserved for future expenses and profits—was the safe and practical rule to follow, and that there would be very great danger in departing from it.

Mr. A. R. BARRAND took exception to the description so often given to the net premium method of valuation, namely, that it was a scientific method. It always seemed to him as essentially an unscientific method under present conditions. He could quite understand that under the circumstances in which the method was first applied, it was properly described as a scientific method; but the conditions now seemed to have so completely changed that it no



longer deserved such a description. In the days when the system was received with so much favour he believed there was some distinct relation between the office premiums and the net premiums used in the valuation. Many offices in the early years used the Carlisle Table for valuation purposes, valuing in many cases at 3 or at  $3\frac{1}{2}$  percent, the office premiums also being based on the Carlisle Table. But now, while the office premiums continued the same, in very many cases being based on the Carlisle Table, the net premium used in the valuation was based on an altogether different table. As from time to time the valuation basis in respect to mortality was changed, they might increase the valuation premium, and thus diminish the loading available for future expenses, while at the same time the expenses might be steadily going up. The net premium method of valuation under such circumstances, so far from making any extra reserve for the increasing expenses, positively produced a lessened reserve in respect to future expenses. It was a real lessening of the reserve for future expenses, because presumably when the mortality basis for valuation purposes was changed and a table involving different valuation premiums was used, it was done not for the same reason that some offices had changed their interest basis, namely, for the purpose of making a concealed reserve or providing reserve for future bonuses, but because it was felt to be called for by the circumstances of the case. This being so, a direct result of the net premium method may be that the margin between the valuation premium used and the office premium receivable is lessened, although at the same time the actual expenses are increasing. Under such circumstances it seemed to him that the net premium method of valuation was not entitled to be called a scientific one.

Mr. T. E. YOUNG said, that so far as he had heard or read, he was not aware that what was usually termed the net premium system of valuation had ever been considered to be universally applicable. He remembered that when the transfer of the European to another company was contemplated the two actuaries who were then employed valued the liabilities of the European on the basis of the office premium, less an adequate percentage for expenses, excluding of course, all negative values. And in 1865, when an effort was made to transfer the business of the old Argus to another company, a similar modification of the pure premium system was adopted as appropriate to the circumstances of the case. With respect to the prominence which the author had assigned to the constant consideration of the office premium, that notion, of course, had always been maintained. In the earlier papers on the subject in the *Journal*, Mr. Jellicoe, one of the most clear-sighted and sagacious actuaries the profession had ever produced, was always insistent upon a due and special regard to the office premium in all comparisons and valuations. He was very glad to observe the halt which Mr. Warner deliberately made at the suggestion of Dr. Sprague, when it proceeded beyond the considerations adduced by Dr. Zillmer—that was to say, that whatever might be the validity of Dr. Zillmer's contention in respect of commuted commission, Dr. Sprague was advancing too far when he extended that judgment to the general expenses of a company. With regard to Dr. Sprague's well-known method, the substitution

of  $P_{x+1}$  or  $P_{x+t}$ , whichever it might be—though it was distinctly to be borne in mind, in justice to Dr. Sprague, that he assigned a limit to the value of  $t$ —he (the speaker) had always objected to it on two grounds. In the first place, it was a distinct perversion of facts, and, in the second place, it possessed the very serious detriment of affording a sanction to the extravagant proceedings of inferior companies. In the hands of Dr. Sprague, his long experience and sagacious judgment would dictate the limitations by which the application of such a formula should be surrounded; but one could readily understand how a company which, simply for purposes of ambition or otherwise, had wasted its reserves, would most gladly hail the ægis of Dr. Sprague behind which to conceal improperly its fallacious and misleading actions. With regard to expenses generally, like Mr. Chisholm, he was a decided advocate of the pure premium method of valuation. The departures, of which he recognized the justice, applied to the transfer of one company's business to another or to the winding-up of a concern and the appropriate reduction of its contracts; but with regard to expenses generally, especially at the present ruinous rates, he would apply to the offices the pungent maxim of the Greeks, *παθηματα μαθηματα*—that is to say, that if offices were desirous, beyond the primary needs of their business (the supplementing of waste which occurred by deaths, surrenders and otherwise), to expend the resources of their constituents upon a mere accession of large figures, he thought they ought not to conceal the fact by any deliberate modification of a legitimate valuation, but ought to bear the punishment of their action in a reduction of profit, and the consequent revelation to their clients of the wasteful burden which a needless competitive extent of new business (procured on extravagant terms) entailed.

Mr. R. TODHUNTER said the discussion so far had been addressed mainly to the question whether it might not be proper in certain circumstances to anticipate a part of the loading in valuation. He thought it would convey an incorrect impression if the idea were to go forth that a prominent member of the Institute had advocated such a course. As he understood the paper, it was not the object of the author to advocate in any way the general anticipation of loading; in fact, it was most clearly stated that "a well established company, stable and prosperous, transacting year by year a steady business, at an average rate of expenditure, which exhibits no undue fluctuations, should deprecate, for practical reasons, the permanent diminution of reserves to provide for initial outlay on new business." Again, in the concluding paragraph, the author said—"The desire and impulse of the writer, as probably of us all, would rather be to maintain them [*i.e.*, the reserves] at such a point as shall permit no possible doubt of their adequacy." The conclusion he drew from those statements was that the author did not wish to advocate the anticipation of loading as a general principle, but that his object was to show there might be some instances in which the net premium method of valuation was not applicable. If it was once admitted that there were circumstances in which the net premium valuation was inadmissible then he thought they would have conceded all that



the author wished them to concede. Then Mr. Warner turned to another point, which was, perhaps, the more important point of the paper, namely, the question not of anticipating any part of the loading, but of reserving a larger amount than the ordinary net premium would provide. A practical question suggested by this part of the paper was whether it would be better to make a  $2\frac{1}{2}$  per-cent valuation, or a  $2\frac{3}{4}$  per-cent valuation with 3 per-cent net premiums. Between the results obtained by those two courses there might be very little difference, but the first method had the effect, which the author deprecated, of throwing the whole reserve for expenses on the interest part, and in the case of non-profit policies, of doing away with the reserve margin of loading. The second course had the advantage of keeping the same reserved loading as in a 3 per-cent net premium valuation, and, at the same time, of making, in some measure, the additional provision which it was desired to make for the decline in the rate of interest.

Mr. J. B. GILLISON said he had had some experience on the other side of the world in connection with what was known as Sprague's method of valuation. It was adopted in the first place in Australia to enable young companies to declare a reasonable rate of bonus. He regarded a valuation from two most important points: the first, as a standard of solvency, and the next, as to the effect it would have on future bonuses. He happened to have had the advantage some years ago of seeing some calculations made by a brother actuary of a hypothetical company—a company very like one that had been and was then doing business in Australia. He had made valuations on a 4 per-cent basis by Sprague's method and also by the pure premium method. The results showed that in the early years the bonuses by Sprague's method were somewhat better than by the other, but in about fifteen or twenty years the bonus by the pure premium method exceeded the bonus by the other method, and at the same time the reserves were much larger. Another fact brought out was that in order to secure an even rate of bonus, if Sprague's method of valuation were adopted, it appeared to be necessary, other things being equal, to do an increasing amount of new business. He did not think any office could be sure it would do an increasing amount of new business; indeed, owing to various circumstances, business in Australia at one time fell off largely. He had, therefore, come to the conclusion that the pure premium method was the more suitable method, both as a standard of solvency and as being likely to secure an increasing rate of bonus in the future. With regard to the author's remarks in connection with the difficulty of the application of the method, he had found it comparatively simple. The practice generally adopted was to value  $P_{x+1}$ , excluding all negative values. With regard to endowment assurances,  $P_{x+1} - P_{x+1}^1$ , or, as was the case in another office with which he was acquainted, a fixed amount of 30s. per-cent was spread over the term of the endowment assurance. The calculations were exceedingly easy. There was only one difficulty, and that was in making estimates between valuation periods of probable surplus—that is to say, it was much easier to make an estimate when the pure

premium method was adopted than when Sprague's method was adopted.

Mr. A. LEVINE thought most of the difficulties had arisen from the fact that the net premium was, so to speak, a constant net premium for all offices. If the method by which the office premiums and net premiums were calculated, and the method by which the bonus was distributed, were in harmony with each other, then the net premium valuation was absolutely scientific. If a net premium valuation of that kind were adopted he thought there was ground for the gradual reductions in the valuation rate of interest. As the rate of interest was reduced they would not alter the premium valued, which would still remain the net premium. He did not think the increase in expense ought to affect the reserve in the first year. If it was found that the expense of new business was going beyond the loading originally put on, the difficulty ought to be met by not giving bonuses to the new policyholders for the first year. There was no doubt that in a great many cases the policyholder himself, although he was not, strictly speaking, an agent of the company, obtained the big commission for the first year; and, therefore, it was no injustice to him if he did not receive a bonus in the first year.

Mr. A. G. HEMMING said that Mr. Warner would appear to consider it an abandonment of the net premium method if an adjustment of any kind were introduced. He pointed out that in the case of limited payment policies it was quite usual now in using the net premium method to make such an adjustment to provide an additional reserve of loading in these cases, and it occurred to him that in making an  $H^M 2\frac{1}{2}$  per-cent valuation the facts might be kept quite free from obscurity if the value of the  $H^M 2\frac{1}{2}$  per-cent net premiums appeared in the ordinary way, and an adjustment, shown separately, was made, bringing the reserve for future expenses and profits up to such a percentage of the office premiums as the actuary considered necessary.

Mr. GEORGE KING, in closing the debate, said that he thought the members could see one important object of the paper, which was indicated at its commencement, namely, that it might elicit a good discussion, and in thanking the author for his work they must congratulate him on that result. Another result of the paper and discussion was that they went to show that a valuation was not a mere mechanical process. In the outside world, people were very apt to imagine that it was all mechanical—that the necessary particulars were metaphorically put into a machine, and the result obtained. But, as a matter of fact, it required a long experience and the soundest judgment to decide on the proper method of valuation in any particular case; and, even if they adhered to the net premium valuation, there were so many ways of making it that this remark did not fall through. There was one curious little contradiction in the paper—not exactly a contradiction on the author's part, but between two statements. Mr. Warner said that the net premium method was "based on scientific principles", and later on it would be found that Mr. Sorley said that the net premium method was one



that "must inevitably disappear before the advance of true actuarial science." He rather tended to agree with Mr. Sorley's view. He did not say that the net premium valuation would be abandoned, but it would be looked at from a very different standpoint. What was a net premium valuation? It had been defined as a valuation by a true table of mortality at a true rate of interest, the net premiums valued being based on that true table; and the author quoted Mr. Manly's paper as giving a kind of algebraic definition of it. It would be noticed, however, that in Mr. Manly's definition there were a great many "ifs", and he thought that that caused it really to fall to the ground. A valuation on the combined  $H^M$  and  $H^{M(5)}$  Tables was not a net premium valuation. He would go farther, and say that a valuation based upon Dr. Sprague's formula, where the premium was  $P_{x+1}$ , was a net premium valuation, although it was not called so. There could be no quite satisfactory definition of a net premium valuation, and, therefore, they were discussing something which was vague and undetermined, although they knew practically what was meant. Although it was well to make a net premium valuation wherever possible, they must beware of setting it up as a fetish. He thought there were many circumstances where it could not be applied, and it was just as well that that statement should go out authoritatively from the Hall, so that blame might not be imputed to all those who departed from it. One or two of the very best companies in the country under no circumstances could make a net premium valuation—those companies that distributed their profits by a percentage reduction of the premium. They could not make a net premium valuation, because, if they did so and ignored the future reductions, they would show a fictitious surplus, whereas if they brought these reductions in as liabilities they would show an equally fictitious deficit. In a paper on Life Assurance Legislation, in the 29th vol. of the *Journal* he (Mr. King) had shown how the net premium valuation might be driven to extremes and do very great mischief, and that it would not do as a test of solvency. In America it had produced great mischief by being made the legal standard of solvency. A young company was perfectly justified in departing from the net premium valuation. It had not had time to create a reserve fund, which, after all, was what a net premium valuation did—by reserve fund he meant something over and above the fund in hand for the calculated liabilities. He had never hesitated when he had been consulted by young companies, mostly in the colonies, to recommend some different system. It also did not apply to another class of business—non-profit business at very low rates of premiums. There were one or two offices of standing in this country which had such low rates that no ordinary net premium valuation would apply; the rates were below the net premium. Something different must be done. Those and various other circumstances might cause them to depart from the net premium valuation, but at the same time he cordially agreed that there were dangers in so doing. Mr. Young had referred to Mr. Jellicoe's early writings, where that giant of the actuarial profession laid great stress upon dealing with the office premiums; but he would remind the members of one sentence in one of his

papers, which he thought showed the great danger of looking at merely the office premiums. Mr. Jellicoe said (*J.I.A.*, vol. i, p. 26) that when one valued the office premiums less a percentage, if that did not show the required surplus they could reduce a little the percentage thrown off, "in order, it may be, to counterbalance any extraordinary fluctuations." That illustrated the great danger of departing from the net premium valuation. The author rather took exception to Dr. Sprague's suggestions for spreading other expenses besides the commuted commissions over the future, and said that these did not cease when the particular policy had been issued, but went on. He thought it was only fair to Dr. Sprague to point out that he assumed that those expenses—advertising and other expenses—incurred for the development of the business, would produce a larger income to bear the expenses, and that, therefore, he was justified in looking at them from that point of view.

The PRESIDENT (Mr. C. D. Higham), after moving a hearty vote of thanks to Mr. Warner, said he was always sorry that the valuation schedules to the Act said anything about net premiums, because he liked to emphasize the two facts—the values of the sums assured and bonuses on the one side, and of the office premiums on the other—and then to show on the liability side the provisions made for profit, commission, and expenses. Although the actuary might not publish the three items, he ought to have clearly in his head how much was for bonus, which depended mostly on sums assured, how much for commission, which was a question principally of premium, and how much for expenses, which was more a matter of the number of policies. Facts must be looked in the face, and he did not think it could be said that it was necessary that such a reserve should be the value of the full loading, but he should be sorry if it ever came to pass that it generally was not. The  $H^M$  and  $H^{N(5)}$  net premium valuation had been a useful spur in the past through the public demand for it. The terrible "man in the street", who was always turning up, thoroughly understood it. He knew that the  $H^M$  Table was His Majesty's Table, and felt perfectly safe. But that spur would be altogether unnecessary if offices generally were to adopt the excellent system laid down by Mr. Manly in the letter which he had read, namely, that the actuary, before a bonus was declared, should put by in reserve future bonuses at the same rate for the whole future existence of the policy. But, whatever was coming, he thought actuaries in the future would have courage to remember that strength in the valuation was the first requirement of all.

The vote of thanks was carried with acclamation.

Mr. WARNER, in reply, said that a few minutes ago he was prepared to rise in a very apologetic manner, because he was deeply conscious of the deficiencies of the paper. It was written somewhat hurriedly, and, in deference to a necessary change of the programme of the session, had to be presented to the Institute a month earlier than had originally been anticipated. Under those circumstances it was not possible to go so fully into such a great subject, or to do it that justice which might otherwise have been attempted, because the time did not allow. Such remarks, however,



did not seem now to come with very good grace from him because of the kind reception given to his paper by the members; he was very glad they had found something in it to discuss. Mr. King was perfectly right in saying that the chief object in bringing the subject forward was that a suggestive and instructive discussion might be elicited; and he certainly thought that object had met with success. He was sorry he did not notice the interesting letter written by Mr. Manly in the *Insurance Record*; probably, as it was under the heading of another controversy, it escaped his notice in preparing the paper somewhat in a hurry. In any case he hoped no injustice had thereby been done to the writer of the letter. He had quoted Mr. Manly's paper not in any way as assuming that thirty-five years afterwards Mr. Manly was an advocate of any particular view he had expressed in his Messenger Prize Essay, but because he had a difficulty in finding anywhere a more lucid or concise definition of the method than he found in the few lines he had quoted. He had been accused, quite good-humouredly, of a certain lack of courage in the expression of his views. It seemed to him that at all events in the closing paragraphs of the paper he had expressed his views pretty plainly; but he would suggest that, although lack of courage was not a very pleasant thing, there was something else which was perhaps on the whole more creditable, and which sometimes had the same practical effect, namely, modesty. He put in a small claim to the members' consideration for having shown that quality rather than lack of courage. The subject dealt with in the paper was a very great one. It had been dealt with by some of the most eminent and illustrious members of the profession; and he had, therefore, felt that a certain guardedness in the expression of one's views and opinions would not, on the whole, be inappropriate. He thought, however, his own views had been made fairly clear. They practically were that the net premium method, as generally understood, had about it a certain rigidity which unfitted it for the complex and changing conditions of their business, and that in those circumstances it might quite conceivably produce arbitrary and artificial results. He thought it was perfectly true to say that it was originally based on scientific principles, as had been pointed out in the discussion. So long as the assumptions involved in the construction of the office premium were exactly carried out in practice and realized in experience, the method was not only a scientific method but the only one. It must be considered, however, that as the conditions continually changed, and must of necessity do so, so the expressions used in valuations must change along with them. Now from the nature of the case there was one thing which did not change, namely, the office premium which they had contracted to receive on the policies actually in force. It had been said that if a valuation were made at a lower rate, and the premiums were not adequate, they must be raised; if the expenses were going up the proper thing to do was to raise the premiums. Well and good, that was a counsel of perfection, and he would be the last to say a word against it; but the premiums could not be raised on the policies which were already running, which constituted the great bulk of the

business. He was very much obliged to Mr. Todhunter for kindly explaining on his behalf that he had not the least desire to advocate any method of anticipating loading. He thought he expressed clearly that he deprecated, in the case of companies which were prosperous and steady going, with an average rate of expenditure which did not vary much from year to year, any attempt by easing the valuation to give a looser rein to initial expenditure. The point was that there were circumstances in which it was perfectly right and fair, circumstances which Dr. Zillmer had described, where a company extinguished a future liability by an immediate payment, in consequence of which it obtained a large accession of new business. The strong point of the argument was not so much the initial payment made, but the future reduction effected. Surely it was fair to take that into account, and to meet exceptional circumstances by exceptional steps. Personally, he strictly limited the applicability of that method, and said that so long as it was not absolutely necessary it should not be resorted to for what he had called politic rather than scientific reasons. In the present day actuaries should be very careful of any suggestion which would in any way countenance, in the hands of those who might not use it quite intelligently or scrupulously, the increase of initial expenditure. That point, however, did not seem to him to be of such great importance as the other two questions of interest and mortality, and their effect upon the so-called net premiums. In his opinion the method of the future would be one which had as its sheet anchor the actual premium of practice—the premium receivable in fact—the premium embodied in their contracts, and which then made whatever modifications in it as were found to be suitable, including a thoroughly adequate deduction for expenses, and the rigid exclusion of all negative values; building up adequate reserves in a way which should clearly show what had actually been done. The net premium was said to be understood by “the man in the street.” It was very questionable whether that ever were so, but it was quite true that the phrase “net premium valuation” had acquired a kind of mystic authority, as if all perfection were summed up in it. That might become very dangerously misleading, because one could easily conceive a net premium valuation at so low a rate of interest as to produce net premiums larger than the premiums actually receivable. In conclusion, he thanked the Institute for the generous reception given to his paper.

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*American Valuation Tables.* By WILLIAM ANDERSON HUTCHESON, F.I.A., F.F.A., Associate Actuary of the Mutual Life Insurance Company of New York.

ALL members of the *Institute* are more or less conversant with the "field" work of American Life Insurance Companies, but comparatively few have had the opportunity of becoming acquainted with the "inside" methods of these Institutions, which, as might be supposed, differ in many respects from the methods of British Companies. One of these differences is that Valuation Tables are much more extensively used there than in Britain, and it is not surprising, therefore, that various Valuation Tables have, from time to time, been published in America. No notice seems to have been taken of them in the *Journal*, so I propose, in the first place, to indicate briefly the contents of these publications, and then to refer to some of the American methods which necessitate the existence of these various Tables.

#### PART I.—VALUATION TABLES.

**Wright's  
Tables.** The first published Tables to which I shall refer are Elizur Wright's "*Valuation Tables on the Combined Experience or Actuaries' Rate of Mortality*"—the mortality Table known in Britain as the "Seventeen Offices' Experience." The first edition of these Tables, published in Boston in 1858, contained the net premiums, and the reserves for uniform premium whole life insurances, at 3 per-cent and at 4 per-cent interest, interpolated for months between the "Initial"\* and "Terminal"\* Reserves.

In 1871, the second, and much enlarged, edition of these Tables was published. In addition to the reserves above-mentioned, it contains the reserves for (1) Limited Payment Life Policies by 5 and 10 annual premiums; (2) Endowment Assurances (known as "Endowments" in America), maturing both at quinquennial ages and at the end of quinary terms of

\* In America, the reserve at the end of any policy-year is designated the *terminal reserve*; this terminal reserve, plus the net premium payable at the commencement of the following Policy-year is called the *initial reserve*; and the mean of the *initial* and *terminal* reserves of any policy-year is known as the *mean reserve* of that year. Thus, e.g., in the case of a life policy effected at the age  $x$ ,  ${}_{n-1}V_x + \pi_x$ ,  ${}_nV_x$ , and  $\frac{1}{2}\{({}_{n-1}V_x + \pi_x) + {}_nV_x\}$  are the *initial*, *terminal*, and *mean* reserves respectively of the  $n$ th year.



years; and (3) Limited Payment "Endowments" by 10 annual premiums. Besides these various reserves, this edition also contains the values of temporary annuities, the single premiums for temporary assurances, the "complements of the reserves", and the "Insurance Values"\* for various classes of insurance.

The additional values given in the second edition are all calculated according to the Actuaries' Table and 4 per-cent interest.

In connection with these Tables, it is interesting to note that State supervision of Life Insurance Companies in America was initiated by the Commonwealth of Massachusetts in 1856, and that Elizur Wright was Insurance Commissioner there when the first valuation of the policies of all the Companies doing business there was made in 1859. This valuation was made according to the "Actuaries' Tables and 4 per-cent interest", and valuations have been made on this basis by the Massachusetts Insurance Department annually since then, although, as will be seen later on, the standard for new business written since 1 January 1901, has now been altered.

On the other hand, the original standard of valuation in the State of New York was the English Life Table No. 3 and 5 per-cent interest. In order to overcome the obvious disadvantages of having their policies valued on one basis by the New York Insurance Department and on another basis by the Massachusetts Insurance Department, an endeavour was made by the Companies to get a uniform basis of valuation adopted in these two States. In this way it was hoped that the necessity of a double valuation might be obviated, but the endeavour to bring about this agreement was not successful. It resulted, however, in New York State adopting a new standard, namely, the American Experience Table of Mortality and  $4\frac{1}{2}$  per-cent interest, which went into effect in 1868. This Table of Mortality was deduced by Mr. Shephard Homans, the Actuary of the Mutual Life Insurance Company of New York, from the mortality experience

\* To quote from the Massachusetts Nonforfeiture Law of 1880, an "Insurance value is the present value of all the normal future yearly costs of insurance which by its terms a policy is exposed to pay in case of its continuance." The "cost of insurance" is the British expected death strain— $q_{x+n}(1-n+1V_x)$  in the case of a "life" policy taken out  $n$  years ago at age  $x$ —so that the "Insurance value", being the present value of all the normal future yearly costs of insurance, would,  $w$  being the limiting age in the Table, in such a case, be

$$\sum_{t=w-(x+n)}^{t=1} \{ v^t \times {}_{t-1}p_{x+n} \times q_{x+n+t-1}(1-n+tV_x) \}.$$



of that Company. While on the subject, it might be well to state that it is the Table on which the present premium rates of a majority of the American Life Insurance Companies are based, and that it has now been adopted by the States having most to do with Insurance as the standard on which the various Companies' policy liabilities are valued. It must not, of course, be confounded with the "Thirty American Offices Table", published in 1881.

The Mutual  
Life's Tables.

This brings me to the first Valuation Tables based upon the American Experience Table of Mortality. Under the title of "*Life Tables, with Appropriate Formulas for the Solution of Questions pertaining to Life Contingencies*", Messrs. Lawton and Griffen, of the Mutual Life Insurance Company of New York, published, in 1873, a very complete set of Tables of net premiums and reserves for (1) Life, (2) Limited Payment Life, and (3) Endowments and Limited Payment "Endowments" (maturing at quinquennial ages, and also at the end of quinary terms of years), and also a set of joint-life commutation columns and annuity-values for every combination of two lives. All the values given in the Tables were computed by the American Table at 4 per-cent interest, and also at  $4\frac{1}{2}$  per-cent interest.

The  
Connecticut  
Mutual's  
Tables.

In 1890, a complete set of tables of net premiums and reserves for Life, Limited Payment Life, Endowment, and Limited Payment Endowment Insurances, based upon the American Table of Mortality and 3 per-cent interest, was published by the Connecticut Mutual Life Insurance Company of Hartford, Connecticut. In addition to these various values, the volume also contains the "costs of insurance" for the policies for which the reserves are given, and also the values of temporary annuities, all on the "American" 3 per-cent basis.

The Spectator's  
Tables.

Probably the most comprehensive set of Tables are those contained in the sixth edition of "*The Principles and Practice of Life Insurance*" (the Spectator Company of New York, 1892), a work which has evolved from a small pamphlet published in 1872 by the late Mr. Nathan Willey. The Tables contained in the various editions have been extended from time to time, and the present edition contains, in addition to Compound Interest Tables, the net premiums and reserves for Life, Limited Payment Life, and "Endowments", computed (1) by the American Table at 3 per-cent,  $3\frac{1}{2}$  per-cent, 4 per-cent,

and  $4\frac{1}{2}$  per-cent interest, and also (2) by the Actuaries' Table and 4 per-cent interest. The "costs of insurance" are also tabulated for the various classes of Insurance according to the Actuaries' Table and 4 per-cent interest. It will thus be seen that this publication contains most of the Tables given in the three published Tables already referred to, and, in addition thereto, the reserves for the same classes of Insurance according to the American Table and  $3\frac{1}{2}$  per-cent. interest. The explanatory text in the present edition was entirely re-written by the late Mr. Henry Worthington Smith, and the Tables were revised under the direction of Mr. John Tatlock, jun., Associate Actuary of the Mutual Life Insurance Company of New York.

The Mutual  
Benefit's  
Tables.

In the year 1900, the Mutual Benefit Life Insurance Company of Newark, New Jersey, published a set of Tables of "*Net Premiums and Reserves according to the American Experience Table of Mortality and 3 per-cent Interest*" for the usual classes of policy issued in America, namely, Life, Limited Payment Life, "Endowment", and Limited Payment "Endowment"—the "endowment" and the "limited payment" periods being, as is usual in America, some multiple of five years. In addition thereto, the volume contains complete Tables of single premiums for "Endowments", Pure Endowments, and Term Insurances (with the interpolated additional premium for each 100 days' insurance between the integral years); the values of temporary annuities; and joint-life commutation columns and annuity-values for all combinations of two lives—all according to the "American" Table of Mortality and 3 per-cent interest.

Dawson's  
Tables.

In addition to the above-mentioned publications containing the reserves for the usual forms of Insurance, a volume, containing the values of various functions which are useful in calculating the reserves on unusual forms of policy and for various other purposes, was published in 1900 by Mr. M. M. Dawson, of New York City. This volume of "*Various Derived Tables*", as they are called, contains the Single Premiums for "Endowments", Pure Endowments, and Temporary Insurances (with the interpolated "daily differences" between the premiums for integral years); the values of immediate

Temporary Annuities, and of the functions  $\frac{D_x}{D_{x+n}}$ ,  $\frac{N_x - N_{x+n}}{D_{x+n}}$ ,  $\frac{M_x - M_{x+n}}{D_{x+n}}$ , and  $\frac{R_x - R_{x+n} - nM_{x+n}}{D_{x+n}}$ . All values in the Tables

are computed by the American Table of Mortality at 3 per-cent, and also at  $3\frac{1}{2}$  per-cent interest.

N. Y. State  
Insurance  
Dept.'s  
Tables.

In view of the method in which the annual valuations are made, the various State Insurance Departments must have the reserves for all classes of Insurance tabulated in order to facilitate the *seriatim* valuations, which they make annually of each Company's policies. In 1887, when the standard of valuation in the State of New York was altered to the "Actuaries' Table of Mortality and 4 per-cent interest", the Insurance Department there issued a set of Tables of Reserves on that basis, and since then that Department has continued to have the reserves for all the various classes of policy which they are called upon to value printed on cards. The standard of valuation in New York State now being the American Table of Mortality and  $3\frac{1}{2}$  per-cent interest for policies issued since 1 January 1901, the Insurance Department is now publishing cards containing the reserves for all classes of policy on this basis also. When we recollect that each Company's total reserve liability, as certified by the Insurance Department, is ascertained by adding together the individual policy reserves posted in the Department's Valuation Registers, and that in these Registers the policies issued in each calendar year are kept separate, it is not surprising to find that the reserves tabulated on these cards—the "mean" reserves in all cases—are grouped according to the duration of the Insurance. Thus, for example, one card contains the "mean" reserves of the *first policy-year* for Ordinary Life Policies, and Limited Payment Life Policies; and others contain respectively the "mean" reserves of the same policy-year for (1) "Endowments" maturing at age " $x+n$ " ( $x$  being the age at issue, and  $n$  various multiples of 5 years); (2) "Endowments" maturing in  $n$  years; (3) Limited Payment "Endowments", and so on. Similar cards, of course, contain the "mean" reserves for subsequent policy-years.

## PART II.—AMERICAN METHODS.

Any note regarding the various Valuation Tables which have been published in America would be incomplete without a statement concerning several American methods and practices which differ from British ones, and which practically necessitate the existence of the various valuation Tables already described.



**Valuations.** The first of these to which I shall refer is the method in which the periodical valuations are made, and in order to bring out the difference in practice, I shall, first of all, outline the British method.

*In Britain.*—The British “Life Assurance Companies’ Act 1870” requires the values of the sums assured and bonuses, and the values of the premiums, to be calculated separately—the reserve being the difference between these values. These valuations are made by the Companies themselves, the usual method being to group together and value, as one policy, all contracts having the same “reversions” and “annuity values” at the date of valuation. The Act does not require the valuations to be made oftener than once in five years; neither does it require them to be made on a net premium basis, nor by any specified table of mortality or rate of interest. What the Act does require is that each Company shall make certain returns to the Board of Trade—within nine months after the date of valuation—stating the basis of the valuation adopted, the results of the valuation, and certain particulars of the Insurances in force at the date of valuation to enable the results to be approximately checked.

As a matter of fact, the Table of Mortality usually employed in these valuations is at present the  $H^M$  (or the  $H^M$  and  $H^{M(5)}$ ) Table, but various rates of interest are assumed by the different offices, extremely low rates being assumed by some of the Companies to enable them to make proper provision for the bonuses under their various systems of division. In many cases the valuations are made by entirely different Tables of Mortality and rates of interest from those on which the premiums were calculated.

*In America.*—Contrast this with the method of valuation pursued in America. Instead of being made by the Companies themselves, the valuations (the results of which are published) are made by the Insurance Department of one or other of the various States of the Union. They are made annually at 31 December, and the method usually pursued is to calculate the net premium reserve for each policy, and take the aggregate of these individual policy reserves as the Company’s Policy reserve liability. To enable the Insurance Department’s Actuaries to make these valuations, they are furnished throughout each year with a description of each policy issued, and of each policy which goes out of force for any reason. The particulars thus



given enable the Insurance Department Actuaries to keep a set of valuation registers for each Company's Policies, and in these registers they insert in the column set aside for each calendar year, the reserve of each policy at the end of the year in question. These individual policy reserves are posted in these valuation registers through the year, and, if the Insurance Departments keep the "totals" up to date, and are furnished at frequent intervals towards the end of the year with lists of "issues" and "terminations", a small adjustment is all that is required to enable them to state, within a few days after the close of each calendar year, the amount of reserves required to be held at 31 December.

*"Mean Reserves."*—In America the age at entry is the then *nearest* age, and not the age at *next* birthday, as in Britain. Further, dividends become due on the anniversary of the policy, and not at the end of the valuation period, as in Britain. This being the case, no reason exists for policies being taken out at the end rather than at any other period of the Company's financial year, as in Britain, and it is therefore assumed that, on the average, policies are effected in the middle of each calendar year, and therefore the "mean" reserve above described is the proper reserve to set aside at the end of each calendar year. In using the "mean" reserve, the assumption is made that annual premiums are paid in every case. This assumption does not, of course, agree with the facts, for premiums may be paid semi-annually, or quarterly, if the insured so desires, on condition, however, that any out-standing semi-annual or quarterly premiums shall be deducted from the sums insured payable at death. In America, then, premiums payable oftener than once a year are considered as instalments of annual premiums, whereas in Britain they are usually regarded as true semi-annual or quarterly premiums, as the case may be. The assumption being made that premiums are payable annually, and reserves being made on this basis, these outstanding instalments are treated as liens, and their amounts, less loadings contained therein, are considered as assets at the end of each year.

*Standards of Valuation.*—The laws of a majority of the States define the mortality table, and the rate of interest at which the reserves are to be calculated. Thus, for example, the Massachusetts Act of 1900 requires that these reserves shall, in the case of all policies issued before 1 January 1901, "be computed upon the basis of the combined experience of Actuaries'

“Table of Mortality, with interest at four per-cent per annum.” For those issued after that date, the reserves “shall be computed upon the basis of the American Experience Table of Mortality, with interest at  $3\frac{1}{2}$  per-cent per annum”, but any office electing to make reserves upon an American 3 per-cent basis, shall have the reserves computed upon that basis.

While Massachusetts has, until the passing of the 1900 Act above referred to, always made valuations according to the Actuaries' 4 per-cent Table, the New York standard, as we have seen, has altered from time to time. The original standard was the English Life Table No. 3 and 5 per-cent interest, but this was changed in 1868 to the American Table and 4 per-cent interest, and again in 1887 the standard was changed to the Actuaries' Table and 4 per-cent interest. The 1893 law required the valuations to be made “according to the standard adopted by the Company”, but, as the standard in many of the other States was the Actuaries' 4 per-cent Table, and, as it is usual for other States to accept a certificate of valuation from the Insurance Department of the State in which any Company has its head office, it was usual for New York Companies to adopt the Actuaries' 4 per-cent Table as the standard under this Act, so as to do away with the necessity of being valued again by other States. In 1901 the law was again changed; it gives the Companies power to adopt different standards for obligations of different dates or classes, but requires that the total reserve computed by any such standards shall not be less than that determined by the legal minimum standard, which it enacts to be the Actuaries' 4 per-cent net premium reserves for contracts issued before 1 January 1901, and the American  $3\frac{1}{2}$  per-cent net premium reserves for subsequent issues.

The laws of Massachusetts do not allow the Insurance Commissioner to accept a certificate of valuation from any other State, and they further require him to make a valuation of *all* policies issued by any Company doing business in Massachusetts, and not merely of the policies issued in Massachusetts. A New York Insurance Company, therefore, if it does business in Massachusetts, requires to make returns of *all* policies “issued” and “terminated” to the Insurance Department of the State of New York, and also to that of the Commonwealth of Massachusetts, to enable these Departments to make their annual valuations of that Company's outstanding policies each 31 December, according to the standards of valuation in these

States. Notwithstanding these two State valuations, if the Company's premiums were calculated according to some other table of mortality or rate of interest (the business of the three large New York Companies was for many years written on an American 4 per-cent basis), the Company itself will make another valuation on the basis on which the premiums were calculated to ascertain its true position as to liabilities and surplus.

Division of  
Surplus.

The next point to which I wish to refer is the distribution of surplus.

*In Britain.*—The usual British practice is to ascertain the amount of surplus at the end of the valuation period, usually five years, and to divide the surplus so found, or some part thereof, amongst those of the policies in force on that date, which, by their terms, were entitled to share in the division. This division of surplus is made according to the method of distribution adopted by the office, which is, in the majority of cases, either the simple reversionary or compound reversionary bonus system. The actual distribution is not made on the anniversary of the policy, as in America, but as soon as possible after the valuation has been completed, and the necessary apportionment of the divisible surplus amongst the individual policies has been made; and the bonus is for the period ending at the date of valuation.

*In America.*—In nearly all American Companies, on the other hand, surplus is divided according to the "Contribution Method." The dividends are distributed on the anniversaries of the policies, and are for policy-years then ending, and not for calendar years ending with the valuation period, as is the usual British custom. The New York law makes this clear: it states that "any domestic Life Insurance Corporation may ascertain at any given time, and from time to time, the proportion of surplus accruing to each policy from the date of the last to the date of the next succeeding premium payment, and may distribute the proportion found to be equitable . . . at the next succeeding date of such payment . . ." The Massachusetts law goes a step further, and requires (of Massachusetts Companies) that "such distribution to policyholders shall be made upon the contribution to surplus plan."

Policy  
Guarantees.

I now come to the third and last point to which I wish to refer, namely, the options which the insured is guaranteed in the event of his wishing to discontinue his policy before its maturity.



*In Britain*, when a policy is discontinued, it is customary to grant either a cash surrender-value or a paid-up policy therefor, but the amounts of these are not usually guaranteed, nor are they stated in the policy. The above-cited Act of 1870 requires the Companies to give, as part of the periodical valuation returns, "a table of minimum values, if any, allowed for the surrender of policies . . . or a statement of the method pursued in calculating such surrender-values . . . ", and the Companies, in their prospectuses, usually give some indication of the minimum surrender-values allowed. It is, therefore, to one or the other of these that the applicant for insurance turns in quest of information as to the surrender-value he may expect on discontinuing his policy.

*The up-to-date American Policy*, on the other hand, guarantees various options in the event of the policyholder deciding, at any time after the policy has been (usually) three years in force, to discontinue payment of further premiums, and the amounts of these guarantees are stated in the policy itself. The usual options are: (1) cash surrender-value; (2) paid-up insurance of a smaller amount but of the same nature as the original insurance; (3) continuance in force of the amount insured by the policy for some stated period of years and months without payment of further premiums. The second, or in some cases the third, option just stated goes automatically into force on default in payment of any premium—the insured having the option of selecting one of the others. In addition to these options, the policy guarantees loan values not exceeding the guaranteed cash values. A consideration of the third option shows why the single premiums for temporary insurances are given in various Valuation Tables.

A brief historical sketch of these various options will, I think, be of interest. In the earliest days of American Insurance, no provision was made in the policy for surrender-values of any kind, but it was the practice nevertheless to give surrender-values in cash. In 1859, the Companies commenced to issue "10-payment life" policies, which provided for a surrender-value in the form of paid-up insurance in case of default after two premiums had been paid—the amount of this paid-up insurance being one-tenth of the original insurance for each premium paid. A few years later, a similar rule was applied to endowment insurances, and, within the next few years, surrender-values in



the form of paid-up insurance were allowed for ordinary life policies. At first these paid-up insurances were granted if the original policies were surrendered within two or three months of default in payment of premium, but this time was subsequently extended to six months.

Meantime, in 1861, the Massachusetts Legislature had passed a non-forfeiture law applicable to Massachusetts Companies requiring that all policies which lapsed for non-payment of premium be kept in force for such time as 80 per-cent of the Actuaries' 4 per-cent reserves would purchase term insurance on a net Actuaries' 4 per-cent basis, the Company being allowed, in paying such claims, to deduct the over-due premiums with 6 per-cent interest thereon.

About the year 1870 several Companies began to issue Tontine Policies, the distinctive features of which were that the holders were entitled neither to surrender-values of any kind, nor to any dividends during the tontine period. The representatives of those who died during the period received simply the sum insured, and the whole of the remainder of the fund arising from the class was accumulated until the end of the tontine period, when its excess over the reserve held for the existing policies was divided amongst the survivors. Another feature of these policies was that the full reserve held therefor—the "American" 4 per-cent reserve—could be drawn in cash at the end of the tontine period, if they were then surrendered.

In 1879, the State of New York passed a Non-Forfeiture Law which applied to all policies issued by "domestic" (*i.e.*, New York State) Companies after 1 January 1880. It required these Companies to grant, after the Policy had been three years in force, a surrender-value in the form of either non-participating term insurance for the same amount, or non-participating paid-up insurance of a smaller amount, but of the same class as the original policy. The "term" in the first case, and the "amount" in the other case was to be ascertained by applying the full "American"  $4\frac{1}{2}$  per-cent reserve at the date of default as a single premium at the Company's published rate at the time the policy was issued, and such rate was not to exceed the "American"  $4\frac{1}{2}$  per-cent net single premium loaded 50 per-cent. In the case of "endowments", if the reserve was more than sufficient to carry the insurance for the remainder of the term, the balance not required for this purpose was to be payable as a

pure endowment at the time the original policy would have matured.

Under the Massachusetts Non-forfeiture Law of 1880, the Massachusetts Companies were required in case of default in payment of premium to give either paid-up insurance for the amount purchaseable by the full reserve according to the "Actuaries' Table" and 4 per-cent interest, less a surrender charge not exceeding "8 per-cent of the *insurance-value* of the policy at the date of default", or (2) a cash surrender-value of such reserve less said charge.

In connection with this surrender charge, Mr. Elizur Wright, in the preface to his "Valuation Tables", stated that a policyholder "cannot be allowed to retire without leaving enough to pay for replacing the *insurance-value* he takes away", and then recommended a surrender-charge based on this "insurance-value." It will thus be seen that his ideas were incorporated in the Massachusetts laws. In the case of policies issued since 1 January 1901, for which reserves on the "American"  $3\frac{1}{2}$  per-cent basis have to be held, the surrender-charge has been altered to 5 per-cent of the present value of the future net premiums on that basis.

Various other States have somewhat similar non-forfeiture laws affecting their own Companies, but sufficient has been written about these laws for our present purpose.

To return to the question of policy guarantees, shortly after the non-forfeiture laws went in force the Companies commenced to issue "semi-tontine" or "deferred dividend" policies. These policies, while entitling their owners to surrender-values in the form of paid-up insurance during the deferred dividend period, did not participate in surplus unless they were kept in force until the end of that period, at which time they could be surrendered for their guaranteed values, the full reserve by the American Table of Mortality and 4 per-cent interest. At first these deferred dividend periods were 20 years, but subsequently policies were issued with shorter dividend periods either 15, 10, or 5 years, the cash values guaranteed at the end of the shortest dividend periods being less than the full reserve held for the policy.

Until a few years ago, the premiums of most of the Companies were calculated according to the American Table of Mortality and 4 per-cent interest, and the reserves were made on the same

basis. The options to which holders of these policies were entitled in the event of discontinuance were: Paid-up Insurance, and (or) extended Insurance, and, in the case of deferred dividend policies, cash values at the end of these dividend periods. The actual amounts of these options were not stated in the policies, but the rules by which they were ascertained were given. Thus, for instance, it was stated that the paid-up policy would be for the amount required by the laws of the State of New York, for example.

Several years ago a general move in the direction of assuming a lower rate of interest in calculating premiums was commenced, and, with the adoption of these new premium rates, the policies were made to provide for cash values on surrender at any time after (usually) three years' premiums had been paid, instead of only at the end of the deferred dividend periods as formerly. Not only was the "cash value" option made annual, but the amounts of all the various guarantees for each year of the policy's existence were inserted in the policy, the result being the above-mentioned "up-to-date American policy."

In conclusion, it is only necessary to call attention to the method in which each Company's total policy reserve liability is ascertained, and the expedition with which the results of these annual valuations are made known, to show that, in the State Insurance Departments at least, the various Valuation Tables which have been published are practically necessary. A consideration of the fact that surplus is divided according to the contribution method, under which excess interest is divided in proportion to the "initial" reserves, makes it apparent that the Companies themselves also require these tables of policy reserves.

When we further consider that the policies now being issued contain the actual amounts of the various guarantees for each policy year, and that these various guarantees have to be prepared beforehand and published in the Company's rate books, it is evident that such extensive tables as those published are absolutely indispensable to the working of any Life Insurance Company in America.

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## REVIEWS.

*Institute of Actuaries' Text-Book, Part II. Life Contingencies (including Life Annuities and Assurances).* Second Edition.  
By GEORGE KING, F.I.A., F.F.A. C. & E. Layton, Farringdon Street, E.C.

THE *Text-Book*, Part II, occupies a unique place in actuarial literature. On the publication of the First Edition it was at once recognized as a masterly codification of the Theory of Life Contingencies, Annuities and Assurances, and accepted as the standard work on the subject in this country; and the 14 or 15 years that have since elapsed have witnessed the establishment of its authority throughout the greater part of the actuarial world. To students it has been a veritable royal road to actuarial knowledge; to practising actuaries an indispensable work of reference. That it is to a great extent a compilation from older sources detracts not at all from its merits, but rather compels admiration for the skill with which the work of so many different hands has been welded in its pages into a homogeneous whole. Seldom has a work of the kind achieved and retained so conspicuous a success. Its exceptional vitality may be attributed partly to the wise limitation of its scope to well-established theory—to the exclusion of debatable matter—partly to the rare appreciation of educational requirements brought by the author to the compilation and arrangement of the work. Actuarial research and original investigations have contributed to fill ten volumes of the *Journal* since 1887, but have not shaken one proposition or conclusion given in the First Edition of the *Text-Book*, Part II. Fourteen years of teaching and examining have not disclosed any very essential educational requirement which—within its limits—it fails to fulfil.

In welcoming a Second Edition of the work the profession generally will, therefore, concur in the view of the Council and the Author, as indicated by the President's prefatory note and by Mr. King's Introduction, to the effect that, unless the fact of the First Edition having run out had necessitated the issue of a Second, there would have been no special occasion for the revision of the book. At the same time there are, as a matter of course, many things in a treatise of this kind that might be done differently, many additions that might be made, and all who have made a special study of it—those especially who have used it in teaching—have looked forward with much interest to see to what extent Mr. King would avail himself of the opportunity of revising his methods or introducing new matter. That alternative proofs and new applications of some of the many propositions in the *Text-Book* should have been devised since the publication of the First Edition is only in the nature of things. It is, in fact, a special merit of Mr. King's work that it puts the student on the track of new ideas; it suggests the possibility of alternative methods, of new applications, by showing



how a problem may be regarded in many cases from different points of view, by exemplifying the scope of general propositions. Whether one method or another is preferable, whether this or that application is worth special notice, is of course a matter of opinion. Some teachers who use the *Text-Book* might probably be in favour of a more general use of symbolical methods in the demonstration and application of Finite Difference and Summation Formulas (*e.g.* in the investigation of the approximate value of  $q_{xy}^1$  Chap. IV, § 6), of a codification and simplification of the applications of the Infinitesimal Calculus, and of a recognition of some modern combinations of benefits exhibiting new features, such as discounted-bonus assurances (the theory of which Mr. King has himself so clearly expounded elsewhere), children's deferred assurances with conditions as to return and limitation of premiums, benefits of the type discussed by Mr. Manly (*Trans. Second International Actuarial Congress*. p. 860), guaranteed investment assurances, &c. From the opening sentence, however, of Mr. King's introduction to the new edition, it may be inferred that after due consideration he came to the conclusion that any considerable changes of this nature were inadvisable. It is possible also that other considerations, such as the fact of the stereotyped plates of the First Edition having been available for the printer, may have had some slight influence in restricting the range of revision, for it is stated later on in the Introduction that "notwithstanding the foregoing changes . . .

"it has been found possible so to plan that each chapter should "begin and end on the same pages as in the First Edition"—a feat which must have been difficult, and which one could almost wish to have been impossible, since it inevitably suggests a doubt whether Mr. King has given himself quite so free a hand as he might otherwise have done. However this may be, the alterations in the body of the work are comparatively few, and include nothing of a fundamentally structural character. Still, the few alterations and additions that have been made are of considerable interest. To begin with, the investigations of the general expressions for the probabilities of survivance of exactly  $r$  and at least  $r$  of  $m$  joint lives have been rewritten and simplified. Without following the strictly algebraical method of expressing  ${}_n p_{\overline{xyz \dots (m)}}^{[r]}$  as a general series in

powers of  $Z$ , determining the coefficients by making  $x, y, z$ , &c., all equal, and deducing  ${}_n p_{\overline{xyz \dots (m)}}^r$  by means of the relation

$$\frac{Z^r}{(1+Z)^{r+1}} + \frac{Z^{r+1}}{(1+Z)^{r+2}} + \&c. ad inf. = \frac{Z^r}{(1+Z)^r},$$

Mr. King has given a proof which is undoubtedly an improvement on the one in the First Edition. In Chapter VII—one of the most effective chapters in the book—no changes appear to have been made beyond a slight abbreviation of the algebra in the derivation of the formulas for  $a_x$ , &c., on De Moivre's hypothesis (an academic investigation which might very well have been cut out in favour of, say, Mr. Lidstone's approximations for Joint Life Endowment Assurance premiums and the values of Increasing Benefits), and none suggest themselves

except, perhaps, that the well-known inequality discussed in § 25 might have been alternatively deduced from the corresponding inequality between  $A_x$  and  $A_{\bar{x}}$ , which follows directly from the fact that the A.M. is greater than the G.M. Chapters VIII, to which one or two examples of possible applications of Conversion Tables have been added, IX and X—which could hardly be altered without being amalgamated (although Mr. King has managed to find room for a new approximation to  $\bar{A}_x$  illustrative of the general method of approximating to the values of definite integrals containing  $\mu$  by substitution of one of the approximate expressions for the force of mortality)—and XI, where the questionable integral  $\int_0^\infty t d\bar{M}$  still

remains as a stumbling block to the unwary, and in which the simple approximation  $\bar{a}_x^{(m)} = v^{\frac{1}{2m}} \bar{a}_x$  might perhaps have deserved a place, call for no special comment. Chapter XII, however, brings the student to one of the most considerable alterations in the new edition, namely, the excision of De Morgan's proof that Simpson's Rule gives Gompertz's Law, in favour of Mr. Levine's generalization of the proof given by Mr. Henderson of the corresponding connection between the Equal Ages Rule and Makeham's Law. The proposition itself may perhaps seem to be purely academic, and to serve no practical purpose, but the elegant and instructive demonstration now given fully justifies its inclusion. In Chap. XII, a new approximation for  $\bar{A}_{xy}^1$ —of the same nature as that given for  $\bar{A}_x$ —is included, and in Chap. XVI the investigation of § 66 is supplemented by a neat verbal proof. It is not quite clear why Mr. King has retained the somewhat complicated expression given in the Errata to the First Edition for the annual premium securing an assurance with return of premiums accumulated at rate  $j$ , in preference to the more elegant formula 
$$a_x - \frac{1+j}{j} \frac{A_x}{(A'_x - A_x)}$$
. The

Chapter on Policy-Values, which has much of the finality and completeness of VII, exhibits no alteration except for the incorporation of the explanatory note appended to the Errata to the First Edition, and could hardly be improved, although room might possibly have been made for a mathematical investigation of the "Expected Death Strain", for one of Herr Altenburger's formulas, and for a short verbal proof of the fact that a decrease in the rate of interest increases the Policy-Value, provided  $a_x$  increases with the age.

This practically exhausts the list of alterations in the body of the work, but at the end of the book, there has been added some new tabular matter which will greatly enhance its value as a work of reference. Priority in importance must be accorded to a complete set of  $2\frac{1}{2}$  per-cent Tables, calculated by Mr. H. J. Baker—a most opportune and useful addition in view of the extent to which  $2\frac{1}{2}$  per-cent is now employed in life-office calculations and of the necessity in many cases of employing a low rate in the valuation of complicated reversionary interests. The most interesting addition,

however, is a Table for finding the substituted age to two places of decimals for two or three joint lives, the extension to three lives being effected by a method specially devised by Mr. King for the present purpose. This little table might alone secure a welcome to the new Edition, and its mode of construction will commend itself as particularly neat. Incidentally it materially facilitates the use of the concluding addition to the book, namely, a reprint from the *Journal* of Messrs. A. S. Hume and W. Stott's Tables of 3 per-cent Annual Premiums for Endowment Assurances on two lives of equal ages.

R. T.

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*A Treatise on the Mathematics of Life Insurance.\**

THE appearance, in September last, of a second edition of Mr. Corneille L. Landré's work testifies at once to the growing interest which is taken on the European Continent in what may be termed the calculus of life insurance, and to the very acceptable form in which the author has clothed his exposition of the subject. In accordance with the strongly-urged advice of Dr. Van Schevichaven, both editions have been translated from the original Dutch into German, and the circle of Mr. Landré's readers thereby enormously widened. But not only have difficulties caused by the less familiar tongue been removed. The present edition will possess for English readers a special lucidity arising from Mr. Landré's employment of the notation which was adopted at the Second International Actuarial Congress of 1898, and we must cordially echo his remark that it is matter for congratulation that no national jealousy hindered the recognition by mathematicians and actuaries of the claims of the English system.

Mr. Landré tells us that he has given his book the simple title *Kapitel* as a means of indicating that "there is much more to be learnt" than is contained in its pages. But he has travelled over his ground with a comprehensiveness, and in portions of the subject, notably that relating to life annuities, with a thoroughness of treatment, not often found in introductory treatises.

The first two chapters of the work are devoted to a brief, but for the author's purpose a sufficient, exposition of the algebra of interest and annuities-certain. In Chapter III the student is introduced within the compass of a few pages to the functions  $l_x$  and  $d_x$ , and to the simple probabilities of life and death for single lives, and for combinations of two and three lives. The Forces of Mortality and kindred functions, as well as Gompertz' and Makeham's formulas, and that of Wittstein, are explained

\* Mathematisch-Technische Kapitel zur Lebensversicherung von Corneille L. Landré, Actuar der Niederländischen Lebensversicherungsgesellschaft in Amsterdam, &c.; Mitglied des Holländischen Vereins von Actuaren in Amsterdam; Corresponding Member of the Institute of Actuaries; Präsident des Mathematischen Vereins in Amsterdam, &c.



with great clearness. Mr. Landré's application of strict methods is well illustrated by his use of the integral calculus to determine the rate of mortality when allowance is made for the movements of persons into and out of observation. The assumptions that are employed in practice in reckoning ages, and the conditions under which  $q_x$  is deduced from observations of various periods, are explained in a very intelligible manner, and after this 29 pages are given to Graduation, a subject that has elsewhere received special attention at the author's hands. He gives a very full description of the methods of Finlaison, Woolhouse, Higham and Karup, comparing the *abrundungskraft*, or smoothing-power, of these methods, by calculating the value in each case of the mean error of  $Z_n$ , the general term in the resulting graduated series. The four formulas being special cases of the general formula,  $Z_n = pu_n + p_1(u_{n+1} + u_{n-1}) + p_2(u_{n+2} + u_{n-2}) + \dots + p_m(u_{n+m} + u_{n-m})$ . Mr. Landré shows how, for a given number of the terms which are embraced in the formula, and a given order of the arithmetical series that is left unaffected by its application, the coefficients  $p, p_1, p_2$ , &c., can be so determined as to render the mean error of  $Z_n$  a minimum. He finds, for example, that if the formula is required to comprise five terms, and to leave a curve of the third degree unaffected, the mean error of  $Z_n$  will be a minimum if  $p = \frac{17}{35}$ ,  $p_1 = \frac{12}{35}$ , and  $p_2 = -\frac{3}{35}$ , but that in this case the value is much greater than in that of Finlaison's formula, which comprises the same number of terms, but will not reproduce curves of a degree higher than the first. Further, very instructive examples are given by the author of the application of the *Théorie des Minimums*, with original theorems, from an article by himself on that subject. Graduation by the Analytic method occupies two or three pages, and the chapter closes with brief sections on Principal Life Tables, Selection, Choice of Mortality Tables, and Rate of Interest, Comparison of Actual with Tabular Mortality, Combined Experience, Probable and Mean Duration, and Expectation of Life.

Chapters IV and V deal with Single and Annual Premiums for the principal forms of benefits involving single lives, and Chapter VI with Insurances and Annuities on more than one life. A very complete view is given of the elements of the subject, commutation symbols being employed throughout, and various points that arise in practice adverted to. The general formulas given in Chapter VII of the Institute *Text-Book* for annuities on the last  $r$  survivors of  $m$  lives, and on a compound status, are not referred to, but a useful method is given of evaluating annuities on combined lives when conditions are introduced respecting variations, to take place as the lives are reduced in number, in the amount of the annual sum payable. The construction and use of conversion tables is not included in these chapters, nor do they embrace the subject of successive lives.

In Chapter VII Return of Premiums is dealt with in connection with a variety of benefits, Mr. Landré's analysis being extended, for



the benefit of the student, to cases where the return of premiums with compound interest is contracted for. Office Premiums occupy a chapter of 23 pages. The theory of loading is discussed very completely as far as regards additions to the net premium designed to meet expenses of administration and commission, though the subject of with-profit premiums loaded with reference to a given rate of bonus is not brought into view. On the question of Extra Premiums, in theory and in practice, Mr. Landré writes in an interesting manner. Among expedients for avoiding the direct rating-up of a life, one is referred to by him as follows: "When the office makes an addition to the candidate's age it is not always necessary to charge the premium corresponding to the higher age; a deferred insurance can be granted, and the normal premium for an immediate insurance exacted. The question to be decided is then, for what period the insurance must be deferred, having regard to the addition to the age." This method would seldom, we imagine, commend itself to a proposer in practice.

One of the merits, and not the least conspicuous of them, to be found in Mr. Landré's work, consists in the references that he gives throughout its pages to publications that have appeared on the Continent, as well as in this country and in America. It is interesting to notice that among the contributors to the literature of his subject is to be found his own daughter. In a section of Chapter VIII on the average premium which may be charged in the case of the insurance of a considerable group of persons, he refers the reader to an article by Miss Henriette F. Landré in the fourth part of the "*Archives*" of Holland. A long chapter on "Premiums payable by Instalments and Continuously", displays the author's thorough acquaintance with the resources of analysis that have been brought to bear on the questions coming under this head. The references on page 242 should, it may be suggested, be supplemented by a citation of Mr. Woolhouse's famous paper in vol. xv of the *J.I.A.* A small section at the end of this chapter may be referred to as an example of the author's method of keeping in view the considerations that arise in practice. He there discusses the difficulties that present themselves when it is proposed to secure, by means of an annual premium, a deferred annuity payable for the lives of two persons, and the life of the survivor of them.

The discussion of "Premium Reserves" occupies 90 of the remaining pages of the work, and is a very thorough one, the Policy Value being looked at from every point of view. Thus, for example, the premium  $P_x$  is analysed into its component parts, the "Risico-Prämie"  $[=v(1-p_{x+m-1})(1-{}_mV)]$  and the "Spar Prämie." Mr. Landré proves that the Policy Value at any time = the accumulation of this "Spar Prämie" at interest from the commencement of the insurance. He discusses gross premium valuations, the reinsurance and other methods, including the " $x+1$  method" known on the Continent as the "Zillmer'sche" method, and touches upon a variety of practical questions connected with office valuations. References to the balance-sheet, as understood

on the Continent, can have little interest for those who live under the blessings of the Life Assurance Companies Act 1870, and a brief section on "Purchase of a reversion (*blosse Eigentum* = *Nuda proprietas*) and of a life-interest", proves how little practical acquaintance with benefits of this description the actuary enjoys on the other side of the channel.

Discontinuance and alterations of policies have a chapter to themselves, and three final chapters are given to Special Problems, Premiums as Analytical Functions, and Approximations respectively. In the first attention is paid to, *inter alia*, "the effect of the interest and mortality basis upon the premiums and reserves", the yearly mathematical risk ( $= \frac{1}{\sqrt{2\pi}} \sqrt{spq}$ , where  $s$  denotes the number of persons exposed to risk, and the other symbols have the usual signification) and Limitation of Risks, these last two sections being of special value to the student of Probability. An outline is also given in this chapter of methods of determining profit and loss under various heads, and of dividing surplus. These are very usefully treated, considering the slender space that is afforded them; but the extracts which Mr. Landré publishes of the life insurance returns which are exacted by the Austrian Government are not likely to put his English readers out of conceit with the statistical requirements of insurance legislation in their own country. A deduction from first principles of the definite integral which represents a continuous annuity on a single life commences the chapter headed Premiums as Analytical Functions. The integrals for  $\bar{A}_x$ , and joint-life functions are given further on, but in between are placed algebraical proofs that, if  $e_x = n$ ,  $\bar{a}_x < a_n$ , and  $\bar{A}_x < v^n$ . These would have found a more appropriate position elsewhere.

A little confusion is, we think, likely to be caused by the use of " $i$ " in section 390 in a different sense from that in which it is employed in the earlier sections, *e.g.*, No. 387. The student having satisfied himself that  $\bar{A} = 1 - i\bar{a}$ , where  $\bar{a} = \frac{\delta}{i} \bar{a}$ , will not take

kindly to  $\bar{A} = 1 - i\bar{a}$  (or  $\bar{A} = 1 - i\bar{a}$ , as Mr. Landré prefers to write it) until he has realised that the effective rate of interest in this latter formula is  $e^i - 1$ , and not  $i$ . The reference in section 390 should be to section No. 9. The attention of students may be directed to a novel method of calculating  $A$  which is given on page 436.

The final chapter gives methods of interpolating premiums, and formulas for approximate summation, the latter based upon Gauss' well known method, and references being given to Jacobi, Boole, Lobatto, and G. F. Hardy. We have also a clear account of the calculation of joint-life annuities by means of Simpson's rule and Makeham's law, and approximate evaluations by means of variation in the rate of interest.

It remains to us to recommend this work warmly, as a companion volume to the Institute *Text-Book*, to all students for whom the

language will not prove a difficulty. In common with most mathematical treatises which hail from the Continent, the work is printed in roomy and luxurious type, and is furnished with a full table of contents. But for convenience of reference the numbers of the chapters should be indicated at the heads of the pages.

F. E. C.

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## THE INSTITUTE OF ACTUARIES.

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### BUST OF DR. SPRAGUE.

The PRESIDENT, Mr. C. D. Higham, at the Sessional Meeting, held in Staple Inn Hall, on the 27 January 1902, said that it was his pleasant duty, on behalf of the subscribers, to ask the Institute to accept the bust of Dr. Sprague to be seen in the hall. When Dr. Sprague's retirement was a settled thing, their northern friends and themselves felt that his services to the profession made it an occasion which ought not to be passed over; and, therefore, not seeking a precedent, and certainly not intending to make one, a small committee was formed for the purpose of receiving subscriptions with the idea of putting a bust of Dr. Sprague in the Hall of the Institute, and also in the Hall of the Faculty of Actuaries in Scotland. He hoped also that they would be able to give the Doctor a small piece of silver, on which a suitable inscription could be engraved, which he might hand down to his family. A good deal of consideration was given to the question of the sculptor to be employed, and, among other reasons, on account of Dr. Sprague's health, it was thought desirable to ask one in Edinburgh to undertake the work. Not unnaturally, therefore, their choice fell on Mr. MacGillivray, at that time an Associate of the Royal Scottish Academy, but who had since become a full Academician, and as well known, not merely as a sculptor but as an artist, for there were his works in marble, his paintings, and verses of no mean order. The result was before them, though he would just remind them that both busts were executed from the same plaster model, so that neither body had the original and the other the replica. They who knew Dr. Sprague needed no memorial of him, but those who only had his writings—and the value of those writings was best estimated by considering the difficulties of preparing for examinations before they were penned—would want in after years to know what the man was

like. Brilliant in his attainments, solid in his learning, varied in his accomplishments, painstaking in his labours, Dr. Sprague left behind him as he retired a name which few in their profession had rivalled ; and while they wished him many years of happy rest, they would always think of him as their great comrade, and those who came after them would see in that portrait that they had done such honour as was in their power to their colleague, their teacher, and their friend.

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# JOURNAL

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*The British Offices Life Tables, 1893; an Investigation of the Rates of Mortality in different Classes of the Assurance Experience, and of the resulting Net Premiums and Policy-Reserves.* By THOMAS G. ACKLAND, Fellow of the Institute of Actuaries.

[Read before the Institute, 24 March 1902.]

I HAVE been invited by the Council of the Institute of Actuaries to read a paper dealing with the general bearings of the New Experience Tables. The Unadjusted Data of the Assurance Experience are set forth in the three volumes published by the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries in Scotland, comprising (1) Whole-Life Assurances—Participating and Non-Participating—Male Lives; (2) Whole-Life Assurances—Participating and Non-Participating—Female Lives; (3) Endowment Assurances and Minor Classes of Assurances—Male and Female Lives. The Joint Committee have also kindly placed at my disposal, in the preparation of this paper, the graduated results, so far as available; and the Monetary Tables which have been computed, and which are in course of being printed for publication.

The ground covered by such an investigation as that here proposed is a very extensive one, and I am most conscious of the imperfections of the present paper, both as to its scope, and as

to the mode of treatment adopted. The time available for development of the subject has been but short, and the results now submitted can only be regarded as a small and imperfect contribution, which may perhaps lead to wider and more adequate treatment by those more qualified than myself to deal with so important an investigation.

The present paper will deal with the graduated Assurance Tables already available—the  $O^M$  and  $O^{M(5)}$  Tables—upon the basis of which Monetary Tables have been, or are in course of being, computed. I propose to include also an investigation into the Endowment Assurance and Temporary Assurance Data, based upon experimental graduations of those Tables; and shall also submit provisional graduations of the Whole-Life—Participating—Female, and the Whole-Life—Non-Participating—Male, Assurance Tables.

I should have liked to have dealt also with the graduated SELECT Table for Whole-Life Participating Assurances—Male Lives; and with the graduated Tables based upon the Life Annuity Experience, for Males and Females. In the case of the former Table, the graduation has been only very recently completed, and the Monetary Tables are not at present available; I have, however, included a few brief Tables setting forth concisely some of the general results of the graduation, and the deduced net premiums and policy-reserves. The full investigation of these Select Assurance Tables would form a worthy subject for a separate paper later on, when the results are more fully available.

As regards the Life Annuity Experience, the Monetary Tables, although in a very forward state, were not fully available when this paper was in course of preparation; and time has not permitted of their adequate consideration. I have, therefore, preferred to leave the Life Annuity Experience for separate treatment as a distinct subject for investigation, later on, by some member of the Institute well qualified for the task.

In dealing with the Classes of Assurances other than the Whole-Life Participating class, it has been necessary, for convenience and lucidity of reference in the text and Tables of the present paper, to employ special symbols to characterize such classes. In the few cases in which this has been done, I have adopted the symbols authorized when the data cards were prepared by the contributing companies. Thus E was adopted for Endowment Assurances; T for Term Assurances, and N for Non-Participating Assurances; these

symbols were ready to hand, and appeared to be convenient and expressive, and were perhaps to some extent familiar. I wish it, however, to be clearly understood that these short titles for the Tables are only tentatively and provisionally suggested, as the Joint Committee have come to no present decision as to the symbols to be employed for classes other than the  $O^M$  and  $O^{M(5)}$  Tables.

I am indebted to Mr. Arthur Herbert Raisin, F.I.A., of the Pelican Life Office, for most valuable assistance in the preparation of the Tables appended to this paper, without which, it would have been quite impossible for me to submit the results to the members of the Institute during the present Session.

### RATES OF MORTALITY.

#### (1) WHOLE-LIFE ASSURANCES.

In Table I of the Appendix, I have set out the rate of mortality at each age, as shown by some of the more important tables. The columns (4) and (5) represent the rates of mortality in respect of the graduated Whole-Life (Male) Participating Assurance Experience, that headed  $O^M$  setting forth the rate as deduced from the full aggregate data, whilst that headed  $O^{M(5)}$  shows the rate as deduced from the aggregate data, after exclusion of the first five years' experience. In columns (2) and (3) are set out the rates under the  $H^M$  and  $H^{M(5)}$  Tables. In Table II a comparison of the rates of mortality under the different Tables above specified is made, by setting out the proportions of one to the other at each age. In columns (2) and (4) the  $O^M$  and  $O^{M(5)}$  Tables are severally compared with the  $H^M$  Table; in column (3) the  $O^{M(5)}$  is compared with the  $H^{M(5)}$ ; whilst in column (11) the  $O^{M(5)}$  is compared with the  $O^M$ . In column (10) the ratio of the  $H^{M(5)}$  to the  $H^M$  Table is shown for purposes of comparison. It will be convenient briefly to discuss the results here shown, before passing to the remaining columns of Tables I and II, which relate to the Endowment Assurance Experience.

Referring to column (2) of Table II, in which the rate of mortality by the  $O^M$  is compared with that by the  $H^M$  Table (taken as unity), the following Table shows the maximum and minimum values of the ratio-curve in passing through successive ages:—

*Comparative Rates of Mortality.* **$O^M$  &  $H^M$** 

Age	RATIO OF $O^M$ ( $H^M=1$ )	
	Minima	Maxima
10	·690	...
14	...	1·283
21	·618	...
44	...	·951
48	·931	...
58	...	·978
63	·961	...
69	...	1·000
74	·937	...
79	...	·960
83	·945	...
89	...	1·101
97	·483	...

The above Table may be read as follows :—

The ratio of the  $O^M$  rate of mortality to that of the  $H^M$  Table (taken as unity) is equal to ·690 at age 10; thence rises steadily to 1·283 at age 14; falls to ·618 at age 21; thence rises to ·951 at age 44; and so on.

It will be seen, by reference to Table II, and that given above, that the  $O^M$  rate is lower than the  $H^M$  at all ages, excepting only 12-16, 69 (where there is identity of rate), and 86-91. Omitting the younger and older ages, where the comparative rates show considerable irregularity, the  $O^M$  rate, between ages 20 and 30, is on the average about 70 per-cent of the  $H^M$  rate; between ages 30 and 40, about 83 per-cent; from 40 to 50, 93 per-cent; from 50 to 70, about 97 per-cent; from 70 to 80, about 95 per-cent; and from 80 to 90, about 97 per-cent.

Turning now again to Table II, we can similarly trace out the comparison of the  $O^{M(5)}$  rate of mortality with that of the  $H^{M(5)}$  Table. The ratios at every age are shown in column (3) of Table II; and in the following Table, the maxima and minima in the ratio-curve are shown as before :—



Comparative Rates of Mortality.  
O<sup>M(5)</sup> & H<sup>M(5)</sup>

Age	RATIO OF O <sup>M(5)</sup> (H <sup>M(5)</sup> =1)	
	Minima	Maxima
10	1·532	...
13	...	2·123
23	·628	...
34	...	·865
38	·827	...
44	...	·936
49	·899	...
59	...	·960
64	·938	...
70	...	·990
74	·927	...
75	...	·931
76	·929	...
80	...	·950
82	·943	...
84	...	·955

Here, the O<sup>M(5)</sup> rate is throughout below that of the H<sup>M(5)</sup> Table, excepting at ages 10-18. The average ratios are roughly as follow :—Ages 20-30, 65 per-cent ; 30-40, 85 per-cent ; 40-60, 93 per-cent ; 60 to 70, 95 per-cent ; 70 to 85, 94 per-cent.

As it has been shown that the O<sup>M</sup> rate of mortality is, on the whole, less than the H<sup>M</sup>, and also that the O<sup>M(5)</sup> rate is, on the whole, less than the H<sup>M(5)</sup>, it may be useful to add a comparison of the O<sup>M(5)</sup> and the H<sup>M</sup> rates of mortality. The ratios (taking the H<sup>M</sup> rate as unity) are given in column (4) of Table II ; and in the following Table the maxima and minima in the ratio-curve are shown at successive ages :—

Comparative Rates of Mortality.  
O<sup>M(5)</sup> and H<sup>M</sup>

Age	RATIO OF O <sup>M(5)</sup> (H <sup>M</sup> =1)	
	Minima	Maxima
10	1·251	...
14	...	2·265
22	·972	...
26	...	1·044
32	·958	...
34	...	·960
38	·936	...
44	...	·991
49	·961	...
54	...	·992
57	·990	...
58	...	·992
64	·968	...
69	...	1·002
74	·938	...
79	...	·960
83	·945	...

The rates by the  $O^{M(5)}$  Table lie, it will be seen, much closer to those of the  $H^M$  Table than those by the  $O^M$  Table; and there is also a closer approximation between the rates of the  $O^{M(5)}$  and  $H^M$  Tables than between the  $O^{M(5)}$  and  $H^{M(5)}$  Tables. The  $O^{M(5)}$  rates are less than those of the  $H^M$  at all ages, excepting 10–20 (where the ratios are very irregular), 24–28, 69, and 70 (at which age the rates of the two tables coincide). It is thus seen that, on the whole, the rate of mortality under the  $O^{M(5)}$  Table, in which the experience of the first five years following selection has been eliminated, is actually less than the rate under the  $H^M$  Table, which includes the full effect of selection. This would appear to indicate, either that the effect of selection is more persistent (after five years) in the new Tables; or, that the lives observed under these Tables are, as a class, apart from selection, of superior vitality to those included in the  $H^M$  and  $H^{M(5)}$  Tables.

In Table III of the Appendix I have set out the curtate expectations of life by the  $H^M$ ,  $H^{M(5)}$ ,  $O^M$ , and  $O^{M(5)}$  Tables; also, in columns (6) to (11), the differences between the expectations by the several compared Mortality Tables. These expectations of life further illustrate the superior vitality of the new to the old Tables at different periods of life.

I am privileged to include, in Table IV of the Appendix, extracts from the valuable and instructive comparative Mortality Tables prepared by Mr. R. P. HARDY (Chairman of the Joint Mortality Committee), and submitted by him at the Annual Meeting of the Institute of Actuaries in June last. A specimen of these Tables has been published in the *Journal of the Institute* (vol. xxxvi, p. 316), but the further extracts here furnished will, I am sure, be most acceptable to the profession generally.

The specimen Table published in the *Journal* (referred to above) dealt with the comparison of the  $O^M$  and  $H^M$ , and of the  $O^{M(5)}$  and  $H^{M(5)}$  Tables. The further extracts now supplied in Table IV show the comparisons between the  $H^M$  and  $H^{M(5)}$  Tables, and also between the  $O^M$  and  $O^{M(5)}$  Tables. The comparisons proceed on the lines of tracing 10,000 lives, entering at the several selected entry ages of 30, 35 and 40, through successive quinary age-groups to the end of life. The whole of the lives so entering necessarily pass out by death during the period of observation, and the total number of deaths is thus 10,000 under each of the two compared Mortality Tables. The differing mortality rates bring out, however, a varying distribution of the total deaths over the quinary age-groups passed through, and

the greater number of deaths in certain age-groups is compensated by the smaller number of deaths in other age-groups. Thus, comparing the  $O^M$  and  $O^{M(5)}$  Tables in respect of lives entering at age 30, we see, on reference to Table IV, that the deaths brought out by the  $O^M$  Table exceed those brought out by the  $O^{M(5)}$  Table by a total of 133·6 deaths, which are distributed entirely over the ages 55 to 100. At the earlier ages, 30 to 54, the deaths by the  $O^M$  Table are less than those by the  $O^{M(5)}$  Table by the same total of 133·6 deaths. If the  $H^M$  and  $H^{M(5)}$  Tables be compared, the distribution of the deaths is similar, but the total number of deaths brought out by the  $H^M$  Table as compared with the  $H^{M(5)}$  shows a smaller number of 205·6 deaths over the age-groups 30–59, and a larger number of 205·6 deaths for the remainder of life.

## (2) ENDOWMENT ASSURANCES.

I now turn to the Endowment Assurance Experience, as deduced by the new Tables. The numbers exposed to risk, the deaths, and the ungraduated rate of mortality at each age are given in the volume of Unadjusted Data ("Endowment Assurances and Minor Classes of Assurance") on page 136 for the full Aggregate Table, and on page 137 for the "truncated" Aggregate Table, excluding the experience of the first five years following selection.

These unadjusted rates of mortality show some irregularities, and I have tentatively graduated the Full Aggregate Table,  $E^M$ , and the Truncated Aggregate Table,  $E^{M(5)}$ , by Woolhouse's formula of interpolation. In order to illustrate the closeness of the graduated results, I append, in Table V, the graduated and ungraduated values of temporary annuities,  ${}_{60-x}a_x$ , at 3 per cent, as computed by both Tables. It will be seen that the graduated and ungraduated results practically coincide, excepting in the neighbourhood of age 20, where, on account of an abnormal rate of mortality shown by the unadjusted data, the graduated results differ somewhat from the ungraduated.

In columns (6) and (7) of Table I are given the rates of mortality at each age from 20 to 60, as shown by the graduated  $E^M$  and  $E^{M(5)}$  Tables. In columns (5) to (9) of Table II these mortality rates are severally compared with those shown by the  $H^M$ ,  $H^{M(5)}$ ,  $O^M$  and  $O^{M(5)}$  Tables. I now proceed to discuss these comparative results seriatim.

## Comparative Rates of Mortality.

## Endowment Assurance Experience.

Age (1)	RATIOS OF EM				RATIOS OF EM <sup>(5)</sup>						Age (12)
	(H <sup>M</sup> =1)		(O <sup>M</sup> =1)		(H <sup>M<sup>(5)</sup></sup> =1)		(O <sup>M<sup>(5)</sup></sup> =1)		(H <sup>M</sup> =1)		
	Minima (2)	Maxima (3)	Minima (4)	Maxima (5)	Minima (6)	Maxima (7)	Minima (8)	Maxima (9)	Minima (10)	Maxima (11)	
20	...	·591	...	·925	...	1·024	1·308	...	...	1·347	20
21	...	...	...	...	...	...	...	1·374	...	...	21
22	·546	...	...	...	...	...	...	...	...	...	22
23	...	·550	...	...	...	...	...	...	...	...	23
24	·547	...	...	...	...	...	...	...	...	...	24
25	...	...	...	...	...	...	...	...	...	...	25
26	...	·568	...	...	...	...	...	...	...	...	26
27	...	...	...	...	·523	...	·734	...	...	...	27
28	·546	...	·720	...	...	...	...	·736	...	...	28
29	...	·553	...	·722	...	...	...	...	...	...	29
30	...	...	...	...	...	·581	...	...	...	...	30
31	·539	...	·689	...	·575	...	·692	...	·665	...	31
32	...	...	...	·697	...	...	...	·697	...	·668	32
33	...	·565	...	...	...	·600	...	...	...	...	33
34	·560	...	·674	...	...	...	·675	...	·648	...	34
35	...	...	...	...	·572	...	...	...	...	·652	35
36	...	...	...	...	...	...	...	...	·650	...	36
37	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	41
42	...	·689	...	·738	...	...	...	...	...	...	42
43	·685	...	·727	...	...	...	...	·798	...	·788	43
44	...	·731	...	·769	...	·742	·793	...	...	...	44
45	...	...	·757	...	...	...	...	...	...	...	45
46	·715	...	...	...	·735	...	...	...	...	...	46
47	...	...	...	...	...	·738	...	·805	...	...	47
48	...	...	...	·776	...	...	...	...	...	...	48
49	...	...	...	...	...	...	...	...	...	...	49
50	...	...	...	...	...	...	...	...	...	...	50
51	...	·729	...	...	...	...	...	...	...	...	51
52	·722	...	·749	...	·697	...	...	...	·747	...	52
53	...	...	...	...	...	...	·755	...	...	...	53
54	...	...	...	...	...	...	...	...	...	...	54
55	...	...	...	...	...	...	...	...	...	...	55
56	...	...	...	...	...	...	...	...	...	...	56
57	...	...	...	...	...	...	...	...	...	...	57
58	...	·902	...	·922	...	...	...	·899	...	·892	58
59	·879	...	·900	...	...	...	·897	...	·886	...	59
60	...	·889	...	·914	...	·866	...	·908	...	894	60

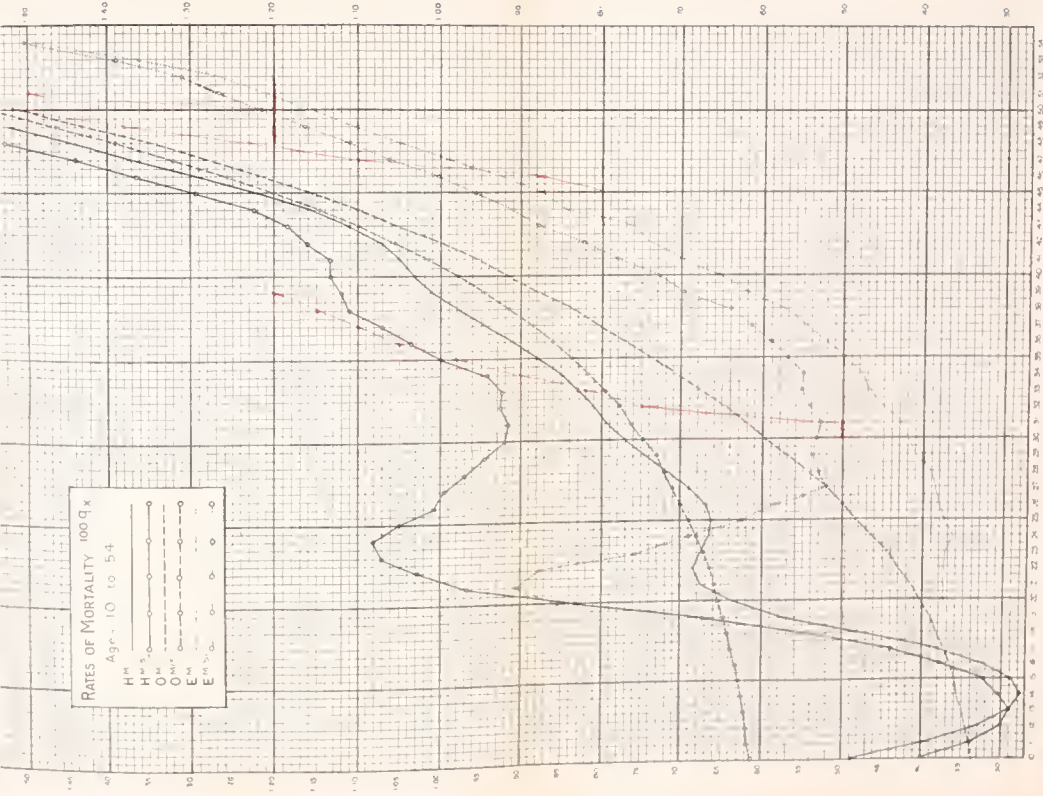
In the appended Table, I have set out the several maxima and minima in the ratio-curves in passing through successive ages, extracted from columns (5) to (9) of Table II, as deduced





# RATES OF MORTALITY 100 q x

Age 10 to 54

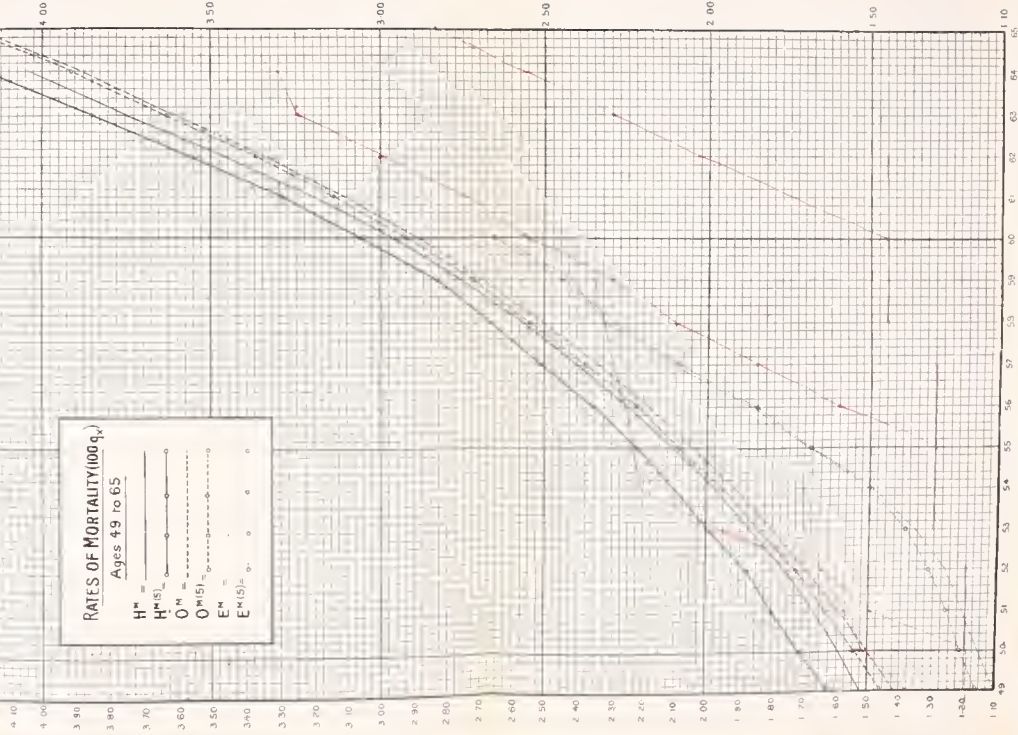




# RATES OF MORTALITY(100 q<sub>x</sub>)

Ages 49 to 65

$H^M =$  ————  
 $H^M(S) =$  ————  
 $O^M =$  ————  
 $O^M(S) =$  ————  
 $E^M =$  ————  
 $E^M(S) =$  ————





from the comparison of the  $E^M$  rate of mortality with (1) the  $H^M$  table; (2) the  $O^M$  Table; also as deduced from the comparison of the  $E^{M(5)}$  rate of mortality with (3) the  $H^{M(5)}$  Table; (4) the  $O^{M(5)}$  Table; (5) the  $H^M$  Table.

It will be observed that the  $E^M$  Table shows rates of mortality throughout less than those of the  $H^M$  and  $O^M$  Tables. The ratios to the  $H^M$  Table, as shown in columns (2) and (3) of the appended Table, fall from 59 per-cent at age 20 to 54 per-cent at age 31, and then rise to about 75 per-cent at age 44, and 90 per-cent. at age 60. The ratios to the  $O^M$  Table, as shown in columns (4) and (5), fall from about 90 per-cent at age 20, to 67 per-cent at age 34, rising to 75 per-cent at age 44, and to 90 per-cent at age 60.

The comparison of the  $E^{M(5)}$  Table with other Tables shows that the rate of mortality is (after age 24) throughout lower than the  $H^{M(5)}$  and  $O^{M(5)}$  Tables, and also than the  $H^M$  Table. As will be seen on reference to columns (6) and (7) of the appended Table, the  $E^{M(5)}$  rate exceeds the  $H^{M(5)}$  at age 20, and then falls to 50 per-cent at age 27, rising to 60 per-cent at age 33, 75 per-cent at age 44, and 86 per-cent at age 60. As compared with the  $O^{M(5)}$  rate, in columns (8) and (9), the  $E^{M(5)}$  Table shows an excess at age 20, falling to about 75 per-cent at age 27, 70 per-cent at age 30, rising to 80 per-cent at age 43, and 90 per-cent at age 60.

The comparison of the  $E^{M(5)}$  Table with the  $H^M$  Table, as set out in columns (10) and (11), shows an excess of about 35 per-cent in the rate of mortality at age 20, falling to identity at age 24. The ratio of the  $E^{M(5)}$  Table then falls to about 65 per-cent of the  $H^M$  at ages 30 to 39, and rises to about 75 per-cent at ages 40 to 54, with a further rise to 90 per-cent at ages 58 to 60.

Speaking generally, it may thus be said that the Tables compared above would range in the following order, from those showing the lightest to those showing the heaviest rate of mortality: (1)  $E^M$ ; (2)  $E^{M(5)}$ ; (3)  $O^M$ ; (4)  $O^{M(5)}$ ; (5)  $H^M$ ; (6)  $H^{M(5)}$ .

I append to this paper four diagrams showing the rates of mortality per-cent under each of the above Tables. The smoothness of the  $O^M$  and  $O^{M(5)}$  curves, as graduated by mathematical formulæ, is very notable in these diagrams, as compared with the  $H^M$  and  $H^{M(5)}$  curves, and the  $E^M$  and  $E^{M(5)}$  provisional curves, graduated by formulæ of interpolation.

## (3) EFFECT OF SELECTION.

The effect of selection during the first five years will be to some extent illustrated by comparing the rates of mortality under (1) the  $H^M$  and  $H^{M(5)}$  Tables; (2) the  $O^M$  and  $O^{M(5)}$  Tables; (3) the  $E^M$  and  $E^{M(5)}$  Tables. The several ratios at each age are set out in columns (10), (11) and (12) of Table II; and the following Table shows the maxima and minima of the ratio-curves at successive ages:—

*Comparative Rates of Mortality.*  
 $H^M$  &  $H^{M(5)}$ ;  $O^M$  &  $O^{M(5)}$ ;  $E^M$  &  $E^{M(5)}$ .

Age	RATIOS OF $H^{M(5)}$ ( $H^M=1$ )		Ratios of $O^{M(5)}$ ( $O^M=1$ )	RATIOS OF $E^{M(5)}$ ( $E^M=1$ )	
	Minima	Maxima		Minima	Maxima
10	·816	...	1·814	...	...
16	...	1·160	1·725	...	...
17	1·134	...	1·701	...	...
20	...	...	1·614	2·281	...
21	...	...	1·584	...	2·420
24	...	1·631	1·469	...	...
27	...	...	1·356	1·352	...
28	...	...	1·325	...	1·355
34	1·109	...	1·156	...	...
35	...	1·140	1·134	...	...
37	1·131	...	1·102	...	...
38	...	1·132	1·092	1·112	...
39	...	...	1·077	...	1·122
40	...	...	1·069	1·108	...
41	...	...	1·062	...	1·119
42	...	...	1·055	1·114	...
43	...	...	1·049	...	1·150
44	1·059	...	1·043	1·076	...
45	...	1·061	1·041	...	1·096
47	1·051	...	1·034	...	...
51	...	1·081	1·022	...	...
58	...	...	1·014	·989	...
59	...	...	1·012	...	1·009
60	...	...	1·012	1·005	...
63	1·027	...	1·008	...	...
64	...	1·032	1·007	...	...
65	1·027	...	1·006	...	...
66	...	1·030	1·005	...	...
67	1·021	...	1·004	...	...
68	...	1·023	1·003	...	...
73	1·008	...	1·000	...	...
76	...	1·016	1·000	...	...
77	1·007	...	1·000	...	...
79	...	1·013	1·000	...	...
81	1·004	...	1·000	...	...
82	...	1·005	1·000	...	...
83	·999	...	1·000	...	...
89	...	1·026	1·000	...	...
90	1·010	...	1·000	...	...
93	...	1·056	1·000	...	...
95	·999	...	1·000	...	...
96	...	1·225	1·000	...	...

It will be seen from this Table that, whilst the ratios of the  $H^{M(5)}$  to the  $H^M$  Table, and the  $E^{M(5)}$  to the  $E^M$  Table, show considerable variations in the contour of the curve, the ratio of the  $O^{M(5)}$  to the  $O^M$  Table shows a steady reduction at successive ages. This, no doubt, arises from the mathematical formulæ employed in the graduation of the two last-named Tables. It will also be observed that, generally speaking, the relative effect of selection appears to be more marked in the  $E^M$  Table than in either the  $O^M$  or  $H^M$  Tables.

The  $O^M$  Table shows the effect of selection to a much more marked extent than the  $H^M$  Table up to age 22, and then falls below the  $H^M$  in this respect to age 28, again rising above to age 34, after which it is below the  $H^M$  ratio to age 85.

The effect of selection is further illustrated by the comparison of the Expectations of life by the  $H^M$  and  $H^{M(5)}$ , and the  $O^M$  and  $O^{M(5)}$  Tables, in columns (6) and (7) of Table III. Mr. R. P. Hardy's comparative Mortality Tables in Table IV of the Appendix also throw useful light upon the same subject.

The above comparisons can only be considered as roughly and partially illustrating the effect of selection, which can hardly be fully investigated until the publication of the graduated Select Tables.

#### (4) TEMPORARY ASSURANCES.

##### (a) *Select Tables.*

I have thought that it might be useful to attempt to deduce, from the tabulated data in the special class of Temporary Assurances, a roughly graduated rate of mortality, from which some indications of the rates of net premium appropriate to that class might be obtained. The data, as set out on pp. 200 to 203 of the volume containing "Endowment Assurances and Minor Classes of Assurance (Unadjusted Data)", are tabulated in grouped ages at entry, and include, in respect of the experience of each of the first ten years of assurance, the numbers exposed to risk, the deaths, and the unadjusted rate of mortality. These data are thus tabulated in a form from which can be deduced the *Select* single and annual net-premiums, at the grouped entry ages tabulated, in respect of assurances for terms of one, two, three, up to ten years.

The available data in the class of term assurances are not very extensive, the total number exposed at entry, between the ages 18 to 62 inclusive, being 9,653, whilst the total number of deaths under observation during the ten years, in respect of the above ages at entry, is 343.





It will be seen from this Table that the rates of mortality at central age at entry 20 are abnormally high at all durations; also that, at ages at entry 25 to 40 inclusive, the rates are on the whole materially below those at age 20, and at the same time show little, if any, real advance as the age at entry increases. Thus the rates at entry ages 35 and 40 are not on the whole higher than those arising at the same durations for entry ages 25 and 30; whilst the abnormal mortality shown at entry age 20 is not attained until entry age 45 is reached.

In order to avoid a fall in the graduated rate of mortality, and in the deduced net premiums, as the entry age increases, and at the same time to give due regard to the marked excess in the rate at entry age 20, I have, after several experiments, decided to include in one group the data in respect of entry ages 18 to 42 inclusive, the central age of which group is approximately 30. I thus finally arrive at the following graduated rates:—

Temporary Assurances.

Graduated Rates of Mortality ( $100q_{[x]+t}$ ).

Duration ( $t$ )	CENTRAL AGE AT ENTRY [ $x$ ]				
	[30]	[45]	[50]	[55]	[60]
0	·50	·80	1·20	1·30	1·45
1	·50	·88	1·50	1·60	1·75
2	·75	1·10	1·64	1·85	2·03
3	·82	1·23	1·95	2·10	2·30
4	·90	1·38	2·20	2·30	2·56
5	·98	1·52	2·23	2·55	2·83
6	1·05	1·58	2·26	2·75	3·10
7	1·10	1·64	2·29	3·00	3·54
8	1·15	1·69	2·32	3·25	3·99
9	1·20	1·70	2·35	3·30	4·44

These graduated rates are not very scientifically deduced, and are perhaps not altogether satisfactory, but they may serve our present purpose. The expected deaths, as brought out by these graduated rates, together with the actual deaths, are set out for each year of duration in grouped ages at entry in the following Table:—

CENTRAL AGE AT ENTRY [x]										ALL AGES	
[30]		[45]		[50]		[55]		[60]		[18]-[62]	
Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths
30	30·9	11	10·2	11	12·7	10	9·4	8	6·1	70	69·
22	21·8	12	8·3	14	12·0	8	8·2	3	4·7	59	55·
25	24·9	9	8·4	14	9·8	7	7·7	4	4·0	59	54·
24	19·6	7	7·3	7	8·8	2	6·3	5	3·2	45	45·
23	17·4	1	7·1	12	8·5	10	5·8	3	2·8	49	41·
12	13·9	5	5·6	7	5·9	4	3·7	1	1·8	29	30·
7	13·0	3	5·0	5	5·1	5	3·4	2	1·7	22	28·
2	2·8	1	0·9	1	0·9	0	0·5	0	0·3	4	5·
1	2·4	0	0·8	1	0·9	0	0·4	0	0·3	2	4·
3	2·2	0	0·8	0	0·7	0	0·4	1	0·2	4	4·
149	148·9	49	54·4	72	65·3	46	45·8	27	25·1	343	339·

This may, perhaps, be considered, having regard to the paucity of the data and the irregularity of the unadjusted rates of mortality, as fairly reproducing the main characteristics of the original observations.

(b) Aggregate Tables.

It would, I think, be of interest to deduce, from the same Select data employed above, the rate of mortality as arising at each age passed through, irrespective of duration. Thus, the numbers exposed to risk at the grouped ages at entry, 18-22, assumed to enter at age 20, would be combined, at ages 25 and onwards, with the entrants at grouped ages 23-27. These would in turn be combined, at ages 30 and onwards, with the cases entering at the grouped ages 28-32; and so on. The same course being followed with the deaths, an ungraduated rate of mortality would be deduced for each age from 20 to 69, inclusive. This would probably have the effect, whilst reproducing the substantial characteristics of the original data, of steadying, as it were, the resulting rates of mortality, and any monetary values deduced therefrom.

I have not had time to work out Tables upon the above lines, but the suggestion may perhaps be worth considering.

## (4) OTHER CLASSES OF ASSURANCES.

In Tables XLII to XLIV of the Appendix, I have included some further comparisons of the rates of mortality experienced in certain special classes. Table XLII shows the rate of mortality at each age in respect of Whole-Life *Non-Participating* Assurances on Male Lives, based on the data for the Full Aggregate Table, and the Truncated Aggregate Table, which I have called  $N^M$  and  $N^{M(5)}$  respectively. These are deduced from the Mortality Tables given on pages 502-3 and 504-5 of the volume of Unadjusted Data—Whole-Life Assurances—Males, the rates of which I have experimentally graduated by Woolhouse's formula.

Table XLIII shows the rate of mortality at each age in respect of Whole-Life Participating Assurances on *Female* lives, based on the data for the Full Aggregate and Truncated Aggregate Tables,  $O^F$  and  $O^{F(5)}$ , given on pages 150 and 151 of the volume of Unadjusted Data—Whole-Life Assurances—Females, which I have also graduated by Woolhouse's formula.

In Table XLIV, I have set out, at every fifth page, the rates of mortality thus deduced in these classes, and have added for comparison the rates of the  $H^M$ ,  $H^{M(5)}$ ,  $O^M$  and  $O^{M(5)}$  Tables. It will be seen that the rates experienced amongst Non-Participating Assurances on Male Lives are throughout higher (up to age 89) than those experienced by the holders of Participating Assurances; also that the Female Lives assured under Participating Policies show a heavier mortality than the corresponding Male lives up to age 44, and a lighter mortality than the Male lives at all higher ages (excepting at ages 81 to 85 and 97 to 99, where the Female rate is somewhat heavier than the Male).

I should have liked to have included in this paper some Tables relative to other classes of Assurance, such as Joint Assurances; Contingent Assurances; Limited Premiums; and Ascending Premiums. The field of this investigation is, however, quite too extensive to be covered by a single paper, and I must leave it to other members of the Institute to carry on the work, and give us the benefit of their labours, in those sections of the Experience into which I have been unable to extend my researches.

## NET PREMIUMS.

## (1) WHOLE-LIFE ASSURANCES.

I now turn to the subject of the Net Premiums deduced from the graduated rates of mortality severally referred to above. Considering, first, the net premiums for Whole-life Assurances, I give, in Table VI of the Appendix, the rates of annual net premium per-cent, at every fifth age from 20 to 60, by the  $H^M$  and  $O^M$  Tables, at  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$ , and 4 per-cent; also the ratios of the  $O^M$  to the  $H^M$  rates, taking the latter as unity. It will be seen that the  $O^M$  premium rates are throughout lower than the  $H^M$  rates; also that the ratio increases steadily from the youngest age to the oldest. At 3 per-cent, the  $O^M$  annual premium at age 20 is  $8\frac{1}{2}$  per-cent below the  $H^M$  premium; at age 40, the difference is only  $2\frac{1}{2}$  per-cent; and at age 60, it has further fallen to slightly under 2 per-cent. The ratios at the other rates of interest tabulated show similar progression from the youngest to the oldest age; whilst the ratios are (as might be anticipated) intrinsically higher as the rate of interest falls, and intrinsically lower as it increases.

## (2) ENDOWMENT ASSURANCES.

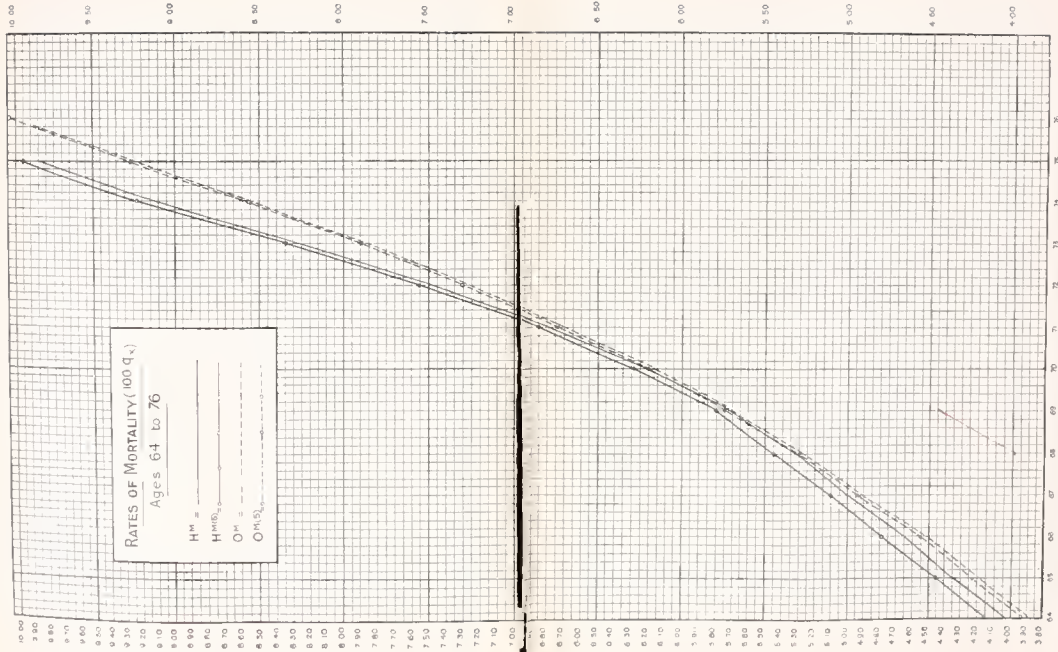
In Table VII are given the net annual premiums for Endowment Assurances payable on the attainment of age 60, or at earlier death. These have been computed for every fifth age at  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$ , and 4 per-cent (1) from the  $O^M$  Table, (2) from the  $E^M$  Table, as given in column (6) of Table I. The  $H^M$  rates of net annual premium have also been inserted in Table VII for purposes of comparison. Finally, the ratios of the  $O^M$  and  $E^M$  premium rates to the  $H^M$ , and also of the  $E^M$  rates to the  $O^M$ , have been calculated and set out in the Table.

The ratios, as before, progressively diminish as the rate of interest increases; and those at 3 per-cent may most conveniently be taken for comparison. At this rate, the relation of the  $O^M$  rate to the  $H^M$  (taken as unity) is, at age 20, equal to  $\cdot937$ ; rises to  $\cdot981$  at age 35; and further to  $\cdot996$  at age 50. The  $E^M$  rate, as compared with the  $H^M$ , is represented by  $\cdot877$  at age 20, rising to  $\cdot930$  at age 35, and to  $\cdot976$  at age 50. Finally, the ratio of the  $E^M$  rate, as compared with the  $O^M$ , is equal to  $\cdot935$  at age 20,  $\cdot948$  at age 35, and  $\cdot979$  at age 50.

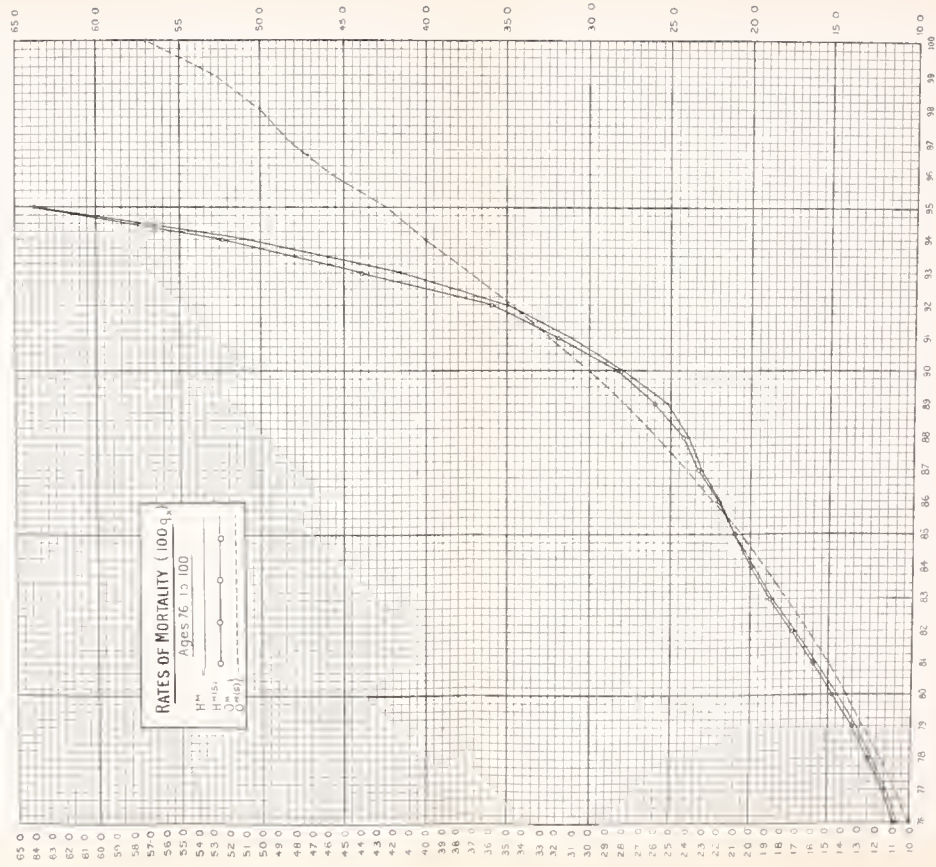
It may thus be said that the annual net premiums, for Endowment Assurances at 60 or death, are at their highest













relative values by the  $H^M$  Table, and at their lowest by the  $E^M$  Table, the  $O^M$  lying between the two. In the case of both of the latter Tables the difference is most marked for young ages and long terms. In the case of the  $O^M$  Table, the 3 per-cent rate is about 6 per-cent below the  $H^M$  for a 40-years' term, and approximates pretty closely to the  $H^M$  rate for terms of 25 years or less. The  $E^M$  rate is again about 6 per-cent less than the  $O^M$  for a term of 40 years, the difference keeping fairly constant until the term falls below 25 years, after which it drops to about 2 per-cent for a ten-year term. The difference between the extreme rates by the  $E^M$  and  $H^M$  Tables reaches  $12\frac{1}{2}$  per-cent (or one-eighth) of the latter for the longest term; falls to 7 per-cent for a 25-year term; and further to  $2\frac{1}{2}$  per-cent for a ten-year term.

### (3) TEMPORARY ASSURANCES.

#### *Select Tables.*

In Table VIII of the Appendix I have set out, for every fifth age at entry from 20 to 60, the 3 per-cent Single and Annual Net Premiums for Term Assurances for 1, 2, 3, up to 10 years. These have been computed (1) from the Unadjusted Data of the Temporary Assurance Experience; (2) from the graduated rates of mortality experimentally deduced from that Experience, as already explained, for the grouped entry ages 18-42, and for every fifth later entry age from 45 to 60. I have also included, for purposes of comparison, the net single and annual premiums for corresponding entry ages and terms, as published by Dr. Sprague in his Select Life Tables, based on the  $H^M$  Extended Mortality Table.

A comparison of these tabulated premiums will show considerable fluctuations as between Dr. Sprague's Select Tables, which I have called  $H^{[M]}$ , and the graduated rate deduced from the Temporary Assurance Data, which I have called  $T^{[M]}$ . At the grouped ages at entry [20] to [40], the  $T^{[M]}$  premium (column 12) is, for a term of 1 year, considerably above the  $H^{[M]}$  rate at the central age 30; but for longer terms the single and annual premiums are lower than the  $H^{[M]}$  rate. At age at entry [45], the  $T^{[M]}$  rate (column 15) is, for terms of one and two years, higher than the  $H^{[M]}$  rate (column 13), but falls below that rate for longer terms. At

age at entry [50], the  $T^{[M]}$  rates (column 18) are materially higher than the  $H^{[M]}$  rates (column 16) for short terms, the excess being as much as 50 per-cent for a one-year term, and falling rapidly to close approximation for a ten-years' term. At age at entry [55], the  $T^{[M]}$  rate (column 21) whilst for one and two-year terms in excess of the  $H^M$  (column 19), falls below the rates of that Table for longer terms. Finally, for age at entry [60], the  $T^{[M]}$  rate (column 24) is for a one-year term practically identical with the  $H^{[M]}$  (column 22), but falls very materially below that Table for longer terms, the difference averaging about 30 per-cent.

It will thus be seen that, generally speaking, the  $T^{[M]}$  Table yields smaller premiums than the  $H^{[M]}$  Table for terms of five years and upwards, and larger premiums for shorter terms. A marked exception arises at entry age 60, where the  $T^{[M]}$  premiums are, for terms exceeding one year, materially below the  $H^{[M]}$  premiums. Considering, however, that the data at entry age 60 comprises only 420 years of risk and 27 deaths, no great confidence can be given to the results at that particular age.

It is probable that too much weight must not be given to these comparisons, as the variations are somewhat erratic, and do not give any clear indication of law. Whilst the Temporary Assurance Data under the New Experience are not very extensive, it must not, however, be overlooked that they represent the actual experience of the contributing Offices in this class; and, perhaps, the conclusion may seem to be justified that the above comparisons lead to grave doubts whether the experience, during the early years of assurance, of a body of lives such as those constituting the  $H^M$  Table, can, in view of our present knowledge, be properly employed as the basis for premiums in respect of Temporary Assurances.

## POLICY-RESERVES.

## (1) WHOLE-LIFE ASSURANCES.

I now turn to the important subject of policy-values. This may perhaps be most fitly introduced by a reference to Table IX of the Appendix, which is reproduced from the 3 per-cent  $O^M$  and  $O^{M(5)}$  Assurance Tables, issued to contributing Offices by the Joint Committee. This Table shows the ratio between the values of  $a_x$  at 3 per-cent according to the  $H^M$ ,  $H^{M(5)}$ ,  $O^M$  and  $O^{M(5)}$  Mortality Tables. As is well known, this ratio is the determining factor in the relation of policy-values by the two compared Tables, and the Note at foot of Table IX illustrates the application of these ratios in determining as to the relative values by the Tables specified in the note. If, for instance, we desire to know whether the  $O^M$  Table will yield at 3 per-cent greater or less policy-values than the  $H^M$  Table, we have only to examine the column marked  $f_x$ , which represents at each age the value of  $a_x^{H^M} \div a_x^{O^M}$ . Beginning with age at entry 20, we take the tabular value of .9720 as our standard of comparison, and observe at what later ages the tabular values are higher or lower than this standard. It will be seen that the values at all ages above 20 are greater than .9720; and it follows that, for all values of  $n$ , the policy-value  ${}_nV_{20}$  is greater by the  $O^M$  than by the  $H^M$  Table at 3 per-cent. Taking now  $x=25$ , we find that the value of  $f_{25+n}$  is always greater than that of  $f_{25}$ , excepting at ages 71 to 77, where the value is less. It follows that the value of  ${}_nV_{25}$  is greater by the  $O^M$  than by the  $H^M$  Table at 3 per-cent, excepting for values of  $n=46$  to 52 inclusive, at which durations the  $O^M$  values are less than the  $H^M$ . By a similar method, the comparison can be applied between any two Mortality Tables, and at any desired entry ages and durations.

The following Tables may be useful to illustrate approximately the points at which the policy-values are greater or less by certain compared Tables, and the general bearing of the relations. They are deduced, by Table IX of annuity ratios at 3 per-cent, for every fifth value of  $x$  from 20 to 75, and of  $n$  from 5 to 60, up to a maximum value of  $(x+n)=80$ . It will be seen that two sets of comparisons are included in each Table, occupying alternate diagonals, the ages at entry being, for the upper diagonal, on the left (reading downwards), and the durations reading horizontally from left to right; whilst, for the lower diagonal, the ages at entry are on the right (reading upwards), and the durations read horizontally from right to left.

3 PER-CENT POLICY VALUES.

(1)  $O^M$  greater (+) or less (−) than  $H^M$

Age at Entry ( $x$ )	DURATION ( $n$ )												Age at Entry ( $x$ )
	5	10	15	20	25	30	35	40	45	50	55	60	
20	+	+	+	+	+	+	+	+	+	+	+	+	75 70 65 60 55 50 45 40 35 30 25 20
25	+	+	+	+	+	+	+	+	+	−	+	5	
30	+	+	+	+	+	+	+	−	−	−	10	+	
35	+	+	+	+	+	+	−	−	−	15	−	−	
40	+	+	+	−	−	−	−	−	20	−	−	−	
45	+	+	−	−	−	−	−	25	−	−	−	−	
50	−	−	−	−	−	−	30	−	−	−	−	−	
55	−	−	−	−	−	35	−	−	−	−	−	−	
60	−	−	−	−	40	−	−	−	−	−	−	−	
65	−	−	−	45	−	−	−	−	−	−	−	−	
70	−	−	50	−	−	−	−	−	−	−	−	−	
75	+	55	−	−	−	−	−	+	+	+	+	−	
	60	−	−	−	+	+	+	+	+	+	+	−	
	+	−	+	+	+	+	+	+	+	+	+	−	
Age at Entry ( $x$ )	60	55	50	45	40	35	30	25	20	15	10	5	Age at Entry ( $x$ )
	DURATION ( $n$ )												

(2) [ $O^M$  and  $O^{M(5)}$ ] greater (+) or less (−) than [ $H^M$  and  $H^{M(5)}$ ]



3 PER-CENT POLICY VALUES.

(3)  $O^{M(5)}$  greater (+) or less (−) than  $H^M$

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )												Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	40	45	50	55	60	
20	−	−	−	−	−	−	−	−	−	−	−	−	75 70 65 60 55 50 45 40 35 30 25 20
25	−	−	−	−	−	−	−	−	−	−	−	5	
30	−	+	=	−	−	−	−	−	−	−	10	+	
35	+	+	+	−	−	−	−	−	−	15	−	−	
40	−	−	−	−	−	−	−	−	20	−	−	−	
45	−	−	−	−	−	−	−	25	−	−	−	=	
50	−	−	−	−	−	−	30	−	−	−	−	−	
55	−	−	−	−	−	35	−	−	−	−	−	+	
60	−	−	−	−	40	−	−	−	−	−	+	+	
65	−	−	−	45	−	−	−	−	+	+	+	+	
70	−	−	50	−	−	−	+	+	+	+	+	+	
75	+	55	−	−	−	+	+	+	+	+	+	+	
	60	+	−	+	+	+	+	+	+	+	+	+	
	+	+	+	+	+	+	+	+	+	+	+	+	
Age at Entry ( <i>x</i> )	60	55	50	45	40	35	30	25	20	15	10	5	Age at Entry ( <i>x</i> )
	DURATION ( <i>n</i> )												

(4) [ $O^M$  and  $O^{M(5)}$ ] greater (+) or less (−) than  $H^M$

## 3 PER-CENT POLICY VALUES.

(5)  $O^M$  greater (+) or less (−) than  $[H^M \text{ and } H^{M(5)}]$ 

Age at Entry ( $x$ )	DURATION ( $n$ )												Age at Entry ( $x$ )
	5	10	15	20	25	30	35	40	45	50	55	60	
20	—	—	—	+	+	+	+	+	+	+	—	+	75 70 65 60 55 50 45 40 35 30 25 20
25	—	—	—	—	+	+	+	+	—	—	—	5	
30	—	—	—	—	—	—	—	—	—	—	10	+	
35	—	—	—	—	—	—	—	—	—	15	—	—	
40	—	—	—	—	—	—	—	—	20	—	—	—	
45	—	—	—	—	—	—	—	25	—	—	—	—	
50	—	—	—	—	—	—	30	—	—	—	—	—	
55	—	—	—	—	—	35	—	—	—	—	—	—	
60	—	—	—	—	40	—	—	—	—	—	—	—	
65	—	—	—	45	—	—	—	—	—	—	—	—	
70	—	—	50	—	—	—	—	—	—	—	—	—	
75	+	55	—	—	—	—	—	—	—	—	—	—	
	60	—	—	—	—	—	—	—	—	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	—	
Age at Entry ( $x$ )	60	55	50	45	40	35	30	25	20	15	10	5	Age at Entry ( $x$ )
	DURATION ( $n$ )												

(6)  $O^{M(5)}$  greater (+) or less (−) than  $[H^M \text{ and } H^{M(5)}]$

It may be explained that in the valuation on the [ $O^M$  and  $O^{M(5)}$ ] combined basis the single premiums and annuities are deducted from the  $O^{M(5)}$ , and the net premiums from the  $O^M$  Table; and similarly with the [ $H^M$  and  $H^{M(5)}$ ] valuations. The valuations by the  $O^{M(5)}$  Table are computed by single premiums, annuities, and *net premiums*, all deducted from the  $O^{M(5)}$  Table.

In the following cases of compared policy-values at 3 per-cent it has not been necessary to set out the results in detail, as the excess is all in one direction, with the few exceptions here specified:—

(7)  $O^{M(5)}$  and  $O^M$ .—The  $O^{M(5)}$  values are smaller for all values of  $x$  to 73\* inclusive. For greater values of  $x$ , the policy-values are equal.

(8) [ $O^M$  and  $O^{M(5)}$ ] and  $O^M$ .—The values by the combined Tables are greater for all values of  $(x+n)$  to 73\* inclusive. For greater values of  $(x+n)$ , the policy-values are equal.

(9) [ $O^M$  and  $O^{M(5)}$ ] and  $O^{M(5)}$ .—The values by the combined Tables are greater for all values of  $x$  to 73\* inclusive. For greater values of  $x$ , the policy-values are equal.

The above Tables and notes, whilst giving plain indications of the trend of the several Tables in the matter of policy-values, give no indication of the extent of the excess or defect in such values by the compared Tables. In Tables X to XXI of the Appendix, I have set out the computed policy-values for Whole-Life assurances of 100, at  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$  and 4 per-cent, for every fifth age from 10 to 80 inclusive, and for every fifth duration from 5 to 75 inclusive, with a maximum attained age of 85. These are tabulated in respect of the  $H^M$ , [ $H^M$  and  $H^{M(5)}$ ],  $O^M$ , [ $O^M$  and  $O^{M(5)}$ ], and  $O^{M(5)}$  Tables; and certain of these values are compared, and the percentages tabulated for each year and duration, namely:—

Tables	X to XIII:	Ratio of $O^M$ to $H^M$ .
„	XIV to XVII:	„ [ $O^M$ and $O^{M(5)}$ ] to [ $H^M$ and $H^{M(5)}$ ].
„	XVIII to XXI:	„ $O^{M(5)}$ to $H^M$ .

\* Theoretically, the point of equality arises at the later age 85, where the  $O^M$  and  $O^{M(5)}$  Tables coincide. But, for all practical purposes, the policy-values may be taken as identical from the earlier age specified.

An examination of the policy-values and percentages thus tabulated will show, in these compared cases, the extent of the difference between the values brought out by the specified Mortality Tables.

These percentages vary considerably, and it is difficult to give any concise and accurate statement of the relation of the policy-values by any two compared Tables. Taking, for example, the 3 per-cent values, an examination of Table XI will show that the  $O^M$  Table, as compared with the  $H^M$ , gives policy-values varying between the extreme limits of 90·8 and 112·8 per-cent of the latter Table. Over the main body of the Table, between entry ages 35 and 60, the difference between the two Tables does not, however, on the average exceed 1 per-cent.

The comparison between [ $O^M$  and  $O^{M(5)}$ ] and [ $H^M$  and  $H^{M(5)}$ ] policy-values at 3 per-cent, as set out in Table XV, shows variations (taking the latter values at 100) ranging between 82·7 and 109·1 per cent. The greatest divergency is shown at the earlier entry ages and the earlier durations, and after 15 or 20 years' duration the policy-values at all entry ages do not differ by more than 1 per-cent on the average.

Considering now the comparative values by the  $O^{M(5)}$  and  $H^M$  Tables, at 3 per-cent, as shown in Table XIX, the percentages of the former to the latter lie between 75·3 and 111·8 per-cent; but, over the main body of the Table, from entry age 30 to 60, at all durations, the difference does not on the average amount to 1 per-cent. The Policy-values by these two Tables appear to show a closer approximation than any other of those selected; out of 120  $O^{M(5)}$  values tabulated, 21 lie between 98 and 99 per-cent of the  $H^M$  values, 55 between 99 and 100 per-cent, 14 between 100 and 101 per cent, and 3 between 101 and 102 per-cent; thus 93 values lie between the limits of 98 and 102 per-cent of the  $H^M$  values. It will be seen that the general tendency of the  $O^{M(5)}$  values is to be *below* those of the  $H^M$  Table.

In the appended Table I have set out in what will, I hope, be a convenient and graphic manner, the relations of the 3 per-cent policy-values by the several compared Tables. In order that the results may be shown in one general conspectus, as it were, I have limited the comparison to selected entry ages and durations, which comprise those most commonly arising in the valuations of Life Offices. The percentages given under the numbers (1), (6) and (9) are respectively extracted



COMPARISON OF 3 PER-CENT POLICY-VALUES,  
*Computed, at certain entry ages and durations, by the specified  
Mortality Tables.*  
*(The value by the last-named Mortality Table or Tables being  
taken as 100).*

Age at Entry	DURATION					DURATION					Age at Entry
	5	15	30	45	60	5	15	30	45	60	
(1) OM, and H <sup>M</sup>						(6) [OM and OM <sup>(5)</sup> ], and [H <sup>M</sup> and H <sup>M(5)</sup> ]					
20	112·8	107·5	103·2	101·0	100·2	96·4	103·4	101·7	100·6	100·1	20
35	104·0	101·5	100·1	99·8	...	96·1	98·9	99·6	99·7	...	35
50	99·6	99·4	99·5	...	...	95·1	98·4	99·3	...	...	50
65	97·5	99·3	...	...	...	95·5	98·9	...	...	...	65
(2) OM, and [H <sup>M</sup> and H <sup>M(5)</sup> ]						(7) [OM and OM <sup>(5)</sup> ], and OM <sup>(5)</sup>					
20	72·1	99·3	101·1	100·5	100·1	154·3	114·0	104·5	101·7	100·6	20
35	89·9	97·8	99·5	99·7	...	110·1	102·5	100·7	100·3	...	35
50	93·3	98·2	99·3	...	...	102·5	100·6	100·2	...	...	50
65	95·4	98·9	...	...	...	100·4	100·1	...	...	...	65
(3) OM, and [OM and OM <sup>(5)</sup> ]						(8) OM <sup>(5)</sup> , and [H <sup>M</sup> and H <sup>M(5)</sup> ]					
20	74·8	96·0	99·4	99·9	100·0	62·5	90·7	97·3	98·9	99·6	20
35	93·5	98·9	99·9	100·0	...	87·3	96·5	98·8	99·4	...	35
50	98·1	99·8	100·0	...	...	92·7	97·8	99·2	...	...	50
65	99·9	100·0	...	...	...	95·1	98·8	...	...	...	65
(4) OM, and OM <sup>(5)</sup>						(9) OM <sup>(5)</sup> , and H <sup>M</sup>					
20	115·4	109·5	103·8	101·6	100·6	97·8	98·1	99·4	99·4	99·6	20
35	103·0	101·4	100·7	100·3	...	101·1	100·0	99·4	99·6	...	35
50	100·6	100·4	100·2	...	...	99·0	99·0	99·3	...	...	50
65	100·3	100·1	...	...	...	97·2	99·2	...	...	...	65
(5) [OM and OM <sup>(5)</sup> ], and H <sup>M</sup>						(10) H <sup>M</sup> , and [H <sup>M</sup> and H <sup>M(5)</sup> ]					
20	150·9	111·9	103·8	101·0	100·2	63·9	92·4	97·9	99·5	99·9	20
35	111·3	102·5	100·2	99·8	...	86·4	96·4	99·4	99·9	...	35
50	101·6	99·6	99·5	...	...	93·6	98·8	99·8	...	...	50
65	97·6	99·3	...	...	...	97·9	99·6	...	...	...	65
Age at Entry	5	15	30	45	60	5	15	30	45	60	Age at Entry
	DURATION					DURATION					

from Tables XI, XV and XIX of the Appendix; in the remaining cases they have been specially computed for the selected entry ages and durations. In the case of each comparison, the value by the Mortality Table or Tables *last* specified is taken as 100.

A careful examination of this Table will show that, on the whole, the adoption of the New Experience Tables does not bring out values differing very materially from those of the Old Experience, especially where analogous Tables are adopted, as in comparisons (1)  $O^M$  and  $H^M$ , and (6) [ $O^M$  and  $O^{M(5)}$ ] and [ $H^M$  and  $H^{M(5)}$ ]. By the Tables compared under (2),  $O^M$  and [ $H^M$  and  $H^{M(5)}$ ], and (3),  $O^M$  and [ $O^M$  and  $O^{M(5)}$ ], where the assumptions as to selection differ, the variations are still not considerable, excepting at the younger entry ages and durations. Under comparison (5) [ $O^M$  and  $O^{M(5)}$ ] and  $H^M$ , the differences are somewhat greater, but tend to diminish as the durations are prolonged. In the remaining comparisons (4),  $O^M$  and  $O^{M(5)}$ , (7) [ $O^M$  and  $O^{M(5)}$ ] and  $O^{M(5)}$ , (8)  $O^{M(5)}$  and [ $H^M$  and  $H^{M(5)}$ ], and (9)  $O^{M(5)}$  and  $H^M$ , the values by factors deducted throughout from the  $O^{M(5)}$  Table are compared with those brought out by the other specified Tables, and the differences are here somewhat considerable, excepting under the last-named comparison, where the  $O^{M(5)}$  values closely approximate throughout to the  $H^M$  values, lying however below them as a general rule. These  $O^{M(5)}$  policy-values are included to complete the comparisons, and I must not be understood as advocating their adoption, although no doubt there are cases where the employment of such a basis for policy-values would be quite appropriate.

## (2) ENDOWMENT ASSURANCES.

I now proceed to investigate the policy-values for Endowment Assurances, and for convenience I have limited my calculations and comparisons to policies payable at 60 or previous death, effected at the ages of 20, 25, 30, 35, 40, 45 and 50, and having durations of 5, 10, 15, 20, 25, 30 and 35 years, with a maximum attained age of 55. I have computed, or extracted from existing Tables, policy-values in respect of the above data at  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$  and 4 per-cent, deduced from the following Mortality Tables: (1)  $H^M$ , (2)  $O^M$ , (3)  $E^M$ , (4) [ $H^M$

and  $H^{M(5)}$ ], (5) [ $O^M$  and  $O^{M(5)}$ ], (6) [ $E^M$  and  $E^{M(5)}$ ]. I have then computed the percentages of the results by these several Tables. The values and ratios (are set out in the following Tables of the Appendix :—

XXII to XXV	$E^M$	compared with $H^M$ .
XXVI to XXIX	$E^M$	„ $O^M$ .
XXX to XXXIII	[ $E^M$ & $E^{M(5)}$ ]	„ [ $H^M$ & $H^{M(5)}$ ].
XXXIV to XXXVII	[ $E^M$ & $E^{M(5)}$ ]	„ [ $O^M$ & $O^{M(5)}$ ].

In Table XXXVIII, I have set out the values at every fifth age of the ratio of the temporary annuity (at 3 per-cent)  ${}_{60-x}a_x$  by the different Tables above specified. These ratios are the factors determining the relation of the policy-values by the compared Tables. In a Note at foot of the Table, its application in determining the relation of policy-values by specified Tables, or combinations of Tables, is illustrated. From this Table of temporary annuity ratios, the relation of 3 per-cent Endowment Assurance (60 or death) policy-values by any two of the compared Tables can readily be determined.

Dealing now with the results tabulated, and taking the 3 per-cent values by way of illustration, we find from Table XXIII that the  $E^M$  Table yields policy-values higher throughout than those brought out by the  $H^M$  Table, the excess varying from 1.1 to 4.6 per-cent. Taking now Table XXVII, we see that the  $E^M$  Table, as compared with the  $O^M$ , gives Endowment Assurance policy-values which are lower, at entry ages 20 and 25, for attained ages younger than 35 and higher for all other entry ages and durations. The percentages of the  $E^M$  values to the  $O^M$  vary from 95.6 to 102.1. Comparing further the percentages under Tables XXIII and XXVII, it appears that the  $H^M$  values lie below the  $O^M$ , which are again (with the exceptions above specified) below the  $E^M$  values, at 3 per-cent.

Turning now to Table XXXI, in which the Endowment Assurance policy-values at 3 per-cent are compared by the [ $E^M$  and  $E^{M(5)}$ ] and [ $H^M$  and  $H^{M(5)}$ ] Tables, it will be seen that, after attained age 30, the former give throughout higher values than the latter Tables. The actual percentages vary from 94.7 (at age 25) to 103.7 of the [ $H^M$  and  $H^{M(5)}$ ] results.

In Table XXXV we have the comparison of 3 per-cent values by the [ $E^M$  and  $E^{M(5)}$ ] and the [ $O^M$  and  $O^{M(5)}$ ] Tables. Here, the former Tables yield lower values at entry ages 20 and 25, up to attained ages 35 and 30 respectively, and subsequently



higher values at all entry ages and durations. The percentages vary between the limits of 95·1 and 103·1.

Comparing now Tables XXXI and XXXV, the 3 per-cent values by the combined [ $H^M$  and  $H^{M(5)}$ ] Tables are found to be, after 5 years' duration, throughout below the values by the combined [ $O^M$  and  $O^{M(5)}$ ] Tables, which are again, with the few exceptions above specified, below those of the [ $E^M$  and  $E^{M(5)}$ ].

It need hardly be added that the [ $E^M$  and  $E^{M(5)}$ ] combined Tables give larger policy-values than the  $E^M$  Table, and that the combined [ $O^M$  and  $O^{M(5)}$ ] Tables similarly give larger policy-values than the  $O^M$  Table, especially at the younger ages attained, where the effect of selection is most in evidence.

The 3 per-cent values will thus, with a few unimportant exceptions, mainly at the early ages, range in the following order, from those giving the greatest to those giving the least policy-values for Endowment Assurances:—(1) [ $E^M$  and  $E^{M(5)}$ ]; (2)  $E^M$ ; (3) [ $O^M$  and  $O^{M(5)}$ ]; (4)  $O^M$ ; (5) [ $H^M$  and  $H^{M(5)}$ ]; (6)  $H^M$ .

## SELECT MORTALITY TABLE.

### WHOLE-LIFE ASSURANCES.

The graduated Mortality Table, in the form of Select Tables, extending over the first ten years following the date of Assurance, and then passing into an Ultimate Table, has been recently completed by Mr. G. F. Hardy, as deduced throughout from the Select Data in respect of Whole-Life Participating Assurances on Male Lives. The results have been anticipated with much interest; and the graduated Tables, thus completed by the mathematical skill and genius of Mr. Hardy, have elicited the warmest expressions of admiration from all who have had the advantage of seeing them.

As already stated in the earlier paragraphs of this paper, the graduation has been completed at so recent a date that the monetary results are not at present available. I am, however, permitted to include in the Appendix to this paper three Tables setting forth results deduced by Mr. G. F. Hardy from his graduated rates. Table XXXIX sets out the graduated rates of Mortality in the form of an Extended Table, in respect



of every fifth age at entry from 15 to 70. Table XL shows the select net annual premiums at 3 per-cent for every fifth age at entry from 20 to 65, as compared with those deduced from Dr. Sprague's Select  $H^{[M]}$  Tables. In Table XLI are given the 3 per-cent policy-values for Whole-Life Assurances of 100, at different entry ages and durations, as deduced from (1) Dr. Sprague's Select ( $H^{[M]}$ ) Life Tables; (2) the new  $O^{[M]}$  Select Tables; (3) the [ $H^M$  and  $H^{M(5)}$ ] Tables. These comparisons will provide much food for thought; and the fuller examination and comparison of the effect of employing these Select Tables must be deferred until the deduced monetary values are available for the purpose.

TABLE I.  
WHOLE-LIFE AND ENDOWMENT ASSURANCE EXPERIENCE.  
*Rates of Mortality.*

Age ( <i>x</i> )	H <sup>M</sup>	H <sup>M(5)</sup>	O <sup>M</sup>	O <sup>M(5)</sup>	E <sup>M</sup>	E <sup>M(5)</sup>	Age ( <i>x</i> )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
10	·00490	·00400	·00338	·00613	...	...	10
11	·00399	·00341	·00341	·00617	...	...	11
12	·00332	·00302	·00345	·00619	...	...	12
13	·00292	·00293	·00350	·00622	...	...	13
14	·00276	·00304	·00354	·00625	...	...	14
15	·00287	·00325	·00360	·00629	...	...	15
16	·00325	·00377	·00367	·00633	...	...	16
17	·00388	·00440	·00375	·00638	...	...	17
18	·00479	·00545	·00383	·00642	...	...	18
19	·00575	·00672	·00392	·00646	...	...	19
20	·00633	·00833	·00404	·00652	·00374	·00853	20
21	·00673	·00966	·00416	·00659	·00374	·00906	21
22	·00684	·01028	·00431	·00665	·00374	·00875	22
23	·00676	·01071	·00446	·00672	·00372	·00758	23
24	·00664	·01083	·00463	·00680	·00363	·00688	24
25	·00663	·01051	·00481	·00689	·00375	·00623	25
26	·00669	·01006	·00500	·00698	·00380	·00583	26
27	·00690	·00994	·00523	·00709	·00385	·00520	27
28	·00717	·00970	·00544	·00721	·00391	·00530	28
29	·00743	·00946	·00569	·00732	·00411	·00537	29
30	·00772	·00920	·00595	·00747	·00420	·00535	30
31	·00792	·00917	·00620	·00762	·00427	·00527	31
32	·00811	·00926	·00648	·00777	·00451	·00542	32
33	·00829	·00923	·00677	·00796	·00468	·00553	33
34	·00850	·00943	·00706	·00816	·00476	·00551	34
35	·00877	·01000	·00738	·00837	·00500	·00572	35
36	·00911	·01035	·00771	·00860	·00524	·00592	36
37	·00946	·01070	·00804	·00886	·00547	·00614	37
38	·00978	·01107	·00838	·00915	·00576	·00641	38
39	·01008	·01119	·00877	·00945	·00621	·00697	39
40	·01031	·01132	·00915	·00978	·00656	·00727	40
41	·01049	·01132	·00956	·01015	·00697	·00780	41
42	·01073	·01158	·01001	·01056	·00739	·00823	42
43	·01113	·01184	·01048	·01099	·00762	·00877	43
44	·01156	·01225	·01099	·01146	·00845	·00909	44
45	·01219	·01294	·01153	·01200	·00873	·00957	45
46	·01294	·01366	·01213	·01256	·00925	·01003	46
47	·01370	·01440	·01277	·01320	·00989	·01062	47
48	·01444	·01532	·01345	·01388	·01044	·01111	48
49	·01522	·01627	·01422	·01463	·01100	·01164	49
50	·01595	·01712	·01504	·01545	·01155	·01215	50
51	·01667	·01801	·01595	·01634	·01215	·01267	51
52	·01755	·01879	·01693	·01731	·01267	·01311	52
53	·01860	·01991	·01799	·01839	·01361	·01389	53
54	·01973	·02094	·01918	·01956	·01494	·01504	54

TABLE I—(continued).

WHOLE-LIFE AND ENDOWMENT ASSURANCE EXPERIENCE.  
*Rates of Mortality.*

Age ( <i>x</i> )	H <sub><i>x</i></sub>	H <sub><i>x</i></sub> (5)	O <sub><i>x</i></sub>	O <sub><i>x</i></sub> (5)	E <sub><i>x</i></sub>	E <sub><i>x</i></sub> (5)	Age ( <i>x</i> )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
55	·02103	·02219	·02045	·02083	·01674	·01676	55
56	·02245	·02351	·02184	·02222	·01867	·01864	56
57	·02399	·02508	·02338	·02375	·02098	·02083	57
58	·02563	·02658	·02505	·02541	·02310	·02285	58
59	·02754	·02836	·02689	·02722	·02420	·02441	59
60	·02968	·03064	·02887	·02921	·02638	·02652	60
61	·03204	·03292	·03105	·03138	...	...	61
62	·03464	·03558	·03344	·03373	...	...	62
63	·03749	·03850	·03603	·03632	...	...	63
64	·04041	·04171	·03886	·03912	...	...	64
65	·04343	·04461	·04196	·04221	...	...	65
66	·04657	·04784	·04532	·04554	...	...	66
67	·04989	·05096	·04900	·04918	...	...	67
68	·05323	·05445	·05299	·05317	...	...	68
69	·05734	·05812	·05735	·05748	...	...	69
70	·06219	·06284	·06207	·06219	...	...	70
71	·06805	·06856	·06723	·06731	...	...	71
72	·07494	·07555	·07281	·07290	...	...	72
73	·08286	·08348	·07892	·07896	...	...	73
74	·09120	·09223	·08548	·08553	...	...	74
75	·09836	·09949	·09264	·09267	...	...	75
76	·10637	·10815	·10043	·10043	...	...	76
77	·11469	·11549	·10882	·10885	...	...	77
78	·12321	·12463	·11795	·11794	...	...	78
79	·13306	·13492	·12782	·12781	...	...	79
80	·14465	·14577	·13844	·13850	...	...	80
81	·15804	·15872	·15000	·15000	...	...	81
82	·17135	·17230	·16240	·16240	...	...	82
83	·18585	·18577	·17573	·17573	...	...	83
84	·19888	·19903	·19014	·19014	...	...	84
85	·20989	·21010	·20569	...	...	...	85
86	·21966	·21995	·22213	...	...	...	86
87	·23123	·23279	·24001	...	...	...	87
88	·23930	·24359	·25887	...	...	...	88
89	·25320	·25989	·27881	...	...	...	89
90	·27945	·28244	·30075	...	...	...	90
91	·31274	·31915	·32258	...	...	...	91
92	·35131	·35938	·34788	...	...	...	92
93	·41578	·43903	·37120	...	...	...	93
94	·50730	·52174	·40000	...	...	...	94
95	·63704	·63636	·42473	...	...	...	95
96	·81633	1·00000	·45794	...	...	...	96
97	1·00000	...	·48276	...	...	...	97
98	...	...	·50000	...	...	...	98
99	...	...	·53333	...	...	...	99
100	...	...	·57143	...	...	...	100

TABLE II.  
WHOLE-LIFE AND ENDOWMENT ASSURANCE EXPERIENCE.  
*Comparison of Rates of Mortality.*

Age (x)	$\frac{OM}{HM}$	$\frac{OM^{(5)}}{HM^{(5)}}$	$\frac{OM^{(5)}}{HM}$	$\frac{EM}{HM}$	$\frac{EM}{OM}$	$\frac{EM^{(5)}}{HM^{(5)}}$	$\frac{EM^{(5)}}{OM^{(5)}}$	$\frac{EM^{(5)}}{HM}$	$\frac{HM^{(5)}}{HM}$	$\frac{OM^{(5)}}{OM}$	$\frac{EM^{(5)}}{EM}$	Age (x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
10	·690	1·532	1·251	...	...	...	...	..	·816	1·814	...	10
11	·855	1·809	1·546	...	...	...	...	...	·855	1·809	...	11
12	1·039	2·050	1·865	...	...	...	...	...	·910	1·794	...	12
13	1·199	2·123	2·130	...	...	...	...	...	1·003	1·777	...	13
14	1·283	2·056	2·265	...	...	...	...	...	1·102	1·766	...	14
15	1·254	1·936	2·192	...	...	...	...	...	1·132	1·744	...	15
16	1·129	1·679	1·948	...	...	...	...	...	1·160	1·725	...	16
17	·967	1·450	1·644	...	...	...	...	...	1·134	1·701	...	17
18	·800	1·178	1·340	...	...	...	...	...	1·138	1·676	...	18
19	·682	·961	1·123	...	...	...	...	...	1·169	1·648	...	19
20	·638	·783	1·030	·591	·925	1·024	1·308	1·347	1·316	1·614	2·281	20
21	·618	·682	·979	·556	·900	·938	1·374	1·346	1·435	1·584	2·420	21
22	·630	·647	·972	·546	·867	·851	1·316	1·279	1·503	1·543	2·343	22
23	·660	·628	·994	·550	·834	·708	1·127	1·121	1·585	1·506	2·038	23
24	·697	·628	1·024	·547	·784	·636	1·013	1·037	1·631	1·469	1·897	24
25	·725	·656	1·039	·565	·779	·593	·904	·940	1·585	1·432	1·663	25
26	·748	·694	1·044	·568	·760	·579	·834	·871	1·504	1·396	1·532	26
27	·758	·713	1·028	·558	·736	·523	·734	·754	1·441	1·356	1·352	27
28	·759	·743	1·006	·546	·720	·547	·736	·740	1·354	1·325	1·355	28
29	·766	·774	·985	·553	·722	·568	·734	·723	1·273	1·286	1·308	29
30	·771	·812	·968	·544	·704	·581	·716	·693	1·192	1·255	1·274	30
31	·783	·831	·962	·539	·689	·575	·692	·665	1·158	1·229	1·234	31
32	·799	·839	·958	·557	·697	·585	·697	·668	1·142	1·199	1·200	32
33	·817	·862	·960	·565	·692	·600	·695	·668	1·113	1·176	1·182	33
34	·831	·865	·960	·560	·674	·584	·675	·648	1·109	1·156	1·158	34
35	·842	·837	·954	·570	·677	·572	·683	·652	1·140	1·134	1·144	35
36	·846	·831	·944	·575	·680	·572	·688	·650	1·136	1·115	1·129	36
37	·850	·828	·937	·578	·680	·574	·694	·650	1·131	1·102	1·124	37
38	·857	·827	·936	·589	·688	·579	·700	·655	1·132	1·092	1·112	38
39	·870	·845	·937	·617	·709	·623	·738	·692	1·110	1·077	1·122	39
40	·887	·864	·948	·637	·718	·643	·744	·705	1·098	1·069	1·108	40
41	·911	·897	·968	·664	·729	·689	·768	·743	1·079	1·062	1·119	41
42	·932	·912	·984	·689	·738	·711	·779	·767	1·079	1·055	1·114	42
43	·942	·928	·987	·685	·727	·740	·798	·788	1·064	1·049	1·150	43
44	·951	·936	·991	·731	·769	·742	·793	·787	1·059	1·043	1·076	44
45	·946	·928	·985	·716	·757	·740	·797	·785	1·061	1·041	1·096	45
46	·938	·919	·971	·715	·762	·735	·799	·776	1·058	1·035	1·085	46
47	·932	·917	·964	·721	·774	·738	·805	·775	1·051	1·034	1·074	47
48	·931	·906	·961	·723	·776	·725	·800	·769	1·061	1·032	1·064	48
49	·934	·899	·961	·723	·774	·715	·796	·765	1·069	1·029	1·058	49
50	·943	·902	·969	·724	·768	·710	·787	·762	1·073	1·027	1·052	50
51	·957	·907	·980	·729	·762	·704	·775	·760	1·081	1·024	1·043	51
52	·965	·921	·986	·722	·749	·697	·757	·747	1·071	1·022	1·034	52
53	·967	·924	·989	·732	·757	·698	·755	·747	1·071	1·022	1·021	53
54	·972	·934	·992	·757	·779	·718	·769	·762	1·061	1·020	1·007	54



TABLE II—(continued).

## WHOLE-LIFE AND ENDOWMENT ASSURANCE EXPERIENCE.

## Comparison of Rates of Mortality.

Age (x)	$\frac{OM}{HM}$	$\frac{OM^{(5)}}{HM^{(5)}}$	$\frac{OM^{(5)}}{HM}$	$\frac{EM}{HM}$	$\frac{EM}{OM}$	$\frac{EM^{(5)}}{HM^{(5)}}$	$\frac{EM^{(5)}}{OM^{(5)}}$	$\frac{EM^{(5)}}{HM}$	$\frac{HM^{(5)}}{HM}$	$\frac{OM^{(5)}}{OM}$	$\frac{EM^{(5)}}{EM}$	Age (x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
55	·973	·939	·991	·796	·818	·755	·805	·797	1·055	1·019	1·002	55
56	·973	·945	·990	·832	·855	·793	·839	·830	1·048	1·017	·998	56
57	·975	·947	·990	·875	·897	·831	·879	·868	1·045	1·016	·993	57
58	·978	·956	·992	·902	·922	·860	·899	·892	1·037	1·014	·989	58
59	·976	·960	·988	·879	·900	·861	·897	·886	1·030	1·012	1·009	59
60	·973	·953	·984	·889	·914	·866	·908	·894	1·032	1·012	1·005	60
61	·969	·953	·980	...	...	...	...	...	1·028	1·011	...	61
62	·965	·948	·974	...	...	...	...	...	1·027	1·009	...	62
63	·961	·943	·969	...	...	...	...	...	1·027	1·008	...	63
64	·962	·938	·968	...	...	...	...	...	1·032	1·007	...	64
65	·966	·946	·972	...	...	...	...	...	1·027	1·006	...	65
66	·973	·952	·978	...	...	...	...	...	1·030	1·005	...	66
67	·982	·965	·986	...	...	...	...	...	1·021	1·004	...	67
68	·995	·977	·999	...	...	...	...	...	1·023	1·003	...	68
69	1·000	·989	1·002	...	...	...	...	...	1·013	1·002	...	69
70	·998	·990	1·000	...	...	...	...	...	1·010	1·002	...	70
71	·988	·982	·989	...	...	...	...	...	1·008	1·001	...	71
72	·972	·965	·973	...	...	...	...	...	1·008	1·001	...	72
73	·953	·946	·953	...	...	...	...	...	1·008	1·000	...	73
74	·937	·927	·938	...	...	...	...	...	1·011	1·000	...	74
75	·942	·931	·942	...	...	...	...	...	1·012	1·001	...	75
76	·944	·929	·944	...	...	...	...	...	1·016	1·000	...	76
77	·948	·942	·949	...	...	...	...	...	1·007	1·000	...	77
78	·958	·947	·957	...	...	...	...	...	1·011	1·000	...	78
79	·960	·947	·960	...	...	...	...	...	1·013	1·000	...	79
80	·957	·950	·957	...	...	...	...	...	1·008	1·001	...	80
81	·949	·945	·949	...	...	...	...	...	1·004	1·000	...	81
82	·948	·943	·948	...	...	...	...	...	1·005	1·000	...	82
83	·945	·946	·945	...	...	...	...	...	·999	1·000	...	83
84	·956	·955	·956	...	...	...	...	...	1·001	1·000	...	84
85	·980	...	...	...	...	...	...	...	1·001	...	...	85
86	1·011	...	...	...	...	...	...	...	1·001	...	...	86
87	1·038	...	...	...	...	...	...	...	1·007	...	...	87
88	1·082	...	...	...	...	...	...	...	1·018	...	...	88
89	1·101	...	...	...	...	...	...	...	1·026	...	...	89
90	1·076	...	...	...	...	...	...	...	1·010	...	...	90
91	1·032	...	...	...	...	...	...	...	1·021	...	...	91
92	·990	...	...	...	...	...	...	...	1·023	...	...	92
93	·893	...	...	...	...	...	...	...	1·056	...	...	93
94	·789	...	...	...	...	...	...	...	1·028	...	...	94
95	·667	...	...	...	...	...	...	...	·999	...	...	95
96	·561	...	...	...	...	...	...	...	1·225	...	...	96
97	·483	...	...	...	...	...	...	...	...	...	...	97

TABLE III.

## WHOLE-LIFE ASSURANCES.

*Comparison of Curtate Expectations of Life.*

Age ( <i>x</i> )	EXPECTATIONS OF LIFE				DIFFERENCES					
	HM	HM <sup>(5)</sup>	OM	OM <sup>(5)</sup>	HM - HM <sup>(5)</sup>	OM - OM <sup>(5)</sup>	OM - HM	OM - HM <sup>(5)</sup>	OM <sup>(5) - HM<sup>(5)</sup></sup>	OM <sup>(5) - HM</sup>
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
10	49.791	47.963	51.459	48.994	1.828	2.465	1.668	3.496	1.031	-.797
15	45.661	43.727	47.323	45.476	1.934	1.847	1.662	3.596	1.749	-.185
20	41.561	39.718	43.182	41.888	1.843	1.294	1.621	3.464	2.170	+ .327
25	37.905	36.648	39.083	38.242	1.257	.841	1.178	2.435	1.594	.337
30	34.181	33.426	35.067	34.555	.755	.512	.886	1.641	1.129	.374
35	30.516	29.922	31.159	30.854	.594	.305	.643	1.237	.932	.338
40	26.899	26.457	27.360	27.169	.442	.191	.461	.903	.712	.270
45	23.292	22.933	23.668	23.541	.359	.127	.376	.735	.608	.249
50	19.806	19.515	20.107	20.022	.291	.085	.301	.592	.507	.216
55	16.462	16.267	16.722	16.669	.195	.053	.260	.455	.402	.207
60	13.330	13.194	13.571	13.543	.136	.028	.241	.377	.349	.213
65	10.512	10.421	10.716	10.704	.091	.012	.204	.295	.283	.192
70	7.995	7.943	8.208	8.205	.052	.003	.213	.265	.262	.210
75	5.876	5.831	6.079	6.079	.045	.000	.203	.248	.248	.203
80	4.219	4.196	4.337	4.336	.023	.001	.118	.141	.140	.117
85	3.011	2.980	2.965	2.965	.031	.000	-.046	-.015	-.015	-.046
90	1.857	1.809	1.931	1.931	.048	.000	+ .074	+ .122	+ .122	+ .074
95	.430	.364	1.188	1.188	.066	.000	.758	.824	.824	.758
100	...	...	.571	.571	...	.000	.571	.571	.571	.571

TABLE IV.  
 BRITISH OFFICES EXPERIENCE (MALE PARTICIPATING ASSURANCES: 1893)  
 and INSTITUTE OF ACTUARIES' EXPERIENCE (H<sup>M</sup> AND H<sup>M(5)</sup>: 1863).  
*Comparison of Full and Truncated Aggregate Tables.*  
*(Compiled by MR. R. P. HARDY, F.I.A.).*  
*Experience of 10,000 Entrants at each age.*

Age Period	Entry-Age = 30						Entry-Age = 35						Entry-Age = 40						Age Period																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
	BRITISH OFFICES EXPERIENCE, 1893			INSTITUTE (I <sup>M</sup> & I <sup>M(5)</sup> ) EXPERIENCE, 1863			BRITISH OFFICES EXPERIENCE, 1893			INSTITUTE (I <sup>M</sup> & I <sup>M(5)</sup> ) EXPERIENCE, 1863			BRITISH OFFICES EXPERIENCE, 1893			INSTITUTE (I <sup>M</sup> & I <sup>M(5)</sup> ) EXPERIENCE, 1863																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
	No. of Survivors at end of period	No. of Deaths	Greater	Less	No. of I <sup>M</sup> Deaths	No. of Survivors at end of period	No. of Deaths	Greater	Less	No. of Survivors at end of period	No. of Deaths	Greater	Less	No. of Survivors at end of period	No. of Deaths	Greater	Less	No. of Survivors at end of period		No. of I <sup>M</sup> Deaths																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
	OM	OM <sup>(5)</sup>				OM	OM <sup>(5)</sup>				HM	HM <sup>(5)</sup>				OM	OM <sup>(5)</sup>					HM	HM <sup>(5)</sup>																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				

TABLE V.

## ENDOWMENT ASSURANCE EXPERIENCE.

*Values of 3 per-cent Term Annuities ( ${}_{60-x}a_x$ ).*

Age ( $x$ )	FULL AGGREGATE TABLE ( $E^M$ )		TRUNCATED AGGREGATE TABLE ( $E^{M(5)}$ )		Age ( $x$ )
	Ungraduated	Graduated	Ungraduated	Graduated	
20	22·008	21·995	21·179	21·322	20
21	21·709	21·706	21·216	21·112	21
22	21·416	21·407	20·974	20·904	22
23	21·092	21·098	20·729	20·682	23
24	20·777	20·778	20·432	20·428	24
25	20·449	20·446	20·144	20·149	25
26	20·109	20·105	19·857	19·847	26
27	19·750	19·753	19·513	19·526	27
28	19·391	19·390	19·179	19·182	28
29	19·026	19·016	18·848	18·827	29
30	18·632	18·634	18·455	18·461	30
31	18·234	18·239	18·070	18·082	31
32	17·836	17·833	17·690	17·688	32
33	17·419	17·416	17·288	17·282	33
34	16·989	16·989	16·870	16·864	34
35	16·554	16·547	16·435	16·431	35
36	16·095	16·094	15·982	15·985	36
37	15·625	15·628	15·522	15·526	37
38	15·154	15·150	15·062	15·055	38
39	14·661	14·659	14·573	14·570	39
40	14·160	14·157	14·079	14·075	40
41	13·635	13·641	13·555	13·566	41
42	13·116	13·111	13·049	13·045	42
43	12·571	12·568	12·509	12·509	43
44	12·008	12·006	11·960	11·959	44
45	11·440	11·433	11·398	11·391	45
46	10·845	10·840	10·813	10·806	46
47	10·231	10·230	10·203	10·203	47
48	9·602	9·602	9·578	9·581	48
49	8·948	8·953	8·932	8·937	49
50	8·291	8·283	8·279	8·272	50
51	7·598	7·589	7·590	7·582	51
52	6·877	6·870	6·872	6·866	52
53	6·131	6·124	6·130	6·123	53
54	5·355	5·350	5·355	5·351	54
55	4·552	4·549	4·552	4·549	55
56	3·719	3·717	3·721	3·718	56
57	2·854	2·852	2·854	2·853	57
58	1·950	1·949	1·950	1·949	58
59	1·000	1·000	1·000	1·000	59



TABLE VI.  
WHOLE-LIFE ASSURANCES.  
*Comparison of Net Annual Premiums (100 P<sub>x</sub>).*

Age ( <i>x</i> )	2½ PER-CENT			3 PER-CENT			3½ PER-CENT			4 PER-CENT			Age ( <i>x</i> )
	Net Premiums		Ratios	Net Premiums		Ratios	Net Premiums		Ratios	Net Premiums		Ratios	
	HM (2)	OM (3)	$\frac{OM}{HM}$ (4)	HM (5)	OM (6)	$\frac{OM}{HM}$ (7)	HM (8)	OM (9)	$\frac{OM}{HM}$ (10)	HM (11)	OM (12)	$\frac{OM}{HM}$ (13)	
20	1·538	1·422	·925	1·427	1·306	·915	1·330	1·204	·905	1·244	1·114	·895	20
25	1·742	1·645	·944	1·625	1·524	·938	1·521	1·416	·931	1·428	1·321	·925	25
30	2·003	1·916	·957	1·880	1·790	·952	1·769	1·677	·948	1·669	1·575	·944	30
35	2·322	2·247	·968	2·193	2·116	·965	2·076	1·997	·962	1·969	1·889	·959	35
40	2·723	2·659	·976	2·589	2·524	·975	2·465	2·400	·974	2·352	2·285	·972	40
45	3·253	3·185	·979	3·114	3·046	·978	2·985	2·917	·977	2·865	2·797	·976	45
50	3·943	3·873	·982	3·800	3·730	·982	3·667	3·597	·981	3·542	3·471	·980	50
55	4·870	4·786	·983	4·725	4·641	·982	4·588	4·501	·982	4·458	4·375	·981	55
60	6·134	6·018	·981	5·987	5·872	·981	5·848	5·733	·980	5·715	5·600	·980	60

TABLE VII.

ENDOWMENT ASSURANCES (DEATH OR 60).

*Comparison of Net Annual Premiums (100 P<sub>x, 60-x</sub>).*

Age <i>x</i>	2½ PER-CENT						3 PER-CENT						3½ PER-CENT						4 PER-CENT						
	Net Premiums			Ratios			Net Premiums			Ratios			Net Premiums			Ratios			Net Premiums			Ratios			
	H <sup>M</sup>	O <sup>M</sup>	E <sup>M</sup>	$\frac{O^M}{H^M}$	$\frac{E^M}{H^M}$	$\frac{E^M}{O^M}$	H <sup>M</sup>	O <sup>M</sup>	E <sup>M</sup>	$\frac{O^M}{H^M}$	$\frac{E^M}{H^M}$	$\frac{E^M}{O^M}$	H <sup>M</sup>	O <sup>M</sup>	E <sup>M</sup>	$\frac{O^M}{H^M}$	$\frac{E^M}{H^M}$	$\frac{E^M}{O^M}$	H <sup>M</sup>	O <sup>M</sup>	E <sup>M</sup>	$\frac{O^M}{H^M}$	$\frac{E^M}{H^M}$	$\frac{E^M}{O^M}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
20	2·008	1·896	1·785	·944	·889	·941	1·864	1·747	1·634	·937	·877	·935	1·733	1·612	1·497	·930	·864	·929	1·615	1·490	1·373	·923	·850	·921	20
25	2·367	2·276	2·140	·962	·904	·940	2·212	2·118	1·979	·958	·895	·934	2·070	1·973	1·829	·953	·884	·927	1·940	1·839	1·693	·948	·873	·921	25
30	2·860	2·781	2·626	·972	·918	·944	2·694	2·613	2·451	·970	·910	·938	2·540	2·456	2·294	·967	·903	·934	2·397	2·309	2·144	·963	·894	·929	30
35	3·544	3·481	3·315	·982	·935	·952	3·366	3·301	3·130	·981	·930	·948	3·199	3·132	2·958	·979	·925	·944	3·041	2·972	2·795	·977	·919	·940	35
40	4·565	4·518	4·348	·990	·952	·962	4·373	4·326	4·151	·989	·949	·960	4·190	4·141	3·964	·988	·946	·957	4·016	3·965	3·786	·987	·943	·955	40
45	6·268	6·224	6·046	·993	·965	·971	6·060	6·016	5·834	·993	·963	·970	5·859	5·815	5·630	·992	·961	·968	5·666	5·620	5·434	·992	·959	·967	45
50	9·619	9·585	9·394	·996	·977	·980	9·389	9·352	9·160	·996	·976	·979	9·165	9·129	8·934	·996	·975	·979	8·947	8·911	8·714	·996	·974	·978	50

TABLE VIII.  
TEMPORARY ASSURANCES.  
*Select Net Single Premiums at 3 per-cent (100 A<sub>[x|n]</sub>).*

Select Net Single Premiums at 3 per cent.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
CENTRAL AGE AT ENTRY [x]																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
[20]		[25]		[30]		[35]		[40]		[45]		[50]		[55]		[60]		Term of Assurance (n)																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	H(M)	T(M)	Term of Assurance (n)																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																			
(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(U)	(G)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
1	·49	·82	·44	·27	·44	·34	·48	·58	·53	·34	·49	·62	·84	·78	·77	1·01	1·17	1·01	1·35	1·26	1·40	1·85	1·41	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
2	1·29	1·31	1·13	·92	1·11	·79	1·19	1·06	1·32	·70	·95	1·54	2·02	1·60	1·91	2·64	2·56	2·55	2·79	2·75	3·59	2·88	3·03	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
3	2·22	2·22	1·95	1·79	1·88	1·06	2·01	2·01	2·23	1·22	1·63	2·61	3·07	2·50	3·29	4·73	4·02	4·45	4·29	4·40	6·41	4·68	4·83	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																									
4	3·14	3·23	2·78	3·50	2·69	1·41	2·89	2·44	3·21	2·15	2·35	3·82	4·09	3·57	4·83	6·04	5·68	6·58	4·86	6·17	9·54	7·68	6·77	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
5	4·05	5·32	3·57	4·07	3·48	2·61	3·80	3·46	4·23	2·90	3·11	5·12	4·25	4·71	6·50	8·56	7·46	8·84	8·08	9·97	15·97	11·08	10·96	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
6	4·89	7·62	4·31	4·85	4·28	3·42	4·71	3·88	5·26	3·28	3·90	6·47	5·33	5·92	8·22	10·58	9·18	11·16	10·19	9·97	15·97	11·08	10·96	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																												
7	5·66	8·56	5·02	5·73	5·08	3·72	5·58	4·35	6·31	3·48	4·72	7·83	6·05	7·11	9·96	12·17	10·83	13·50	13·09	11·96	19·16	13·72	13·17	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
8	6·40	8·56	5·71	5·73	5·88	3·72	6·44	4·35	7·36	5·36	5·54	9·18	7·39	8·30	11·71	13·82	12·41	15·87	13·09	14·02	22·31	13·72	15·55	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																														
9	7·08	8·56	6·37	5·73	6·67	5·05	7·28	4·35	8·44	5·36	6·36	10·54	7·39	9·47	13·17	15·52	13·94	18·28	13·09	16·09	25·40	13·72	18·06	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
10	7·73	16·01	7·02	5·73	7·44	6·54	8·11	4·35	9·53	6·52	7·19	11·91	7·39	10·60	15·25	15·52	15·40	20·71	13·09	18·09	28·43	26·82	20·66	10																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															

G = Graduated.

U = Ungraduated.





TABLE IX.

## RATIOS OF ANNUITIES-DUE

 $H^M, H^{M(5)}, O^M, O^{M(5)}.$ *For Comparison of Policy-Values*

3 PER-CENT.

 $\frac{a_x}{a'_x}$ 

Age	$\frac{H^M}{O^M}$	$\frac{H^M}{O^{M(5)}}$	$\frac{H^{M(5)}}{O^M}$	$\frac{H^{M(5)}}{O^{M(5)}}$	$\frac{O^M}{O^{M(5)}}$	Age	$\frac{H^M}{O^M}$	$\frac{H^M}{O^{M(5)}}$	$\frac{H^{M(5)}}{O^M}$	$\frac{H^{M(5)}}{O^{M(5)}}$	$\frac{O^M}{O^{M(5)}}$
$x$	$f_x$	$g_x$	$h_x$	$j_x$	$k_x$	$x$	$f_x$	$g_x$	$h_x$	$j_x$	$k_x$
20	·9720	·9961	·9385	·9618	1·0248	65	·9860	·9870	·9793	·9802	1·0009
21	·9731	·9958	·9398	·9618	1·0233	66	·9859	·9866	·9795	·9803	1·0008
22	·9744	·9957	·9424	·9630	1·0219	67	·9855	·9860	·9796	·9802	1·0006
23	·9757	·9957	·9455	·9648	1·0205	68	·9846	·9851	·9791	·9795	1·0005
24	·9769	·9955	·9490	·9671	1·0191	69	·9827	·9832	·9777	·9782	1·0005
25	·9778	·9952	·9526	·9695	1·0178	70	·9804	·9806	·9754	·9757	1·0003
26	·9785	·9946	·9558	·9715	1·0165	71	·9775	·9778	·9726	·9728	1·0003
27	·9792	·9941	·9586	·9732	1·0152	72	·9749	·9752	·9698	·9700	1·0003
28	·9798	·9936	·9612	·9747	1·0141	73	·9732	·9733	·9678	·9679	1·0002
29	·9806	·9933	·9635	·9760	1·0130	74	·9728	·9728	·9670	·9670	1·0000
30	·9813	·9931	·9654	·9769	1·0120	75	·9739	·9739	·9682	·9682	1·0000
31	·9822	·9930	·9668	·9775	1·0110	76	·9751	·9751	·9695	·9695	1·0000
32	·9830	·9930	·9681	·9779	1·0101	77	·9764	·9764	·9716	·9716	1·0000
33	·9838	·9929	·9692	·9781	1·0093	78	·9778	·9778	·9728	·9728	1·0000
34	·9844	·9928	·9700	·9782	1·0086	79	·9785	·9785	·9739	·9739	1·0000
35	·9850	·9929	·9707	·9784	1·0080	80	·9792	·9792	·9755	·9755	1·0000
36	·9856	·9928	·9717	·9789	1·0074	81	·9808	·9808	·9772	·9772	1·0000
37	·9862	·9930	·9728	·9794	1·0068	82	·9848	·9848	·9810	·9810	1·0000
38	·9869	·9932	·9739	·9801	1·0064	83	·9908	·9908	·9870	·9870	1·0000
39	·9875	·9934	·9751	·9809	1·0060	84	1·0003	1·0003	·9952	·9952	1·0000
40	·9881	·9937	·9760	·9816	1·0056	<div>NOTE.</div> <div><div><div><math>{}_nV_x</math> by the</div><div>HM Table,</div><div>„ „</div><div>„ „</div><div>HM<sup>(5)</sup> „</div><div>„ „</div><div>„ „</div><div>HM &amp; HM<sup>(5)</sup></div><div>„ „</div><div>„ „</div><div>OM Table,</div><div>„ „</div><div>OM<sup>(5)</sup> „</div><div>OM<sup>(5)</sup> „</div><div>OM &amp; OM<sup>(5)</sup> „</div><div>OM<sup>(5)</sup> Table „</div><div>„ „</div><div>OM<sup>(5)</sup> „</div></div><div><div><math>{}_nV'_x</math> by the</div><div>OM Table, as <math>f_x</math></div><div>OM<sup>(5)</sup> „ „ <math>g_x</math></div><div>OM &amp; OM<sup>(5)</sup> „ „ <math>f_x</math></div><div>OM Table, „ „ <math>h_x</math></div><div>OM<sup>(5)</sup> „ „ „ <math>j_x</math></div><div>OM &amp; OM<sup>(5)</sup> „ „ <math>h_x</math></div><div>OM Table, „ „ <math>f_x</math></div><div>OM<sup>(5)</sup> „ „ „ <math>g_x</math></div><div>OM &amp; OM<sup>(5)</sup> „ „ <math>f_x</math></div><div>OM<sup>(5)</sup> Table „ „ <math>k_x</math></div><div>OM &amp; OM<sup>(5)</sup> „ „ 1</div><div>„ „ „ „ 1</div></div><div><div><math>f_{x+n}</math></div><div><math>g_{x+n}</math></div><div><math>g_{x+n}</math></div><div><math>h_{x+n}</math></div><div><math>j_{x+n}</math></div><div><math>j_{x+n}</math></div><div><math>h_{x+n}</math></div><div><math>h_{x+n}</math></div><div><math>j_{x+n}</math></div><div><math>k_{x+n}</math></div><div><math>k_{x+n}</math></div><div><math>k_x</math></div></div></div>					
41	·9886	·9939	·9768	·9820	1·0053						
42	·9889	·9938	·9772	·9821	1·0050						
43	·9890	·9937	·9774	·9821	1·0048						
44	·9889	·9934	·9773	·9818	1·0046						
45	·9888	·9931	·9772	·9814	1·0044						
46	·9887	·9928	·9771	·9812	1·0042						
47	·9888	·9928	·9771	·9810	1·0040						
48	·9891	·9928	·9773	·9810	1·0038						
49	·9892	·9928	·9776	·9811	1·0036						
50	·9895	·9930	·9780	·9814	1·0035						
51	·9897	·9930	·9785	·9818	1·0034						
52	·9897	·9928	·9790	·9822	1·0032						
53	·9895	·9925	·9793	·9823	1·0030						
54	·9893	·9921	·9796	·9824	1·0028						
55	·9890	·9916	·9797	·9823	1·0026						
56	·9888	·9911	·9799	·9822	1·0024						
57	·9884	·9906	·9798	·9820	1·0022						
58	·9880	·9901	·9798	·9819	1·0022						
59	·9875	·9895	·9795	·9814	1·0020						
60	·9870	·9887	·9791	·9807	1·0018						
61	·9866	·9881	·9788	·9804	1·0016						
62	·9863	·9877	·9786	·9800	1·0014						
63	·9860	·9873	·9786	·9798	1·0013						
64	·9860	·9871	·9787	·9798	1·0011						

TABLE X.  
WHOLE-LIFE ASSURANCES.  
*Policy-Values* ( $100 {}_nV_x$ ).

OM : HM.

**2½ PER-CENT.**

Age at Entry (x)	DURATION (n)								Age at Entry (x)
	5	10	15	20	25	30	35	40	
10	4'091 4'435 92'2	8'650 9'329 92'7	13'648 13'760 99'2	19'016 18'812 101'1	24'737 24'254 102'0	30'820 30'149 102'2	37'292 36'644 101'8	44'121 43'498 101'4	10
15	4'753 5'120 92'8	9'965 9'758 102'1	15'562 15'044 103'4	21'527 20'738 103'8	27'869 26'907 103'6	24'618 33'703 102'7	41'737 40'876 102'1	49'099 48'370 101'5	15
20	5'472 4'888 112'0	11'348 10'459 108'5	17'611 16'461 107'0	24'270 22'963 105'7	31'355 30'125 104'1	38'830 37'685 103'0	46'558 45'581 102'1	54'346 53'609 101'4	20
25	6'216 5'858 106'1	12'841 12'168 105'5	19'886 19'001 104'6	27'381 26'535 103'2	35'289 34'483 102'3	43'465 42'787 101'6	51'703 51'225 100'9	59'733 59'374 100'6	25
30	7'065 6'702 105'4	14'576 13'964 104'4	22'568 21'963 102'6	31'000 30'406 102'0	39'718 39'227 101'3	48'502 48'190 100'7	57'064 56'847 100'4	65'099 65'125 100'0	30
35	8'082 7'783 103'8	16'682 16'357 102'0	25'755 25'407 101'4	35'135 34'861 100'8	44'587 44'468 100'3	53'800 53'747 100'1	62'446 62'620 99'7	70'234 70'566 99'5	35
40	9'356 9'297 100'6	19'226 19'111 100'6	29'432 29'363 100'2	39'715 39'781 99'8	49'738 49'843 99'8	59'144 59'465 99'5	67'617 68'081 99'3	74'927 75'143 99'7	40
45	10'889 10'820 100'6	22'148 22'123 100'1	33'493 33'608 99'7	44'550 44'701 99'7	54'927 55'310 99'3	64'274 64'809 99'2	72'339 72'595 99'6	78'970 78'480 100'6	45
50	12'634 12'675 99'7	25'365 25'551 99'3	37'774 37'992 99'4	49'419 49'888 99'1	59'909 60'540 99'0	68'959 69'271 99'6	76'401 75'870 100'7	84'938 80'481 100'6	Age at Entry (x) 40
55	14'572 14'748 98'8	28'775 28'992 99'3	42'104 42'615 98'8	54'111 54'813 98'7	64'470 64'810 99'5	72'988 72'367 100'9	82'479 82'000 100'6	90'954 88'078 99'8	35
60	16'626 16'708 99'5	32'228 32'688 98'6	46'283 46'995 98'5	58'410 58'723 99'5	68'380 67'587 101'2	78'582 78'614 100'0	88'559 88'773 100'4	98'516 98'984 101'0	30
65	18'713 19'185 97'5	35'571 36'363 97'8	50'116 50'443 99'4	62'075 61'085 101'6	72'988 72'367 100'9	82'479 82'000 100'6	92'454 92'078 99'8	102'429 102'052 101'2	25
70	20'739 21'256 97'6	38'632 38'678 99'9	53'344 51'847 102'9	65'564 64'963 100'7	76'642 76'670 100'0	86'736 86'760 100'2	96'736 96'760 100'4	106'736 106'760 100'6	20
75	22'575 22'125 102'0	41'136 38'848 105'9	56'250 55'733 100'6	67'597 67'704 99'9	77'597 77'704 99'9	87'597 87'704 99'9	97'597 97'704 99'9	107'597 107'704 99'9	15
80	23'973 21'475 111'6	75 86'813 86'366 100'5	70 82'655 82'637 100'0	65 77'597 77'704 99'9	60 71'736 71'686 100'1	55 64'965 64'965 100'4	50 57'937 57'937 100'6	45 51'181 50'660 101'0	10
Age at Entry (x)	5	75	70	65	60	55	50	45	Age at Entry (x)





TABLE XII.  
WHOLE-LIFE ASSURANCES.  
*Policy-Values* ( $100 {}_nV_x$ ).

**3½ PER-CENT.**

Age at Entry (x)	DURATION (n)								Age at Entry (x)
	5	10	15	20	25	30	35	40	
10	3'134 3'505 89'4	6'776 7'534 89'9	10'906 11'132 98'0	15'494 15'415 100'5	20'529 20'168 101'8	26'055 25'492 102'2	32'130 31'571 101'8	38'742 38'192 101'4	10
15	3'759 4'175 90'0	8'023 7'904 101'5	12'759 12'342 103'4	17'957 17'268 104'0	23'662 22'785 103'9	29'934 29'085 102'9	36'760 35'947 102'3	44'038 43'346 101'6	15
20	4'430 3'892 113'8	9'352 8'523 109'7	14'753 13'664 108'0	20'680 19'421 106'5	27'197 25'995 104'6	34'290 33'156 103'4	41'853 40'878 102'4	49'688 48'952 101'5	20
25	5'149 4'819 106'9	10'801 10'168 106'2	17'003 16'158 105'2	23'822 22'999 103'6	31'244 30'449 102'6	39'157 38'484 101'8	47'356 46'885 101'0	55'566 55'211 100'6	25
30	5'958 5'620 106'0	12'498 11'913 104'9	19'686 19'100 103'1	27'511 26'928 102'2	35'854 35'369 101'4	44'498 44'196 100'7	53'153 52'944 100'4	61'474 61'524 99'9	30
35	6'954 6'668 104'3	14'598 14'283 102'2	22'919 22'577 101'5	31'790 31'521 100'9	40'981 40'873 100'3	50'185 50'142 100'1	59'033 59'233 99'7	67'178 67'559 99'4	35
40	8'215 8'159 100'7	17'158 17'045 100'7	26'692 26'628 100'2	36'571 36'649 99'8	46'462 46'579 99'7	55'972 56'321 99'4	64'725 65'241 99'2	72'419 72'681 99'6	40
45	9'743 9'676 100'7	20'131 20'110 100'1	30'893 31'021 99'6	41'670 41'834 99'6	52'031 52'440 99'2	61'567 62'153 99'1	69'950 70'254 99'6	76'966 76'458 100'7	45
50	11'509 11'552 99'6	23'433 23'632 99'2	35'374 35'603 99'4	46'853 47'346 99'0	57'419 58'099 98'8	66'706 67'068 99'5	74'480 73'936 100'7	45	Age at Entry (x)
55	13'475 13'657 98'7	26'969 27'192 99'2	39'941 40'469 98'7	51'881 52'626 98'6	62'376 62'767 99'4	71'161 70'532 100'9	78'859 78'379 100'6	40	
60	15'595 15'676 99'5	30'587 31'052 98'5	44'387 45'133 98'4	56'517 56'877 99'4	66'670 65'871 101'2	55	80'329 79'820 100'6	74'337 74'503 99'8	35
65	17'762 18'235 97'4	34'111 34'933 97'6	48'483 48'861 99'2	60'512 59'526 101'7	60	81'501 80'954 100'7	75'866 75'936 99'9	69'133 69'382 99'6	30
70	19'880 20'422 97'3	37'356 37'456 99'7	51'982 50'500 102'9	65	82'454 81'872 100'7	77'109 77'095 100'0	70'723 70'857 99'8	63'458 63'378 100'1	25
75	21'812 21'406 101'9	40'068 37'797 106'0	70	83'231 82'578 100'8	78'123 77'987 100'2	72'020 71'991 100'0	65'077 64'804 100'4	57'534 56'954 101'0	20
80	23'349 20'855 112'0	75	83'861 83'305 100'7	78'945 78'906 100'1	73'072 73'161 99'9	66'390 66'273 100'2	59'130 58'751 100'6	51'579 51'083 101'0	15
		84'367 83'890 100'6	79'605 79'645 99'9	73'916 74'102 99'7	67'443 67'455 100'0	60'411 60'197 100'4	53'097 52'798 100'6	45'792 45'332 101'0	10
Age at Entry (x)	5	75	70	65	60	55	50	45	Age at Entry (x)



$$O^M : H^M.$$

4 PER-CENT.

Age at Entry (x)	DURATION (n)								Age at Entry (x)
	5	10	15	20	25	30	35	40	
10	2'757 3'130 88'1	6'015 6'798 88'5	9'768 10'039 97'3	14'000 13'976 100'2	18'713 18'408 101'7	23'964 23'447 102'2	29'819 29'303 101'8	36'290 35'777 101'4	10
15	3'350 3'787 88'5	7'210 7'132 101'1	11'562 11'197 103'3	16'409 15'772 104'0	21'808 20'974 104'0	27'829 27'019 103'0	34'484 33'702 102'3	41'684 41'017 101'6	15
20	3'994 3'477 114'9	8'496 7'702 110'3	13'511 12'457 108'5	19'098 17'864 106'9	25'328 24'147 104'9	32'212 31'093 103'6	39'662 38'695 102'5	47'490 46'757 101'6	20
25	4'690 4'376 107'2	9'913 9'303 106'6	15'733 14'905 105'6	22'221 21'414 103'8	29'393 28'610 102'7	37'152 36'487 101'8	45'306 44'839 101'0	53'568 53'217 100'7	25
30	5'481 5'152 106'4	11'586 11'010 105'2	18'394 17'817 103'2	25'918 25'343 102'3	34'060 33'580 101'4	42'615 42'314 100'7	51'283 51'076 100'4	59'717 59'776 99'9	30
35	6'459 6'176 104'6	13'662 13'353 102'3	21'623 21'288 101'6	30'236 29'972 100'9	39'287 39'181 100'3	48'458 48'419 100'1	57'381 57'591 99'6	65'681 66'083 99'4	35
40	7'700 7'649 100'7	16'210 16'106 100'6	25'419 25'362 100'2	35'095 35'177 99'8	44'899 45'023 99'7	54'438 54'799 99'3	63'311 63'851 99'2	71'177 71'463 99'6	40
45	9'220 9'157 100'7	19'197 19'180 100'1	29'680 29'808 99'6	40'302 40'470 99'6	50'637 51'055 99'2	60'250 60'856 99'0	68'772 69'100 99'5	75'966 75'452 100'7	45
50	10'990 11'033 99'6	22'538 22'732 99'1	34'239 34'469 99'3	45'624 46'121 98'9	56'213 56'910 98'8	65'601 65'985 99'4	73'525 72'977 100'8		Age at Entry (x)
55	12'974 13'150 98'7	26'119 26'342 99'2	38'910 39'440 98'7	50'806 51'567 98'5	61'354 61'767 99'3	70'257 69'626 100'9		77'817 77'330 100'6	40
60	15'105 15'190 99'4	29'803 30'271 98'5	43'472 44'234 98'3	55'592 55'978 99'3	65'823 65'027 101'2		79'250 78'730 100'7	73'039 73'226 99'7	35
65	17'312 17'782 97'4	33'414 34'246 97'6	47'691 48'093 99'2	59'741 58'764 101'7		80'387 79'826 100'7	74'516 74'605 99'9	67'562 67'831 99'6	30
70	19'473 20'025 97'3	36'738 36'867 99'7	51'312 49'845 102'9		81'307 80'709 100'7	75'712 75'717 100'0	69'083 69'238 99'8	61'606 61'536 100'1	25
75	21'440 21'059 101'8	39'538 37'287 106'0		82'053 81'379 100'8	76'682 76'561 100'2	70'318 70'308 100'0	63'140 62'874 100'4	55'422 54'844 101'1	20
80	23'037 20'557 112'1		82'655 82'084 100'7	77'463 77'449 100'0	71'312 71'432 99'8	64'375 64'280 100'2	56'916 56'554 100'6	49'250 48'773 101'0	15
		83'133 82'645 100'6	78'084 78'154 99'9	72'103 72'327 99'7	65'357 65'397 99'9	58'103 57'914 100'3	50'649 50'376 100'5	43'201 42'862 101'0	10
Age at Entry (x)	5	75	70	65	60	55	50	45	Age at Entry (x)
DURATION (n)									











TABLE XVII.  
WHOLE-LIFE ASSURANCES.  
Policy-Values (100  $_nV_x$ ).

4 PER-CENT.

Age at Entry ( $x$ )	DURATION ( $n$ )								Age at Entry ( $x$ )	
	5	10	15	20	25	30	35	40		
10	5'544 5'775 96'0	8'135 9'780 83'2	11'250 12'199 92'2	14'957 15'258 98'0	19'316 19'498 99'1	24'364 24'308 100'2	30'108 30'073 100'1	36'499 36'478 100'1	10	
15	5'531 6'865 80'6	8'734 9'362 93'3	12'546 12'521 100'2	17'028 16'898 100'8	22'220 21'863 101'6	28'126 27'814 101'1	34'699 34'425 100'8	41'833 41'538 100'7	15	
20	5'570 5'795 96'2	9'514 9'078 104'8	14'152 13'627 103'9	19'523 18'788 103'9	25'635 24'973 102'7	32'435 31'845 101'9	39'816 39'237 101'5	47'580 47'163 100'9	20	
25	5'750 5'802 99'1	10'581 10'515 100'6	16'176 15'862 102'0	22'541 22'270 101'2	29'623 29'389 100'8	37'313 37'048 100'7	45'399 45'260 100'3	53'608 53'525 100'2	25	
30	6'181 6'420 96'3	12'051 12'011 100'3	18'730 18'712 100'1	26'162 26'158 100'0	34'228 34'167 100'2	42'712 4'2755 99'9	51'326 51'398 99'9	59'728 59'967 99'6	30	
35	6'951 7'232 96'1	14'018 14'297 98'1	21'880 22'147 98'8	30'414 30'591 99'4	39'391 39'645 99'4	48'503 48'758 99'5	57'393 57'793 99'3	65'681 66'274 99'1	35	
40	8'080 8'655 93'4	16'486 17'022 96'9	25'609 26'022 98'4	35'205 35'672 98'7	44'947 45'385 99'0	54'451 55'014 99'0	63'311 64'054 98'8	71'183 71'568 99'5	40	
45	9'518 10'149 93'8	19'403 19'895 97'5	29'800 30'344 98'2	40'354 40'862 98'8	50'651 51'288 98'8	60'250 61'076 98'6	68'779 69'213 99'4	75'966 75'609 100'5	45	
50	11'217 11'820 94'9	22'670 23'322 97'2	34'296 34'900 98'3	45'639 46'378 98'4	56'213 57'153 98'4	65'608 66'110 99'2	73'525 73'150 100'5	45	Age at Entry ( $x$ )	
55	13'122 13'813 95'0	26'184 26'827 97'6	38'927 39'728 98'0	50'806 51'839 98'0	61'362 61'907 99'1	70'257 69'820 100'0	77'817 77'475 100'4	40	Age at Entry ( $x$ )	
60	15'180 15'748 96'4	29'822 30'602 97'5	43'472 44'547 97'6	55'601 56'139 99'0	65'823 65'251 100'9	79'250 78'866 100'5	73'044 73'324 99'6	35	Age at Entry ( $x$ )	
65	17'335 18'173 95'4	33'414 34'616 96'5	47'701 48'284 98'8	59'741 59'027 101'2	60	80'387 79'955 100'5	74'522 74'698 99'8	67'562 68'012 99'3	30	Age at Entry ( $x$ )
70	19'473 20'474 95'1	36'752 37'099 99'1	51'312 50'166 102'3	65	81'307 80'832 100'6	75'717 75'806 99'9	69'083 69'411 99'5	61'617 61'719 99'8	25	Age at Entry ( $x$ )
75	21'457 21'349 100'5	39'538 37'688 104'9	70	82'053 81'498 100'7	76'686 76'647 100'1	70'318 70'475 99'8	63'150 63'050 100'2	55'461 55'142 100'6	20	Age at Entry ( $x$ )
80	23'037 21'065 109'3	75	82'655 82'199 100'6	77'467 77'531 99'9	71'312 71'593 99'6	64'384 64'450 99'9	56'953 56'840 100'2	49'336 49'164 100'4	15	Age at Entry ( $x$ )
		83'133 82'756 100'5	78'089 78'235 99'8	72'103 72'482 99'5	65'366 65'562 99'7	58'140 58'191 99'9	50'733 50'755 100'0	43'436 43'368 100'2	10	Age at Entry ( $x$ )
Age at Entry ( $x$ )	5	75	70	65	60	55	50	45	Age at Entry ( $x$ )	
DURATION ( $n$ )										

TABLE XVIII.  
WHOLE-LIFE ASSURANCES.  
*Policy-Values* ( $100 {}_nV_x$ ).

$$O^{M(5)} : H^M$$

**2½ PER-CENT.**

Age at Entry (x)	DURATION (n)								Age at Entry (x)
	5	10	15	20	25	30	35	40	
10	3'433 4'435 77'4	7'351 9'329 78'8	11'794 13'760 85'7	16'787 18'812 89'2	22'348 24'254 92'1	28'457 30'149 94'4	35'068 36'614 95'7	42'081 43'498 96'7	10
15	4'058 5'120 79'3	8'658 9'758 88'7	13'829 15'044 91'9	19'588 20'738 94'5	25'913 26'907 96'3	32'760 33'703 97'2	40'022 40'876 97'9	47'559 48'370 98'3	15
20	4'795 4'888 98'1	10'185 10'459 97'4	16'187 16'461 98'3	22'780 22'963 99'2	29'916 30'125 99'3	37'486 37'685 99'5	45'341 45'584 99'5	53'261 53'609 99'4	20
25	5'661 5'858 96'6	11'966 12'168 98'3	18'891 19'004 99'4	26'386 26'535 99'4	34'337 34'483 99'6	42'588 42'787 99'5	50'907 51'225 99'4	59'033 59'374 99'4	25
30	6'683 6'702 99'7	14'023 13'964 100'4	21'968 21'963 100'0	30'397 30'406 100'0	39'143 39'227 99'8	47'961 48'190 99'5	56'575 56'847 99'5	64'682 65'125 99'3	30
35	7'866 7'783 101'1	16'380 16'357 100'1	25'412 25'407 100'0	34'785 34'861 99'8	44'234 44'468 99'5	53'465 53'747 99'5	62'153 62'620 99'3	69'996 70'566 99'2	35
40	9'241 9'297 99'4	19'044 19'111 99'7	29'217 29'363 99'5	39'473 39'781 99'2	49'492 49'843 99'3	58'922 59'465 99'1	67'434 68'081 99'0	74'782 75'143 99'5	40
45	10'801 10'820 99'8	22'010 22'123 99'5	33'310 33'608 99'1	44'349 44'701 99'2	54'739 55'310 99'0	64'119 64'809 98'9	72'214 72'595 99'5	78'875 78'480 99'5	45
50	12'566 12'675 99'1	25'235 25'554 98'8	37'611 37'992 99'0	49'259 49'888 98'7	59'774 60'540 98'7	68'850 69'271 99'4	76'317 75'870 100'6	45 80'827 80'481 100'4	Age at Entry (x) 40
55	14'489 14'748 98'2	28'644 28'992 98'8	41'966 42'615 98'5	53'992 54'813 98'5	64'373 64'810 99'3	72'914 72'367 100'8	50 82'336 82'000 100'5	76'766 77'078 99'6	35
60	16'553 16'708 99'1	32'133 32'688 98'3	46'197 46'995 98'3	58'336 58'723 99'3	68'324 67'587 101'1	55 83'516 83'207 100'4	82'336 82'000 100'5	76'766 77'078 99'6	30
65	18'670 19'185 97'3	35'524 36'363 97'7	50'071 50'443 99'3	62'041 61'085 101'6	60 84'449 84'190 100'3	83'516 83'207 100'4	78'318 78'614 99'6	72'001 72'538 99'3	25
70	20'722 21'256 97'5	38'610 38'678 99'8	53'327 51'847 102'9	65 85'195 84'963 100'3	84'449 84'190 100'3	79'546 79'867 99'6	73'586 74'147 99'2	66'682 67'168 99'3	20
75	22'563 22'125 102'0	41'126 38'848 105'9	70 85'796 85'733 100'1	85'195 84'963 100'3	80'527 80'851 99'6	74'853 75'411 99'3	68'279 68'773 99'3	60'998 61'360 99'4	15
80	23'973 21'475 111'6	75 86'283 86'366 99'9	85'796 85'733 100'1	81'317 81'832 99'4	75'873 76'670 99'0	69'567 70'372 98'9	62'580 63'339 98'8	55'157 55'984 98'5	10
Age at Entry (x)	5	75	70	65	60	55	50	45	Age at Entry (x)

TABLE XIX.  
WHOLE-LIFE ASSURANCES.  
*Policy-Values* ( $100 {}_nV_x$ ).

0M(5) : HM.

3 PER-CENT.

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )								Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	40	
10	2'965 3'938 75'3	6'421 8'374 76'7	10'412 12'369 84'2	14'992 17'023 88'1	20'191 22'114 91'3	26'004 27'724 93'8	32'404 34'018 95'3	39'310 40'766 96'4	10
15	3'562 4'618 77'1	7'675 8'776 87'5	12'395 13'622 91'0	17'753 18'921 93'8	23'743 24'761 95'9	30'339 31'313 96'9	37'455 38'338 97'7	44'951 45'800 98'1	15
20	4'265 4'360 97'8	9'159 9'440 97'0	14'716 14'996 98'1	20'927 21'119 99'1	27'766 27'987 99'2	35'146 35'353 99'4	42'918 43'175 99'4	50'873 51'238 99'3	20
25	5'112 5'311 96'3	10'916 11'121 98'2	17'404 17'523 99'3	24'548 24'705 99'4	32'256 32'406 99'5	40'375 40'585 99'5	48'684 49'016 99'3	56'912 57'265 99'4	25
30	6'116 6'135 99'7	12'954 12'897 100'4	20'483 20'481 100'0	28'606 28'614 100'0	37'162 37'252 99'8	45'920 46'156 99'5	54'590 54'868 99'5	62'842 63'309 99'3	30
35	7'283 7'203 101'1	15'303 15'284 100'1	23'955 23'948 100'0	33'069 33'151 99'8	42'396 42'636 99'4	51'632 51'918 99'4	60'421 60'910 99'2	68'451 69'054 99'1	35
40	8'650 8'708 99'3	17'982 18'045 99'7	27'811 27'962 99'5	37'872 38'183 99'2	47'832 48'186 99'3	57'313 57'876 99'0	65'972 66'652 99'0	73'523 73'909 99'5	40
45	10'216 10'228 99'9	20'976 21'091 99'5	31'989 32'287 99'1	42'893 43'243 99'2	53'271 53'858 98'9	62'751 63'471 98'9	71'016 71'420 99'4	77'874 77'468 100'5	45
50	11'984 12'100 99'0	24'250 24'573 98'7	36'395 36'777 99'0	47'954 48'601 98'7	58'512 59'309 98'7	67'718 68'164 99'3	75'357 74'901 100'6	45 79'788 79'430 100'5	Age at Entry ( <i>x</i> )
55	13'936 14'189 98'2	27'735 28'074 98'8	40'867 41'526 98'4	52'863 53'708 98'4	63'322 63'781 99'3	72'001 71'446 100'8	50 81'260 80'912 100'4	75'451 75'788 99'6	40
60	16'033 16'180 99'1	31'292 31'857 98'2	45'230 46'053 98'2	57'383 57'792 99'3	67'467 66'725 101'1	55 82'406 82'083 100'4	76'953 77'274 99'6	70'380 70'953 99'2	35
65	18'172 18'703 97'2	34'773 35'640 97'6	49'246 49'645 99'2	61'256 60'301 101'6	60 83'306 83'035 100'3	82'406 82'083 100'4	76'953 77'274 99'6	70'380 70'953 99'2	30
70	20'287 20'833 97'4	37'975 38'060 99'8	52'651 51'168 102'9	65 84'018 83'774 100'3	83'306 83'035 100'3	78'131 78'481 99'6	71'895 72'496 99'2	64'742 65'258 99'2	25
75	22'189 21'761 102'0	40'601 38'318 106'0	70 84'587 84'524 100'1	84'018 83'774 100'3	79'064 79'419 99'6	73'093 73'695 99'2	66'246 66'772 99'2	58'749 59'128 99'4	20
80	23'662 21'162 111'8	75 85'044 85'133 99'9	84'587 84'524 100'1	79'809 80'369 99'3	74'052 74'909 98'9	67'448 68'307 98'7	60'219 61'016 98'7	52'623 53'490 98'4	15
			80'408 81'142 99'1	74'821 75'897 98'6	68'413 69'555 98'4	61'398 62'551 98'2	54'027 55'322 97'7	46'583 47'934 97'2	10
Age at Entry <i>x</i>	5	75	70	65	60	55	50	45	Age at Entry ( <i>x</i> )
DURATION ( <i>n</i> )									



TABLE XX.  
WHOLE-LIFE ASSURANCES.

$$O^{M(5)} : H^M.$$

*Policy-Values* ( $100 \text{ } _nV_x$ ).

**3½ PER-CENT.**

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )								Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	40	
10	2'562	5'603	9'193	13'390	18'235	23'752	29'930	36'702	10
	3'505	7'534	11'132	15'415	20'168	25'492	31'571	38'192	
	73'1	74'4	82'6	86'9	90'4	93'2	94'8	96'1	
15	3'121	6'806	11'113	16'085	21'747	28'088	35'038	42'465	15
	4'175	7'904	12'342	17'268	22'785	29'085	35'947	43'346	
	74'8	86'1	90'0	93'1	95'4	96'6	97'5	98'0	
20	3'803	8'249	13'382	19'226	25'772	32'945	40'612	48'576	20
	3'892	8'523	13'664	19'421	25'995	33'156	40'878	48'952	
	97'7	96'8	97'9	99'0	99'1	99'4	99'3	99'2	
25	4'622	9'957	16'023	22'837	30'294	38'264	46'542	54'839	25
	4'819	10'168	16'158	22'999	30'449	38'484	46'885	55'211	
	95'9	97'9	99'2	99'3	99'5	99'4	99'3	99'3	
30	5'594	11'964	19'098	26'916	35'272	43'952	52'651	61'038	30
	5'620	11'913	19'100	26'928	35'369	44'196	52'944	61'524	
	99'5	100'4	100'0	100'0	99'7	99'4	99'4	99'2	
35	6'748	14'304	22'586	31'437	40'631	49'845	58'729	66'930	35
	6'668	14'283	22'577	31'521	40'873	50'142	59'233	67'559	
	101'2	100'2	100'0	99'7	99'4	99'4	99'1	99'1	
40	8'103	16'984	26'476	36'335	46'216	55'743	64'537	72'274	40
	8'159	17'045	26'628	36'649	46'579	56'321	65'241	72'681	
	99'3	99'6	99'4	99'1	99'2	99'0	98'9	99'4	
45	9'664	19'993	30'721	41'474	51'841	61'410	69'829	76'869	45
	9'676	20'110	31'021	41'834	52'440	62'153	70'254	76'458	
	99'9	99'4	99'0	99'1	98'9	98'8	99'4	100'5	
50	11'433	23'310	35'213	46'689	57'282	66'601	74'394		Age at Entry ( <i>x</i> )
	11'552	23'632	35'603	47'346	58'099	67'068	73'936	45	
	99'0	98'6	98'9	98'6	98'6	99'3	100'6		
55	13'410	26'849	39'806	51'767	62'290	71'089		78'743	40
	13'657	27'192	40'469	52'626	62'767	70'532	50	78'379	
	98'2	98'7	98'4	98'4	99'2	100'8		100'5	
60	15'521	30'485	44'298	56'450	66'611		80'177	74'145	35
	15'676	31'052	45'133	56'877	65'871	55	79'820	74'503	
	99'0	98'2	98'2	99'2	101'1		100'4	99'5	
65	17'713	34'064	48'449	60'477		81'286	75'591	68'780	30
	18'235	34'933	48'861	59'526	60	80'954	75'936	69'382	
	97'1	97'5	99'2	101'6		100'4	99'5	99'1	
70	19'871	37'352	51'969		82'151	76'719	70'223	62'839	25
	20'422	37'456	50'500	65	81'872	77'095	70'857	63'378	
	97'3	99'7	102'9		100'3	99'5	99'1	99'1	
75	21'816	40'058		82'830	77'605	71'356	64'252	56'557	20
	21'406	37'797	70	82'578	77'987	71'991	64'804	56'954	
	101'9	106'0		100'3	99'5	99'1	99'1	99'3	
80	23'333		83'366	78'303	72'250	65'368	57'913	50'180	15
	20'855	75	83'305	78'906	73'161	66'273	58'751	51'083	
	111'9		100'1	99'2	98'8	98'6	98'6	98'2	
		83'792	78'859	72'960	66'255	58'991	51'457	43'939	10
		83'890	79'645	74'102	67'455	60'197	52'798	45'332	
		99'9	99'0	98'5	98'2	98'0	97'5	96'9	
Age at Entry ( <i>x</i> )	5	75	70	65	60	55	50	45	Age at Entry ( <i>x</i> )
DURATION ( <i>n</i> )									













TABLE XXVIII.  
ENDOWMENT ASSURANCE (*Death or 60*).

$$\text{Policy-Values } 100 \left( 1 - \frac{|60-x-n a_{x+n}|}{|60-x a_x|} \right)$$

EM: 0M

3½ PER-CENT.

Age at Entry (x)	DURATION (n)							Age at Entry (x)
	5	10	15	20	25	30	35	
20	6.386	14.039	23.046	33.589	45.867	60.386	78.012	20
	6.726	14.452	23.327	33.615	45.695	60.083	77.627	
	94.9	97.1	98.8	99.9	100.4	100.5	100.5	
25	8.175	17.797	29.059	42.174	57.683	76.512	...	25
	8.283	17.798	28.828	41.779	57.204	76.013	...	
	98.7	100.0	100.8	100.9	100.8	100.7	...	
30	10.478	22.743	37.026	53.916	74.421	...	...	30
	10.375	22.400	36.521	53.339	73.847	...	...	
	101.0	101.5	101.4	101.1	100.8	...	...	
35	13.700	29.656	48.522	71.427	...	...	...	35
	13.418	29.173	47.938	70.820	...	...	...	
	102.1	101.7	101.2	100.9	...	...	...	
40	18.488	40.350	66.801	...	...	...	...	40
	18.197	39.870	66.298	...	...	...	...	
	101.6	101.2	100.9	...	...	...	...	
45	26.820	59.382	...	...	...	...	...	45
	26.494	58.801	...	...	...	...	...	
	101.2	101.0	...	...	...	...	...	
50	44.495	...	...	...	...	...	...	50
	43.951	...	...	...	...	...	...	
	101.2	...	...	...	...	...	...	
Age at Entry (x)	5	10	15	20	25	30	35	Age at Entry (x)

DURATION (n)

EM: 0M

## TABLE XXIX.

4 PER-CENT.

Age at Entry (x)	DURATION (n)							Age at Entry (x)
	5	10	15	20	25	30	35	
20	5.787	12.882	21.413	31.621	43.761	58.447	76.692	20
	6.136	13.324	21.744	31.695	43.631	58.172	76.309	
	94.3	96.7	98.5	99.8	100.3	100.5	100.5	
25	7.530	16.586	27.421	40.306	55.895	75.261	...	25
	7.658	16.628	27.230	39.946	55.437	74.760	...	
	98.3	99.7	100.7	100.9	100.8	100.7	...	
30	9.793	21.511	35.445	52.303	73.246	...	...	30
	9.714	21.196	34.966	51.742	72.667	...	...	
	100.8	101.5	101.4	101.1	100.8	...	...	
35	12.990	28.437	47.125	70.341	...	...	...	35
	12.717	27.969	46.550	69.726	...	...	...	
	102.2	101.7	101.2	100.9	...	...	...	
40	17.753	39.231	65.914	...	...	...	...	40
	17.474	38.762	65.315	...	...	...	...	
	101.6	101.2	100.9	...	...	...	...	
45	26.114	58.556	...	...	...	...	...	45
	25.796	57.971	...	...	...	...	...	
	101.2	101.0	...	...	...	...	...	
50	43.908	...	...	...	...	...	...	50
	43.360	...	...	...	...	...	...	
	101.3	...	...	...	...	...	...	
Age at Entry (x)	5	10	15	20	25	30	35	Age at Entry (x)

DURATION (n)

TABLE XXX.  
ENDOWMENT ASSURANCE (*Death or 60*).

$[E^M \text{ \& } E^{M(5)}]$  Policy-Values  $100 \left( 1 - \frac{60-x-n a_{x+n}}{60-x a_x} \right)$   
 $[H^M \text{ \& } H^{M(5)}]$   $2\frac{1}{2}$  PER-CENT.

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )							Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	
20	9'124	17'398	27'113	38'129	50'404	64'355	80'608	20
	9'575	17'147	26'450	36'977	49'237	63'307	79'775	
	95'3	101'5	102'5	103'1	102'4	101'7	101'0	
25	10'457	20'989	32'930	46'236	61'360	78'979	...	25
	10'457	20'512	31'889	45'138	60'345	78'142	...	
	100'0	102'3	103'3	102'4	101'7	101'1	...	
30	12'592	25'802	40'523	57'254	76'745	...	...	30
	12'362	24'905	39'513	56'279	75'901	...	...	
	101'9	103'6	102'6	101'7	101'1	...	...	
35	15'719	32'440	51'444	73'584	...	...	...	35
	15'203	31'699	50'630	72'787	...	...	...	
	103'4	102'3	101'6	101'1	...	...	...	
40	20'307	42'724	68'841	...	...	...	...	40
	20'052	42'212	68'147	...	...	...	...	
	101'3	101'2	101'0	...	...	...	...	
45	28'392	61'043	...	...	...	...	...	45
	28'158	60'400	...	...	...	...	...	
	100'8	101'1	...	...	...	...	...	
50	45'675	...	...	...	...	...	...	50
	45'158	...	...	...	...	...	...	
	101'1	...	...	...	...	...	...	
Age at Entry ( <i>x</i> )	5	10	15	20	25	30	35	Age at Entry ( <i>x</i> )

$[E^M \text{ \& } E^{M(5)}]$  TABLE XXXI. 3 PER-CENT.  
 $[H^M \text{ \& } H^{M(5)}]$

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )							Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	
20	8'393	16'066	25'297	36'007	48'211	62'391	79'318	20
	8'864	15'862	24'699	34'912	47'084	61'366	78'476	
	94'7	101'3	102'4	103'1	102'4	101'7	101'1	
25	9'708	19'638	31'160	44'288	59'542	77'751	...	25
	9'727	19'208	30'165	43'225	58'549	76'906	...	
	99'8	102'2	103'3	102'5	101'7	101'1	...	
30	11'824	24'466	38'871	55'608	75'588	...	...	30
	11'611	23'599	37'886	54'651	74'735	...	...	
	101'8	103'7	102'6	101'8	101'1	...	...	
35	14'939	31'160	50'009	72'508	...	...	...	35
	14'442	30'442	49'216	71'707	...	...	...	
	103'4	102'4	101'6	101'1	...	...	...	
40	19'538	41'569	67'867	...	...	...	...	40
	19'291	41'075	67'172	...	...	...	...	
	101'3	101'2	101'0	...	...	...	...	
45	27'648	60'211	...	...	...	...	...	45
	27'430	59'569	...	...	...	...	...	
	100'8	101'1	...	...	...	...	...	
50	45'080	...	...	...	...	...	...	50
	44'569	...	...	...	...	...	...	
	101'2	...	...	...	...	...	...	
Age at Entry ( <i>x</i> )	5	10	15	20	25	30	35	Age at Entry ( <i>x</i> )

TABLE XXXII.  
ENDOWMENT ASSURANCE (*Death or 60*).

[ $E^M$  &  $E^{M(5)}$ ] *Policy-Values*  $100 \left( 1 - \frac{{}_{60-x-n}a_{x+n}}{{}_{60-x}a_x} \right)$   $3\frac{1}{2}$  PER-CENT.  
[ $H^M$  &  $H^{M(5)}$ ]

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )							Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	
20	7'713	14'812	23'572	33'964	46'063	60'445	78'008	20
	8'215	14'670	23'048	32'930	44'975	59'440	77'162	
	93'9	101'0	102'3	103'1	102'4	101'7	101'1	
25	9'000	18'359	29'459	42'383	57'747	76'507	...	25
	9'051	17'980	28'513	41'351	56'769	75'658	...	
	99'4	102'1	103'3	102'5	101'7	101'1	...	
30	11'090	23'179	37'253	53'985	74'416	...	...	30
	10'908	22'348	36'294	53'041	73'559	...	...	
	101'7	103'7	102'6	101'8	101'2	...	...	
35	14'188	29'909	48'599	71'421	...	...	...	35
	13'714	29'210	47'819	70'619	...	...	...	
	103'5	102'4	101'6	101'1	...	...	...	
40	18'782	40'439	66'885	...	...	...	...	40
	18'543	39'956	66'192	...	...	...	...	
	101'3	101'2	101'1	...	...	...	...	
45	26'930	59'373	...	...	...	...	...	45
	26'718	58'738	...	...	...	...	...	
	100'8	101'1	...	...	...	...	...	
50	44'484	...	...	...	...	...	...	50
	43'978	...	...	...	...	...	...	
	101'2	...	...	...	...	...	...	
Age at Entry ( <i>x</i> )	5	10	15	20	25	30	35	Age at Entry ( <i>x</i> )

[ $E^M$  &  $E^{M(5)}$ ]

TABLE XXXIII.

4 PER-CENT.

[ $H^M$  &  $H^{M(5)}$ ]

Age at Entry ( <i>x</i> )	DURATION ( <i>n</i> )							Age at Entry ( <i>x</i> )
	5	10	15	20	25	30	35	
20	7'088	13'643	21'942	32'003	43'965	58'509	76'692	20
	7'607	13'549	21'473	31'019	42'919	57'522	75'834	
	93'2	100'7	102'2	103'2	102'4	101'7	101'1	
25	8'338	17'147	27'826	40'523	55'961	75'261	...	25
	8'412	16'806	26'920	39'527	54'998	74'398	...	
	99'1	102'0	103'4	102'5	101'8	101'2	...	
30	10'400	21'949	35'679	52'374	73'246	...	...	30
	10'239	21'150	34'753	51'445	72'377	...	...	
	101'6	103'8	102'7	101'8	101'2	...	...	
35	13'475	28'697	47'204	70'341	...	...	...	35
	13'010	28'017	46'433	69'525	...	...	...	
	103'6	102'4	101'7	101'2	...	...	...	
40	18'052	39'322	65'914	...	...	...	...	40
	17'831	38'852	65'213	...	...	...	...	
	101'2	101'2	101'1	...	...	...	...	
45	26'225	58'556	...	...	...	...	...	45
	26'016	57'710	...	...	...	...	...	
	100'8	101'1	...	...	...	...	...	
50	43'908	...	...	...	...	...	...	50
	43'393	...	...	...	...	...	...	
	101'2	...	...	...	...	...	...	
Age at Entry ( <i>x</i> )	5	10	15	20	25	30	35	Age at Entry ( <i>x</i> )







## TABLE XXXVIII.

## RATIOS OF TERM ANNUITIES DUE.

$$H^M, H^{M(5)}, O^M, O^{M(5)}, E^M, E^{M(5)}.$$

*For Comparison of Endowment Assurance Policy-Values.*

$$\frac{60-x \cdot a_x}{60-x \cdot a'_x}$$

3 PER-CENT.

Age ( $x$ )	$\frac{H^M}{O^M}$	$\frac{H^M}{O^{M(5)}}$	$\frac{H^M}{E^M}$	$\frac{H^M}{E^{M(5)}}$	$\frac{H^{M(5)}}{O^M}$	$\frac{H^{M(5)}}{O^{M(5)}}$	$\frac{H^{M(5)}}{E^M}$	$\frac{H^{M(5)}}{E^{M(5)}}$	$\frac{O^M}{O^{M(5)}}$	$\frac{O^M}{E^M}$	$\frac{O^M}{E^{M(5)}}$	$\frac{O^{M(5)}}{E^M}$	$\frac{O^{M(5)}}{E^{M(5)}}$	$\frac{E^M}{E^{M(5)}}$	Age ( $x$ )
	$a_x$	$b_x$	$c_x$	$f_x$	$g_x$	$h_x$	$j_x$	$k_x$	$n_x$	$r_x$	$s_x$	$t_x$	$v_x$	$w_x$	
20	·9756	·9978	·9518	·9818	·9453	·9669	·9223	·9514	1·0228	·9756	1·0064	·9539	·9840	1·0315	20
25	·9816	·9973	·9543	·9684	·9598	·9752	·9331	·9469	1·0160	·9722	·9866	·9569	·9710	1·0148	25
30	·9855	·9956	·9571	·9661	·9733	·9833	·9453	·9541	1·0103	·9712	·9803	·9613	·9703	1·0094	30
35	·9896	·9959	·9625	·9693	·9796	·9858	·9527	·9594	1·0063	·9726	·9794	·9665	·9733	1·0071	35
40	·9935	·9974	·9696	·9752	·9863	·9901	·9625	·9681	1·0039	·9759	·9816	·9721	·9778	1·0058	40
45	·9951	·9977	·9748	·9781	·9891	·9917	·9690	·9725	1·0026	·9796	·9832	·9771	·9807	1·0037	45
50	·9971	·9988	·9814	·9827	·9920	·9937	·9765	·9778	1·0017	·9843	·9856	·9826	·9839	1·0013	50
55	·9989	·9995	·9927	·9927	·9967	·9973	·9906	·9905	1·0006	·9938	·9938	·9932	·9932	1·0000	55

**NOTE.**
 ${}_nV_{x:\overline{60-x}|}$  by the

 $H^M$  Table,

" "

" "

" "

 $[H^M \text{ \& } H^{M(5)}]$  Tables,

" "

" "

" "

 $O^M$  Table,

" "

" "

 $[O^M \text{ \& } O^{M(5)}]$  Tables,

" "

 $E^M$  Table,

 ${}_nV_{x:\overline{60-x}|}$  by the

 $O^M$  Table,
as  $a_x$ 
 $[O^M \text{ \& } O^{M(5)}]$  Tables,
"  $a_x$ 
 $E^M$  Table,
"  $c_x$ 
 $[E^M \text{ \& } E^{M(5)}]$  Tables,
"  $c_x$ 
 $O^M$  Table,
"  $a_x$ 
 $[O^M \text{ \& } O^{M(5)}]$  Tables,
"  $a_x$ 
 $E^M$  Table,
"  $c_x$ 
 $[E^M \text{ \& } E^{M(5)}]$  Tables,
"  $c_x$ 
 $[O^M \text{ \& } O^{M(5)}]$  Tables,

" 1

 $E^M$  Table,
"  $r_x$ 
 $[E^M \text{ \& } E^{M(5)}]$  Tables,
"  $r_x$ 
 $E^M$  Table,
"  $r_x$ 
 $[E^M \text{ \& } E^{M(5)}]$  Tables,
"  $v_x$ 
 $[E^M \text{ \& } E^{M(5)}]$  Tables,

" 1

 $a_{x+n}$  $b_{x+n}$  $c_{x+n}$  $f_{x+n}$  $g_{x+n}$  $h_{x+n}$  $j_{x+n}$  $k_{x+n}$  $n_{x+n}$  $r_{x+n}$  $s_{x+n}$  $t_{x+n}$  $v_{x+n}$  $w_{x+n}$

TABLE XXXIX.  
BRITISH OFFICES LIFE TABLES, 1893.  
WHOLE-LIFE ASSURANCES. MALES. WITH PROFITS.  
*Select Tables joining Truncated Ten-year Table (Select Data).*  
*Probability of Dying in the Tables as under:*  
(Compiled by MR. G. F. HARDY, F.I.A.)

0[M]

Log e = .039

Age at Date of Assurance	YEARS ELAPSED SINCE DATE OF ASSURANCE										Ultimate Table	Age Attained
	0	1	2	3	4	5	6	7	8	9		
15	.00248	.00421	.00482	.00516	.00544	.00570	.00597	.00625	.00653	.00681	.00700	25
20	.00261	.00437	.00502	.00539	.00571	.00602	.00633	.00665	.00699	.00733	.00757	30
25	.00281	.00463	.00532	.00576	.00614	.00651	.00690	.00739	.00771	.00814	.00846	35
30	.00312	.00502	.00580	.00633	.00680	.00728	.00778	.00829	.00883	.00940	.00986	40
35	.00361	.00564	.00655	.00722	.00789	.00849	.00916	.00986	.01060	.01138	.01205	45
40	.00441	.00660	.00773	.00862	.00948	.01038	.01132	.01231	.01336	.01447	.01546	50
45	.00558	.00811	.00956	.01080	.01204	.01334	.01469	.01613	.01766	.01929	.02079	55
50	.00746	.01047	.01244	.01422	.01603	.01794	.01995	.02210	.02433	.02680	.02907	60
55	.01040	.01415	.01692	.01955	.02226	.02511	.02814	.03136	.03479	.03845	.04192	65
60	.01499	.01765	.02390	.02783	.03193	.03625	.04084	.04571	.05089	.05642	.06169	70
65	.02213	.02887	.03474	.04068	.04689	.05344	.06039	.06776	.07558	.08390	.09187	75
70	.03322	.04270	.05149	.06046	.06986	.07977	.09023	.10128	.11298	.12536	.13720	80

TABLE XL.

WHOLE-LIFE PARTICIPATING ASSURANCES. MALES.

Select Net Premiums at 3 per-cent.

(100  $P_{[x]}$ )

(Compiled by MR. G. F. HARDY, F.I.A.)

Age at Entry [ <i>x</i> ]	Dr. Sprague's Select Tables $H^{[M]}$	New Select Tables $O^{[M]}$
20	1·563	1·365
25	1·703	1·551
30	1·925	1·785
35	2·218	2·081
40	2·602	2·457
45	3·106	2·940
50	3·755	3·564
55	4·635	4·377
60	5·827	5·446
65	7·433	6·854

TABLE XLI.

WHOLE-LIFE PARTICIPATING ASSURANCES. MALES.

Comparison of Select Policy Values at 3 per-cent.

(Compiled by MR. G. F. HARDY, F.I.A.)

Age at date of Assur- ance	Mortality Table	Number of Years in Force				
		10	20	30	40	50
20	Sprague, $H^{[M]}$ . . .	8·122	19·642	34·099	50·114	65·902
	New, $O^{[M]}$ . . . .	10·160	21·722	35·722	51·240	66·443
	$H^M$ and $H^{M(5)}$ . . .	10·915	22·085	36·103	51·631	66·938
30	Sprague, $H^{[M]}$ . . .	13·146	28·772	46·081	63·146	77·146
	New, $O^{[M]}$ . . . .	14·031	29·407	46·450	63·146	77·103
	$H^M$ and $H^{M(5)}$ . . .	13·963	29·443	46·589	63·492	77·361
40	Sprague, $H^{[M]}$ . . .	18·797	38·530	57·984	73·345	...
	New, $O^{[M]}$ . . . .	19·310	38·790	57·875	73·328	...
	$H^M$ and $H^{M(5)}$ . . .	18·996	38·681	58·087	74·009	...
50	Sprague, $H^{[M]}$ . . .	25·688	49·206	68·601	...	...
	New, $O^{[M]}$ . . . .	26·177	49·195	68·435	...	...
	$H^M$ and $H^{M(5)}$ . . .	25·180	48·858	68·286	...	...
60	Sprague, $H^{[M]}$ . . .	33·424	58·714	...	...	...
	New, $O^{[M]}$ . . . .	34·433	59·264	...	...	...
	$H^M$ and $H^{M(5)}$ . . .	32·198	57·954	...	...	...



## TABLE XLII.

## WHOLE-LIFE NON-PARTICIPATING ASSURANCES. MALES.

*Rates of Mortality.**Full Aggregate, and Truncated Aggregate Table.*

Age ( <i>x</i> )	Full Aggregate Table NM	Truncated Aggregate Table NM <sup>(5)</sup>	Age ( <i>x</i> )	Full Aggregate Table NM	Truncated Aggregate Table NM <sup>(5)</sup>
20	·00668	·00261	60	·03044	·03084
21	·00711	·00379	61	·03256	·03301
22	·00686	·00538	62	·03469	·03511
23	·00614	·00659	63	·03757	·03818
24	·00620	·00835	64	·04039	·04101
25	·00636	·00831	65	·04273	·04349
26	·00613	·00896	66	·04653	·04725
27	·00620	·00885	67	·05061	·05141
28	·00663	·00907	68	·05423	·05489
29	·00657	·00833	69	·05842	·05914
30	·00665	·00835	70	·06450	·06508
31	·00699	·00812	71	·06945	·07003
32	·00726	·00834	72	·07510	·07563
33	·00740	·00826	73	·08237	·08299
34	·00779	·00866	74	·09054	·09100
35	·00817	·00900	75	·09788	·09851
36	·00850	·00933	76	·10695	·10841
37	·00904	·00971	77	·11722	·11687
38	·00961	·01033	78	·13213	·12700
39	·01016	·01072	79	·13700	·13741
40	·01075	·01143	80	·14848	·14856
41	·01138	·01196	81	·16086	·16083
42	·01176	·01219	82	·17358	·17355
43	·01219	·01246	83	·18775	·18760
44	·01273	·01292	84	·20145	·20122
45	·01304	·01325	85	·21752	·21701
46	·01329	·01340	86	·23311	·23258
47	·01384	·01413	87	·24544	·24468
48	·01439	·01477	88	·26439	·26345
49	·01487	·01532	89	·28231	·28181
50	·01582	·01633	90	·29779	·29309
51	·01698	·01759	91	·30934	·31124
52	·01803	·01860	92	·33876	·33931
53	·01833	·01987	93	·33990	·34074
54	·02048	·02127	94	·35448	·35206
55	·02172	·02247	95	·36416	·37572
56	·02302	·02374	96	·45455	·44445
57	·02461	·02535	97	·45000	·45000
58	·02625	·02679	98	·60607	·57576
59	·02821	·02862	99	·76924	·78571
			100	1·00000	1·00000

## TABLE XLIII.

## WHOLE-LIFE PARTICIPATING ASSURANCES. FEMALES.

*Rates of Mortality.**Full Aggregate, and Truncated Aggregate Table.*

Age ( <i>x</i> )	Full Aggregate Table OF	Truncated Aggregate Table OF <sup>(5)</sup>	Age ( <i>x</i> )	Full Aggregate Table OF	Truncated Aggregate Table OF <sup>(5)</sup>
20	·00532	·00550	60	·02449	·02496
21	·00588	·00543	61	·02695	·02692
22	·00646	·00536	62	·02812	·02868
23	·00683	·00589	63	·03008	·03072
24	·00684	·00565	64	·03241	·03296
25	·00701	·00611	65	·03543	·03591
26	·00727	·00638	66	·03847	·03911
27	·00784	·00697	67	·04278	·04321
28	·00823	·00727	68	·04701	·04725
29	·00851	·00788	69	·05175	·05193
30	·00900	·00848	70	·05661	·05674
31	·00907	·00873	71	·06181	·06183
32	·00873	·00882	72	·06682	·06689
33	·00869	·00910	73	·07280	·07295
34	·00910	·00961	74	·07914	·07934
35	·00893	·00976	75	·08598	·08618
36	·00934	·01031	76	·09361	·09384
37	·01008	·01115	77	·10254	·10271
38	·01059	·01169	78	·11275	·11290
39	·01071	·01174	79	·12379	·12384
40	·01108	·01198	80	·13635	·13634
41	·01117	·01194	81	·15027	·15016
42	·01100	·01162	82	·16460	·16463
43	·01109	·01163	83	·17817	·17815
44	·01124	·01180	84	·19705	·19242
45	·01118	·01180	85	·20646	·20645
46	·01151	·01222	86	·21873	·21876
47	·01196	·01306	87	·23179	·23161
48	·01230	·01311	88	·24356	·24347
49	·01272	·01361	89	·25631	·25645
50	·01370	·01451	90	·26871	·26876
51	·01434	·01509	91	·28129	·28102
52	·01500	·01572	92	·29200	·29180
53	·01579	·01649	93	·31693	·31729
54	·01687	·01742	94	·34323	·34361
55	·01762	·01821	95	·38388	·38254
56	·01859	·01920	96	·45550	·45652
57	·01993	·02039	97	·56731	·56999
58	·02148	·02187	98	·75556	·74418
59	·02286	·02338	99	1·00000	1·00000

TABLE XLIV.  
*Comparison of Rates of Mortality ( $q_x$ ).*

Age	INSTITUTE OF ACTUARIES' LIFE TABLES, 1863			BRITISH OFFICES LIFE TABLES, 1893						Age
				Whole-Life Assurances :—Males.				Whole-Life Assurances :—Females		
	Males		Females	Participating		Non-Participating		Participating		
	HM	HM(5)	HF	OM	OM(5)	NM	NM(5)	OF	OF(5)	
20	·00633	·00833	·00688	·00404	·00652	·00668	·00261	·00532	·00550	20
25	·00663	·01051	·01081	·00481	·00689	·00636	·00831	·00701	·00611	25
30	·00772	·00920	·01185	·00595	·00747	·00665	·00835	·00900	·00848	30
35	·00877	·01000	·01176	·00738	·00837	·00817	·00900	·00893	·00976	35
40	·01031	·01132	·01255	·00915	·00978	·01075	·01143	·01108	·01198	40
45	·01219	·01294	·01362	·01153	·01200	·01304	·01325	·01118	·01180	45
50	·01595	·01712	·01445	·01504	·01545	·01582	·01633	·01370	·01451	50
55	·02103	·02219	·01827	·02045	·02083	·02172	·02247	·01762	·01821	55
60	·02968	·03064	·02368	·02887	·02921	·03044	·03084	·02449	·02496	60
65	·04343	·04461	·03738	·04196	·04221	·04273	·04349	·03543	·03591	65
70	·06219	·06284	·05564	·06207	·06219	·06450	·06508	·05661	·05674	70
75	·09836	·09949	·09485	·09264	·09267	·09788	·09851	·08598	·08618	75
80	·14465	·14577	·12179	·13844	·13850	·14848	·14856	·13635	·13634	80
85	·20989	·21010	·18564	·20569	·20569	·21752	·21701	·20646	·20645	85
90	·27945	·28244	·27504	·30075	·30075	·29779	·29309	·26871	·26876	90
95	·63704	·63636	·20858	·42473	·42473	·36416	·37572	·38388	·38254	95

DISCUSSION.

The PRESIDENT (Mr. C. D. Higham) noticed that Mr. Ackland had what he was disposed to call the hardihood to say that he hoped his paper might lead someone better qualified than himself to deal with the matter; but he (the President) hoped the members would be able to learn from Mr. Ackland what other person was in his mind, as it would be very difficult to say who was better qualified than Mr. Ackland himself.

Mr. L. F. HOVIL said that this paper gave them all the comparisons and data necessary to enable them to see the general bearings of the New Mortality Experience. He should have liked to have found rather more of Mr. Ackland in the paper, and his conclusions from the figures, but it had been largely left to the members to draw their own. Some of the teachings were probably comparatively obvious, but it would be a long time before they would exhaust all that the figures had to reveal. It would have added to the interest of the diagrams if the author had given the mortality curve for the old Seventeen Offices Table, for, after all, the relationship which the H<sup>M</sup> Table bore to the old Seventeen Offices Table had been to some extent the basis on which they had framed their anticipation of what the



$O^M$  would be. The Seventeen Offices curve lay very evenly over the  $H^M$  curve; and just as the  $O^M$  showed very much lighter mortality than the  $H^M$  at the early parts of the table, so was the  $H^M$  better than the old Seventeen Offices Table. The most noticeable feature in the  $O^M$  curve was the great saving in life at the early part of the table, and that was no doubt due to selection. It was shown in all the subsequent tables and figures in the paper, especially in Mr. R. P. Hardy's table,\* in which he gave the survivors from 10,000 entrants at different ages, as well as in Mr. G. F. Hardy's select tables.† The improvement in mortality at the younger ages was after all exactly what might have been expected, as the whole machinery of selection in offices, quite apart from any advances in medical science, had been wonderfully improved of late years. The  $E^M$  mortality was, as had also been anticipated, very low throughout, but there was such a very large proportion of recently admitted lives in these figures that he did not know how far they could be acted on. In the volume containing the unadjusted data, out of the 109,000 entrants, only 21,000 were of ten years' duration and over, compared with which the whole-life figures gave 331,000 of ten years' duration and over, out of a total number of 551,000 entrants. In fact, it might be said that the endowment assurance business had not reached a sufficiently advanced stage. There was quite a noticeable improvement in the female mortality. The author did not refer to it in his paper, but apparently the female mortality had improved to a much greater extent than the male, that was to say, the difference between the  $H^F$  and the  $O^F$  was much greater than the difference between the  $H^M$  and the  $O^M$ . That was shown by the age at which the female mortality became better than the male; it was about 45 in the  $O^M$  table, compared with about 50 in the  $H^M$ . Why that was so he did not know. Perhaps it had something to do with the publication of the  $H^F$  table, the offices after the publication of that table having been more careful in the selection of their female lives; or it may be there was a different class of female applicants coming in, income-earning women assuring their own lives. In the old days, policies on the lives of females were probably effected almost entirely for business purposes. The non-profit rates were rather dismal reading. Some day it might be possible to have rates based on discounted bonus policies, and it would be interesting to see if they showed the same high mortality. The main interest of the paper really settled around the question of the policy reserves, and the first question asked when the new tables were published was, how they would affect the policy reserves. The answer roughly was that when changing from one table to another, as long as they were analagous tables, the  $H^M$  to the  $O^M$ , or the  $H^M$  and  $H^{M(5)}$  to the  $O^M$  and  $O^{M(5)}$ , the change was not very great. He had not seen any published figures showing the effect of the change from  $H^M$  to  $O^M$ , but he had himself valued a rather large group of whole-life policies, where the total liability was something over three millions, and there was only an increase of £7,000 by

\* *J.I.A.* xxxvi, 316, and Table IV of the present paper.

† Tables XXXIX to XLI of the present paper.



using the  $O^M$  instead of the  $H^M$ . He imagined there would be plenty of food for reflection in Mr. Ackland's figures for those interested in the discussion on Net Premium Valuations at the last meeting. One of the strong points of the Net Premium Valuation was its effect at the time of a change of valuation basis. The paper proved that the new mortality was a great deal lighter, and it also showed that it would be necessary to keep equally heavy reserves, if not heavier. What did that mean but that under a net premium valuation they kept intact for the future all the sources of profit enjoyed in the past, and what was lost by valuing under the lighter mortality table was gained by increased loading reserve. There was a comparison made of the reserves of the  $O^{M(5)}$  and the  $H^M$  tables, and he did not quite know why it was given. Possibly it was that, seeing the mortality curves were very closely allied, Mr. Ackland thought it was well to give some warning that the reserves were not necessarily the same. The  $E^M$  valuations were very interesting, and he had made some calculations on the same lines as Mr. Ackland's Table No. 35, showing what the effect would be of valuing by interest alone, and the differences were very slight indeed—in fact, out of twenty-eight cases, in ten the values were less by the interest table, and in about eighteen they were more. Totalling up the twenty-eight valuations made in that way, the  $E^M$  table provided for a reserve of £1,154 against a reserve of £1,160 valuing by interest only, which showed really that the  $E^M$  table was becoming an interest table pure and simple.

Mr. H. E. NIGHTINGALE said that everyone knew that mortality tables, in addition to being required for the calculation of premiums and reserves, were necessary for the testing of the individual experience of offices from time to time—as, for example, the comparison of the actual with the expected claims, and the death strain. For those purposes the use of select tables appeared essential, and the value of results and comparisons based on aggregate tables depended on the degree to which they approximated to the correct figures based on select tables. Therefore, the calculations in Mr. Ackland's paper based upon select tables were those which interested him in the largest degree. The New Mortality Experience was given separately for all the chief classes of assurances, but the author did not discuss the question upon which portion of the Experience the prospectus premiums should be based. The  $O^M$  exposed to risk was enormously greater than in either the non-participating or endowment assurance experience; and, moreover, the average duration of the policies was longer. Under those circumstances, it would seem that for estimating office premiums the whole-life participating experience should form the basis of calculation, except for term assurances, and that any deviation in mortality should, if desired, be given effect to in the loading. One great lesson taught by the New Experience was the influence exerted on the rate of mortality by the motive inducing assurance. With regard to rates of mortality and premiums, comparing the  $O^M$  and  $O^{M(5)}$  tables with the  $H^M$  and  $H^{M(5)}$ , it was interesting to notice that the rate of mortality in the former tables increased constantly from

age 10 upwards, whereas in the latter tables the rates decreased to age 14 and then increased onwards. The improvement in mortality shown by the new  $O^M$  experience was remarkable, and had a notable effect on the premiums. At age 20 the new  $3\frac{1}{2}$  per-cent select premium was about 12 per-cent less than Dr. Sprague's; at age 30, 7·8 per-cent; at age 40, 6 per-cent; and at age 50 nearly 5 per-cent. The non-participating rates of mortality were considerably heavier throughout than those of the participating experience, due, he thought, to the fact that a large number of policies included were effected in connection with loan business. The difference between the  $3\frac{1}{2}$  per-cent. participating and non-participating (aggregate) net premiums was roughly about 1s. 9d. per-cent at age 25, increasing to 3s. per-cent at age 60. It was noticeable that the  $O^F$  mortality rates exceeded the  $O^M$  up to the age of 44, and that after age 45 the  $O^F$  rates were less. Hence for term risks during the child-bearing period that fact should be given effect to in the office premiums. With regard to endowment assurances, the light mortality experienced by persons assured under those policies was remarkable, due partly to the small average duration of the experience. The net premiums were consequently affected very considerably. For many reasons he thought it would be unwise to base the office premiums on those tables. Temporary assurances and ascending premium policies exhibited a much higher rate of mortality than the  $O^M$ . The author's graduated table of rates was, he feared, not reliable, and his grouping of the ages at entry completely blinded the real character of the mortality rates below age 50. The New Experience presented the same curious fall in mortality from age 20 to age 30 shown by Dr. Sprague's select tables. He (Mr. Nightingale) had prepared a comparative table of  $3\frac{1}{2}$  per-cent premiums, which showed that the new figures did not differ materially from those of Dr. Sprague, except as regarded ages at entry below 30 and also for all ages when the term was one year only. In such cases the new rates were considerably less than the old ones. When discussing the question of policy reserves, most actuaries would be disposed to consider first what would be the effect of changing the present valuation basis to the corresponding tables of the New Experience, and therefore the comparison of the  $O^M$  and  $H^M$ ,  $O^{M(5)}$  and  $H^{M(5)}$ , &c., would interest them more than the comparisons of like with unlike. Mr. Ackland deserved thanks for calculating such a large number of policy values; but it was difficult to express a decided opinion on the effect on reserves without actual application in practice. In the case of an office of moderate age and sustained business, the change from  $H^M$  to  $O^M$ , or from  $H^M$  and  $H^{M(5)}$  to  $O^M$  and  $O^{M(5)}$ , would appear to have only a very moderate effect, probably a slight increase of reserve in the former case and a decrease in the latter. In regard to endowment assurances, it was noticeable that for an office to change its 3 per-cent basis from  $H^M$  and  $H^{M(5)}$  to  $E^M$  and  $E^{M(5)}$  would require an increase of between 3 and 4 per-cent in the reserve, or from the old combined basis to the new combined  $O^M$  tables—an increase of about 2 per-cent. Mr. Hardy's interesting table of Select Policy-Values\* showed that,

\* Table XLI of the present paper.



for policies of ten years or more in force, the reserve would not differ materially from an  $H^M$  and  $H^{M(5)}$  valuation. It was important to note, however, that for contracts under five years in force, the reserve by the New Select Tables was from 20 to 30 per-cent in excess of the  $H^M$  results. That fact was of great consequence, as showing that the Combined Aggregate Tables, while providing a safe valuation basis, furnished to a considerable extent the elasticity necessary for meeting a rapidly expanding business with its accompanying pressure of expenditure. As to the effect of selection, the comparative table on page 122 was interesting, but the true effect of selection was largely disguised by it. In the first place, the selection was considerably in operation after five years; and, again, a comparison of aggregate tables with the truncated experience gave a false impression as regarded the relative degree of selection at the various ages. He had prepared a table of mortality rates and annuities based on select experience throughout, and the results demonstrated without doubt that the effect of selection increased in intensity with the age at entry, and, moreover, endured for a longer period as the entry age increased. At entry age 25 the effect disappeared after about 15 years; at entry age 40 it was more intense, and was still traceable after 25 years. Hence it would appear to be more correct that the premiums should be based upon select tables throughout, rather than upon select tables for a term of years coupled with a truncated experience. The new select tables (in common with the old ones) compared with the Registrar-General's Healthy Districts Tables, showed a lighter mortality than the latter during the first few years, then a heavier mortality for a term of years depending on withdrawals; afterwards a change occurred, and for the remainder of life the select tables were superior in vitality. Those comparisons proved that the true mortality of assured lives was less than that of the Healthy Districts Tables throughout life, but the effect of withdrawals—numerous shortly after entry, and decreasing rapidly with the years of duration—very soon counteracted the medical examination and forced the mortality above that of the Healthy Districts Table for a long period, after which it gradually subsided to its true rate.

Mr. G. F. HARDY considered that those gentlemen who had had the privilege of sitting on the Mortality Experience Committee with Mr. Ackland, and of reading his periodical reports on that investigation, would be best able to appreciate the further debt under which he had placed the Institute by the present paper. One point which would strike everyone in the present mortality investigation was, the thoroughness with which the extensive data collected by the Institute and Faculty had been analysed. In the Twenty Offices' Experience very little analysis was attempted; there was a distinction made between healthy and diseased lives, and male and female lives; but now a large number of most interesting tables were given. On the present occasion the data had reference only to healthy lives, but in addition to the classification by sex, various classes of assurance, and profit and non-profit assurances, had been dealt with separately. Those analyses raised a great many very interesting points which were not produced by the Institute's Experience; among others, the

important question which had been referred to by Mr. Nightingale, as to whether it was possible to use one standard table for the whole of their assurance work, or whether several tables were required, according to the class of assurance dealt with. For example, not only was there a considerable difference in the mortality between whole-life assurances with profits and the same class without profits, but the important classes of endowment assurances and term assurances showed a very marked divergence from whole-life assurance mortality, and in opposite directions. He thought, however, the members would agree with Mr. Nightingale that (excepting the class of survivorship assurances, and perhaps term assurances) one standard should, as far as possible, be employed, taking into account the facts with regard to the mortality of the special classes in the loadings placed upon the net premiums. It was important that too much account should not be taken of the light mortality shown in the endowment assurance class. The proportion which that class represented of the whole business was a constantly increasing one. Up to recent years it had been a comparatively select class: only a small portion of the total business had gone into that class, and that portion had shown a very light mortality. But if a larger proportion of business was to consist of endowment assurances, it was clear that the extremely light mortality shown in the present tables was not likely to continue, and that the same difference in future must not be looked for between the whole-life and the endowment assurance mortality, as was shown by the experience of the last thirty years. The practical bearing of the new tables mainly centred about the change in the net premium resulting from the new select tables. Policy-values were, of course, an important point, and the actuary had always to consider how much money for reserves any particular standard would require; but, after all, the difference between those various tables hardly amounted to more than a fraction per-cent on the total reserve; and was not equivalent to more than a change of about  $\frac{1}{8}$  per-cent in the rate of interest forming the basis of the valuation. Whether the valuation should be at  $2\frac{1}{2}$  per-cent,  $2\frac{3}{4}$  per-cent, or 3 per-cent, was a much more important point financially, than whether the  $H^M$  table should be used, or the  $O^M$ , or the  $O^{M(5)}$  table. The question of the lower net premiums was however of practical importance, in that it enabled offices to go more easily to lower rates of interest in their valuations. The main practical difficulty in assuming a low rate of interest, such as  $2\frac{1}{2}$  per-cent, has been that the net premiums, when based on the  $H^M$  table, were very nearly equal to the office premiums for many ages at entry; and the loadings, which formed the ostensible reserve for expenses, were thus cut down to almost vanishing point. But, with the new basis for net premiums, that difficulty would be largely got over, and it would be possible to value at a low rate of interest, and still to show a fair margin for future expenses. A minor point was, the strength of selection in the new tables as compared with the old. He was quite in agreement with Mr. Nightingale, that the effect of selection was more powerful in the New Experience than in the Old Twenty Offices. That could hardly be shown satisfactorily by comparing the  $H^{M(5)}$  and the  $H^M$  tables with the  $O^{M(5)}$  and the  $O^M$  tables, because there



was a distinction between the two Experiences. In the  $H^M$  Experience the whole of the data went back to the commencement of the insurances, but with the new Experience the tables started at a fixed point, in 1863, with a large number of non-select lives already on the books, and these came into both the  $O^M$  and the  $O^{M(5)}$  tables. The effect of that was to lessen the difference between these two tables, but on comparing the "select" rates of mortality with the "ultimate" rate of mortality (after say 10 years) by the Select Tables, or with the rates of mortality in the  $O^{M(5)}$  table, it would be seen there was a greater difference between those select rates and the ultimate rates, than between Dr. Sprague's select rates during the first five years of insurance and the  $H^{M(5)}$  rates. If those differences were accumulated at any age at entry, they gave some measure of the strength of the selection; and that cumulation of the difference of the values of  $q_x$  in Dr. Sprague's table amounted to a little over one year's mortality at the age of entry, and in the new tables to about two years of mortality. In other words, to go back to the old phrase "suspended mortality", the "suspended mortality" in one case was represented by one year's death rate, and in the other by two years. There was an incidental result of that fact, in the larger premiums that would be required under the new tables for the option of insurance at a future date without fresh medical examination. These premiums would probably be nearly twice as great under the new tables as under Dr. Sprague's tables. Such premiums had been quoted rather freely in the last few years in connection with various loan transactions, and that point might be one of some little importance. There were one or two important questions in connection with the new Experience which would no doubt have to be dealt with in the future. One of these was the advisability, or otherwise, of using two mortality tables in combination for the purpose of valuations. Was it desirable to use a combination like the  $O^M$  and  $O^{M(5)}$ , or the  $H^M$  and  $H^{M(5)}$ ? The use of such a combination had become rather a fashion. It was found that the  $H^{M(5)}$  and  $H^M$  combination gave higher reserves than the  $H^M$  table alone, and it was at once concluded that it must be a better basis for valuation. That was a *non sequitur*. The most suitable basis of valuation was not necessarily that which brought out the highest reserve. It might be seriously considered whether, from many points of view, it was not desirable to go back to the use of a single table for valuation purposes. It was long ago said that the effect of selection might very conveniently be set against the initial cost of obtaining business. Although the effect of selection was so considerable in the new Experience, in these times, when competition was so strong, the saving to an office by the light mortality of the first few years of assurance was generally less than the cost which the office had to incur in getting the business; and it seemed to him there was nothing in principle against the use of a table like the  $O^{M(5)}$  for all purposes of the office, both for the calculation of premiums and for the calculation of reserves. The effect of that was to provide from the outset for a rate of mortality which would ultimately prevail; the premiums and the valuations being both based on that assumption; and this provision

for a heavier rate of mortality in the early years than that actually experienced, enabled reserves to be set upon an expanding business, which could not always be done conveniently when a combination such as the  $H^M$  and  $H^{M(5)}$  was employed as the basis of valuation.

Mr. GEO. KING thought that the only criticism that could be passed upon the paper must take the form of commendation of Mr. Ackland's industry, because he had given a vast mass of material which would serve for future reference. At the moment, however, it rather confused than helped one to grasp the whole subject, and it would take time to find out what it all meant. He mentioned that, to excuse himself for not entering into a minute discussion of the many points that were raised. Many of those points would form very admirable subjects for future papers by some of the younger members, who could, with great profit to themselves and to the profession, expand them; and when the whole of the facts were analyzed in that way, taking one portion at a time, their bearings would be better known. There was a very interesting table of comparisons of premiums, and he thought that was one of the most important questions that arose in connection with the new mortality tables. One matter connected with it rather surprised him, namely, the extraordinarily low premiums which were brought out for short term assurances. His own experience of those assurances had not been favourable. He had found that the lives had fallen in certainly more rapidly than among the whole-life assured, and he was surprised that in the experience of the combined offices that fact should not be apparent. Mr. Hovil had remarked of the assured female mortality, that it had improved much more than the male mortality, and had put forward two possible reasons, one, that the lives were examined more carefully, and another, that a different class of female lives might now be insuring. But it was curious that the same peculiarity was found in the new annuity experience. Female annuitant longevity had improved much more than that of the males, and therefore it looked as if there were really an increase in the length of female life generally, as compared with the life of males.

Referring to Tables X to XIII, it was possible for any age at entry, or any duration, to find what was the difference between the policy-values by different tables and rates of interest; but in any of these tables, taking the horizontal lines first, it would be found, for any age at entry, that the ratio between, for instance, the  $O^M$  and  $H^M$  reserve jumped up and down—sometimes it was above unity, and sometimes below. In the same way, looking up and down the columns, taking the same duration, that fact would also be found; and it was not possible to tell from a table of that kind what the general effect would be upon the valuations of a life office. He had thought, therefore, that it would be interesting to the Institute if he were to go into that question a little, and make a valuation of his own model office by the new tables, so as to judge what the effect would be upon the valuation of an ordinary office; and of this he submitted a summary table. The model office was constructed on the average of the twenty offices contributing to the  $H^M$  Experience, the average not only with regard to mortality, but also with regard to the numbers entering at the various ages, and the numbers

withdrawing. A long experience of that model office had shown how extraordinarily accurate an average office of that kind was as a measure of the effect of a change of valuation basis in any company. He did not think he ever knew a difference between the reserve estimated by means of the model office and the actual reserve made by complete valuation, of more than 1 in 1,000, and usually the fraction was smaller. Therefore he thought it was possible with great confidence, even under considerably changing conditions, still to use his model office to make estimates.

*Comparison of Reserves at 3 per-cent interest by various Tables, with the 3 per-cent Reserves by the  $H^M$  Table, and by the combined  $H^M$  and  $H^{M(5)}$  Tables, respectively.*

1st.— $H^M$ 3 per-cent Reserve taken as 1,000						
Age of Office	$H^M$ 1st 5 Years, $H^M$ & $H^{M(5)}$ after	$H^M$	$O^M$	$O^{M(5)}$ throughout	$O^M$ 1st 5 Years, $O^M$ & $O^{M(5)}$ after	$O^M$ & $O^{M(6)}$ throughout
5 Years	1,000	1,000	1,031	990	1,031	1,209
10 "	1,060	1,000	1,026	992	1,054	1,109
15 "	1,052	1,000	1,022	993	1,044	1,072
20 "	1,042	1,000	1,019	994	1,036	1,053
25 "	1,034	1,000	1,016	994	1,029	1,042
30 "	1,029	1,000	1,014	994	1,025	1,034
35 "	1,025	1,000	1,012	994	1,021	1,030
40 "	1,023	1,000	1,011	994	1,019	1,026
45 "	1,021	1,000	1,010	994	1,017	1,024
50 "	1,020	1,000	1,010	994	1,017	1,023
2nd.—Combined $H^M$ and $H^{M(5)}$ 3 per-cent Reserve taken as 1,000						
5 Years	1,000	1,000	1,031	990	1,031	1,209
10 "	1,000	943	968	934	994	1,046
15 "	1,000	951	972	944	993	1,020
20 "	1,000	960	978	954	994	1,011
25 "	1,000	967	982	961	995	1,007
30 "	1,000	972	985	966	996	1,005
35 "	1,000	975	987	969	996	1,004
40 "	1,000	978	988	971	996	1,003
45 "	1,000	979	989	973	996	1,003
50 "	1,000	980	990	974	996	1,003

Taking that model office, and looking at the first quinquennium of insurance, policies issued within the last five years, the difference of reserve between the  $H^M$  and the  $O^M$  was an increase of fully 3 per-cent on the  $H^M$ —the  $O^M$  table required fully 3 per-cent higher reserve for the recent policies. But the difference in the reserve rapidly diminished with the duration of the policies, and after 25 years it was barely more than  $1\frac{1}{2}$  per-cent. Taking an office of 50 years' standing, it was only 1 per-cent; therefore, for an old-established office, it might be said that 1 per-cent was about the difference to be



provided in order to pass from the  $H^M$  to the  $O^M$  table. An office doing a very large new business must remember that, for the first periods of insurance, the difference was greater, and some allowance must be made for that; but even some variation in that respect would not have very much effect in the estimate of the reserves by the model office. He had also prepared a table for the  $O^{M(5)}$  alone, taking the  $O^{M(5)}$  premiums throughout; and it would be found a little short of the  $H^M$  alone; and the combined  $O^M$  and  $O^{M(5)}$  would fall a little short of the combined  $H^M$  and  $H^{M(5)}$ . With regard to the remarks of Mr. Hardy, it seemed to the speaker that what it was desirable to ascertain was, what table with the least trouble gave the closest reserve to that by a select mortality table, because, after all, a select mortality valuation gave the nearest to the true results. Such a table was very troublesome to use, and therefore it was not customary to apply it in practice, and the object in making various combinations of tables was to get the nearest result. To pass from some of the tables to others made a little more difference than Mr. Hardy seemed to think. To pass from the  $H^M$  to the combined  $H^M$  and  $H^{M(5)}$ , meant a quarter per-cent in the valuation rate of interest, which was rather a large margin.

Mr. LEVINE asked whether Mr. King in his model office calculations had taken bonuses into account?

Mr. KING said he had not, and had dealt with the whole-life policies only, excluding endowment assurances and special classes. A careful investigation showed that the amount of endowment assurances made very little difference in the estimates given by the model office. Bonuses were not taken into account, but even that did not make much difference, and they could easily be otherwise dealt with. It was extraordinary how closely the model office applied to offices of all kinds.

Mr. LEVINE said that he had recently taken out the actual sums assured and bonuses, and net premiums for quinquennial ages, in a business in which there were few reversionary bonuses, but large bonus-reductions of premium; and he had found that the difference in reserve was a good deal more than the 1 per-cent which Mr. King had mentioned—nearly 3 per-cent.

Mr. JAS. CHATHAM said that he himself had read a paper in Edinburgh a short time previously, in which he had dealt entirely with net premiums deduced from select tables of all the various classes in the Experience—whole-life, ascending scale, limited payment, endowment assurance, contingent survivorship, and temporary assurance. One very prominent feature brought out in that paper was the effect which the proposer exercised in choosing his class. It was very marked indeed, and he thought it was not possible to apply one table—namely, whole-life—to all classes of assurance, and allow for it in the loading. For instance, not only in temporary insurance, but in contingent survivorship, the mortality was very high indeed. With regard to the low mortality in endowment assurances, he agreed with what Mr. Hardy had said, that it was only a very short time that endowment assurances had been in existence. At first the policyholders adopting that class were a very good set of lives indeed; but now the option of the few had become



the fashion of the many, and a higher rate of mortality might be expected in the future. He knew one office which had made an investigation into its endowment assurance experience, dividing it into two periods, and had found that the mortality in the later period was heavier than the mortality in the earlier period. In his paper he went into the select net premiums deduced from the unadjusted facts. He employed no graduation at all, and in the case of the whole-life participating and non-participating experience he calculated his net premiums for each age of entry from the observations relating to the first and each subsequent year of insurance for that particular age at entry only. He compared them with the net premiums calculated by joining on the select tables after ten years to an aggregate table excluding the first ten years, and found that the difference was very small indeed, about  $1\frac{1}{2}$  per-cent; so that, although he had no doubt the period of selection did vary, still he thought that method might be adopted, as after ten years it made very little difference. Like others, he was much interested in Mr. Ackland's table of policy-values, and thought it would interest the meeting to give the particulars of a valuation he had himself made of the whole-life participating class in a young office by the  $O^M$  table. He did so with bonus and without bonus. At 3 per-cent interest, comparing it with the  $H^M$ , he found the  $O^M$  was 1.22 per-cent greater, leaving out of account bonus; that was, if the  $H^M$  reserve was 100, the  $O^M$  reserve was 101.22. If, however, bonus was taken into account, the  $O^M$  reserve was reduced from 101.22 to 100.46, which was a very material reduction. It showed that old offices with large bonuses would have very little difficulty indeed in passing to the  $O^M$  table. He quite agreed with what Mr. Nightingale had said with regard to Mr. Ackland's figures for temporary assurances at young ages. In deducing office rates he had himself omitted age 20 altogether, as being one requiring to be specially dealt with, and started with age 25. As the rates ran somewhat irregularly he had adjusted them. He found that after that age the differences between Mr. Ackland's figures and his were not so very great after all, as the following table would show. His premiums were calculated at  $3\frac{1}{2}$  per-cent, but that would not affect the comparison materially.

*Short Term Net Annual Premiums  $3\frac{1}{2}$  per-cent.*

Age.	1 Year	3 Years	5 Years	10 Years
30	.42	.60	.72	.86
45	.67	.94	1.10	1.38
50	.91	1.22	1.50	1.90
55	1.24	1.68	2.10	2.61

When the above rates were loaded with a constant of 5s., and 10 per-cent, they would be found to agree very well, on the whole, with the average rates of premium charged by offices for that class of business.

Mr. ALFRED MOORHOUSE said that, with regard to comparisons

already made of the  $O^M$  and  $H^M$  valuations, he might mention that a short time ago he had occasion to make a valuation in an old-established office with a net liability of about  $1\frac{3}{4}$  millions, and the difference between the two valuations was about £9,000.

Mr. H. W. MANLY considered that the paper contained an immense amount of work, and was exceedingly valuable, as Mr. Ackland, being so intimately acquainted with the whole of the experience investigation from its commencement to the present, was more fitted to deal with it than anyone else. He had, indeed, treated it microscopically, and the difficulty was to see the main differences in the whole. The paper had led to a most interesting discussion, and brought out some very interesting facts. Mr. King deserved the thanks of the members for the labour he had undertaken in making a valuation of his model office, by which a full comparison was obtained of the effect of the new Experience when used in valuations, as compared with the old  $H^M$  or  $H^{M(5)}$ . The conclusion he had drawn roughly in his own mind with regard to the effect of the new Experience was, that those offices which had been using the  $H^M$  table, instead of the  $H^M$  and  $H^{M(5)}$  combined, had been using the better standard of the two, and in that respect he congratulated himself on having always refused to use the  $H^M$  and  $H^{M(5)}$  combination. Mr. Hovil's doubts as to the endowment assurance experience had been confirmed by Mr. G. F. Hardy. It was known that the effect of selection lasted more than ten years—in fact it might be traced for more than fifteen years—and naturally, in the  $E^M$  table, we should expect to find the full effect of selection.

The PRESIDENT imagined that the wits and energy of many would be set to work upon the mine of information provided by the new tables, and there might be overlapping. Therefore he trusted that any gentleman who was proposing to undertake an enquiry would think proper to communicate with him, and then, if a second appeared in the field, he would take care that the two were put into communication, so that there might be either some division of labour, or joint authorship.

A vote of thanks was unanimously accorded to Mr. ACKLAND, who, in responding, said that he felt that the main purpose which he had in writing the paper, that of eliciting a valuable discussion which would provide much food for thought, had been fully realized; and he felt deeply indebted to those gentlemen who had taken part therein. It would be especially interesting to see the results of the tables and calculations which Mr. King and Mr. Chatham had kindly promised to complete for publication. He had understood Mr. Nightingale to remark that he was not satisfied that the term insurance figures in the paper were altogether reliable; but, as Mr. Nightingale did not give any reason for expressing that opinion, he found himself unable at the moment to make any response to it.

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[We have pleasure in complying with Mr. Ackland's desire to supplement his reply in the discussion on his paper by the following interesting comparison of the rates of mortality in

the Classes of Term Assurances, and Assurances with Ascending Scale of Premiums.—ED. *J.I.A.*]

#### NOTE AS TO TERM ASSURANCE PREMIUMS.

Since the above discussion took place, Mr. Nightingale has sent me the Table of Net Annual Premiums, as deduced by him, at  $3\frac{1}{2}$  per-cent interest, for Term Assurances. These are, as stated by him in the discussion, based upon the combined experience of Assurances with Ascending Scale of Premiums, and Temporary Assurances. It will, I think, be found that the rates of mortality obtaining in these two classes are quite distinct. In order to test this, I have calculated the expected deaths which would have arisen in the class of Assurances with Ascending Scale of Premiums, had the rates of mortality actually experienced in the class of Temporary Assurances been in operation. The Table on the next page shows the results, in respect of each group of ages at entry from 18 to 57 inclusive, in each of the first ten years of assurance.

It will be seen that, in each of the first six years of assurance, (and also in the tenth year), the rate experienced in the class of Ascending Premiums is materially below that experienced in the class of Temporary Assurances, whilst, in the seventh, eighth, and ninth years, the rate experienced considerably exceeds that of the Term Assurances. During the first six years the rate of mortality is 16 per-cent below, and during the remaining years 37 per-cent above, that experienced in the class of Term Assurances. It appears to me, therefore, that, in combining the data in respect of these two classes, for the purpose of deducing Term Assurance premiums, the mortality is understated during the first six years of assurance, and subsequently overstated, with a corresponding effect upon the net premiums deduced from such combined data. Any comparisons of net premiums so deduced, with those computed by Dr. Sprague, or those published in my paper, would thus, I fear, prove unreliable.

I have also had an opportunity of reading that portion of Mr. Chatham's paper, read before the Faculty of Actuaries on 10 March 1902, and referred to by him in the above discussion, which deals with the subject of rates of premium for Term Assurances. I find that, in respect of the early ages at entry, Mr. Chatham has adopted a plan somewhat similar to that followed in my paper, he having deduced a uniform net premium from the grouped data at central ages 25, 30, and 35 (23–37 inclusive), whilst, in my calculations, the data for central ages 20, 25, 30, 35, and 40 (18–42 inclusive), were grouped. Allowing for these different methods of grouping, and also for the different rate of interest at which the premiums are computed, it will be found, as stated by Mr. Chatham, that his resulting net premiums do not differ very materially from those published in my paper, and loaded office premiums deduced from the latter will be found to show a fairly close agreement with those arrived at by Mr. Chatham, and with the average rates charged by representative offices.

T. G. A.



## ASSURANCES WITH ASCENDING SCALE OF PREMIUMS.

Table showing number of Actual Deaths, in Grouped Ages at Date of Assurance, and in Years of Assurance; also number of Expected Deaths, if the Rates of Mortality respectively obtaining had been identical with those experienced in the class of Temporary Assurances.

Grouped Ages at Date of Assurance	YEARS OF ASSURANCE																		GROUPED YEARS OF ASSURANCE				Grouped Ages at Date of Assurance				
	1		2		3		4		5		6		7		8		9		10		1-6			7-10		TOTALS	
	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual	Expected Deaths	Actual		Expected Deaths	Actual	Expected Deaths	Actual
18-22	11.8	3	6.2	5	10.7	6	10.9	7	21.4	9	21.8	9	8.6	5	...	6	...	8	60.8	5	82.8	39	69.4	24	152.2	63	18-22
23-27	12.3	16	26.6	28	33.2	20	61.6	16	19.7	16	24.3	21	25.6	17	...	19	...	16	...	21	177.7	117	25.6	73	203.3	190	23-27
28-32	17.3	12	21.3	17	11.6	23	13.8	19	45.5	17	27.4	28	9.1	18	...	21	37.4	16	40.5	14	136.9	116	87.0	69	223.9	185	28-32
33-37	22.0	13	16.6	19	29.6	23	12.5	18	27.8	25	10.1	10	10.6	16	...	19	...	17	...	16	118.6	108	10.6	68	129.2	176	33-37
38-42	9.6	15	8.8	21	11.9	17	19.9	18	14.8	21	6.6	15	3.2	9	28.3	16	...	14	15.9	14	71.6	107	47.4	53	119.0	160	38-42
43-47	15.6	9	19.9	11	16.3	18	14.4	17	2.1	16	11.5	12	7.0	14	12.1	13	...	14	...	10	79.8	83	19.1	51	98.9	134	43-47
48-52	11.4	13	16.7	10	19.4	13	11.4	12	20.4	17	14.2	8	10.2	10	10.3	11	10.2	10	...	16	93.5	73	30.7	47	124.2	120	48-52
53-57	6.3	1	5.8	6	5.3	3	1.9	8	9.3	6	5.3	4	6.8	7	...	4	...	5	...	6	33.9	28	6.8	22	40.7	50	53-57
18-57	106.3	82	121.9	117	138.0	123	146.4	115	161.0	127	121.2	107	81.1	96	50.7	109	47.6	100	117.2	102	794.8	671	296.6	407	1,091.4	1,078	18-57
Actual Deaths															84 %		137 %		98.8 %		of Expected Deaths.						



*On the Valuation of Staff Pension Funds.* By HENRY WILLIAM MANLY, *Actuary of The Equitable Life Assurance Society, and Ex-President of the Institute of Actuaries.* With Tables by ERNEST CHARLES THOMAS, *of The Gresham Life Assurance Company, Fellow of the Institute of Actuaries,* and JOHN NORMAN LEWIS, *of The London Assurance Corporation, Fellow of the Institute of Actuaries.*

(Continued from vol. xxxvi, page 276.)

#### ADDITIONS AND ALTERATIONS.

ON reading over my paper after it was published, I did not feel quite satisfied as to the accuracy of my approximate formula, given on page 235, for the adjustment of  $N_x^s$  in respect of the excess of contributions included in the Table as constructed, so I have had the values of  ${}^dC_{xsx}$ ,  ${}^wC_{xsx}$  and  ${}^rC_{xsx}$ , calculated. The constant summation of these columns, namely,  $\Sigma {}^dC_{xsx}$ ,  $\Sigma {}^wC_{xsx}$  and  $\Sigma {}^rC_{xsx}$ , will give the columnar values to be used for making accurate adjustments. These latter values, divided by  $D_x^s$ , are the present values of a full year's salary at the end of the year of death, withdrawal, or resignation, equated to a salary of 1 at  $x$ ; and the columns  $\Sigma {}^dC_{xsx}$ ,  $\Sigma {}^wC_{xsx}$  and  $\Sigma {}^rC_{xsx}$ , will therefore be appropriately represented by the symbols  ${}^dM_x^{ls}$ ,  ${}^wM_x^{ls}$  and  ${}^rM_x^{ls}$  respectively. These may, on some occasion, be found useful for other purposes; such as when one year's salary is given on death or retirement in lieu of other compensation.

When the correct values are substituted for those used in Section XI of the Valuation Schedule on page 248 (see page 196), it will be seen that a closer approximation would have been

$$\cdot 74({}^dM_x^s + {}^wM_x^s + {}^rM_x^s)$$

which does not differ much from my estimate on page 235.

Another alteration which I have seen reason to make is more important, and affects Problems IIIA, XB and XIB. In those problems (pages 221, 238 and 239), I have assumed that the annuity commences at the end of the year of retirement, that the full year's salary in the year of retirement is received, and that the annuity-value is then  $a'_{x+1}$ ; but if we assume that retirement on the average takes place in the middle of the year, the basis on which the pension is calculated is too large by half the last year's salary. Further, on the assumption that retirement on the average occurs in the middle of the year, the value of the annuity will be  $a'_{x+\cdot 5}$  instead of  $a'_{x+1}$ , and the continuous annuity will be, approximately,  $a'_{x+\cdot 5} + \cdot 5$ .

I have recalculated the Tables for valuing the benefits described in the above-mentioned three Problems by substituting  $a'_{x+5} + \cdot 5$  for  $a'_{x+1}$ ; and these Tables will be found on pages 207 and 208. The values  $\frac{{}^ra R_x^s}{D_x^s}$  and  $\frac{{}^ra R_x^{ls}}{D_x^s}$  will now be too great by the present value of an annuity of  $a'_{x+5} + \cdot 5$  based on half the full salary receivable in the year of retirement, that is, by  $\frac{1}{2} \sum r_x s_x (a'_{x+5} + \cdot 5) v^{x+1} \div D_x^s$  or  $\frac{{}^ra M_x^{ls}}{2D_x^s}$ ; and the adjusted values will be too small by half a year's interest.

#### FURTHER COMMENTS ON ADJUSTMENTS.

It may be useful to give here the results of my more mature judgment respecting adjustments.

I think, as regards the question of discount, a distinction should be made between the accumulation of capital and the reserve for liabilities. As I have already explained (page 235), the periodical contributions and interest on investments are not invested directly they are received, and consequently, on the assumption that the investments are made, on the average, at the end of every quarter, my values of the future contributions (which assume that the investments are made at the end of the year), are over-discounted by  $\frac{3}{8}$ ths of a year, and have therefore to be multiplied by  $(1 + \frac{3}{8}i)$ . In those Funds where the master contributes half the contributions, the more common rule is for him to pay his share at the end of the year, and my values in that case are consequently over-discounted to the extent of only one-fifth of a year, and have therefore to be multiplied by  $(1 + \cdot 2i)$ . On the other hand, the liabilities have to be met immediately the events provided for happen. Now my values assume that the events happen at the end of the year in which they occur, and that the last year's salary will be received in full—conditions which only apply to the value of the pension on the attainment of the pension age. With that exception, all my values of liabilities, on the assumption that the events will happen in the middle of the year, are too great by the value of half the last year's salary, and are over-discounted by exactly half a year. The correction for half the last year's salary must therefore be first made, and the result multiplied by  $(1 + \frac{1}{2}i)$ .

The proper multiplier for finding the value of the future contributions will therefore be

$$\left\{ \mathbb{N}_x^s - \frac{1}{2} ({}^d M_x^{ls} + {}^w M_x^{ls} + {}^r M_x^{ls}) \right\} \div D_x^s$$

multiplied by  $(1 + \frac{3}{8}i)$  or  $(1 + \cdot 2i)$  according as the full contributions are received equally over the year and invested quarterly, or half received over the year and half paid in a lump sum at the end of the year.

The multiplier for finding the value of the return of contributions without interest on death will be

$$(1 + \frac{1}{2}i)(^dR_x^s - \frac{1}{2}^dM_x^{ls}) \div D_x^s;$$

and, for return with interest,  $(1 + \frac{1}{2}i)(^dN_x^s - \frac{1}{2}^dM_x^{ls}) \div D_x^s$ .

Similarly for the return on other modes of exit.

The multipliers for the value of a pension on early retirement will be

$(1 + \frac{1}{2}i)(^{ra}R_x^s - \frac{1}{2}^{ra}M_x^{ls}) \div D_x^s$  for pension based on average salary,  
 $(1 + \frac{1}{2}i)(^{ra}R_x^{ls} - \frac{1}{2}^{ra}M_x^{ls}) \div D_x^s$  for pension based on last salary.

In Problem IIIA, the formula for the correct value is evidently

$$(1 + \frac{1}{2}i)(^{ra}R_x - \frac{1}{2}^{ra}M_x) \div D_x.$$

It may be thought that I am now aiming at too much refinement; but on consideration it will be seen that my proposals are all on the side of safety.

VALUATION SCHEDULE.

Sections VI, X and XI of the Valuation Schedule, pages 245 and 248, will now be altered as follows:—

REVISED VALUATION SCHEDULE—(see page 245).

		SECTION VI				
		Annuity of 1 per-cent of Total Salary on Retirement before 65				
Present Age	Number of Members	$\frac{^{ra}R_x^s}{D_x^s}$	$\frac{^{ra}M}{D_x}$	Present Value in respect of Future Contributions (3) $\times$ (28)	Present Value in respect of Past Contributions (8) $\times$ (26)	Total of last two Col.s. (30) + (31)
(1)	(2)	(28)	(29)	(30)	(31)	(32)
20	50	26·782	·271	602·59	16·19	618·78
30	50	28·736	·680	1293·12	254·30	1547·42
40	25	28·299	1·241	919·72	530·28	1450·00
50	15	21·537	1·930	572·88	913·21	1486·09
60	6	7·003	2·185	73·53	576·40	649·93
Totals	146	...	...	3461·84	2290·38	5752·22

REVISED VALUATION SCHEDULE—(see page 248).

SECTION X						
Annuity of 1 per-cent of Last Salary and Number of Years of Service, on Retirement before 65						
Present Age	Number of Members	$\frac{raR_x^{ls}}{D_x^s}$	$\frac{raM_x^{ls}}{D_x^s}$	Present Value in respect of Future Service (3)×(55)	Present Value in respect of Past Service (44)×(56)	Present Value in respect of Total Service (57)+(58)
(1)	(2)	(55)	(56)	(57)	(58)	(59)
20	50	42·607	1·136	958·66	88·88	1047·54
30	50	39·426	1·430	1774·17	788·00	2562·17
40	25	34·453	1·863	1119·72	1215·98	2335·70
50	15	23·761	2·315	632·04	1725·60	2357·64
60	6	7·178	2·270	75·37	1014·69	1090·06
Totals	146	...	...	4559·96	4833·15	9393·11

REVISED VALUATION SCHEDULE—(see page 248).

SECTION XI—Adjustments									
Present Age	Number of Members	$\frac{wM_x^{ls}}{2D_x^s}$	(3)×(60)	$\frac{M_x^{ls}}{2D_x^s}$	(3)×(62)	$\frac{rM_x^{ls}}{2D_x^s}$	(3)×(64)	$\frac{raM_x^{ls}}{2D_x^s}$	(3)×(66)
(1)	(2)	(60)	(61)	(62)	(63)	(64)	(65)	(66)	(67)
20	50	·307	6·91	·116	2·61	·064	1·44	·568	12·78
30	50	·118	5·31	·120	5·40	·080	3·60	·715	32·18
40	25	·041	1·33	·127	4·13	·104	3·38	·932	30·29
50	15	·003	·08	·120	3·19	·129	3·43	1·158	30·80
60	6	...	...	·071	·75	·130	1·37	1·135	11·92
Totals	146	...	13·63	...	16·08	...	13·22	...	117·97

As an example of the effect of the alterations I will re-value Fund VIII on page 254.



FUND VIII.		Pension as a Percentage of Last Salary for every Year of Service	
BENEFITS :—		Without	With
(i) Pension on attainment of age 65 based on last Salary.			
(ii) Pension on retirement before 65 based on last Salary.			
(iii) Return of Contributions, with interest, on death before 65.		Return of Difference between Total Contributions and Annuity Payments	
(iv) Return of Half Contributions, without interest, on withdrawal.			
Present Value of 1 per-cent of Future Salaries	2250·92		
Deduct :			
Adjustment for $\frac{1}{2}$ years' contributions on withdrawal, death and early retirement, over-estimated = 13·63 + 16·08 + 13·22	42·93		
	2207·99		
Add : Adjustment for over-discount ( $1\frac{1}{2}$ per-cent = $\frac{3}{8}i$ )	33·12		
	2241·11		
Deduct :			
1 per-cent of Salaries, with interest, on death before 65	1144·36		
$\frac{1}{2}$ per-cent of Salaries, without interest, on withdrawal	147·32		
	1291·68		
Deduct adjustment for over-estimate of contributions (6·82 + 16·08)	22·90		
	1268·78		
Add adjustment for over-discount (2 per-cent = $\frac{1}{2}i$ )	25·37		
	1294·15		
	946·96		
	5		
	4734·80		
Add : Fund	12000·00		
Total Asset to provide Pensions	16734·80		
This has to be divided by—			
Present Value of Pension of 1 per-cent of last Salary on attainment of age 65	10850·26		
Present Value of Pension of 1 per-cent of last Salary on retirement before age 65	9393·11		
Deduct adjustment for over-estimate of contribution	117·97		
	9275·14		
Add adjustment for over-discount (2 per-cent)	185·50		
	9460·64		
	20310·90		
Which will give a Pension of		·824	·784

## A CORRECTION.

Mr. W. A. Robertson, of the Scottish Union and National Office, Edinburgh, points out an error which I am very pleased to acknowledge.

In Problem XIII<sub>A</sub>, page 227,

$${}^r\mathbb{N}_x - {}^r\mathbb{N}_{x+15} - 15 {}^rD_{x+15}$$

is given as the value to be used in the formula to provide for the return of the premiums paid with compound interest on retirement from invalidity before 15 years' service. The correct value is

$${}^r\mathbb{N}_x - {}^r\mathbb{N}_{x+16} - a_{(16)} {}^rD_{x+15}$$

or 
$${}^r\mathbb{N}_x - {}^r\mathbb{N}_{x+15} - (1+i) a_{(15)} {}^rD_{x+15}$$

This follows from Problem II<sub>A</sub> (page 220), where it is shown that the deferred value for the return on death after 5 years is

$$\frac{1}{v^x l_x} [\{(1+i)^5 + (1+i)^4 + \dots + (1+i) + 1\} vD'_{x+5} + vD'_{x+6} + \dots]$$

which is the same as

$$\frac{1}{v^x l_x} [(1+i)\{(1+i)^4 + (1+i)^3 + (1+i)^2 + (1+i) + 1\} vD'_{x+5} + vD'_{x+5} + vD'_{x+6} + \dots]$$

that is, 
$$(a_{(6)} {}^dD_{x+5} + {}^d\mathbb{N}_{x+6}) \div D_x$$

or 
$$\{(1+i)a_{(5)} {}^dD_{x+5} + {}^d\mathbb{N}_{x+5}\} \div D_x$$

and the value of the temporary benefit, for the return on death within 5 years, would therefore be the difference between the immediate benefit ( ${}^d\mathbb{N}_x \div D_x$ ) and the deferred benefit, namely,

$$({}^d\mathbb{N}_x - {}^d\mathbb{N}_{x+6} - a_{(6)} {}^dD_{x+5}) \div D_x$$

or 
$$({}^d\mathbb{N}_x - {}^d\mathbb{N}_{x+5} - (1+i)a_{(5)} {}^dD_{x+5}) \div D_x$$

By substituting  $r$  for  $d$ , the same formula will apply for the return on retirement within 5 years.

A similar error crept into Problem XII<sub>B</sub> (page 241), where

$${}^r\mathbb{N}_x^s - {}^r\mathbb{N}_{x+15}^s - (as)_{15} {}^rD_{x+15}$$

should read

$${}^r\mathbb{N}_x^s - {}^r\mathbb{N}_{x+16}^s - as_{(16)} {}^rD_{x+15}$$

or 
$${}^r\mathbb{N}_x^s - {}^r\mathbb{N}_{x+15}^s - (1+i)(as)_{(15)} {}^rD_{x+15}$$

Mr. Robertson obtained the same results by the direct method of summing the values on pages 219 and 237 for 15 years.

#### NEW TABLES AT 3 AND 4 PER-CENT.

In a postscript which I added to my paper when it was published, I had the satisfaction of stating that, in response to my invitation, a gentleman had kindly volunteered to assist me by undertaking to calculate additional Tables. I have great pleasure in stating now that the gentleman is Mr. John Norman Lewis, at that time in the Scottish Widows' Fund, but now of the London Assurance; and I desire to express the indebtedness, not only of myself, but of the Institute and the profession, to him for his conscientious labours and for the skilful manner in which he has performed the work. Mr. Lewis has calculated the whole of the Tables at 3 per-cent, and Mr. Thomas has completed the Tables at 4 per-cent. It has given me great pleasure to have had these two gentlemen associated with me, and I feel sure that both will, in due time, receive their reward.

In view of the claims on the Editor of the *Journal*, I have reduced the Tables to the smallest number for efficient use. I had to decide between giving simply the Tables of multipliers, or publishing such of the Tables as would be useful for other investigations than those I have made; and after consulting many of my friends engaged in this class of work, who all wished to have the full Tables, I decided to publish those now given.

When we come to construct Tables for Pensions commencing at age 60, it will be found that all the figures in Table 4, after age 59, are altered. As all remaining in the service at age 60 are pensioned, it follows that all the  $r$ 's after 59 disappear; the  $d$ 's after 59 are increased by the deaths that occur amongst those who would otherwise retire between 60 and 65; and, of course, the  $l$ 's are also changed. In these circumstances, I have assumed that Table 4 holds good up to age 60, and that from that age to age 65 the mortality follows my Table 2, after which it merges into the English Life Table No. 3. As a consequence, the  $N$ 's,  $M$ 's and  $R$ 's, in Table 7, no longer apply, and have to be recalculated.

My Table 2 gives a slightly smaller annuity for age 60 than the English Life Table, the reason being that the heavy mortality

assumed to prevail amongst those who retire early, in Table 8, col.  $q_x^{(r)}$ , brings the total mortality, from age 57 to 64, slightly above the English Life rate. If I were to recalculate all the Tables, I should modify  $q_x^{(r)}$  after age 50 so as to make the total mortality, after age 56, the same as the English Life rate. As it is, the difference is very small and is not likely to affect a valuation, and certainly would not justify the trouble and labour of recalculating all the Tables.

All the D's, C's and  $(a')$ 's, remain the same for the same rate of interest, so these have not been repeated in the Tables for Pension age 60.

#### VALUATION OF FUNDS FOR PENSION AGE 60.

One question arises here for consideration, namely, the annuity-value which is to be used for calculating the pensions at age 60. I have used the value according to my Table No. 2, in order that my valuations shall be consistent throughout, but I should not be disposed to recommend it in all cases; for, although the after-life of persons who retire at age 65 may be fairly represented by the English Life Table, I do not think it is a good measure of the vitality of persons who retire at age 60. A man at 60 can adapt himself more readily to new conditions of life than a man at 65 can, so that in most cases I should be inclined to use a Table giving higher annuity-values for valuing the Pensions at 60 and over; but the selection of the Table must be left to individual judgment. On the other hand it must not be forgotten that there will always be a set-off by reason of many of the members remaining in the service after 60 years of age.

#### VALUATION OF FUNDS AT 3 PER-CENT.

On the assumption that the Fund is only making 3 per-cent interest on its investments, and that in those benefits where the contributions are to be returned with compound interest, the same rate is to be allowed, col. (6) in my imaginary particulars will be changed, and the amount of the invested capital will be different.

The following figures must, therefore, be substituted for those in col. (6), page 242 :



## TOTAL PAST SALARY, WITH 3 PER-CENT COMPOUND INTEREST

Present Age								
20			30			40		
Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest	Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest	Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest
10	5	1,722	5	15	3,905	4	25	7,674
10	4	1,246	5	14	3,756	3	24	5,391
5	4	676	11	13	3,400	4	23	8,867
5	4	777	4	13	3,129	6	22	11,900
10	3	1,154	15	12	13,230	3	20	6,016
5	2	380	4	11	2,963	2	19	3,191
5	1	250	1	11	1,091	1	18	1,848
...	...	...	3	10	2,435	1	17	3,434
...	...	...	1	10	1,644	1	10	3,608
...	...	...	1	8	719	...	...	...
...	...	6,205	...	...	42,272	...	...	51,929
...	...	...	...	...	...	...	...	63,568
...	...	...	...	...	...	...	...	44,122

And I shall assume that the Fund has an invested capital of £10,500.

Strictly speaking, the capital would not be the same in each of the Funds if the respective benefits had been in operation in past years; because the payment out in each case would have been different, so that to be quite accurate it should be understood that my valuations apply to a single existing Fund now possessing a capital of £12,000 or £10,500, according as the interest assumed is 4 per-cent or 3 per-cent; and the results produced show the changes which would have to be made in the scale of pension caused by a change in the benefits.

In order to avoid repetition, I propose in future to refer to the benefits by numbers.

## TABLE OF BENEFITS.

- I. Pension on attainment of fixed age 60 or 65.
- II. Ditto with return of excess of contributions, without interest, over annuity payments.
- III. Return of half the contributions, without interest, on withdrawal.

- IV. Return of total contributions, without interest, on withdrawal.
- V. Return of total contributions, without interest, on death before pension age.
- VI. Return of total contributions, with interest, on death before pension age.
- VII. Return of total contributions, without interest, on retirement before pension age.
- VIII. Return of total contributions, with interest, on retirement before pension age.
- IX. Pension according to scale on retirement before pension age.

TABLE G.

*Showing the alterations in the Pension Benefit as the result of changing existing or introducing other Benefits in an established Fund: Contributions being 5 per-cent of Salary. The Funds referred to in the Table are those on pages 251-4.*

Pension Age . . .		65		60					
Rate of guaranteed Interest . . .		4 per-cent	3 per-cent	4 per-cent	3 per-cent				
Amount of Fund .		12,000	10,500	12,000	10,500				
FUND	BENEFITS other than Pension on attainment of Pension Age	SCALE OF PENSION, being the percentage of Average Salary or of Last Salary for every year of Service							
		I	II	I	II	I	II	I	II
<i>Pension based on Average Salary</i>									
I	—	3·568	3·530	2·665	2·624	1·630	1·583	1·247	1·208
II	V	3·057	3·017	2·213	2·166	1·465	1·426	1·099	1·055
III	V, VII	2·552	2·509	1·767	1·713	1·377	1·337	1·018	·971
IV	IV, V, VII	2·347	2·301	1·589	1·529	1·271	1·227	·929	·877
V	IV, V, IX	1·506	1·442	1·094	1·012	1·165	1·119	·868	·813
VI	III, VI, IX	1·366	1·297	1·009	·920	1·112	1·064	·846	·790
<i>Pension based on Last Salary</i>									
VII	III, V, VIII	1·188	1·157	·833	·794	·940	·919	·563	·533
VIII	III, VI, IX	·824	·784	·603	·551	·810	·786	·512	·478

STANDARD SCALE OF BENEFITS WHICH CAN BE GIVEN FOR  
5 PER-CENT OF SALARY.

The above Funds, however, do not give us all the information we require. What we frequently want to know is:—Assuming certain benefits to be agreed upon, what is the pension which ought to be given for a certain fixed contribution of salary? For this purpose I think we might take 20 as a fair average age at entry on which to base a standard pension, and the following Table will give the desired information:—

TABLE H.

*Showing the Standard Pension Benefit for 5 per-cent of Salary  
after allowing for various other Benefits.*

Pension Age .	65		60		65		60	
Rate of Interest	4%	3%	4%	3%	4%	3%	4%	3%
BENEFITS other than Pen- sion on attain- ment of Pension Age	SCALE OF PENSION; being the percentage of <i>Average</i> Salary for every year of Service				SCALE OF PENSION; being the percentage of <i>Last</i> Salary for every year of Service			
	I	I	I	I	I	I	I	I
0	4·588	3·301	2·115	1·564	2·795	2·011	1·310	·969
V	3·873	2·711	1·852	1·346	2·359	1·652	1·148	·834
V, VII	3·316	2·220	1·733	1·242	2·020	1·352	1·074	·770
III, V, VII	2·971	1·982	1·568	1·124	1·810	1·207	·971	·696
IV, V, VII	2·626	1·744	1·403	1·006	1·599	1·062	·869	·623
III, VI, VII	2·422	1·670	1·395	1·024	1·475	1·017	·864	·634
IV, VI, VII	2·078	1·498	1·230	·906	1·265	·912	·762	·561
III, VI, VIII	1·877	1·346	1·299	·967	1·143	·820	·804	·599
IV, VI, VIII	1·532	1·108	1·133	·849	·933	·675	·702	·526
IX	2·287	1·693	1·727	1·293	1·410	1·039	1·077	·804
V, IX	1·931	1·390	1·512	1·113	1·190	·853	·943	·692
III, V, IX	1·759	1·268	1·378	1·015	1·084	·779	·859	·631
IV, V, IX	1·587	1·146	1·243	·917	·978	·704	·775	·570
III, VI, IX	1·485	1·108	1·236	·932	·915	·680	·771	·580
IV, VI, IX	1·313	·986	1·102	·835	·809	·605	·687	·519

To ascertain the Pension when the excess of the accumulated contributions without interest over the annuity payments is returned, the formula in Problem VIIb must be applied to the above figures as explained on page 255.

I will conclude with the investigation of three problems about which inquiry is sometimes made.

*Problem XIIIb.*—What proportion of salary, commencing at age 20, is required to provide a pension of two-thirds of the last salary, to those only who reach the age of 65?

Equating contributions to liability, we have

$$P \left\{ N_{20}^s - \frac{1}{2} \left( {}^dM_{20}^{ls} + {}^wM_{20}^{ls} + {}^rM_{20}^{ls} \right) \right\} (1 + \frac{3}{8}i) = \frac{2}{3}s_{64}(N_{65} + \frac{1}{2}D_{65})$$

$$P = \frac{\frac{2}{3}s_{64}(N_{65} + \frac{1}{2}D_{65})}{(1 + \frac{3}{8}i) \left\{ N_{20}^s - \frac{1}{2} \left( {}^dM_{20}^{ls} + {}^wM_{20}^{ls} + {}^rM_{20}^{ls} \right) \right\}}$$

If interest is guaranteed at 4 per-cent, free of income tax, and there are no expenses, then  $P = .026505$ : say 2.65 per-cent.

If interest is guaranteed at 3 per-cent, with the same conditions, then  $P = .03683$ : say 3.683 per-cent.

*Problem XIVb.*—What proportion of salary, commencing at age 20, is required to provide a pension of  $\frac{1}{60}$ th of last salary for every year of service not exceeding 40, on retirement from ill-health before the age of 65, and on compulsory retirement at 65? This is practically the Government scale.

This is the same as Problem XIIIb with an addition to the numerator of a provision for early retirement.

As no more than 40 years' service is to be reckoned, we shall have to divide the provision for early retirement into two parts, namely, before and after 40 years' service.

The provision for early retirement during the first 40 years is  $\frac{1}{60}({}^raR_{20}^{ls} - {}^raR_{60}^{ls} - 40{}^raM_{60}^{ls})$ .

For the last 5 years it is the insurance of an annuity of  $\frac{40}{60}$ ths of the last salary  $= \frac{40}{60}{}^raM_{60}^{ls}$ .

And these, being added together, will give  $\frac{1}{60}({}^raR_{20}^{ls} - {}^raR_{60}^{ls})$ .

The correction, beyond the allowance for over-discount, will be  $-\frac{1}{2}{}^raM_{20}^{ls}$ , because the above formula provides, whenever retirement takes place before 65, whether before or after 40 years' service, an excess of an annuity of half the last year's salary.

The complete formula, therefore, is

$$P = \frac{\frac{2}{3}s_{64}(N_{65} + \frac{1}{2}D_{65}) + \frac{1}{60}({}^raR_{20}^{ls} - {}^raR_{60}^{ls} - \frac{1}{2}{}^raM_{20}^{ls})(1 + \frac{1}{2}i)}{(1 + \frac{3}{8}i) \left\{ N_{20}^s - \frac{1}{2} \left( {}^dM_{20}^{ls} + {}^wM_{20}^{ls} + {}^rM_{20}^{ls} \right) \right\}}$$



If 4 per-cent interest, free of income tax, is guaranteed, and there are no expenses, then  $P = .054616$ : say 5.462 per-cent.

If 3 per-cent is guaranteed, then  $P = .073944$ : say 7.394 per-cent.

*Problem XV<sub>B</sub>.*—The same as Problem XIV<sub>B</sub>, only substituting “average salary for 40 years or for term of service if less than 40 years” for “last salary.”

If the number of years' service exceeds 40, the employee will certainly select the last 40 years on which to base his average, and consequently the portion of the formula on the liability side representing the value of retirement at age 65 will be

$$\frac{1}{60} \sum s_{25} (N_{65} + \frac{1}{2} D_{65}).$$

When we come to consider the terms in the formula for retirement before 65, we have to divide it, as before, into two parts, and consider retirement before and after 40 years separately.

For the first 40 years' service we have—

$$\frac{1}{60} \{ {}^a R_{20}^s - {}^a R_{60}^s - (s_{20} + s_{21} + \dots + s_{59}) {}^a M_{60} \}$$

and for service after 40 years,

$$\frac{1}{60} \{ (s_{21} + \dots + s_{60}) {}^a C_{60} + (s_{22} + \dots + s_{61}) {}^a C_{61} + (s_{23} + \dots + s_{62}) {}^a C_{62} \\ + (s_{24} + \dots + s_{63}) {}^a C_{63} + (s_{25} + \dots + s_{64}) {}^a C_{64} \}$$

Now we have—

$$(s_{21} + \dots + s_{60}) {}^a C_{60} = (s_{20} + \dots + s_{59}) {}^a C_{60} \\ + (s_{60} - s_{20}) {}^a C_{60}$$

$$(s_{22} + \dots + s_{61}) {}^a C_{61} = (s_{20} + \dots + s_{59}) {}^a C_{61} \\ + (s_{60} + s_{61} - s_{20} - s_{21}) {}^a C_{61}$$

$$(s_{23} + \dots + s_{62}) {}^a C_{62} = (s_{20} + \dots + s_{59}) {}^a C_{62} \\ + (s_{60} + s_{61} + s_{62} - s_{20} - s_{21} - s_{22}) {}^a C_{62}$$

$$(s_{24} + \dots + s_{63}) {}^a C_{63} = (s_{20} + \dots + s_{59}) {}^a C_{63} \\ + (s_{60} + s_{61} + s_{62} + s_{63} - s_{20} - s_{21} - s_{22} - s_{23}) {}^a C_{63}$$

$$(s_{25} + \dots + s_{64}) {}^a C_{64} = (s_{20} + \dots + s_{59}) {}^a C_{64} \\ + (s_{60} + s_{61} + s_{62} + s_{63} + s_{64} - s_{20} - s_{21} - s_{22} - s_{23} - s_{24}) {}^a C_{64}$$

Summing the values on the right-hand side, we have—

$$(s_{20} + \dots + s_{59}) {}^a M_{60} + {}^a R_{60}^s - ({}^a M_{60} s_{20} + {}^a M_{61} s_{21} + \dots + {}^a M_{64} s_{24})$$

so that those terms of the formula which represent the pension on early retirement will be—

$$\frac{1}{60}\{ {}^aR_{20}^s - ({}^aM_{60s_{20}} + {}^aM_{61s_{21}} + \dots + {}^aM_{64s_{64}}) \}$$

which, on consideration, will be found to be self-evident; for it is  $\frac{1}{60}$ th of—

The present value of an annuity based on the total salary from age 20 to 65;

*Less* the present value of an annuity of the salary at age 20 to all those whose service will exceed 40 years; the present value of an annuity of the salary at age 21 to all those whose service will exceed 41 years; and so on:

thus cutting off the first year's salary for those who retire in the 41st year of service, the first two years' salary for those who retire in the 42nd year of service, &c.; so that each of those whose service exceeds 40 years will receive a pension based on  $\frac{1}{60}$ th of his total salary dating back for 40 years from the end of the year of his retirement.

The complete formula, therefore, is—

$$P = \frac{\frac{1}{60} \Sigma_{25} (N_{65} + \frac{1}{2} D_{65}) + \frac{1}{60} \{ {}^aR_{20}^s - ({}^aM_{60s_{20}} + {}^aM_{61s_{21}} + \dots + {}^aM_{64s_{64}}) - \frac{1}{2} {}^aM_{20}^{ls} \} (1 + \frac{1}{2} i)}{(1 + \frac{3}{8} i) \{ N_{20}^s - \frac{1}{2} ({}^dM_{20}^{ls} + {}^wM_{20}^{ls} + {}^rM_{20}^{ls}) \}}$$

If interest is guaranteed at 4 per-cent, free of income tax, and there are no expenses, then  $P = \cdot 035347$ , say 3·535 per-cent. If interest is guaranteed at 3 per-cent with the same conditions, then  $P = \cdot 04773$ , say 4·773 per-cent.

I should like to direct attention to the "Actuarial Note" by Mr. H. T. Adlard, in vol. xxxvi, page 389, as affording the means of solving the very difficult problem of a return of contributions on death, withdrawal, or retirement, with compound interest at a different rate to that used in the valuation.

## Hypothetical Experience of Staff Pension Fund.

TABLE 18.

*Commutation Columns for finding the Value of one Year's Salary on Death, Withdrawal, and Retirement; the Value of a Pension of 1 for each Year of Service on Early Retirement; and the Value of a Pension on Early Retirement based on Average Salary or Last Salary and Number of Years' Service.*

PENSION AGE 65.

INTEREST 4 PER-CENT.

Age (x)	$dM_x^{ls}$ = $\sum^d C_x s_x$	$wM_x^{ls}$ = $\sum^w C_x s_x$	$rM_x^{ls}$ = $\sum^r C_x s_x$	$raM_x^*$	$raR_x$ = $\sum^{ra} M_x$	$raM_x^s$ = $raM_x \times s_x$	$raR_x^s$ = $\sum^{ra} M_x^s$	$raM_x^{ls} \dagger$	$raR_x^{ls}$ = $\sum^{ra} M_x^{ls}$	Age (x)
15	64,737	259,869	33,017	1,572	64,373	31,436	7,218,757	296,255	12,590,444	15
16	63,969	243,849	33,017	1,572	62,801	39,295	7,187,321	296,255	12,294,189	16
17	63,096	222,749	33,017	1,572	61,229	47,154	7,148,026	296,255	11,997,934	17
18	62,148	200,219	33,017	1,572	59,657	55,013	7,100,872	296,255	11,701,679	18
19	61,168	179,149	33,017	1,572	58,085	62,872	7,045,859	296,255	11,405,424	19
20	60,164	159,909	33,017	1,572	56,513	70,731	6,982,987	296,255	11,109,169	20
21	59,138	142,404	33,017	1,572	54,941	78,591	6,912,256	296,255	10,812,914	21
22	58,103	126,754	33,017	1,572	53,369	86,450	6,833,665	296,255	10,516,659	22
23	57,052	112,784	33,017	1,572	51,797	94,309	6,747,215	296,255	10,220,404	23
24	55,996	100,304	33,017	1,572	50,225	102,168	6,652,906	296,255	9,924,149	24
25	54,949	89,124	33,017	1,572	48,653	110,027	6,550,738	296,255	9,627,894	25
26	53,892	79,114	33,017	1,572	47,081	116,314	6,440,711	296,255	9,331,639	26
27	52,812	70,234	33,017	1,572	45,509	122,601	6,324,397	296,255	9,035,384	27
28	51,743	62,278	33,017	1,572	43,937	128,888	6,201,796	296,255	8,739,129	28
29	50,693	55,226	32,965	1,567	42,365	134,765	6,072,908	295,864	8,442,874	29
30	49,635	48,862	32,911	1,562	40,798	140,613	5,938,143	295,462	8,147,000	30
31	48,564	43,192	32,831	1,556	39,236	146,225	5,797,530	294,851	7,851,548	31
32	47,492	38,116	32,724	1,547	37,680	151,583	5,651,305	294,022	7,556,697	32
33	46,414	33,608	32,621	1,538	36,133	156,892	5,499,722	293,213	7,262,675	33
34	45,343	29,528	32,491	1,528	34,595	161,937	5,342,830	292,188	6,969,462	34
35	44,272	25,818	32,357	1,518	33,067	166,927	5,180,893	291,108	6,677,274	35
36	43,172	22,518	32,196	1,506	31,549	171,645	5,013,966	289,804	6,386,166	36
37	42,078	19,554	32,037	1,494	30,043	176,310	4,842,321	288,492	6,096,362	37
38	40,957	16,840	31,850	1,481	28,549	180,681	4,666,011	286,940	5,807,870	38
39	39,822	14,400	31,639	1,466	27,068	184,771	4,485,330	285,160	5,520,930	39
40	38,663	12,258	31,402	1,450	25,602	188,555	4,300,559	283,142	5,235,770	40
41	37,493	10,338	31,142	1,433	24,152	192,046	4,112,004	280,901	4,952,628	41
42	36,300	8,566	30,858	1,415	22,719	195,227	3,919,958	278,423	4,671,727	42
43	35,072	7,048	30,552	1,395	21,304	198,106	3,724,731	275,721	4,393,304	43
44	33,837	5,770	30,224	1,375	19,909	200,677	3,526,625	272,795	4,117,583	44
45	32,567	4,602	29,873	1,353	18,534	202,932	3,325,948	269,638	3,844,788	45
46	31,232	3,552	29,479	1,329	17,181	204,661	3,123,016	266,052	3,575,150	46
47	29,861	2,628	29,040	1,303	15,852	205,868	2,918,355	262,032	3,309,098	47
48	28,471	1,838	28,560	1,275	14,549	206,530	2,712,487	257,594	3,047,066	48
49	27,029	1,190	28,038	1,245	13,274	206,657	2,505,957	252,743	2,789,472	49
50	25,535	692	27,454	1,212	12,029	206,035	2,299,300	247,274	2,536,729	50
51	24,022	352	26,809	1,176	10,817	204,671	2,093,265	241,204	2,289,455	51
52	22,491	178	26,036	1,137	9,641	202,379	1,888,594	234,364	2,048,251	52
53	20,907	...	25,285	1,094	8,504	199,170	1,686,215	226,779	1,813,887	53
54	19,287	...	24,387	1,048	7,410	194,844	1,487,045	218,264	1,587,108	54
55	17,632	...	23,398	997	6,362	189,432	1,292,201	208,863	1,368,844	55
56	15,922	...	22,300	942	5,365	182,767	1,102,769	198,430	1,159,981	56
57	14,157	...	21,056	881	4,423	174,517	920,002	186,654	961,551	57
58	12,375	...	19,652	805	3,542	162,648	745,485	173,431	774,897	58
59	10,557	...	18,034	730	2,737	150,458	582,837	158,319	601,466	59
60	8,724	...	16,075	642	2,007	134,887	432,379	140,177	443,147	60
61	6,876	...	13,715	539	1,365	115,301	297,492	118,603	302,976	61
62	5,057	...	10,781	416	826	90,647	182,191	92,286	184,367	62
63	3,269	...	7,299	276	410	61,181	91,544	61,718	92,081	63
64	1,582	...	3,636	134	134	30,363	30,363	30,363	30,363	64

\*  $raM_x = \sum \{ rC_x \times (a' - x + .5) \} + raM_x^{ls} = \sum \{ rC_x \times (a' - x + .5) \}$



## Hypothetical Experience of Staff Pension Fund.

TABLE 19—(continuation of Table 17).

*Multipliers for use in a Valuation.*

PENSION AGE 65.

INTEREST 4 PER-CENT.

Age ( <i>x</i> )	$dM_x^{ls}$ $\div D_x^s$	$wM_x^{ls}$ $\div D_x^s$	$rM_x^{ls}$ $\div D_x^s$	$raM_x$ $\div D_x$	$raR_x$ $\div D_x$	$raR_x^s$ $\div D_x^s$	$raM_x^{ls}$ $\div D_x^s$	$raR_x^{ls}$ $\div D_x^s$	Age ( <i>x</i> )
15	·291	1·170	·149	·142	5·796	32·499	1·334	56·683	15
16	·260	0·991	·134	·160	6·382	29·214	1·204	49·971	16
17	·245	·865	·128	·183	7·135	27·764	1·151	46·598	17
18	·238	·765	·126	·210	7·987	27·163	1·133	44·764	18
19	·233	·684	·126	·240	8·867	26·889	1·131	43·524	19
20	·231	·613	·127	·271	9·754	26·782	1·136	42·607	20
21	·229	·552	·128	·305	10·650	26·797	1·149	41·919	21
22	·228	·498	·130	·340	11·532	26·847	1·164	41·318	22
23	·228	·450	·132	·376	12·406	26·935	1·183	40·800	23
24	·227	·407	·134	·415	13·252	27·006	1·203	40·285	24
25	·227	·369	·137	·455	14·082	27·086	1·225	39·809	25
26	·230	·338	·141	·497	14·880	27·580	1·265	39·855	26
27	·233	·310	·146	·541	15·650	27·882	1·306	39·834	27
28	·235	·283	·150	·586	16·388	28·210	1·348	39·752	28
29	·238	·259	·155	·632	17·090	28·485	1·388	39·602	29
30	·240	·236	·159	·680	17·769	28·736	1·430	39·426	30
31	·242	·225	·164	·730	18·403	28·928	1·471	39·179	31
32	·244	·196	·168	·780	18·992	29·066	1·512	38·866	32
33	·246	·178	·173	·832	19·542	29·161	1·555	38·509	33
34	·248	·161	·178	·886	20·055	29·220	1·598	38·116	34
35	·250	·146	·182	·941	20·500	29·200	1·641	37·633	35
36	·251	·131	·188	·997	20·893	29·127	1·684	37·099	36
37	·252	·117	·192	1·057	21·247	29·022	1·729	36·537	37
38	·253	·104	·197	1·117	21·530	28·843	1·774	35·902	38
39	·254	·092	·202	1·178	21·742	28·592	1·818	35·195	39
40	·254	·081	·207	1·241	21·900	28·299	1·863	34·453	40
41	·255	·070	·212	1·305	21·996	27·948	1·909	33·661	41
42	·255	·060	·217	1·371	22·015	27·525	1·955	32·803	42
43	·255	·051	·222	1·438	21·963	27·012	2·002	31·896	43
44	·254	·043	·227	1·507	21·830	26·486	2·049	30·924	44
45	·253	·036	·232	1·577	21·601	25·843	2·095	29·874	45
46	·251	·029	·237	1·647	21·290	25·129	2·141	28·768	46
47	·249	·022	·243	1·719	20·913	24·368	2·188	27·630	47
48	·246	·015	·247	1·788	20·405	23·484	2·230	26·380	48
49	·243	·011	·253	1·861	19·841	22·566	2·276	25·118	49
50	·239	·006	·257	1·930	19·154	21·537	2·315	23·761	50
51	·234	·003	·261	1·994	18·334	20·391	2·350	22·302	51
52	·228	·002	·265	2·056	17·434	19·186	2·381	20·809	52
53	·222	...	·269	2·117	16·449	17·920	2·410	19·278	53
54	·215	...	·272	2·169	15·342	16·552	2·429	17·666	54
55	·206	...	·273	2·211	14·106	15·080	2·437	15·974	55
56	·196	...	·275	2·254	12·835	13·599	2·447	14·305	56
57	·185	...	·275	2·278	11·429	12·064	2·436	12·549	57
58	·172	...	·273	2·262	9·950	10·367	2·412	10·776	58
59	·158	...	·269	2·247	8·422	8·706	2·365	8·984	59
60	·141	...	·260	2·185	6·827	7·003	2·270	7·178	60
61	·122	...	·244	2·049	5·190	5·286	2·107	5·383	61
62	·101	...	·215	1·808	3·591	3·634	1·841	3·677	62
63	·075	...	·167	1·399	2·081	2·093	1·411	2·105	63
64	·042	...	·097	·809	·809	·809	·809	·809	64



## Hypothetical Experience of Staff Pension Fund.

TABLE 20.

*Simple Commutation Columns (according to Table 4), and Commutation Columns for valuing Benefits on Death and Early Retirement.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age ( <i>x</i> )	$N_x^{(4)}$	$M_x^{(4)}$	$R_x^{(4)}$	${}^dM_x$ $=\sum_x {}^{59}C_x$	${}^dR_x$ $=\sum {}^dM_x$	${}^rM_x$ $=\sum_x {}^{59}rC_x$	${}^rR_x$ $=\sum {}^rM_x$	Age ( <i>x</i> )
15	104,449	785·6	20777·6	603·4	10481·6	100·84	3582·49	15
16	94,608	747·2	19992·0	565·0	9878·2	100·84	3481·65	16
17	86,026	712·3	19244·8	530·1	9313·2	100·84	3380·81	17
18	78,557	680·7	18532·5	498·5	8783·1	100·84	3279·97	18
19	72,006	652·7	17851·8	470·5	8284·6	100·84	3179·13	19
20	66,212	627·6	17199·1	445·4	7814·1	100·84	3078·29	20
21	61,053	604·8	16571·5	422·6	7368·7	100·84	2977·45	21
22	56,425	584·1	15966·7	401·9	6946·1	100·84	2876·61	22
23	52,250	565·0	15382·6	382·8	6544·2	100·84	2775·77	23
24	48,460	547·4	14817·6	365·2	6161·4	100·84	2674·93	24
25	45,005	531·3	14270·2	349·1	5796·2	100·84	2574·09	25
26	41,841	516·2	13738·9	334·0	5447·1	100·84	2473·25	26
27	38,933	501·6	13222·7	319·4	5113·1	100·84	2372·41	27
28	36,252	487·9	12721·1	305·7	4793·7	100·84	2271·57	28
29	33,773	475·1	12233·2	292·9	4488·0	100·20	2170·73	29
30	31,477	462·8	11758·1	280·6	4195·1	99·58	2070·53	30
31	29,345	450·9	11295·3	268·7	3914·5	98·69	1970·95	31
32	27,361	439·5	10844·4	257·3	3645·8	97·55	1872·26	32
33	25,512	428·5	10404·9	246·3	3388·5	96·45	1774·71	33
34	23,787	418·0	9976·4	235·8	3142·2	95·13	1678·26	34
35	22,174	407·9	9558·4	225·7	2906·4	93·86	1583·13	35
36	20,664	397·9	9150·5	215·7	2680·7	92·40	1489·27	36
37	19,250	388·3	8752·6	206·1	2465·0	91·00	1396·87	37
38	17,924	378·8	8364·3	196·6	2258·9	89·42	1305·87	38
39	16,679	369·5	7985·5	187·3	2062·3	87·69	1216·45	39
40	15,510	360·3	7616·0	178·1	1875·0	85·81	1128·76	40
41	14,412	351·3	7255·7	169·1	1696·9	83·81	1042·95	41
42	13,380	342·4	6904·4	160·2	1527·8	81·69	959·14	42
43	12,410	333·5	6562·0	151·3	1367·6	79·47	877·45	43
44	11,498	324·8	6228·5	142·6	1216·3	77·16	797·98	44
45	10,640	316·1	5903·7	133·9	1073·7	74·76	720·82	45
46	9,833	307·2	5587·6	125·0	939·8	72·13	646·06	46
47	9,075	298·3	5280·4	116·1	814·8	69·28	573·93	47
48	8,362	289·5	4982·1	107·3	698·7	66·24	504·65	48
49	7,693	280·6	4692·6	98·4	591·4	63·02	438·41	49
50	7,065	271·6	4412·0	89·4	493·0	59·50	375·39	50
51	6,475	262·7	4140·4	80·5	403·6	55·71	315·89	51
52	5,922	253·9	3877·7	71·7	323·1	51·55	260·18	52
53	5,405	245·0	3623·8	62·8	251·4	47·05	208·63	53
54	4,922	236·1	3378·8	53·9	188·6	42·12	161·58	54
55	4,471	227·2	3142·7	45·0	134·7	36·80	119·46	55
56	4,053	218·2	2915·5	36·0	89·7	31·02	82·66	56
57	3,666	209·1	2697·3	26·9	53·7	24·61	51·64	57
58	3,310	200·1	2488·2	17·9	26·8	17·52	27·03	58
59	2,985	191·1	2288·1	8·9	8·9	9·51	9·51	59
60	2,691	182·2	2097·0	...	...	...	...	60

NOTE.— $D_x^{(4)}$  and  $C_x^{(4)}$  are the same as in Table 7.  ${}^dC_x$ ,  ${}^wC_x$ ,  ${}^wM_x$ ,  ${}^wR_x$  and  ${}^rC_x$  are the same as in Table 9.

Hypothetical Experience of Staff Pension Fund.

TABLE 21.

Commutation Tables for finding the Values of the Return of Contributions of 1 per annum, with Compound Interest at 4 per-cent, on Death or Early Retirement, and for finding the Value of a Pension of 1 for each Year of Service on Early Retirement.

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age (x)	${}^dD_x$ $=l'_x \times v^{x+1}$	${}^dN_x$ $=\Sigma {}^dD_x$	${}^rD_x$ $=\Sigma r_x \times v^{x+1}$	${}^rN_x$ $=\Sigma {}^rD_x$	${}^raM_x$ $=\Sigma \{ {}^rC_x \times (a'_{x+.5} + .5) \}$	${}^raR_x$ $=\Sigma {}^raM_x$	Age (x)
15	1,333	19,575	410	8,128	930	33,469	15
16	1,245	18,242	394	7,718	930	32,539	16
17	1,163	16,997	379	7,324	930	31,609	17
18	1,088	15,834	364	6,945	930	30,679	18
19	1,020	14,746	350	6,581	930	29,749	19
20	956	13,726	337	6,231	930	28,819	20
21	898	12,770	324	5,894	930	27,889	21
22	843	11,872	311	5,570	930	26,959	22
23	792	11,029	299	5,259	930	26,029	23
24	745	10,237	288	4,960	930	25,099	24
25	701	9,492	277	4,672	930	24,169	25
26	659	8,791	266	4,395	930	23,239	26
27	620	8,132	256	4,129	930	22,309	27
28	583	7,512	246	3,873	930	21,379	28
29	548	6,929	236	3,627	925	20,449	29
30	515	6,381	226	3,391	920	19,524	30
31	484	5,866	217	3,165	913	18,604	31
32	454	5,382	207	2,948	905	17,691	32
33	426	4,928	198	2,741	896	16,786	33
34	400	4,502	189	2,543	885	15,890	34
35	375	4,102	181	2,354	875	15,005	35
36	351	3,727	172	2,173	863	14,130	36
37	328	3,376	164	2,001	852	13,267	37
38	306	3,048	157	1,837	839	12,415	38
39	286	2,742	149	1,680	824	11,576	39
40	266	2,456	141	1,531	808	10,752	40
41	247	2,190	134	1,390	791	9,944	41
42	229	1,943	127	1,256	772	9,153	42
43	212	1,714	120	1,129	753	8,381	43
44	195	1,502	113	1,009	732	7,628	44
45	179	1,307	106	896	711	6,896	45
46	164	1,128	100	790	687	6,185	46
47	149	964	93	690	661	5,498	47
48	135	815	87	597	633	4,837	48
49	121	680	80	510	603	4,204	49
50	108	559	74	430	570	3,601	50
51	95	451	67	356	534	3,031	51
52	83	356	61	289	495	2,497	52
53	71	273	54	228	452	2,002	53
54	60	202	47	174	405	1,550	54
55	49	142	40	127	355	1,145	55
56	38	93	33	87	300	790	56
57	28	55	26	54	239	490	57
58	18	27	18	28	163	251	58
59	9	9	10	10	88	88	59

## Hypothetical Experience of Staff Pension Fund.

TABLE 22.

*Table of Average Salaries and various combinations thereof.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age ( $x$ )	$s_x$	$\Sigma s_x$	$\frac{\Sigma s_x}{\div s_x}$	$\frac{s_{59}}{\div s_x}$	$s_x v^x$	$\Sigma s_x v^x$	$(1+i)^{x-1}$	Age ( $x$ )
15	20	5,255	262.75	10.300	11.11	1083.39	1.732	15
16	25	5,235	209.40	8.240	13.35	1072.28	1.801	16
17	30	5,210	173.67	6.867	15.40	1058.93	1.873	17
18	35	5,180	148.00	5.886	17.28	1043.53	1.948	18
19	40	5,145	128.63	5.150	18.98	1026.25	2.026	19
20	45	5,105	113.44	4.578	20.54	1007.27	2.107	20
21	50	5,060	101.20	4.120	21.94	986.73	2.191	21
22	55	5,010	91.09	3.746	23.21	964.79	2.279	22
23	60	4,955	82.58	3.433	24.34	941.58	2.370	23
24	65	4,895	75.31	3.169	25.36	917.24	2.465	24
25	70	4,830	69.00	2.943	26.26	891.88	2.563	25
26	74	4,760	64.33	2.784	26.69	865.62	2.666	26
27	78	4,686	60.08	2.641	27.05	838.93	2.773	27
28	82	4,608	56.20	2.512	27.35	811.88	2.883	28
29	86	4,526	52.63	2.395	27.58	784.53	2.999	29
30	90	4,440	49.33	2.289	27.75	756.95	3.119	30
31	94	4,350	46.28	2.192	27.87	729.20	3.243	31
32	98	4,256	43.43	2.102	27.94	701.33	3.373	32
33	102	4,158	40.76	2.020	27.96	673.39	3.508	33
34	106	4,056	38.26	1.943	27.94	645.43	3.648	34
35	110	3,950	35.91	1.873	27.87	617.49	3.794	35
36	114	3,840	33.68	1.807	27.78	589.62	3.946	36
37	118	3,726	31.58	1.746	27.65	561.84	4.104	37
38	122	3,608	29.57	1.689	27.49	534.19	4.268	38
39	126	3,486	27.67	1.635	27.29	506.70	4.439	39
40	130	3,360	25.85	1.585	27.08	479.41	4.616	40
41	134	3,230	24.11	1.537	26.84	452.33	4.801	41
42	138	3,096	22.44	1.493	26.58	425.49	4.993	42
43	142	2,958	20.83	1.451	26.30	398.91	5.193	43
44	146	2,816	19.29	1.411	25.99	372.61	5.401	44
45	150	2,670	17.80	1.373	25.68	346.62	5.617	45
46	154	2,520	16.36	1.338	25.35	320.94	5.841	46
47	158	2,366	14.97	1.304	25.01	295.59	6.075	47
48	162	2,208	13.63	1.272	24.66	270.58	6.318	48
49	166	2,046	12.33	1.241	24.29	245.92	6.571	49
50	170	1,880	11.06	1.212	23.92	221.63	6.833	50
51	174	1,710	9.83	1.184	23.54	197.71	7.107	51
52	178	1,536	8.63	1.157	23.16	174.17	7.391	52
53	182	1,358	7.46	1.132	22.77	151.01	7.687	53
54	186	1,176	6.32	1.108	22.38	128.24	7.994	54
55	190	990	5.21	1.084	21.98	105.86	8.314	55
56	194	800	4.12	1.062	21.57	83.88	8.646	56
57	198	606	3.06	1.040	21.17	62.31	8.992	57
58	202	408	2.02	1.020	20.77	41.14	9.352	58
59	206	206	1.00	1.000	20.37	20.37	9.726	59



## Hypothetical Experience of Staff Pension Fund.

TABLE 23.

*Commutation Columns for finding the Present Values of Future Salary, and Return of Contributions on Death and Early Retirement, without Interest.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age (x)	$\text{I}_x^s$ $= \sum_x^{59} (vD_x^s)$	${}^dM_x^s$ $= {}^dM_x \times s_x$	${}^dR_x^s$ $= \sum {}^dM_x^s$	${}^rM_x^s$ $= {}^rM_x \times s_x$	${}^rR_x^s$ $= \sum {}^rM_x^s$	Age (x)
15	7,354,555	12,068	851,190	2,017	362,839	15
16	7,140,978	14,125	839,122	2,521	360,822	16
17	6,904,416	15,903	824,997	3,025	358,301	17
18	6,656,858	17,448	809,094	3,529	355,276	18
19	6,405,497	18,820	791,646	4,034	351,747	19
20	6,153,535	20,043	772,826	4,538	347,713	20
21	5,902,833	21,130	752,783	5,042	343,175	21
22	5,654,804	22,105	731,653	5,546	338,133	22
23	5,410,054	22,968	709,548	6,050	332,587	23
24	5,169,189	23,738	686,580	6,555	326,537	24
25	4,932,314	24,437	662,842	7,059	319,982	25
26	4,699,766	24,716	638,405	7,463	312,923	26
27	4,474,635	24,913	613,689	7,866	305,460	27
28	4,256,535	25,067	588,776	8,269	297,594	28
29	4,045,148	25,189	563,709	8,617	289,325	29
30	3,840,154	25,254	538,520	8,962	280,708	30
31	3,641,462	25,258	513,266	9,277	271,746	31
32	3,448,762	25,215	488,008	9,560	262,469	32
33	3,261,808	25,123	462,793	9,838	252,909	33
34	3,080,464	24,995	437,670	10,084	243,071	34
35	2,904,647	24,827	412,675	10,325	232,987	35
36	2,734,041	24,590	387,848	10,534	222,662	36
37	2,568,522	24,320	363,258	10,738	212,128	37
38	2,408,087	23,985	338,938	10,909	201,390	38
39	2,252,537	23,600	314,953	11,049	190,481	39
40	2,101,700	23,153	291,353	11,155	179,432	40
41	1,955,575	22,659	268,200	11,231	168,277	41
42	1,814,102	22,108	245,541	11,273	157,046	42
43	1,677,164	21,485	223,433	11,285	145,773	43
44	1,544,722	20,820	201,948	11,265	134,488	44
45	1,416,691	20,085	181,128	11,214	123,223	45
46	1,292,941	19,250	161,043	11,108	112,009	46
47	1,173,443	18,344	141,793	10,946	100,901	47
48	1,058,285	17,383	123,449	10,731	89,955	48
49	947,222	16,334	106,066	10,461	79,224	49
50	840,439	15,198	89,732	10,115	68,763	50
51	737,785	14,007	74,534	9,694	58,648	51
52	639,073	12,763	60,527	9,176	48,954	52
53	544,425	11,430	47,764	8,563	39,778	53
54	453,950	10,025	36,334	7,834	31,215	54
55	367,567	8,550	26,309	6,992	23,381	55
56	285,173	6,984	17,759	6,018	16,389	56
57	207,200	5,326	10,775	4,873	10,371	57
58	133,521	3,616	5,449	3,539	5,498	58
59	64,375	1,833	1,833	1,959	1,959	59

NOTE.—The values of  $D_x^s$  will be found in Table 13, and the values of  ${}^wM_x^s$  and  ${}^wR_x^s$  are the same as in Table 14.



## Hypothetical Experience of Staff Pension Fund.

TABLE 24.

*Commutation Columns for finding the Values of Return of Contributions, with Compound Interest, on Death or Early Retirement.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age ( $x$ )	${}^dD_x^s$ $= {}^dD_x \times s_x$	${}^dN_x^s$ $= \sum {}^dD_x^s$	${}^rD_x^s$ $= {}^rD_x \times s_x$	${}^rN_x^s$ $= \sum {}^rD_x^s$	Age ( $x$ )
15	26,660	1,435,361	8,200	678,082	15
16	31,125	1,408,701	9,850	669,882	16
17	34,890	1,377,576	11,370	660,032	17
18	38,080	1,342,686	12,740	648,662	18
19	40,800	1,304,606	14,000	635,922	19
20	43,020	1,263,806	15,165	621,922	20
21	44,900	1,220,786	16,200	606,757	21
22	46,365	1,175,886	17,105	590,557	22
23	47,520	1,129,521	17,940	573,452	23
24	48,425	1,082,001	18,720	555,512	24
25	49,070	1,033,576	19,390	536,792	25
26	48,766	984,506	19,684	517,402	26
27	48,360	935,740	19,968	497,718	27
28	47,806	887,380	20,172	477,750	28
29	47,128	839,574	20,296	457,578	29
30	46,350	792,446	20,340	437,282	30
31	45,496	746,096	20,398	416,942	31
32	44,492	700,600	20,286	396,544	32
33	43,452	656,108	20,196	376,258	33
34	42,400	612,656	20,034	356,062	34
35	41,250	570,256	19,910	336,028	35
36	40,014	529,006	19,608	316,118	36
37	38,704	488,992	19,352	296,510	37
38	37,332	450,288	19,154	277,158	38
39	36,036	412,956	18,774	258,004	39
40	34,580	376,920	18,330	239,230	40
41	33,098	342,340	17,956	220,900	41
42	31,602	309,242	17,526	202,944	42
43	30,104	277,640	17,040	185,418	43
44	28,470	247,536	16,498	168,378	44
45	26,850	219,066	15,900	151,880	45
46	25,256	192,216	15,400	135,980	46
47	23,542	166,960	14,694	120,580	47
48	21,870	143,418	14,094	105,886	48
49	20,086	121,548	13,280	91,792	49
50	18,360	101,462	12,580	78,512	50
51	16,530	83,102	11,658	65,932	51
52	14,774	66,572	10,858	54,274	52
53	12,922	51,798	9,828	43,416	53
54	11,160	38,876	8,742	33,588	54
55	9,310	27,716	7,600	24,846	55
56	7,372	18,406	6,402	17,246	56
57	5,544	11,034	5,148	10,844	57
58	3,636	5,490	3,636	5,696	58
59	1,854	1,854	2,060	2,060	59

## Hypothetical Experience of Staff Pension Fund.

TABLE 25.

*Commutation Columns for finding the Present Value of the Last Year's Salary on Death or Retirement; and the Present Value of a Pension based on Average Salary and Last Salary.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age ( $x$ )	$^dM_x^{ls}$ $= \Sigma(^dC_x s_x)$	$^rM_x^{ls}$ $= \Sigma(^rC_x s_x)$	$^{ra}M_x^s$ $= ^{ra}M_x \times s_x$	$^{ra}R_x^s$ $= \Sigma^{ra}M_x^s$	$^{ra}M_x^{ls}$ $= \Sigma\{^rC_x s_x \times (a'_{x+5} + .5)\}$	$^{ra}R_x^{ls}$ $= \Sigma^{ra}M_x^{ls}$	Age ( $x$ )
15	56,013	16,942	18,600	3,411,828	156,078	5,839,331	15
16	55,245	16,942	23,250	3,393,228	156,078	5,683,253	16
17	54,372	16,942	27,900	3,369,978	156,078	5,527,175	17
18	53,424	16,942	32,550	3,342,078	156,078	5,371,097	18
19	52,444	16,942	37,200	3,309,528	156,078	5,215,019	19
20	51,440	16,942	41,850	3,272,328	156,078	5,058,941	20
21	50,414	16,942	46,500	3,230,478	156,078	4,902,863	21
22	49,379	16,942	51,150	3,183,978	156,078	4,746,789	22
23	48,328	16,942	55,800	3,132,828	156,078	4,590,707	23
24	47,272	16,942	60,450	3,077,028	156,078	4,434,629	24
25	46,225	16,942	65,100	3,016,578	156,078	4,278,551	25
26	45,168	16,942	68,820	2,951,478	156,078	4,122,473	26
27	44,088	16,942	72,540	2,882,658	156,078	3,966,395	27
28	43,019	16,942	76,260	2,810,118	156,078	3,810,317	28
29	41,969	16,890	79,550	2,733,858	155,687	3,654,239	29
30	40,911	16,836	82,800	2,654,308	155,285	3,498,552	30
31	39,840	16,756	85,822	2,571,508	154,674	3,343,267	31
32	38,768	16,649	88,690	2,485,686	153,845	3,188,593	32
33	37,690	16,546	91,392	2,396,996	153,036	3,034,748	33
34	36,619	16,416	93,810	2,305,604	152,011	2,881,712	34
35	35,548	16,282	96,250	2,211,794	150,931	2,729,701	35
36	34,448	16,121	98,382	2,115,544	149,627	2,578,770	36
37	33,354	15,961	100,536	2,017,162	148,315	2,429,143	37
38	32,233	15,775	102,358	1,916,626	146,770	2,280,828	38
39	31,098	15,564	103,824	1,814,268	144,983	2,134,058	39
40	29,939	15,327	105,040	1,710,444	142,965	1,989,075	40
41	28,769	15,067	105,994	1,605,404	140,723	1,846,110	41
42	27,576	14,783	106,536	1,499,410	138,246	1,705,387	42
43	26,348	14,477	106,926	1,392,874	135,544	1,567,141	43
44	25,113	14,149	106,872	1,285,948	132,618	1,431,597	44
45	23,843	13,798	106,650	1,179,076	129,461	1,298,979	45
46	22,508	13,404	105,798	1,072,426	125,875	1,169,518	46
47	21,137	12,965	104,438	966,628	121,854	1,043,643	47
48	19,747	12,484	102,546	862,190	117,416	921,789	48
49	18,305	11,963	100,098	759,644	112,565	804,373	49
50	16,811	11,379	96,900	659,546	107,096	691,808	50
51	15,298	10,734	92,916	562,646	101,027	584,712	51
52	13,767	10,010	88,110	469,730	94,187	483,685	52
53	12,183	9,209	82,264	381,620	86,602	389,498	53
54	10,563	8,312	75,330	299,356	78,086	302,896	54
55	8,908	7,323	67,450	224,026	68,686	224,810	55
56	7,198	6,224	58,200	156,576	58,253	156,124	56
57	5,433	4,981	47,322	98,376	46,477	97,871	57
58	3,651	3,577	32,926	51,054	33,253	51,394	58
59	1,833	1,959	18,128	18,128	18,141	18,141	59

NOTE.—The values of  $^wM_x^{ls}$  will be found in Table 18.

## Hypothetical Experience of Staff Pension Fund.

TABLE 26.

*Multipliers for use in a Valuation.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age (x)	$(N_{60} + \frac{1}{2} D_{60})$ $\div D_x$	$N_x^s$ $\div D_x^s$	$dM_x$ $\div D_x$	$dR_x^s$ $\div D_x^s$	$rM_x$ $\div D_x$	$rR_x^s$ $\div D_x^s$	$dD_x$ $\div D_x$	$dN_x^s$ $\div D_x^s$	Age (x)
15	·255	33·111	·054	3·832	·009	1·633	·120	6·462	15
16	·288	29·026	·057	3·411	·010	1·467	·127	5·726	16
17	·331	26·818	·062	3·201	·012	1·392	·136	5·351	17
18	·380	25·465	·067	3·095	·014	1·359	·146	5·136	18
19	·433	24·444	·072	3·021	·015	1·342	·156	4·979	19
20	·490	23·601	·077	2·964	·017	1·334	·165	4·847	20
21	·550	22·883	·082	2·918	·020	1·330	·174	4·733	21
22	·613	22·216	·087	2·874	·022	1·328	·182	4·620	22
23	·680	21·597	·092	2·833	·024	1·328	·190	4·509	23
24	·749	20·983	·096	2·787	·027	1·326	·197	4·392	24
25	·821	20·394	·101	2·741	·029	1·323	·203	4·274	25
26	·897	20·073	·106	2·727	·032	1·337	·208	4·205	26
27	·976	19·728	·110	2·706	·035	1·347	·213	4·126	27
28	1·058	19·362	·114	2·678	·038	1·354	·217	4·037	28
29	1·144	18·974	·118	2·644	·040	1·357	·221	3·938	29
30	1·236	18·584	·122	2·606	·043	1·359	·224	3·835	30
31	1·331	18·170	·126	2·561	·046	1·356	·227	3·723	31
32	1·430	17·738	·130	2·510	·049	1·350	·229	3·603	32
33	1·534	17·295	·133	2·454	·052	1·341	·230	3·479	33
34	1·645	16·847	·137	2·394	·055	1·329	·232	3·351	34
35	1·759	16·370	·140	2·326	·058	1·313	·232	3·214	35
36	1·879	15·883	·143	2·253	·061	1·294	·232	3·073	36
37	2·006	15·394	·146	2·177	·064	1·271	·232	2·931	37
38	2·140	14·886	·148	2·095	·067	1·245	·231	2·784	38
39	2·279	14·359	·150	2·008	·070	1·214	·230	2·633	39
40	2·427	13·830	·152	1·917	·073	1·181	·228	2·480	40
41	2·584	13·291	·154	1·823	·076	1·144	·225	2·327	41
42	2·749	12·738	·155	1·724	·079	1·103	·222	2·171	42
43	2·925	12·176	·156	1·622	·082	1·058	·219	2·016	43
44	3·111	11·601	·156	1·517	·085	1·010	·214	1·859	44
45	3·307	11·008	·156	1·407	·087	·957	·209	1·702	45
46	3·516	10·404	·155	1·296	·089	·901	·203	1·547	46
47	3·743	9·798	·153	1·184	·091	·842	·197	1·394	47
48	3·979	9·162	·150	1·069	·093	·779	·189	1·242	48
49	4·241	8·529	·147	·955	·094	·713	·181	1·095	49
50	4·518	7·872	·142	·841	·095	·644	·172	·950	50
51	4·809	7·187	·136	·726	·094	·571	·161	·809	51
52	5·130	6·492	·130	·615	·093	·497	·150	·676	52
53	5·487	5·786	·121	·508	·091	·423	·137	·550	53
54	5·874	5·053	·112	·404	·087	·347	·124	·433	54
55	6·290	4·290	·100	·307	·082	·273	·109	·323	55
56	6·787	3·517	·086	·219	·074	·202	·091	·227	56
57	7·331	2·704	·070	·141	·064	·135	·072	·144	57
58	7·969	1·857	·050	·076	·049	·076	·051	·076	58
59	8·729	·962	·027	·027	·029	·029	·027	·027	59

NOTE.— $N_{60} = D_{60} \times a_{60}$  (No. 2 Table).  ${}^wM_x \div D_x$  and  ${}^wR_x^s \div D_x^s$  are the same as in Table 17.



## Hypothetical Experience of Staff Pension Fund.

TABLE 26—(continued).

*Multipliers for use in a Valuation.*

PENSION AGE 60.

INTEREST 4 PER-CENT.

Age ( <i>x</i> )	$rD_x$ ÷ $D_x$	$rN_x^s$ ÷ $D_x^s$	$r^aM_x$ ÷ $D_x$	$r^aR_x^s$ ÷ $D_x^s$	$r^aM_x^{ls}$ ÷ $D_x^s$	$r^aR_x^{ls}$ ÷ $D_x^s$	$dM_x^{ls}$ ÷ $D_x^s$	$rM_x^{ls}$ ÷ $D_x^s$	Age ( <i>x</i> )
15	·037	3·053	·084	15·360	·703	26·289	·252	·076	15
16	·040	2·723	·035	13·792	·634	23·100	·225	·069	16
17	·044	2·564	·108	13·089	·606	21·468	·211	·066	17
18	·049	2·481	·125	12·784	·597	20·546	·204	·065	18
19	·053	2·427	·142	12·630	·596	19·902	·200	·065	19
20	·058	2·385	·161	12·550	·598	19·403	·197	·065	20
21	·063	2·352	·180	12·524	·605	19·007	·195	·066	21
22	·067	2·320	·201	12·509	·613	18·649	·194	·067	22
23	·072	2·289	·223	12·506	·623	18·326	·193	·068	23
24	·076	2·255	·245	12·491	·634	18·001	·192	·069	24
25	·080	2·220	·269	12·473	·645	17·691	·191	·070	25
26	·084	2·210	·294	12·606	·667	17·607	·193	·072	26
27	·088	2·194	·320	12·709	·688	17·487	·194	·075	27
28	·092	2·173	·347	12·782	·710	17·332	·196	·077	28
29	·095	2·146	·373	12·824	·730	17·140	·197	·079	29
30	·098	2·116	·401	12·845	·752	16·931	·198	·082	30
31	·102	2·080	·428	12·831	·772	16·682	·199	·084	31
32	·104	2·040	·456	12·785	·791	16·400	·199	·086	32
33	·107	1·995	·485	12·710	·811	16·091	·200	·088	33
34	·110	1·947	·513	12·609	·831	15·760	·200	·090	34
35	·112	1·894	·543	12·466	·851	15·385	·200	·092	35
36	·114	1·836	·572	12·289	·869	14·981	·200	·095	36
37	·116	1·777	·603	12·090	·889	14·558	·200	·096	37
38	·118	1·713	·633	11·848	·907	14·099	·199	·098	38
39	·120	1·645	·662	11·566	·924	13·604	·198	·099	39
40	·121	1·574	·691	11·255	·941	13·089	·197	·101	40
41	·122	1·501	·720	10·911	·956	12·547	·196	·102	41
42	·123	1·425	·748	10·528	·971	11·975	·194	·104	42
43	·124	1·346	·776	10·113	·984	11·377	·191	·105	43
44	·124	1·265	·803	9·658	·996	10·752	·189	·106	44
45	·124	1·180	·829	9·161	1·006	10·093	·185	·107	45
46	·124	1·094	·851	8·629	1·013	9·410	·181	·108	46
47	·123	1·007	·872	8·071	1·017	8·714	·176	·108	47
48	·122	·917	·888	7·464	1·017	7·980	·171	·108	48
49	·120	·827	·901	6·841	1·014	7·243	·165	·108	49
50	·118	·735	·908	6·178	1·003	6·480	·157	·107	50
51	·114	·642	·905	5·481	·984	5·696	·149	·105	51
52	·110	·551	·895	4·772	·957	4·914	·140	·102	52
53	·104	·461	·874	4·056	·920	4·139	·129	·098	53
54	·097	·374	·839	3·332	·869	3·372	·118	·093	54
55	·089	·290	·787	2·614	·802	2·624	·104	·086	55
56	·079	·213	·718	1·931	·718	1·925	·089	·077	56
57	·065	·142	·618	1·284	·607	1·277	·071	·065	57
58	·051	·079	·458	·710	·462	·715	·051	·050	58
59	·029	·029	·271	·271	·271	·271	·027	·029	59

NOTE.— $^wM_x^{ls} \div D_x^s$  will be found in Table 19.



## Hypothetical Experience of Staff Pension Fund.

TABLE 27.

Simple Commutation Columns (according to Table 2).

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	$D_x^{(2)}$	$N_x^{(2)}$	$C_x^{(2)}$	$M_x^{(2)}$	$R_x^{(2)}$	Age ( <i>x</i> )
15	12,837	303,898	44.9	3612.3	135941.9	15
16	12,418	291,480	44.8	3567.4	132329.6	16
17	12,012	279,468	44.1	3522.6	128762.2	17
18	11,618	267,850	43.9	3478.5	125239.6	18
19	11,236	256,614	43.7	3434.6	121761.1	19
20	10,865	245,749	43.0	3390.9	118326.5	20
21	10,505	235,244	42.8	3347.9	114935.6	21
22	10,157	225,087	42.6	3305.1	111587.7	22
23	9,818	215,269	41.8	3262.5	108282.6	23
24	9,491	205,778	41.6	3220.7	105020.1	24
25	9,172	196,606	40.8	3179.1	101799.4	25
26	8,864	187,742	41.4	3138.3	98620.3	26
27	8,565	179,177	40.6	3096.9	95482.0	27
28	8,275	170,902	40.3	3056.3	92385.1	28
29	7,993	162,909	40.4	3016.0	89328.8	29
30	7,720	155,189	40.4	2975.6	86312.8	30
31	7,455	147,734	41.2	2935.2	83337.2	31
32	7,197	140,537	41.1	2894.0	80402.0	32
33	6,946	133,591	41.0	2852.9	77508.0	33
34	6,703	126,888	41.6	2811.9	74655.1	34
35	6,466	120,422	42.1	2770.3	71843.2	35
36	6,235	114,187	41.9	2728.2	69072.9	36
37	6,012	108,175	43.3	2686.3	66344.7	37
38	5,794	102,381	43.9	2643.0	63658.4	38
39	5,581	96,800	43.8	2599.1	61015.4	39
40	5,375	91,425	45.5	2555.3	58416.3	40
41	5,172	86,253	45.1	2509.8	55861.0	41
42	4,977	81,276	47.4	2464.7	53351.2	42
43	4,784	76,492	47.4	2417.3	50886.5	43
44	4,598	71,894	49.2	2369.9	48469.2	44
45	4,414	67,480	50.6	2320.7	46099.3	45
46	4,235	63,245	51.8	2270.1	43778.6	46
47	4,060	59,185	52.5	2218.3	41508.5	47
48	3,889	55,296	54.3	2165.8	39290.2	48
49	3,722	51,574	56.3	2111.5	37124.4	49
50	3,557	48,017	56.9	2055.2	35012.9	50
51	3,397	44,620	57.6	1998.3	32957.7	51
52	3,240	41,380	58.9	1940.7	30959.4	52
53	3,087	38,293	60.0	1881.8	29018.7	53
54	2,937	35,356	61.6	1821.8	27136.9	54
55	2,790	32,566	63.0	1760.2	25315.1	55
56	2,646	29,920	65.3	1697.2	23554.9	56
57	2,503	27,417	66.4	1631.9	21857.7	57
58	2,364	25,053	68.0	1565.5	20225.8	58
59	2,227	22,826	68.7	1497.5	18660.3	59
60	2,093	20,733	70.4	1428.8	17162.8	60
61	1,962	18,771	70.9	1358.4	15734.0	61
62	1,834	16,937	71.5	1287.5	14375.6	62
63	1,709	15,228	71.5	1216.0	13088.1	63
64	1,588	13,640	69.8	1144.5	11872.1	64
65	{ 1,472 } { (×8.266) }	12,168	67.2	1074.7	10727.6	65

## Hypothetical Experience of Staff Pension Fund.

TABLE 28.

*Simple Commutation Columns (according to Table 3).*

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	$D_x^{(3)}$	$N_x^{(3)}$	$C_x^{(3)}$	$M_x^{(3)}$	$R_x^{(3)}$	Age ( <i>x</i> )
15	12,837	139,365	44.9	1304.1	41772.2	15
16	11,484	127,881	41.1	1259.2	40468.1	16
17	10,113	117,768	37.6	1218.1	39208.9	17
18	8,888	108,880	33.6	1180.5	37990.8	18
19	7,872	101,008	30.5	1146.9	36810.3	19
20	7,029	93,979	28.0	1116.4	35663.4	20
21	6,320	87,659	25.6	1088.4	34547.0	21
22	5,723	81,936	23.8	1062.8	33458.6	22
23	5,215	76,721	22.1	1039.0	32395.8	23
24	4,779	71,942	20.5	1016.9	31356.8	24
25	4,400	67,542	19.5	996.4	30339.9	25
26	4,069	63,473	18.9	976.9	29343.5	26
27	3,775	59,698	17.9	958.0	28366.6	27
28	3,514	56,184	17.0	940.1	27408.6	28
29	3,280	52,904	16.5	923.1	26468.5	29
30	3,070	49,834	16.0	906.6	25545.4	30
31	2,880	46,954	15.9	890.6	24638.8	31
32	2,706	44,248	15.5	874.7	23748.2	32
33	2,548	41,700	15.0	859.2	22873.5	33
34	2,403	39,297	14.9	844.2	22014.3	34
35	2,269	37,028	14.8	829.3	21170.1	35
36	2,145	34,883	14.4	814.5	20340.8	36
37	2,031	32,852	14.6	800.1	19526.3	37
38	1,925	30,927	14.5	785.5	18726.2	38
39	1,826	29,101	14.4	771.0	17940.7	39
40	1,733	27,368	14.6	756.6	17169.7	40
41	1,646	25,722	14.4	742.0	16413.1	41
42	1,564	24,158	14.9	727.6	15671.1	42
43	1,487	22,671	14.7	712.7	14943.5	43
44	1,415	21,256	15.1	698.0	14230.8	44
45	1,346	19,910	15.4	682.9	13532.8	45
46	1,281	18,629	15.7	667.5	12849.9	46
47	1,219	17,410	15.7	651.8	12182.4	47
48	1,161	16,249	16.2	636.1	11530.6	48
49	1,105	15,144	16.7	619.9	10894.5	49
50	1,051	14,093	16.8	603.2	10274.6	50
51	1,001	13,092	17.0	586.4	9671.4	51
52	952	12,140	17.3	569.4	9085.0	52
53	906	11,234	17.6	552.1	8515.6	53
54	862	10,372	18.1	534.5	7963.5	54
55	818	9,554	18.5	516.4	7429.0	55
56	776	8,778	19.1	497.9	6912.6	56
57	734	8,044	19.4	478.8	6414.7	57
58	693	7,351	19.9	459.4	5935.9	58
59	653	6,698	20.2	439.5	5476.5	59
60	614	6,084	20.6	419.3	5037.0	60
61	576	5,508	20.8	398.7	4617.7	61
62	538	4,970	21.0	377.9	4219.0	62
63	501	4,469	21.0	356.9	3841.1	63
64	466	4,003	20.5	335.9	3484.2	64
65	{ 432 } { ×8.266 }	3,571	19.8	315.4	3148.3	65

## Hypothetical Experience of Staff Pension Fund.

TABLE 29.

*Simple Commutation Columns (according to Table 4).*

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	$D_x^{(4)}$	$N_x^{(4)}$	$C_x^{(4)}$	$M_x^{(4)}$	$R_x^{(4)}$	Age ( <i>x</i> )
15	12,837	136,322	44.9	1092.3	30684.9	15
16	11,484	124,838	41.1	1047.4	29592.6	16
17	10,113	114,725	37.6	1006.3	28545.2	17
18	8,888	105,837	33.6	968.7	27538.9	18
19	7,872	97,965	30.5	935.1	26570.2	19
20	7,029	90,936	28.0	904.6	25635.1	20
21	6,320	84,616	25.6	876.6	24730.5	21
22	5,723	78,893	23.8	851.0	23853.9	22
23	5,215	73,678	22.1	827.2	23002.9	23
24	4,779	68,899	20.5	805.1	22175.7	24
25	4,400	64,499	19.5	784.6	21370.6	25
26	4,069	60,430	18.9	765.1	20586.0	26
27	3,775	56,655	17.9	746.2	19820.9	27
28	3,514	53,141	17.0	728.3	19074.7	28
29	3,279	49,862	16.5	711.3	18346.4	29
30	3,068	46,794	16.0	694.8	17635.1	30
31	2,877	43,917	15.5	678.8	16940.3	31
32	2,702	41,215	15.1	663.3	16261.5	32
33	2,543	38,672	14.6	648.2	15598.2	33
34	2,396	36,276	14.2	633.6	14950.0	34
35	2,262	34,014	14.1	619.4	14316.4	35
36	2,137	31,877	13.7	605.3	13697.0	36
37	2,022	29,855	13.7	591.6	13091.7	37
38	1,914	27,941	13.6	577.9	12500.1	38
39	1,814	26,127	13.5	564.3	11922.2	39
40	1,720	24,407	13.4	550.8	11357.9	40
41	1,631	22,776	13.3	537.4	10807.1	41
42	1,548	21,228	13.5	524.1	10269.7	42
43	1,470	19,758	13.3	510.6	9745.6	43
44	1,395	18,363	13.5	497.3	9235.0	44
45	1,325	17,038	13.9	483.8	8737.7	45
46	1,258	15,780	14.0	469.9	8253.9	46
47	1,194	14,586	14.0	455.9	7784.0	47
48	1,133	13,453	14.3	441.9	7328.1	48
49	1,075	12,378	14.6	427.6	6886.2	49
50	1,019	11,359	14.6	413.0	6458.6	50
51	965	10,394	14.6	398.4	6045.6	51
52	913	9,481	14.8	383.8	5647.2	52
53	863	8,618	15.0	369.0	5263.4	53
54	814	7,804	15.2	354.0	4894.4	54
55	766	7,038	15.5	338.8	4540.4	55
56	718	6,320	15.8	323.3	4201.6	56
57	671	5,649	15.8	307.5	3878.3	57
58	623	5,026	15.9	291.7	3570.8	58
59	575	4,451	16.0	275.8	3279.1	59
60	525	3,926	15.8	259.8	3003.3	60
61	474	3,452	15.5	244.0	2743.5	61
62	419	3,033	15.1	228.5	2499.5	62
63	363	2,670	14.2	213.4	2271.0	63
64	307	2,363	13.0	199.2	2057.6	64
65	255	2,108	11.7	186.2	1858.4	65



## Hypothetical Experience of Staff Pension Fund.

TABLE 30.

*Commutation Columns for Valuing the Return of Contributions of 1 per annum on Death, Withdrawal, or Early Retirement.*

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	${}^dC_x$ = $C_x^{(4)}$	${}^dM_x$ = $\Sigma {}^dC_x$	${}^dR_x$ = $\Sigma {}^dM_x$	${}^wC_x^*$	${}^wM_x$ = $\Sigma {}^wC_x$	${}^wR_x$ = $\Sigma {}^wM_x$	${}^rC_x^\dagger$	${}^rM_x$ = $\Sigma {}^rC_x$	${}^rR_x$ = $\Sigma {}^rM_x$	Age ( <i>x</i> )
15	44.9	906.1	19,517	935	7,102	46,205	...	300.2	12,482	15
16	41.1	861.2	18,610	995	6,167	39,103	...	300.2	12,182	16
17	37.6	820.1	17,749	893	5,172	32,936	...	300.2	11,882	17
18	33.6	782.5	16,929	723	4,279	27,764	...	300.2	11,582	18
19	30.5	748.9	16,147	584	3,556	23,485	...	300.2	11,281	19
20	28.0	718.4	15,398	477	2,972	19,929	...	300.2	10,981	20
21	25.6	690.4	14,679	387	2,495	16,957	...	300.2	10,681	21
22	23.8	664.8	13,989	317	2,108	14,462	..	300.2	10,381	22
23	22.1	641.0	13,324	263	1,791	12,354	...	300.2	10,081	23
24	20.5	618.9	12,683	219	1,528	10,563	...	300.2	9,780	24
25	19.5	598.4	12,064	184	1,309	9,035	...	300.2	9,480	25
26	18.9	578.9	11,466	156	1,125	7,726	...	300.2	9,180	26
27	17.9	560.0	10,887	133	969	6,601	...	300.2	8,880	27
28	17.0	542.1	10,327	114	836	5,632	.85	300.2	8,580	28
29	16.5	525.1	9,785	98	722	4,796	.82	299.4	8,279	29
30	16.0	508.6	9,260	85	624	4,074	1.20	298.5	7,980	30
31	15.5	492.6	8,751	74	539	3,450	1.55	297.3	7,682	31
32	15.1	477.1	8,259	64	465	2,911	1.51	295.8	7,384	32
33	14.6	462.0	7,781	56	401	2,446	1.83	294.3	7,088	33
34	14.2	447.4	7,319	49	345	2,045	1.78	292.4	6,794	34
35	14.1	433.2	6,872	43	296	1,700	2.07	290.7	6,502	35
36	13.7	419.1	6,439	38	253	1,404	2.01	288.6	6,211	36
37	13.7	405.4	6,020	33	215	1,151	2.28	286.6	5,923	37
38	13.6	391.7	5,614	28	182	936	2.53	284.3	5,636	38
39	13.5	378.1	5,223	25	154	754	2.76	281.8	5,352	39
40	13.4	364.6	4,845	22	129	600	2.98	279.0	5,070	40
41	13.3	351.2	4,480	19	107	471	3.18	276.0	4,791	41
42	13.5	337.9	4,129	17	88	364	3.37	272.9	4,515	42
43	13.3	324.4	3,791	14	71	276	3.54	269.5	4,242	43
44	13.5	311.1	3,466	12	57	205	3.70	265.9	3,972	44
45	13.9	297.6	3,155	11	45	148	4.11	262.2	3,707	45
46	14.0	283.7	2,858	9	34	103	4.49	258.1	3,444	46
47	14.0	269.7	2,574	7	25	69	4.84	253.6	3,186	47
48	14.3	255.7	2,304	6	18	44	5.17	248.8	2,933	48
49	14.6	241.4	2,049	5	12	26	5.70	243.6	2,684	49
50	14.6	226.8	1,807	3	7	14	6.20	237.9	2,440	50
51	14.6	212.2	1,580	2	4	7	6.88	231.7	2,202	51
52	14.8	197.6	1,368	1	2	3	7.52	224.9	1,970	52
53	15.0	182.8	1,171	1	1	1	8.31	217.3	1,746	53
54	15.2	167.8	988	...	...	...	9.05	209.0	1,528	54
55	15.5	152.6	820	...	...	...	9.93	200.0	1,319	55
56	15.8	137.1	667	...	...	...	11.13	190.0	1,119	56
57	15.8	121.3	530	...	...	...	12.42	178.9	929	57
58	15.9	105.5	409	...	...	...	14.16	166.5	750	58
59	16.0	89.6	304	...	...	...	16.97	152.3	584	59
60	15.8	73.6	214	...	...	...	20.27	135.4	432	60
61	15.5	57.8	140	...	...	...	24.96	115.1	296	61
62	15.1	42.3	83	...	...	...	29.36	90.1	181	62
63	14.2	27.2	40	...	...	...	30.61	60.8	91	63
64	13.0	13.0	13	...	...	...	30.16	30.2	30	64

\*  ${}^wC_x = w_x \times v^{x+1}$ †  ${}^rC_x = r_x \times v^{x+1}$



## Hypothetical Experience of Staff Pension Fund.

TABLE 31.

Table for Valuing Return of Contributions of 1 per annum with Compound Interest at 3 per-cent on Death or Early Retirement; the Values of Annuities of 1 per annum on Early Retirement from Ill-health; and for finding the Value of a Pension of 1 for each Year of Service on Early Retirement.

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age (x)	${}^dD_x^*$	${}^dN_x$ $=\sum {}^dD_x$	${}^rD_x^\dagger$	${}^rN_x$ $=\sum {}^rD_x$	$D_x^{(r)}$	$N_x^{(r)}$	$a'_x$	${}^{ra}C_x^\ddagger$	${}^{ra}M_x$ $=\sum {}^{ra}C_x$	${}^{ra}R_x$ $=\sum {}^{ra}M_x$	Age (x)
15	1,851	33,337	1,025	25,171	...	...	...	...	2,908	120,856	15
16	1,753	31,486	995	24,146	...	...	...	...	2,908	117,948	16
17	1,662	29,733	966	23,151	...	...	...	...	2,908	115,040	17
18	1,577	28,071	938	22,185	...	...	...	...	2,908	112,132	18
19	1,499	26,494	910	21,247	...	...	...	...	2,908	109,225	19
20	1,426	24,995	884	20,337	...	...	...	...	2,908	106,317	20
21	1,357	23,569	858	19,453	...	...	...	...	2,908	103,409	21
22	1,293	22,212	833	18,595	...	...	...	...	2,908	100,501	22
23	1,232	20,919	809	17,762	...	...	...	...	2,908	97,594	23
24	1,174	19,687	785	16,953	...	...	...	...	2,908	94,686	24
25	1,120	18,513	762	16,168	...	...	...	...	2,908	91,778	25
26	1,069	17,393	740	15,406	...	...	...	...	2,908	88,870	26
27	1,019	16,324	719	14,666	...	...	...	...	2,908	85,963	27
28	972	15,305	698	13,947	...	...	...	6.9	2,908	83,055	28
29	972	14,333	676	13,249	8,487	64,788	7.63	6.7	2,901	80,147	29
30	884	13,406	656	12,573	7,416	57,372	7.74	10.0	2,894	77,246	30
31	843	12,522	636	11,917	6,487	50,885	7.84	13.0	2,884	74,352	31
32	803	11,679	616	11,281	5,681	45,204	7.96	12.9	2,871	71,468	32
33	765	10,876	596	10,665	4,983	40,221	8.07	15.8	2,858	68,596	33
34	729	10,111	577	10,069	4,378	35,843	8.19	15.6	2,843	65,738	34
35	694	9,382	559	9,492	3,853	31,990	8.30	18.3	2,827	62,896	35
36	660	8,688	540	8,933	3,397	28,593	8.42	18.1	2,809	60,069	36
37	627	8,028	523	8,393	2,999	25,594	8.53	20.7	2,791	57,260	37
38	596	7,401	505	7,870	2,653	22,941	8.65	23.3	2,770	54,469	38
39	565	6,805	488	7,365	2,350	20,591	8.76	25.8	2,747	51,700	39
40	536	6,240	471	6,877	2,086	18,505	8.87	28.1	2,721	48,953	40
41	507	5,704	455	6,406	1,854	16,651	8.98	30.3	2,693	46,232	41
42	479	5,197	438	5,951	1,650	15,001	9.09	32.5	2,662	43,540	42
43	452	4,718	422	5,513	1,472	13,529	9.19	34.5	2,630	40,877	43
44	426	4,266	406	5,091	1,314	12,215	9.29	36.4	2,595	38,247	44
45	401	3,840	391	4,685	1,176	11,039	9.39	40.8	2,559	35,652	45
46	376	3,439	376	4,294	1,054	9,985	9.47	45.0	2,518	33,093	46
47	351	3,063	360	3,918	946	9,039	9.56	48.9	2,473	30,575	47
48	327	2,712	345	3,558	850	8,189	9.63	52.5	2,424	28,102	48
49	304	2,385	330	3,213	766	7,423	9.69	58.3	2,372	25,677	49
50	281	2,081	315	2,883	691	6,732	9.75	63.7	2,314	23,306	50
51	258	1,800	300	2,568	624	6,108	9.79	70.9	2,250	20,992	51
52	237	1,542	284	2,268	565	5,543	9.82	77.6	2,179	18,742	52
53	215	1,305	269	1,984	512	5,031	9.83	85.8	2,101	16,563	53
54	195	1,090	253	1,715	465	4,566	9.83	93.4	2,015	14,462	54
55	174	895	237	1,462	422	4,144	9.81	102.2	1,922	12,447	55
56	154	721	220	1,225	385	3,759	9.77	114.0	1,820	10,525	56
57	134	567	203	1,005	351	3,408	9.72	126.4	1,706	8,705	57
58	115	433	185	802	320	3,088	9.64	142.9	1,579	6,999	58
59	96	318	166	617	293	2,795	9.53	169.2	1,437	5,419	59
60	78	222	145	451	269	2,526	9.40	199.1	1,267	3,983	60
61	60	144	121	306	247	2,279	9.24	240.9	1,068	2,715	61
62	43	84	93	185	227	2,052	9.05	277.2	828	1,647	62
63	28	41	62	92	209	1,843	8.82	281.3	550	819	63
64	13	13	30	30	193	1,650	8.56	269.0	269	269	64
65	...	...	...	...	178	1,472	8.27	...	...	...	65

\*  ${}^dD_x = l'_x \times v^{x+1}$

†  ${}^rD_x = \sum r_x \times v^{x+1}$

‡  ${}^{ra}C_x = {}^rC_x \times (a'_{x+.5} + .5)$

## Hypothetical Experience of Staff Pension Fund.

TABLE 32.

Table for finding the Accumulation of Average Salary at 3 per-cent Compound Interest, and for finding the Value of the last year's Full Salary on Death, Withdrawal, or Early Retirement.

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age ( $x$ )	$s_x v^x$	$\Sigma s_x v^x$	$(1+i)^{x+1}$	$dC_x^s$ $= dC_x \times s_x$	$dM_x^{ls}$ $= \Sigma dC_x^s$	$wC_x^s$ $= wC_x \times s_x$	$wM_x^{ls}$ $= \Sigma wC_x^s$	$rC_x^s$ $= rC_x \times s_x$	$rM_x^{ls}$ $= \Sigma rC_x^s$	Age ( $x$ )
15	12·84	1740·60	1·5126	898	99,718	18,700	329,829	...	57,736	15
16	15·58	1727·76	1·5580	1,028	98,820	24,875	311,129	...	57,736	16
17	18·15	1712·18	1·6047	1,128	97,792	26,790	286,254	...	57,736	17
18	20·56	1694·03	1·6528	1,176	96,664	25,305	259,464	...	57,736	18
19	22·81	1673·47	1·7024	1,220	95,488	23,360	234,159	...	57,736	19
20	24·92	1650·66	1·7535	1,260	94,268	21,465	210,799	...	57,736	20
21	26·88	1625·74	1·8061	1,280	93,008	19,350	189,334	...	57,736	21
22	28·70	1598·86	1·8603	1,309	91,728	17,435	169,984	...	57,736	22
23	30·40	1570·16	1·9161	1,326	90,419	15,780	152,549	...	57,736	23
24	31·98	1539·76	1·9736	1,333	89,093	14,235	136,769	...	57,736	24
25	33·43	1507·78	2·0328	1,365	87,760	12,880	122,534	...	57,736	25
26	34·31	1474·35	2·0938	1,399	86,395	11,544	109,654	...	57,736	26
27	35·11	1440·04	2·1566	1,396	84,996	10,374	98,110	...	57,736	27
28	35·84	1404·93	2·2213	1,394	83,600	9,348	87,736	70	57,736	28
29	36·49	1369·09	2·2879	1,419	82,206	8,428	78,388	71	57,667	29
30	37·08	1332·60	2·3566	1,440	80,787	7,650	69,960	108	57,596	30
31	37·60	1295·52	2·4273	1,457	79,347	6,956	62,310	146	57,488	31
32	38·06	1257·92	2·5001	1,480	77,890	6,272	55,354	148	57,343	32
33	38·46	1219·86	2·5751	1,489	76,410	5,712	49,082	187	57,195	33
34	38·80	1181·40	2·6523	1,505	74,921	5,194	43,370	189	57,008	34
35	39·09	1142·60	2·7319	1,551	73,416	4,730	38,176	230	56,819	35
36	39·33	1103·51	2·8139	1,562	71,865	4,332	33,446	229	56,589	36
37	39·53	1064·18	2·8983	1,617	70,303	3,894	29,114	269	56,360	37
38	39·68	1024·65	2·9852	1,659	68,686	3,416	25,220	309	56,091	38
39	39·79	984·97	3·0748	1,710	67,027	3,150	21,804	348	55,783	39
40	39·85	945·18	3·1670	1,742	65,317	2,860	18,654	387	55,435	40
41	39·88	905·33	3·2620	1,782	63,575	2,546	15,794	426	55,047	41
42	39·88	865·45	3·3599	1,863	61,793	2,346	13,248	465	54,621	42
43	39·84	825·57	3·4607	1,889	59,930	1,988	10,902	503	54,156	43
44	39·77	785·73	3·5645	1,971	58,041	1,752	8,914	540	53,653	44
45	39·67	745·96	3·6715	2,085	56,070	1,650	7,162	617	53,113	45
46	39·54	706·29	3·7816	2,156	53,985	1,386	5,512	692	52,497	46
47	39·38	666·75	3·8950	2,212	51,829	1,106	4,126	765	51,805	47
48	39·20	627·37	4·0119	2,317	49,617	972	3,020	838	51,041	48
49	39·00	588·17	4·1323	2,424	47,300	830	2,048	946	50,203	49
50	38·78	549·17	4·2562	2,482	44,876	510	1,218	1,054	49,257	50
51	38·53	510·39	4·3839	2,540	42,394	348	708	1,197	48,203	51
52	38·27	471·86	4·5154	2,634	39,854	178	360	1,339	47,006	52
53	37·99	433·59	4·6509	2,730	37,220	182	182	1,512	45,667	53
54	37·70	395·60	4·7904	2,827	34,490	...	...	1,683	44,155	54
55	37·39	357·90	4·9341	2,945	31,663	...	...	1,887	42,471	55
56	37·06	320·51	5·0821	3,065	28,718	...	...	2,159	40,585	56
57	36·72	283·45	5·2346	3,128	25,653	...	...	2,459	38,426	57
58	36·37	246·73	5·3917	3,212	22,525	...	...	2,860	35,966	58
59	36·01	210·36	5·5534	3,296	19,313	...	...	3,496	33,106	59
60	35·64	174·35	5·7200	3,318	16,017	...	...	4,257	29,610	60
61	35·27	138·71	5·8916	3,317	12,699	...	...	5,341	25,354	61
62	34·88	103·44	6·0684	3,292	9,382	...	...	6,401	20,012	62
63	34·48	68·56	6·2504	3,152	6,090	...	...	6,795	13,612	63
64	34·08	34·08	6·4379	2,938	2,938	...	...	6,816	6,816	64



## Hypothetical Experience of Staff Pension Fund.

TABLE 33.

*Commutation Columns for finding the Present Value of Future Salary, and Return of Contributions at Death, without Interest.*

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age ( $x$ )	$D_x^s$ = $D_x^{(4)} \times s_x$	$vD_x^s$ = $D_x^s \div 1.03$	$N_x^s$ = $\Sigma(v \cdot D_x^s)$	${}^dM_x^s$ = ${}^dM_x \times s_x$	${}^dR_x^s$ = $\Sigma {}^dM_x^s$	Age ( $x$ )
15	256,740	249,262	10,654,124	18,122	1,779,340	15
16	287,100	278,738	10,404,862	21,530	1,761,218	16
17	303,390	294,553	10,126,124	24,603	1,739,688	17
18	311,080	302,020	9,831,571	27,388	1,715,085	18
19	314,880	305,709	9,529,551	29,956	1,687,697	19
20	316,305	307,093	9,223,842	32,328	1,657,741	20
21	316,000	306,796	8,916,749	34,520	1,625,413	21
22	314,765	305,598	8,609,953	36,564	1,590,893	22
23	312,900	303,786	8,304,355	38,460	1,554,329	23
24	310,635	301,587	8,000,569	40,229	1,515,869	24
25	308,000	299,030	7,698,982	41,888	1,475,640	25
26	301,106	292,337	7,399,952	42,839	1,433,752	26
27	294,450	285,874	7,107,615	43,680	1,390,913	27
28	288,148	279,756	6,821,741	44,452	1,347,233	28
29	281,994	273,781	6,541,985	45,159	1,302,781	29
30	276,120	268,078	6,268,204	45,774	1,257,622	30
31	270,438	262,561	6,000,126	46,304	1,211,848	31
32	264,796	257,083	5,737,565	46,756	1,165,544	32
33	259,386	251,831	5,480,482	47,124	1,118,788	33
34	253,976	246,579	5,228,651	47,424	1,071,664	34
35	248,820	241,573	4,982,072	47,652	1,024,240	35
36	243,618	236,523	4,740,499	47,777	976,588	36
37	238,596	231,647	4,503,976	47,837	928,811	37
38	233,508	226,707	4,272,329	47,787	880,974	38
39	228,564	221,907	4,045,622	47,641	833,187	39
40	223,600	217,087	3,823,715	47,398	785,546	40
41	218,554	212,189	3,606,628	47,061	738,148	41
42	213,624	207,402	3,394,439	46,630	691,087	42
43	208,740	202,660	3,187,037	46,065	644,457	43
44	203,670	197,738	2,984,377	45,421	598,392	44
45	198,750	192,961	2,786,639	44,640	552,971	45
46	193,732	188,090	2,593,678	43,691	508,331	46
47	188,652	183,157	2,405,588	42,613	464,640	47
48	183,546	178,200	2,222,431	41,423	422,027	48
49	178,450	173,253	2,044,231	40,072	380,604	49
50	173,230	168,185	1,870,978	38,556	340,532	50
51	167,910	163,019	1,702,793	36,923	301,976	51
52	162,514	157,781	1,539,774	35,173	265,053	52
53	157,066	152,491	1,381,993	33,270	229,880	53
54	151,404	146,994	1,229,502	31,211	196,610	54
55	145,540	141,301	1,082,508	28,994	165,399	55
56	139,292	135,235	941,207	26,597	136,405	56
57	132,858	128,988	805,972	24,017	109,808	57
58	125,846	122,181	676,984	21,311	85,791	58
59	118,450	115,000	554,803	18,458	64,480	59
60	110,250	107,039	439,803	15,456	46,022	60
61	101,436	98,482	332,764	12,369	30,566	61
62	91,342	88,682	234,282	9,221	18,197	62
63	80,586	78,239	145,600	6,038	8,976	63
64	69,382	67,361	67,361	2,938	2,938	64

## Hypothetical Experience of Staff Pension Fund.

TABLE 34.

*Commutation Columns for finding the Present Values of Return of Contributions on Withdrawal and Early Retirement.*

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age ( $x$ )	${}^wM_x^s$ $= {}^wM_x \times s_x$	${}^wR_x^s$ $= \sum {}^wM_x^s$	${}^rM_x^s$ $= {}^rM_x \times s_x$	${}^rR_x^s$ $= \sum {}^rM_x^s$	Age ( $x$ )
15	142,040	2,148,868	6004'0	1413732'1	15
16	154,175	2,006,828	7505'0	1407728'1	16
17	155,160	1,852,653	9006'0	1400223'1	17
18	149,765	1,697,493	10507'0	1391217'1	18
19	142,240	1,547,728	12008'0	1380710'1	19
20	133,740	1,405,488	13509'0	1368702'1	20
21	124,750	1,271,748	15010'0	1355193'1	21
22	115,940	1,146,998	16511'0	1340183'1	22
23	107,460	1,031,058	18012'0	1323672'1	23
24	99,320	923,598	19513'0	1305660'1	24
25	91,630	824,278	21014'0	1286147'1	25
26	83,250	732,648	22214'8	1265133'1	26
27	75,582	649,398	23415'6	1242918'3	27
28	68,552	573,816	24616'4	1219502'7	28
29	62,092	505,264	25744'1	1194886'3	29
30	56,160	443,172	26867'7	1169142'2	30
31	50,666	387,012	27949'0	1142274'5	31
32	45,570	336,346	28986'4	1114325'5	32
33	40,902	290,776	30015'5	1085339'1	33
34	36,570	249,874	30998'6	1055323'6	34
35	32,560	213,304	31972'6	1024325'0	35
36	28,842	180,744	32899'3	992352'4	36
37	25,370	151,902	33816'4	959453'1	37
38	22,204	126,532	34684'6	925636'7	38
39	19,404	104,328	35503'0	890952'1	39
40	16,770	84,924	36271'3	855449'1	40
41	14,338	68,154	36988'0	819177'8	41
42	12,144	53,816	37653'3	782189'8	42
43	10,082	41,672	38266'2	744536'5	43
44	8,322	31,590	38827'2	706270'3	44
45	6,750	23,268	39336'0	667443'1	45
46	5,236	16,518	39752'0	628107'1	46
47	3,950	11,282	40075'1	588355'1	47
48	2,916	7,332	40305'6	548280'0	48
49	1,992	4,416	40442'6	507974'4	49
50	1,190	2,424	40448'1	467531'8	50
51	696	1,234	40321'0	427083'7	51
52	356	538	40023'3	386762'7	52
53	182	182	39554'1	346739'4	53
54	...	...	38877'7	307185'3	54
55	...	...	37994'3	268307'6	55
56	...	...	36867'8	230313'3	56
57	...	...	35424'2	193445'5	57
58	...	...	33631'0	158021'3	58
59	...	...	31380'0	124390'3	59
60	...	...	28425'6	93010'3	60
61	...	...	24629'3	64584'7	61
62	...	...	19648'3	39955'4	62
63	...	...	13490'9	20307'1	63
64	...	...	6816'2	6816'2	64



## Hypothetical Experience of Staff Pension Fund.

TABLE 35.

*Commutation Columns for finding the Values of Return of Contributions, with Compound Interest at 3 per-cent on Death or Early Retirement.*

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age (x)	${}^dD_x^s$ $= {}^dD_x \times s_x$	${}^dN_x^s$ $= \sum {}^dD_x^s$	${}^rD_x^s$ $= {}^rD_x \times s_x$	${}^rN_x^s$ $= \sum {}^rD_x^s$	Age (x)
15	37,020	2,755,427	20,500	2,395,105	15
16	43,825	2,718,407	24,875	2,374,605	16
17	49,860	2,674,582	28,980	2,349,730	17
18	55,195	2,624,722	32,830	2,320,750	18
19	59,960	2,569,527	36,400	2,287,920	19
20	64,170	2,509,567	39,780	2,251,520	20
21	67,850	2,445,397	42,900	2,211,740	21
22	71,115	2,377,547	45,815	2,168,840	22
23	73,920	2,306,432	48,540	2,123,025	23
24	76,310	2,232,512	51,025	2,074,485	24
25	78,400	2,156,202	53,340	2,023,460	25
26	79,106	2,077,802	54,760	1,970,120	26
27	79,482	1,998,696	56,082	1,915,360	27
28	79,704	1,919,214	57,236	1,859,278	28
29	79,722	1,839,510	58,136	1,802,042	29
30	79,560	1,759,788	59,040	1,743,906	30
31	79,242	1,680,228	59,784	1,684,866	31
32	78,694	1,600,986	60,368	1,625,082	32
33	78,030	1,522,292	60,792	1,564,714	33
34	77,274	1,444,262	61,162	1,503,922	34
35	76,340	1,366,988	61,490	1,442,760	35
36	75,240	1,290,648	61,560	1,381,270	36
37	73,986	1,215,408	61,714	1,319,710	37
38	72,712	1,141,422	61,610	1,257,996	38
39	71,190	1,068,710	61,488	1,196,386	39
40	69,680	997,520	61,230	1,134,898	40
41	67,938	927,840	60,970	1,073,668	41
42	66,102	859,902	60,444	1,012,698	42
43	64,184	793,800	59,924	952,254	43
44	62,196	729,616	59,276	892,330	44
45	60,150	667,420	58,650	833,054	45
46	57,904	607,270	57,904	774,404	46
47	55,458	549,366	56,880	716,500	47
48	52,974	493,908	55,890	659,620	48
49	50,464	440,934	54,780	603,730	49
50	47,770	390,470	53,550	548,950	50
51	44,892	342,700	52,200	495,400	51
52	42,186	297,808	50,552	443,200	52
53	39,130	255,622	48,958	392,648	53
54	36,270	216,492	47,058	343,690	54
55	33,060	180,222	45,030	296,632	55
56	29,876	147,162	42,680	251,602	56
57	26,532	117,286	40,194	208,922	57
58	23,230	90,754	37,370	168,728	58
59	19,776	67,524	34,196	131,358	59
60	16,380	47,748	30,450	97,162	60
61	12,840	31,368	25,894	66,712	61
62	9,374	18,528	20,274	40,818	62
63	6,216	9,154	13,764	20,544	63
64	2,938	2,938	6,780	6,780	64

## Hypothetical Experience of Staff Pension Fund.

TABLE 36.

*Commutation Columns for finding the Values of Pensions on Early Retirement, based on number of Years' Service and*

(a) *Average Salary.*

PENSION AGE 65.

(b) *Last Salary.*

INTEREST 3 PER-CENT.

Age (x)	AVERAGE SALARY		LAST SALARY			Age (x)
	${}^raM_x^s$ $= {}^raM_x \times s_x$	${}^raP_x^s$ $= \sum {}^raM_x^s$	${}^raC_x^{ls}$ $= {}^raC_x \cdot s_x (a'_{x+ \cdot 5} + \cdot 5)$	${}^raM_x^{ls}$ $= \sum {}^raC_x^{ls}$	${}^raR_x^{ls}$ $= \sum {}^raM_x^{ls}$	
15	58,155	13,683,338	...	559,028	24,017,405	15
16	72,694	13,625,183	...	559,028	23,458,377	16
17	87,233	13,552,489	...	559,028	22,899,349	17
18	101,772	13,465,256	...	559,028	22,340,321	18
19	116,311	13,363,484	...	559,028	21,781,293	19
20	130,850	13,247,173	...	559,028	21,222,265	20
21	145,389	13,116,323	...	559,028	20,663,237	21
22	159,927	12,970,934	...	559,028	20,104,209	22
23	174,466	12,811,007	...	559,028	19,545,181	23
24	189,005	12,636,541	...	559,028	18,986,153	24
25	203,544	12,447,536	...	559,028	18,427,125	25
26	215,175	12,243,992	...	559,028	17,868,097	26
27	226,806	12,028,817	...	559,028	17,309,069	27
28	238,437	11,802,011	563.2	559,028	16,750,041	28
29	249,477	11,563,574	577.4	558,465	16,191,013	29
30	260,476	11,314,097	895.3	557,887	15,632,548	30
31	271,118	11,053,621	1223.9	556,992	15,074,661	31
32	281,379	10,782,503	1261.0	555,768	14,517,669	32
33	291,551	10,501,124	1611.2	554,507	13,961,901	33
34	301,310	10,209,573	1651.1	552,896	13,407,394	34
35	310,967	9,908,263	2036.0	551,245	12,854,498	35
36	320,184	9,597,296	2057.3	549,209	12,303,253	36
37	329,288	9,277,112	2445.2	547,151	11,754,044	37
38	337,922	8,947,824	2843.1	544,706	11,206,893	38
39	346,064	8,609,902	3241.5	541,863	10,662,187	39
40	353,703	8,263,838	3653.2	538,622	10,120,324	40
41	360,820	7,910,135	4065.0	534,968	9,581,702	41
42	367,404	7,549,315	4483.6	530,903	9,046,734	42
43	373,440	7,181,911	4896.3	526,420	8,515,831	43
44	378,925	6,808,471	5315.6	521,524	7,989,411	44
45	383,846	6,429,546	6121.8	516,208	7,467,887	45
46	387,797	6,045,700	6928.8	510,086	6,951,679	46
47	390,761	5,657,903	7723.5	503,157	6,441,593	47
48	392,735	5,267,142	8509.0	495,434	5,938,436	48
49	410,312	4,874,407	9670.2	486,925	5,443,002	49
50	398,297	4,464,095	10824.6	477,255	4,956,077	50
51	391,472	4,065,798	12342.1	466,430	4,478,822	51
52	387,846	3,674,326	13814.4	454,088	4,012,392	52
53	382,437	3,286,480	15623.1	440,274	3,558,304	53
54	374,876	2,904,043	17371.7	424,650	3,118,030	54
55	365,191	2,529,167	19414.1	407,279	2,693,380	55
56	353,057	2,163,976	22110.2	387,865	2,286,101	56
57	337,770	1,810,919	25034.7	365,754	1,898,236	57
58	319,053	1,473,149	28860.4	340,720	1,532,482	58
59	295,940	1,154,096	34853.1	311,859	1,191,762	59
60	266,150	858,156	41800.8	277,006	879,903	60
61	228,629	592,000	51544.5	235,205	602,897	61
62	180,395	363,371	60420.7	183,661	367,692	62
63	122,175	182,976	62449.7	123,240	184,031	63
64	60,801	60,801	60790.5	60,791	60,791	64

## Hypothetical Experience of Staff Pension Fund.

TABLE 37.

*Multipliers for use in a Valuation.*

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age (x)	$\frac{N_{65} + \frac{1}{2}D_{65}}{D_x}$	$\frac{N_x^s}{D_x^s}$	$\frac{dM_x}{D_x}$	$\frac{dR_x^s}{D_x^s}$	$\frac{wM_x}{D_x}$	$\frac{wR_x^s}{D_x^s}$	$\frac{rM_x}{D_x}$	$\frac{rR_x^s}{D_x^s}$	$\frac{dD_x}{D_x}$	$\frac{dN_x^s}{D_x^s}$	Age (x)
15	·174	41·498	·071	6·931	·553	8·370	·023	5·506	·144	10·732	15
16	·195	36·241	·075	6·135	·537	6·990	·026	4·903	·153	9·468	16
17	·221	33·377	·081	5·734	·511	6·107	·030	4·615	·164	8·816	17
18	·252	31·605	·088	5·513	·481	5·457	·034	4·472	·177	8·437	18
19	·284	30·264	·095	5·360	·452	4·915	·038	4·385	·190	8·160	19
20	·318	29·161	·102	5·241	·423	4·443	·043	4·327	·203	7·934	20
21	·354	28·218	·109	5·144	·395	4·025	·048	4·289	·215	7·739	21
22	·391	27·354	·116	5·054	·368	3·644	·052	4·258	·226	7·553	22
23	·429	26·540	·123	4·968	·343	3·295	·058	4·230	·236	7·371	23
24	·468	25·756	·130	4·880	·320	2·973	·063	4·203	·246	7·187	24
25	·508	24·997	·136	4·791	·298	2·676	·068	4·176	·255	7·001	25
26	·549	24·575	·142	4·762	·276	2·433	·074	4·202	·263	6·901	26
27	·592	24·139	·148	4·724	·257	2·205	·080	4·221	·270	6·788	27
28	·636	23·674	·154	4·675	·238	1·991	·085	4·232	·277	6·661	28
29	·682	23·199	·160	4·620	·220	1·792	·091	4·237	·283	6·523	29
30	·729	22·701	·166	4·555	·203	1·605	·097	4·234	·288	6·373	30
31	·777	22·187	·171	4·481	·187	1·431	·103	4·224	·293	6·213	31
32	·827	21·668	·177	4·402	·172	1·270	·109	4·208	·297	6·046	32
33	·879	21·129	·182	4·313	·158	1·121	·116	4·184	·301	5·869	33
34	·933	20·587	·187	4·220	·144	·984	·122	4·155	·304	5·687	34
35	·988	20·023	·192	4·116	·131	·857	·129	4·117	·307	5·494	35
36	1·046	19·459	·196	4·009	·118	·742	·135	4·073	·309	5·298	36
37	1·106	18·877	·201	3·893	·106	·637	·142	4·021	·310	5·094	37
38	1·168	18·296	·205	3·773	·095	·542	·149	3·964	·311	4·888	38
39	1·232	17·700	·208	3·645	·085	·456	·155	3·898	·311	4·676	39
40	1·300	17·101	·212	3·513	·075	·380	·162	3·826	·312	4·461	40
41	1·370	16·502	·215	3·377	·066	·312	·169	3·748	·311	4·245	41
42	1·444	15·890	·218	3·235	·057	·252	·176	3·662	·309	4·025	42
43	1·521	15·268	·221	3·087	·048	·200	·183	3·567	·308	3·803	43
44	1·602	14·652	·223	2·938	·041	·155	·191	3·468	·305	3·582	44
45	1·687	14·021	·225	2·782	·034	·117	·198	3·358	·303	3·358	45
46	1·777	13·388	·226	2·624	·027	·085	·205	3·242	·299	3·135	46
47	1·872	12·751	·226	2·463	·021	·060	·212	3·119	·294	2·912	47
48	1·973	12·108	·226	2·299	·016	·040	·220	2·987	·289	2·691	48
49	2·080	11·455	·225	2·133	·011	·025	·227	2·847	·283	2·471	49
50	2·194	10·801	·223	1·966	·007	·014	·234	2·699	·276	2·254	50
51	2·317	10·141	·220	1·798	·004	·007	·240	2·543	·267	2·041	51
52	2·448	9·475	·216	1·631	·002	·003	·246	2·380	·260	1·833	52
53	2·590	8·799	·212	1·464	·001	·001	·252	2·208	·249	1·628	53
54	2·746	8·121	·206	1·299	...	...	·257	2·029	·240	1·430	54
55	2·918	7·438	·199	1·136	...	...	·261	1·844	·227	1·238	55
56	3·114	6·757	·191	·979	...	...	·265	1·653	·214	1·057	56
57	3·332	6·066	·181	·827	...	...	·267	1·456	·200	·883	57
58	3·588	5·379	·169	·682	...	...	·267	1·256	·185	·721	58
59	3·888	4·684	·156	·544	...	...	·265	1·050	·167	·570	59
60	4·258	3·989	·140	·417	...	...	·258	·844	·149	·433	60
61	4·716	3·281	·122	·301	...	...	·243	·637	·127	·309	61
62	5·335	2·565	·101	·199	...	...	·215	·437	·104	·203	62
63	6·158	1·807	·075	·111	...	...	·167	·252	·077	·114	63
64	7·282	·971	·042	·042	...	...	·098	·098	·042	·042	64



## Hypothetical Experience of Staff Pension Fund.

TABLE 37—continued.

*Multipliers for use in a Valuation.*

PENSION AGE 65.

INTEREST 3 PER-CENT.

Age (x)	$\frac{rD_x}{D_x}$	$\frac{rI_x^s}{D_x}$	$\frac{raM_x^s}{D_x}$	$\frac{raR_x^s}{D_x}$	$\frac{raM_x^{ls}}{D_x}$	$\frac{raR_x^{ls}}{D_x}$	$\frac{dM_x^{ls}}{D_x}$	$\frac{wM_x^{ls}}{D_x}$	$\frac{rM_x^{ls}}{D_x}$	Age (x)
15	·080	9·329	·227	53·296	2·177	93·548	·388	1·285	·225	15
16	·087	8·271	·253	47·458	1·947	81·708	·344	1·084	·201	16
17	·096	7·745	·288	44·670	1·843	75·478	·322	·944	·190	17
18	·106	7·460	·327	43·286	1·797	71·815	·311	·834	·186	18
19	·116	7·266	·369	42·440	1·775	69·173	·303	·744	·183	19
20	·126	7·118	·414	41·881	1·767	67·094	·298	·666	·183	20
21	·136	6·999	·460	41·507	1·769	65·390	·294	·599	·183	21
22	·146	6·890	·508	41·169	1·774	63·811	·291	·540	·183	22
23	·155	6·785	·558	40·943	1·787	62·465	·289	·488	·184	23
24	·164	6·678	·608	40·680	1·800	61·121	·287	·440	·186	24
25	·173	6·570	·661	40·414	1·815	59·828	·285	·398	·187	25
26	·182	6·543	·715	40·663	1·856	59·342	·287	·364	·192	26
27	·190	6·505	·770	40·852	1·902	58·785	·289	·333	·196	27
28	·199	6·453	·827	40·958	1·940	58·130	·290	·304	·200	28
29	·206	6·390	·885	41·007	1·981	57·416	·292	·278	·205	29
30	·214	6·316	·943	40·975	2·020	56·615	·293	·253	·209	30
31	·221	6·230	1·003	40·873	2·060	55·742	·293	·230	·213	31
32	·228	6·137	1·063	40·720	2·098	54·826	·294	·209	·217	32
33	·234	6·032	1·124	40·485	2·138	53·827	·295	·189	·221	33
34	·241	5·908	1·186	40·199	2·176	52·790	·295	·171	·224	34
35	·247	5·785	1·250	39·821	2·215	51·662	·295	·153	·228	35
36	·253	5·670	1·314	39·395	2·254	50·502	·295	·137	·232	36
37	·259	5·531	1·380	38·881	2·294	49·262	·295	·122	·236	37
38	·264	5·387	1·447	38·319	2·333	47·994	·294	·108	·240	38
39	·269	5·234	1·514	37·670	2·371	46·649	·293	·095	·244	39
40	·274	5·076	1·582	36·958	2·409	45·261	·292	·083	·248	40
41	·279	4·913	1·651	36·193	2·448	43·841	·291	·072	·252	41
42	·283	4·741	1·715	35·339	2·485	42·349	·289	·062	·256	42
43	·287	4·562	1·789	34·406	2·522	40·796	·287	·052	·259	43
44	·291	4·381	1·860	33·429	2·561	39·227	·285	·041	·263	44
45	·295	4·191	1·931	32·350	2·597	37·574	·282	·036	·267	45
46	·299	3·997	2·002	31·207	2·633	35·883	·279	·028	·271	46
47	·302	3·798	2·071	29·991	2·667	34·145	·275	·022	·275	47
48	·305	3·594	2·140	28·697	2·699	32·354	·270	·016	·278	48
49	·307	3·383	2·299	27·315	2·729	30·502	·265	·011	·281	49
50	·309	3·169	2·299	25·770	2·755	28·610	·259	·007	·284	50
51	·311	2·948	2·331	24·214	2·778	26·674	·252	·004	·287	51
52	·311	2·727	2·387	22·609	2·794	24·690	·245	·002	·289	52
53	·311	2·500	2·435	20·924	2·803	22·655	·237	·001	·291	53
54	·311	2·270	2·476	19·181	2·805	20·594	·228	...	·292	54
55	·309	2·038	2·509	17·378	2·799	18·506	·218	...	·292	55
56	·306	1·806	2·535	15·535	2·785	16·412	·206	...	·291	56
57	·303	1·573	2·542	13·630	2·753	14·288	·193	...	·289	57
58	·297	1·341	2·535	11·706	2·707	12·177	·179	...	·286	58
59	·289	1·109	2·498	9·743	2·633	10·061	·163	...	·279	59
60	·276	·881	2·414	7·784	2·513	7·981	·145	...	·269	60
61	·255	·658	2·254	5·836	2·319	5·944	·125	...	·250	61
62	·222	·447	1·975	3·978	2·011	4·025	·103	...	·219	62
63	·171	·255	1·516	2·271	1·529	2·284	·076	...	·169	63
64	·098	·098	·876	·876	·876	·876	·042	...	·098	64



## Hypothetical Experience of Staff Pension Fund.

TABLE 38.

*Simple Commutation Columns (according to Table 4), and Commutation Columns for valuing Benefits on Death and Early Retirement.*

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	$N_x^{(4)}$	$M_x^{(4)}$	$R_x^{(4)}$	${}^dM_x$ $=\sum_x {}^{59}C_x$	${}^dR_x$ $=\sum {}^dM_x$	${}^rM_x$ $=\sum_x {}^rC_x$	${}^rR_x$ $=\sum {}^rM_x$	Age ( <i>x</i> )
15	137,597	1190.9	36423.6	832.5	15990.6	164.84	5959.55	15
16	126,113	1146.0	35232.7	787.6	15158.1	164.84	5794.71	16
17	116,000	1104.9	34086.7	746.5	14370.5	164.84	5629.87	17
18	107,112	1067.3	32981.8	708.9	13624.0	164.84	5465.03	18
19	99,240	1033.7	31914.5	675.3	12915.1	164.84	5300.19	19
20	92,211	1003.2	30880.8	644.8	12239.8	164.84	5135.35	20
21	85,891	975.2	29877.6	616.8	11595.0	164.84	4970.51	21
22	80,168	949.6	28902.4	591.2	10978.2	164.84	4805.67	22
23	74,953	925.8	27952.8	567.4	10387.0	164.84	4640.83	23
24	70,174	903.7	27027.0	545.3	9819.6	164.84	4475.99	24
25	65,774	883.2	26123.3	524.8	9274.3	164.84	4311.15	25
26	61,705	863.7	25240.1	505.3	8749.5	164.84	4146.31	26
27	57,930	844.8	24376.4	486.4	8244.2	164.84	3981.47	27
28	54,416	826.9	23531.6	468.5	7757.8	164.84	3816.63	28
29	51,137	809.9	22704.7	451.5	7289.3	163.99	3651.79	29
30	48,069	793.4	21894.8	435.0	6837.8	163.17	3487.80	30
31	45,192	777.4	21101.4	419.0	6402.8	161.97	3324.63	31
32	42,490	761.9	20324.0	403.5	5983.8	160.42	3162.66	32
33	39,947	746.8	19562.1	388.4	5580.3	158.91	3002.24	33
34	37,551	732.2	18815.3	373.8	5191.9	157.08	2843.33	34
35	35,289	718.0	18083.1	359.6	4818.1	155.30	2686.25	35
36	33,152	703.9	17365.1	345.5	4458.5	153.23	2530.95	36
37	31,130	690.2	16661.2	331.8	4113.0	151.22	2377.72	37
38	29,216	676.5	15971.0	318.1	3781.2	148.94	2226.50	38
39	27,402	662.9	15294.5	304.5	3463.1	146.41	2077.56	39
40	25,682	649.4	14631.6	291.0	3158.6	143.65	1931.15	40
41	24,051	636.0	13982.2	277.6	2867.6	140.67	1787.50	41
42	22,503	622.7	13346.2	264.3	2590.0	137.49	1646.83	42
43	21,033	609.2	12723.5	250.8	2325.7	134.12	1509.34	43
44	19,638	595.9	12114.3	237.5	2074.9	130.58	1375.22	44
45	18,313	582.4	11518.4	224.0	1837.4	126.88	1244.64	45
46	17,055	568.5	10936.0	210.1	1613.4	122.77	1117.76	46
47	15,861	554.5	10367.5	196.1	1403.3	118.28	994.99	47
48	14,728	540.5	9813.0	182.1	1207.2	113.44	876.71	48
49	13,653	526.2	9272.5	167.8	1025.1	108.27	763.27	49
50	12,634	511.6	8746.3	153.2	857.3	102.57	655.00	50
51	11,669	497.0	8234.7	138.6	704.1	96.37	552.43	51
52	10,756	482.4	7737.7	124.0	565.5	89.49	456.06	52
53	9,893	467.6	7255.3	109.2	441.5	81.97	366.57	53
54	9,079	452.6	6787.7	94.2	332.3	73.66	284.60	54
55	8,313	437.4	6335.1	79.0	238.1	64.61	210.94	55
56	7,595	421.9	5897.7	63.5	159.1	54.68	146.33	56
57	6,924	406.1	5475.8	47.7	95.6	43.55	91.65	57
58	6,301	390.3	5069.7	31.9	47.9	31.13	48.10	58
59	5,726	374.4	4679.4	16.0	16.0	16.97	16.97	59
60	5,201	358.4	4305.0	...	...	...	...	

NOTE.— $D_x^{(4)}$  and  $C_x^{(4)}$  are the same as in Table 29.  ${}^dC_x$ ,  ${}^wC_x$ ,  ${}^rC_x$ ,  ${}^wM_x$ , and  ${}^wR_x$  are the same as in Table 30.

Hypothetical Experience of Staff Pension Fund.

TABLE 39.

Commutation Tables for finding the Values of the Return of Contributions of 1 per annum, with Compound Interest at 3 per-cent, on Death or Early Retirement, and for finding the Value of a Pension of 1 for each year of service on Early Retirement.

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	${}^dD_x$ $=l'_x \times v^{x+1}$	${}^dH_x$ $=\Sigma {}^dD_x$	${}^rD_x$ $=\Sigma r_x \times v^{x+1}$	${}^rH_x$ $=\Sigma {}^rD_x$	${}^raM_x$ $=\Sigma \{ {}^rC_x \times (a'_{x+5} + .5) \}$	${}^raR_x$ $=\Sigma {}^raM_x$	Age ( <i>x</i> )
15	1,556	25,676	478	10,912	1,641	59,843	15
16	1,467	24,120	464	10,434	1,641	58,202	16
17	1,385	22,653	451	9,970	1,641	56,562	17
18	1,308	21,268	437	9,519	1,641	54,921	18
19	1,237	19,960	425	9,082	1,641	53,281	19
20	1,171	18,723	412	8,657	1,641	51,640	20
21	1,110	17,552	400	8,245	1,641	50,000	21
22	1,053	16,442	389	7,845	1,641	48,359	22
23	999	15,389	377	7,456	1,641	46,719	23
24	949	14,390	366	7,079	1,641	45,078	24
25	901	13,441	356	6,713	1,641	43,438	25
26	856	12,540	345	6,357	1,641	41,797	26
27	813	11,684	335	6,012	1,641	40,157	27
28	771	10,871	325	5,677	1,641	38,516	28
29	733	10,100	315	5,352	1,634	36,876	29
30	695	9,367	305	5,037	1,627	35,242	30
31	659	8,672	295	4,732	1,617	33,615	31
32	625	8,013	285	4,437	1,604	31,999	32
33	592	7,388	275	4,152	1,591	30,395	33
34	561	6,796	265	3,877	1,575	28,804	34
35	531	6,235	256	3,612	1,560	27,229	35
36	501	5,704	247	3,356	1,541	25,669	36
37	474	5,203	237	3,109	1,523	24,128	37
38	446	4,729	228	2,872	1,503	22,604	38
39	420	4,283	219	2,644	1,479	21,102	39
40	395	3,863	210	2,425	1,453	19,623	40
41	370	3,468	201	2,215	1,425	18,169	41
42	347	3,098	192	2,014	1,395	16,744	42
43	324	2,751	183	1,822	1,363	15,349	43
44	301	2,427	175	1,639	1,328	13,987	44
45	279	2,126	166	1,464	1,292	12,659	45
46	258	1,847	157	1,298	1,251	11,367	46
47	237	1,589	148	1,141	1,206	10,116	47
48	216	1,352	139	993	1,157	8,910	48
49	196	1,136	130	854	1,104	7,753	49
50	176	940	121	724	1,046	6,649	50
51	157	764	111	603	982	5,603	51
52	138	607	101	492	912	4,621	52
53	120	469	91	391	834	3,709	53
54	102	349	80	300	748	2,875	54
55	84	247	69	220	655	2,127	55
56	66	163	57	151	553	1,472	56
57	49	97	45	94	439	920	57
58	32	48	32	49	312	481	58
59	16	16	17	17	169	169	59

## Hypothetical Experience of Staff Pension Fund.

TABLE 40.

*Table for finding Accumulation of Salary at Compound Interest; and Commutation Columns for finding the Present Value of the Return of Contributions on Death and Early Retirement Without Interest.*

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( $x$ )	$s_x v^x$	$\Sigma s_x v^x$	$(1+i)^{x-1}$	$\begin{matrix} dM_x^s \\ = dM_x \times s_x \end{matrix}$	$\begin{matrix} dR_x^s \\ = \Sigma^d M_x^s \end{matrix}$	$\begin{matrix} rM_x^s \\ = rM_x \times s_x \end{matrix}$	$\begin{matrix} rR_x^s \\ = \Sigma^r M_x^s \end{matrix}$	Age ( $x$ )
15	12.84	1566.25	1.5126	16,650	1,346,547	3,297	609,405	15
16	15.58	1553.41	1.5580	19,690	1,329,897	4,121	606,108	16
17	18.15	1537.83	1.6047	22,395	1,310,207	4,945	601,987	17
18	20.56	1519.68	1.6528	24,812	1,287,812	5,769	597,042	18
19	22.81	1499.12	1.7024	27,012	1,263,000	6,594	591,273	19
20	24.92	1476.31	1.7535	29,016	1,235,988	7,418	584,679	20
21	26.88	1451.39	1.8061	30,840	1,206,972	8,242	577,261	21
22	28.70	1424.51	1.8603	32,516	1,176,132	9,066	569,019	22
23	30.40	1395.81	1.9161	34,044	1,143,616	9,890	559,953	23
24	31.98	1365.41	1.9736	35,445	1,109,572	10,715	550,063	24
25	33.43	1333.43	2.0328	36,736	1,074,127	11,539	539,348	25
26	34.31	1300.00	2.0938	37,392	1,037,391	12,198	527,809	26
27	35.11	1265.69	2.1566	37,939	999,999	12,858	515,611	27
28	35.84	1230.58	2.2213	38,417	962,060	13,517	502,754	28
29	36.49	1194.74	2.2879	38,829	923,643	14,103	489,237	29
30	37.08	1158.25	2.3566	39,150	884,814	14,685	475,134	30
31	37.60	1121.17	2.4273	39,386	845,664	15,225	460,448	31
32	38.06	1083.57	2.5001	39,543	806,278	15,721	445,223	32
33	38.46	1045.51	2.5751	39,617	766,735	16,209	429,502	33
34	38.80	1007.05	2.6523	39,623	727,118	16,651	413,293	34
35	39.09	968.25	2.7319	39,556	687,495	17,083	396,643	35
36	39.33	929.16	2.8139	39,387	647,939	17,468	379,560	36
37	39.53	889.83	2.8983	39,152	608,552	17,844	362,092	37
38	39.68	850.30	2.9852	38,808	569,400	18,171	344,248	38
39	39.79	810.62	3.0748	38,367	530,592	18,448	326,077	39
40	39.85	770.83	3.1670	37,830	492,225	18,675	307,629	40
41	39.88	730.98	3.2620	37,198	454,395	18,850	288,955	41
42	39.88	691.10	3.3599	36,473	417,197	18,974	270,105	42
43	39.84	651.22	3.4607	35,614	380,724	19,045	251,131	43
44	39.77	611.38	3.5645	34,675	345,110	19,065	232,086	44
45	39.67	571.61	3.6715	33,600	310,435	19,032	213,022	45
46	39.54	531.94	3.7816	32,355	276,835	18,907	193,990	46
47	39.38	492.40	3.8950	30,984	244,480	18,688	175,083	47
48	39.20	453.02	4.0119	29,500	213,496	18,377	156,395	48
49	39.00	413.82	4.1323	27,855	183,996	17,973	138,017	49
50	38.78	374.82	4.2562	26,044	156,141	17,437	120,045	50
51	38.53	336.04	4.3839	24,116	130,097	16,768	102,608	51
52	38.27	297.51	4.5154	22,072	105,981	15,929	85,839	52
53	37.99	259.24	4.6509	19,874	83,909	14,919	69,910	53
54	37.70	221.25	4.7904	17,521	64,035	13,701	54,992	54
55	37.39	183.55	4.9341	15,010	46,514	12,276	41,291	55
56	37.06	146.16	5.0821	12,319	31,504	10,608	29,015	56
57	36.72	109.10	5.2346	9,445	19,185	8,623	18,407	57
58	36.37	72.38	5.3917	6,444	9,740	6,288	9,784	58
59	36.01	36.01	5.5534	3,296	3,296	3,496	3,496	59



## Hypothetical Experience of Staff Pension Fund.

TABLE 41.

*Commutation Columns for finding the Present Values of Future Salary, and of Return of Contributions, with Compound Interest, on Death or Early Retirement.*

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( $x$ )	$\Pi_x^s$ $= \sum_x^{59} (vD_x^s)$	${}^dD_x^s$ $= {}^dD_x \times s_x$	${}^d\Pi_x^s$ $= \sum {}^dD_x^s$	${}^rD_x^s$ $= {}^rD_x \times s_x$	${}^r\Pi_x^s$ $= \sum {}^rD_x^s$	Age ( $x$ )
15	10,214,321	31,120	1,988,978	9,560	963,768	15
16	9,965,059	36,675	1,957,858	11,600	954,208	16
17	9,686,321	41,550	1,921,183	13,530	942,608	17
18	9,391,768	45,780	1,879,633	15,295	929,078	18
19	9,089,748	49,480	1,833,853	17,000	913,783	19
20	8,784,039	52,695	1,784,373	18,540	896,783	20
21	8,476,946	55,500	1,731,678	20,000	878,243	21
22	8,170,150	57,915	1,676,178	21,395	858,243	22
23	7,864,552	59,940	1,618,263	22,620	836,848	23
24	7,560,766	61,685	1,558,323	23,790	814,228	24
25	7,259,179	63,070	1,496,638	24,920	790,438	25
26	6,960,149	63,344	1,433,568	25,530	765,518	26
27	6,667,812	63,414	1,370,224	26,130	739,988	27
28	6,381,938	63,222	1,306,810	26,650	713,858	28
29	6,102,182	63,038	1,243,588	27,090	687,208	29
30	5,828,401	62,550	1,180,550	27,450	660,118	30
31	5,560,323	61,946	1,118,000	27,730	632,668	31
32	5,297,762	61,250	1,056,054	27,930	604,938	32
33	5,040,679	60,384	994,804	28,050	577,008	33
34	4,788,848	59,466	934,420	28,090	548,958	34
35	4,542,269	58,410	874,954	28,160	520,868	35
36	4,300,696	57,114	816,544	28,158	492,708	36
37	4,064,173	55,932	759,430	27,966	464,550	37
38	3,832,526	54,412	703,498	27,816	436,584	38
39	3,605,819	52,920	649,086	27,594	408,768	39
40	3,383,912	51,350	596,166	27,300	381,174	40
41	3,166,825	49,580	544,816	26,934	353,874	41
42	2,954,636	47,886	495,236	26,496	326,940	42
43	2,747,234	46,008	447,350	25,986	300,444	43
44	2,544,574	43,946	401,342	25,550	274,458	44
45	2,346,836	41,850	357,396	24,900	248,908	45
46	2,153,875	39,732	315,546	24,178	224,008	46
47	1,965,785	37,446	275,814	23,384	199,830	47
48	1,782,628	34,992	238,368	22,518	176,446	48
49	1,604,428	32,536	203,376	21,580	153,928	49
50	1,431,175	29,920	170,840	20,570	132,348	50
51	1,262,990	27,318	140,920	19,314	111,778	51
52	1,099,971	24,564	113,602	17,978	92,464	52
53	942,190	21,840	89,038	16,562	74,486	53
54	789,699	18,972	67,198	14,880	57,924	54
55	642,705	15,960	48,226	13,110	43,044	55
56	501,404	12,804	32,266	11,058	29,934	56
57	366,169	9,702	19,462	8,910	18,876	57
58	237,181	6,464	9,760	6,464	9,966	58
59	115,000	3,296	3,296	3,502	3,502	59



## Hypothetical Experience of Staff Pension Fund.

TABLE 42.

*Commutation Columns for finding the Present Value of the Last Year's Salary on Death or Early Retirement; and the Present Value of a Pension based on Average Salary and Last Salary.*

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( $x$ )	$dM_x^{ls}$ $=\Sigma(dC_x^{s_x})$	$rM_x^{ls}$ $=\Sigma(rC_x^{s_x})$	$raM_x^s$ $=raM_x \times s_x$	$raR_x^s$ $=\Sigma raM_x^s$	$raM_x^{ls}$ $=\Sigma\{rC_x^{s_x} \times (a'_{x+\cdot 5} + \cdot 5)\}$	$raR_x^{ls}$ $=\Sigma raM_x^{ls}$	Age ( $x$ )
15	83,701	28,126	32,810	6,143,619	282,022	10,672,226	15
16	82,803	28,126	41,013	6,110,809	282,022	10,390,204	16
17	81,775	28,126	49,215	6,069,796	282,022	10,108,182	17
18	80,647	28,126	57,418	6,020,581	282,022	9,826,160	18
19	79,471	28,126	65,620	5,963,163	282,022	9,544,138	19
20	78,251	28,126	73,823	5,897,543	282,022	9,262,116	20
21	76,991	28,126	82,025	5,823,720	282,022	8,980,094	21
22	75,711	28,126	90,228	5,741,695	282,022	8,698,072	22
23	74,402	28,126	98,430	5,651,467	282,022	8,416,050	23
24	73,076	28,126	106,633	5,553,037	282,022	8,134,028	24
25	71,743	28,126	114,835	5,446,404	282,022	7,852,006	25
26	70,378	28,126	121,397	5,331,569	282,022	7,569,984	26
27	68,979	28,126	127,959	5,210,172	282,022	7,287,962	27
28	67,583	28,126	134,521	5,082,213	282,022	7,005,940	28
29	66,189	28,057	140,490	4,947,692	281,458	6,723,918	29
30	64,770	27,986	146,421	4,807,202	280,881	6,442,460	30
31	63,330	27,878	151,989	4,660,781	279,986	6,161,579	31
32	61,873	27,732	157,182	4,508,792	278,762	5,881,593	32
33	60,393	27,584	162,282	4,351,610	277,501	5,602,831	33
34	58,904	27,398	166,971	4,189,328	275,890	5,325,330	34
35	57,399	27,209	171,556	4,022,357	274,239	5,049,440	35
36	55,848	26,979	175,708	3,850,801	272,203	4,775,201	36
37	54,286	26,750	179,738	3,675,093	270,145	4,502,998	37
38	52,669	26,481	183,305	3,495,355	267,700	4,232,853	38
39	51,010	26,172	186,379	3,312,050	264,857	3,965,153	39
40	49,300	25,825	188,942	3,125,671	261,615	3,700,296	40
41	47,558	25,437	190,990	2,936,729	257,962	3,438,681	41
42	45,776	25,011	192,510	2,745,739	253,897	3,180,719	42
43	43,913	24,546	193,475	2,553,229	249,414	2,926,822	43
44	42,024	24,043	193,888	2,359,754	244,517	2,677,408	44
45	40,053	23,503	193,740	2,165,866	239,202	2,432,891	45
46	37,968	22,887	192,623	1,972,126	233,080	2,193,689	46
47	35,812	22,195	190,516	1,779,503	226,151	1,960,609	47
48	33,600	21,430	187,418	1,588,987	218,428	1,734,458	48
49	31,283	20,593	183,330	1,401,569	209,919	1,516,030	49
50	28,859	19,647	177,837	1,218,239	200,248	1,306,111	50
51	26,377	18,593	170,938	1,040,402	189,424	1,105,863	51
52	23,837	17,396	162,247	869,464	177,082	916,439	52
53	21,203	16,057	151,770	707,217	163,267	739,357	53
54	18,473	14,545	139,147	555,447	147,644	576,090	54
55	15,646	12,861	124,393	416,300	130,273	428,446	55
56	12,701	10,975	107,185	291,907	110,858	298,173	56
57	9,636	8,815	86,823	184,722	88,748	187,315	57
58	6,508	6,356	63,044	97,899	63,714	98,567	58
59	3,296	3,496	34,855	34,855	34,853	34,853	59

NOTE.— ${}^wM_x^{ls}$  will be found in Table 32.

## Hypothetical Experience of Staff Pension Fund.

TABLE 43.

*Multipliers for use in a Valuation.*

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( $x$ )	$N_{60} + \frac{1}{2}D_{60}$ $D_x$	$\frac{N_x^s}{D_x^s}$	$\frac{dM_x}{D_x}$	$\frac{dR_x^s}{D_x^s}$	$\frac{rM_x}{D_x}$	$\frac{rR_x^s}{D_x^s}$	$\frac{dD_x}{D_x}$	$\frac{dN_x^s}{D_x^s}$	Age ( $x$ )
15	·426	39·785	·065	5·245	·013	2·374	·121	7·747	15
16	·476	34·709	·069	4·632	·014	2·111	·128	6·820	16
17	·540	31·927	·074	4·319	·016	1·984	·137	6·332	17
18	·615	30·191	·080	4·140	·019	1·919	·147	6·042	18
19	·694	28·867	·086	4·011	·021	1·878	·157	5·824	19
20	·777	27·771	·092	3·908	·023	1·848	·167	5·641	20
21	·864	26·826	·098	3·820	·026	1·827	·176	5·480	21
22	·955	25·956	·103	3·737	·029	1·808	·184	5·325	22
23	1·048	25·134	·109	3·655	·032	1·790	·192	5·172	23
24	1·143	24·340	·114	3·572	·034	1·771	·199	5·017	24
25	1·242	23·569	·119	3·487	·037	1·751	·205	4·859	25
26	1·343	23·115	·124	3·445	·041	1·753	·210	4·761	26
27	1·447	22·645	·129	3·396	·044	1·751	·215	4·653	27
28	1·555	22·148	·133	3·339	·047	1·745	·219	4·535	28
29	1·666	21·639	·138	3·275	·050	1·735	·224	4·410	29
30	1·781	21·108	·142	3·204	·053	1·721	·227	4·276	30
31	1·899	20·560	·146	3·127	·056	1·703	·229	4·134	31
32	2·022	20·007	·149	3·045	·059	1·681	·231	3·988	32
33	2·148	19·433	·153	2·956	·062	1·656	·233	3·835	33
34	2·280	18·856	·156	2·863	·066	1·627	·234	3·679	34
35	2·415	18·255	·159	2·763	·069	1·594	·235	3·516	35
36	2·556	17·654	·162	2·660	·072	1·558	·234	3·352	36
37	2·702	17·034	·164	2·551	·075	1·518	·234	3·183	37
38	2·854	16·413	·166	2·438	·078	1·474	·233	3·013	38
39	3·012	15·776	·168	2·321	·081	1·427	·232	2·840	39
40	3·176	15·134	·169	2·201	·084	1·376	·230	2·666	40
41	3·350	14·490	·170	2·079	·086	1·322	·227	2·493	41
42	3·529	13·831	·171	1·953	·088	1·264	·224	2·318	42
43	3·716	13·161	·171	1·824	·091	1·203	·220	2·143	43
44	3·916	12·494	·170	1·694	·094	1·140	·216	1·971	44
45	4·123	11·808	·169	1·562	·096	1·072	·211	1·798	45
46	4·343	11·118	·167	1·429	·098	1·001	·205	1·629	46
47	4·575	10·420	·164	1·296	·099	·928	·199	1·462	47
48	4·822	9·712	·161	1·163	·100	·852	·191	1·299	48
49	5·082	8·991	·156	1·031	·101	·773	·182	1·140	49
50	5·361	8·262	·150	·901	·101	·693	·173	·986	50
51	5·661	7·522	·144	·775	·100	·611	·163	·839	51
52	5·984	6·768	·136	·652	·098	·528	·151	·699	52
53	6·330	5·999	·127	·534	·095	·445	·139	·567	53
54	6·711	5·216	·116	·423	·090	·363	·125	·444	54
55	7·132	4·416	·103	·320	·084	·284	·110	·331	55
56	7·609	3·600	·088	·226	·076	·208	·092	·232	56
57	8·142	2·756	·071	·144	·065	·139	·073	·146	57
58	8·769	1·885	·051	·077	·050	·078	·051	·078	58
59	9·501	·971	·028	·028	·030	·030	·028	·028	59

## Hypothetical Experience of Staff Pension Fund.

TABLE 43—(continued).

*Multipliers for use in a Valuation.*

PENSION AGE 60.

INTEREST 3 PER-CENT.

Age ( <i>x</i> )	$\frac{rD_x}{D_x}$	$\frac{rR_x^s}{D_x^s}$	$\frac{raM_x}{D_x}$	$\frac{raR_x^s}{D_x^s}$	$\frac{raM_x^{ls}}{D_x^s}$	$\frac{raR_x^{ls}}{D_x^s}$	$\frac{dM_x^{ls}}{D_x^s}$	$\frac{rM_x^{ls}}{D_x^s}$	Age ( <i>x</i> )
15	·037	3·754	·128	23·929	1·098	41·568	·326	·110	15
16	·040	3·324	·143	21·285	·983	36·190	·288	·098	16
17	·045	3·107	·162	20·007	·929	33·318	·270	·093	17
18	·049	2·987	·185	19·354	·907	31·587	·259	·090	18
19	·054	2·902	·208	18·938	·896	30·310	·252	·089	19
20	·059	2·835	·233	18·645	·892	29·282	·247	·089	20
21	·063	2·779	·260	18·430	·892	28·418	·244	·089	21
22	·068	2·727	·287	18·241	·895	27·608	·240	·089	22
23	·072	2·675	·315	18·062	·901	26·897	·238	·090	23
24	·077	2·621	·343	17·876	·908	26·185	·235	·091	24
25	·081	2·566	·373	17·683	·916	25·494	·233	·091	25
26	·085	2·512	·403	17·707	·937	25·141	·234	·093	26
27	·089	2·513	·435	17·695	·958	24·751	·234	·096	27
28	·092	2·477	·467	17·638	·979	24·314	·235	·098	28
29	·096	2·437	·498	17·545	·998	23·844	·235	·100	29
30	·099	2·391	·530	17·410	1·017	23·332	·235	·101	30
31	·103	2·339	·562	17·234	1·035	22·784	·234	·103	31
32	·105	2·285	·594	17·027	1·053	22·212	·234	·105	32
33	·108	2·225	·626	16·777	1·070	21·600	·233	·106	33
34	·111	2·162	·657	16·495	1·086	20·968	·232	·108	34
35	·113	2·093	·690	16·166	1·102	20·294	·231	·109	35
36	·116	2·023	·721	15·807	1·117	19·601	·229	·111	36
37	·117	1·947	·753	15·403	1·132	18·872	·228	·112	37
38	·119	1·870	·785	14·969	1·146	18·127	·226	·113	38
39	·121	1·788	·815	14·491	1·159	17·348	·223	·114	39
40	·122	1·705	·845	13·979	1·170	16·549	·220	·115	40
41	·123	1·619	·874	13·437	1·180	15·734	·218	·116	41
42	·124	1·530	·901	12·853	1·189	14·889	·214	·117	42
43	·124	1·439	·927	12·232	1·195	14·021	·210	·118	43
44	·125	1·348	·952	11·586	1·200	13·146	·206	·118	44
45	·125	1·252	·975	10·897	1·204	12·241	·202	·118	45
46	·125	1·156	·994	10·180	1·203	11·323	·196	·118	46
47	·124	1·059	1·010	9·433	1·199	10·393	·190	·118	47
48	·123	·961	1·021	8·657	1·190	9·450	·183	·117	48
49	·121	·863	1·027	7·854	1·176	8·496	·175	·115	49
50	·119	·764	1·027	7·033	1·156	7·540	·167	·113	50
51	·115	·666	1·018	6·196	1·128	6·586	·157	·111	51
52	·111	·569	·999	5·350	1·090	5·639	·147	·107	52
53	·105	·474	·966	4·503	1·039	4·707	·135	·102	53
54	·098	·383	·919	3·669	·975	3·805	·122	·096	54
55	·090	·296	·855	2·860	·895	2·944	·108	·088	55
56	·079	·215	·770	2·096	·796	2·141	·091	·079	56
57	·067	·142	·654	1·390	·668	1·410	·073	·066	57
58	·051	·079	·501	·778	·506	·783	·052	·051	58
59	·030	·030	·294	·294	·294	·294	·028	·030	59

NOTE.—The values of  ${}^wM_x^{ls} \div D_x^s$  will be found in Table 37.



## APPENDIX I.

## EXPLANATION OF THE NOTATION USED IN THE PAPER.

In all the simple investigations throughout the paper I have used the Institute notation with one exception, namely, the symbol to represent the amount of an annuity-certain of 1 per annum for  $n$  years. That symbol in the Institute notation is  $s_{\overline{n}|}$ ; but as I wished to use  $s$  to represent salary, and the similarity might lead to confusion, I decided to substitute  $a_{(n)}$  for  $s_{\overline{n}|}$ .

*The new elementary symbols are :—*

$w_x$  = the number of persons withdrawing (by resignation or dismissal) between the ages of  $x$  and  $x+1$ .

$r_x$  = the number of persons retiring before the pension age (on pension or with compensation) from ill-health between the ages of  $x$  and  $x+1$ .

$s_x$  = the salary receivable between the ages  $x$  and  $x+1$ .

$ls$  = last salary; that is, the full year's salary receivable in the year of death, withdrawal, or retirement. This is only used in combination with commutation symbols.

$(as)_{(n)}$  = the amount of annual salary for  $n$  years accumulated at compound interest.

$a'_x$  = the present value of an annuity of 1 on the life of an invalid of the age  $x$  who has retired from ill-health.

$ra$  = annuity on early retirement (before the pension age is reached). This is only used in combination with commutation symbols.

An index consisting of a figure or letter in a bracket indicates the Experience Table from which the values are taken, or on which they are based. These tables are called 2, 3, 4, and ( $r$ ). Nos. 2, 3, and 4 are different arrangements of the same experience, and ( $r$ ) is the experience representing the mortality prevailing amongst those who retire from ill-health before the pension age, and is to be found in Table No. 8. Thus—

$\left. \begin{array}{l} l_x^{(2)} \\ d_x^{(2)} \\ D_x^{(2)} \\ N_x^{(2)} \\ \&c. \end{array} \right\}$	are the values of	$\left. \begin{array}{l} l_x \\ d_x \\ D_x \\ N_x \\ \&c. \end{array} \right\}$	by Experience Table No. 2.
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$$\left. \begin{matrix} l_x^{(r)} \\ q_x^{(r)} \\ d_x^{(r)} \\ D_x^{(r)} \\ N_x^{(r)} \\ \&c. \end{matrix} \right\} \text{ are the values of } \left. \begin{matrix} l_x \\ q_x \\ d_x \\ D_x \\ N_x \\ \&c. \end{matrix} \right\} \text{ by the Invalid Mortality Table No. 8.}$$

Where no index is used the values are based on Experience Table No. 4, which is the basis of all the working formulas.

The only successful way of producing tables for solving the problems which are dealt with in this paper is by the use of the columnar method, and I therefore looked to the columnar symbols to represent the several values to be tabulated.

To the student the columnar symbols should be essentially pictorial, and this will be so if the origin of the symbols is kept in mind. I think there is little doubt that when Griffith Davies adopted D and N as the titles of his columns he selected the initial letters of the words denominator and numerator, so that  $\frac{N}{D} = \frac{\text{Numerator}}{\text{Denominator}} = a$ . The column for finding the value of an increasing annuity is formed by the constant summation of the N column, hence the use of the letter S to represent the sum of N. Subsequently, when corresponding columns were formed for finding the assurance values, the preceding letters in the alphabet were used, namely, C, M, R. The open N has since been added to represent the numerator for an annuity-due, so that  $N_x \div D_x = 1 + a_x$ .

The notation in the paper will appear simple if the mental retina receives the fixed impression that—

$\frac{N_x}{D_x}$  is the present value of an annuity-due, and the present value of the amount of an annuity-due accumulated at compound interest when interest and discount are at the same rate.

$\frac{M_x}{D_x}$  is the present value of the assurance of 1 payable at the end of the year of death.

$\frac{R_x}{D_x}$  is the present value of the assurance of 1 increasing by 1 every year up to the year of death.

As the pension age is the *limiting age* in the working tables, it will be necessary sometimes to have a distinguishing symbol to represent the pension age. For this I propose that the pension age shall be placed in a half square as a left-hand suffix. Thus—

$$\left. \begin{array}{l} {}_{60}\overline{N}_x \\ {}_{60}\overline{M}_x \\ {}_{60}\overline{R}_x \\ \text{\&c.} \end{array} \right\} \text{ are the values of } \left. \begin{array}{l} \overline{N}_x \\ \overline{M}_x \\ \overline{R}_x \\ \text{\&c.} \end{array} \right\} \text{ when the pension age is 60.}$$

Throughout this work the tables are calculated on the assumption that all events occur at the end of the year of assurance; that is to say, that deaths, withdrawals, and retirements happen at the end of the year in which they occur, and that the yearly contributions, although due at the beginning of the year, are payable at the end of the year. A full year's contribution is therefore supposed to be received in the year of death, withdrawal, or retirement.

#### COMMUTATION SYMBOLS NOT INVOLVING SALARY.

##### *Contributions.*

$D_x = l_x v^x = \sum_x^{\omega} d_x \times v^x$ . This is the common denominator for all columnar values not involving salary.

${}_{65}\overline{N}_x = v(N_{x-1} - N_{64}) = v(D_x + D_{x+1} + \dots + D_{64})$ . A separate column has not been calculated for this value. This symbol is not used in the paper except in combination with salary.

$\frac{{}_{65}\overline{N}_x}{D_x}$  = the present value, at age  $x$ , of contributions of 1 per annum, ceasing at the end of the year of death, withdrawal, early retirement, or retirement at the pension age, 65.

Columns are required for each of the various modes of exit, and these are distinguished by placing the letter representing the elementary function as a top prefix to the columnar symbol.

##### *Return of Contributions with Interest.*

$$\frac{d}{65}D_x = v^{x+1} \sum_x^{64} d_x, \text{ or } l'_x v^{x+1} \text{ where } l'_x = \sum_x^{64} d_x$$

$$\frac{d}{65}\overline{N}_x = \frac{d}{65}D_x + \frac{d}{65}D_{x+1} + \dots + \frac{d}{65}D_{64}$$

$$\frac{r}{65}D_x = v^{x+1} \sum_x^{64} r_x$$

$$\frac{r}{65}\overline{N}_x = \frac{r}{65}D_x + \frac{r}{65}D_{x+1} + \dots + \frac{r}{65}D_{64}$$

so that

$$\left. \begin{aligned} \frac{{}^dN_x}{D_x} &= \left\{ \begin{array}{l} \text{The present value of an} \\ \text{annuity-due of 1 per} \\ \text{annum deferred one year;} \\ \text{which is the same as} \end{array} \right\} \begin{array}{l} \text{to the end of} \\ \text{the year of} \\ \text{death.} \end{array} \\ \\ \frac{{}^rN_x}{D_x} &= \left\{ \begin{array}{l} \text{The present value of the accu-} \\ \text{mulation of an annuity-} \\ \text{due of 1 per annum,} \\ \text{deferred one year, at com-} \\ \text{pound interest at the same} \\ \text{rate as used in the valua-} \\ \text{tion,} \end{array} \right\} \begin{array}{l} \text{to the end of} \\ \text{the year of} \\ \text{early retire-} \\ \text{ment.} \end{array} \end{aligned}$$

*Return of Contributions without Interest.*

$${}^dC_x = v^{x+1}d_x$$

$${}^wC_x = v^{x+1}w_x$$

$${}^rC_x = v^{x+1}r_x$$

$$\frac{{}^d}{65}M_x = {}^dC_x + {}^dC_{x+1} + \dots + {}^dC_{64}$$

$$\frac{{}^w}{65}M_x = {}^wC_x + {}^wC_{x+1} + \dots + {}^wC_{64}$$

$$\frac{{}^r}{65}M_x = {}^rC_x + {}^rC_{x+1} + \dots + {}^rC_{64}$$

$$\frac{{}^d}{65}R_x = \frac{{}^d}{65}M_x + \frac{{}^d}{65}M_{x+1} + \dots + \frac{{}^d}{65}M_{64}$$

$$\frac{{}^w}{65}R_x = \frac{{}^w}{65}M_x + \frac{{}^w}{65}M_{x+1} + \dots + \frac{{}^w}{65}M_{64}$$

$$\frac{{}^r}{65}R_x = \frac{{}^r}{65}M_x + \frac{{}^r}{65}M_{x+1} + \dots + \frac{{}^r}{65}M_{64}$$

so that

$$\left. \begin{aligned} \frac{{}^dM_x}{D_x} &= \\ \frac{{}^wM_x}{D_x} &= \\ \frac{{}^rM_x}{D_x} &= \end{aligned} \right\} \begin{array}{l} \text{the present value at age } x \text{ of} \\ \text{the assurance of 1 at the} \\ \text{end of the year of} \end{array} \left\{ \begin{array}{l} \text{death.} \\ \text{withdrawal.} \\ \text{early retirement.} \end{array} \right.$$

and

$$\left. \begin{aligned} \frac{{}^dR_x}{D_x} &= \\ \frac{{}^wR_x}{D_x} &= \\ \frac{{}^rR_x}{D_x} &= \end{aligned} \right\} \begin{array}{l} \text{the present value at age } x \text{ of} \\ \text{the assurance of 1 increasing} \\ \text{by 1 per annum; in other} \\ \text{words, the present value of} \\ \text{the assurance of the return} \\ \text{of contributions of 1 per} \\ \text{annum without interest} \end{array} \left\{ \begin{array}{l} \text{on death.} \\ \text{on withdrawal.} \\ \text{on early retire-} \\ \text{ment.} \end{array} \right.$$

*Annuity on Early Retirement.*

$${}^raC_x = {}^rC_x \times (a'_{x+.5} + .5)$$

$$\frac{{}^ra}{.65}M_x = {}^raC_x + {}^raC_{x+1} + \dots + {}^raC_{64}$$

$$\frac{{}^ra}{.65}R_x = \frac{{}^ra}{.65}M_x + \frac{{}^ra}{.65}M_{x+1} + \dots + \frac{{}^ra}{.65}M_{64}$$

so that

$\frac{{}^raC_x}{D_x}$  = the present value at age  $x$  of an annuity of 1 per annum on retirement before age  $x+1$ .

$\frac{{}^raM_x}{D_x}$  = the present value at age  $x$  of the assurance of an annuity of 1 per annum on retirement before the pension age.

$\frac{{}^raR_x}{D_x}$  = the present value at age  $x$  of the assurance of an annuity on retirement before the pension age of 1 increasing by 1 per annum; in other words, the present value of the assurance on early retirement of an annuity of the contributions of 1 per annum.

## COMMUTATION SYMBOLS INVOLVING SALARY.

In all cases where salary is introduced into the values, the letter  $s$  is used as an index to the columnar symbol.

*Contributions.*

$D_x^s = D_x \times s_x$ . This is the common denominator for all columnar values involving salary, when such values have to be equated to a salary of 1 at  $x$ .

$$\frac{{}^raN_x^s}{.65} = v(D_x^s + D_{x+1}^s + \dots + D_{64}^s).$$

so that

$\frac{{}^raN_x^s}{D_x}$  = the present value at age  $x$  of total salary receivable until death, withdrawal, early retirement, or attainment of pension age.

$\frac{{}^raN_x^s}{D_x^s}$  = the present value at age  $x$  of total salary receivable until death, withdrawal, early retirement, or attainment of pension age, equated to 1 of salary at age  $x$ .



## (1) BENEFITS BASED ON AVERAGE SALARY.

*Return of Contributions with Interest.*

$$\frac{d}{65}D_x^s = \frac{d}{65}D_x \times s_x = v^{x+1} \sum_x^{64} d_x \times s_x$$

$$\frac{d}{65} \sum_x^{\infty} s_x = \frac{d}{65}D_x^s + \frac{d}{65}D_{x+1}^s + \dots + \frac{d}{65}D_{64}^s$$

$$\frac{r}{65}D_x^s = \frac{r}{65}D_x \times s_x = v^{x+1} \sum_x^{64} r_x \times s_x$$

$$\frac{r}{65} \sum_x^{\infty} s_x = \frac{r}{65}D_x^s + \frac{r}{65}D_{x+1}^s + \dots + \frac{r}{65}D_{64}^s$$

so that

$$\left. \begin{aligned} \frac{d}{65} \sum_x^{\infty} s_x &= \frac{d}{65}D_x^s \\ \frac{r}{65} \sum_x^{\infty} s_x &= \frac{r}{65}D_x^s \end{aligned} \right\} \begin{array}{l} \text{The present value of the accu-} \\ \text{mulations of future salary} \\ \text{compounded at the same rate} \\ \text{of interest as used in the valu-} \\ \text{ation, equated to a salary of 1} \\ \text{at age } x \end{array} \left. \begin{array}{l} \text{to the end of the} \\ \text{year of death.} \\ \text{to the end of the} \\ \text{year of early} \\ \text{retirement.} \end{array} \right\}$$

*Return of Contributions without Interest.*

$$\frac{d}{65}M_x^s = \frac{d}{65}M_x \times s_x$$

$$\frac{w}{65}M_x^s = \frac{w}{65}M_x \times s_x$$

$$\frac{r}{65}M_x^s = \frac{r}{65}M_x \times s_x$$

$$\frac{d}{65}R_x^s = \frac{d}{65}M_x^s + \frac{d}{65}M_{x+1}^s + \dots + \frac{d}{65}M_{64}^s$$

$$\frac{w}{65}R_x^s = \frac{w}{65}M_x^s + \frac{w}{65}M_{x+1}^s + \dots + \frac{w}{65}M_{64}^s$$

$$\frac{r}{65}R_x^s = \frac{r}{65}M_x^s + \frac{r}{65}M_{x+1}^s + \dots + \frac{r}{65}M_{64}^s$$

so that

$$\left. \begin{aligned} \frac{d}{65} \frac{M_x^s}{D_x} &= \frac{d}{65} \frac{M_x}{D_x} \\ \frac{w}{65} \frac{M_x^s}{D_x} &= \frac{w}{65} \frac{M_x}{D_x} \\ \frac{r}{65} \frac{M_x^s}{D_x} &= \frac{r}{65} \frac{M_x}{D_x} \end{aligned} \right\} \begin{array}{l} \text{The present value, at age } x, \text{ of} \\ \text{1 year's salary at age } x, \\ \text{payable at the end of the} \\ \text{year of} \end{array} \left. \begin{array}{l} \text{death.} \\ \text{withdrawal.} \\ \text{early retirement.} \end{array} \right\}$$
  

$$\left. \begin{aligned} \frac{d}{65} \frac{M_x^s}{D_x^s} &= \frac{d}{65} \frac{M_x}{D_x} \\ \frac{w}{65} \frac{M_x^s}{D_x^s} &= \frac{w}{65} \frac{M_x}{D_x} \\ \frac{r}{65} \frac{M_x^s}{D_x^s} &= \frac{r}{65} \frac{M_x}{D_x} \end{aligned} \right\} \begin{array}{l} \text{The present value, at} \\ \text{age } x, \text{ of 1 year's salary} \\ \text{at age } x \text{ equated to 1} \\ \text{of salary at age } x; \text{ in} \\ \text{other words, the pre-} \\ \text{sent value of the assur-} \\ \text{ance of 1 at the end} \\ \text{of the year of} \end{array} \left. \begin{array}{l} \text{death.} \\ \text{withdrawal.} \\ \text{early retirement.} \end{array} \right\}$$

$$\left. \begin{aligned}
 \frac{{}^d R_x^s}{D_x} &= \\
 \frac{{}^w R_x^s}{D_x} &= \\
 \frac{{}^r R_x^s}{D_x} &=
 \end{aligned} \right\} \begin{array}{l} \text{The present value, at age } x, \text{ of} \\ \text{the assurance of the total} \\ \text{salary to the end of the} \\ \text{year of} \end{array} \left. \begin{array}{l} \text{death.} \\ \text{withdrawal.} \\ \text{early retirement.} \end{array} \right.$$
  

$$\left. \begin{aligned}
 \frac{{}^d R_x^s}{D_x^s} &= \\
 \frac{{}^w R_x^s}{D_x^s} &= \\
 \frac{{}^r R_x^s}{D_x^s} &=
 \end{aligned} \right\} \begin{array}{l} \text{The present value, at age } x, \text{ of} \\ \text{the return of total salary,} \\ \text{equated to salary of 1 at age} \\ \text{}x, \text{ to the end of the year of} \end{array} \left. \begin{array}{l} \text{death.} \\ \text{withdrawal.} \\ \text{early retirement.} \end{array} \right.$$

*Pension on Early Retirement.*

$$\frac{{}^r a}{\overline{65}} M_x^s = \frac{{}^r a}{\overline{65}} M_x \times s_x$$

$$\frac{{}^r a}{\overline{65}} R_x^s = \frac{{}^r a}{\overline{65}} M_x^s + \frac{{}^r a}{\overline{65}} M_{x+1}^s + \dots + \frac{{}^r a}{\overline{65}} M_{64}^s$$

so that

$$\frac{{}^r a M_x^s}{D_x} = \text{the present value at age } x \text{ of the assurance of an annuity of one year's salary at age } x \text{ on retirement before the pension age.}$$

$$\frac{{}^r a M_x^s}{D_x^s} = \frac{{}^r a M_x}{D_x} = \text{the above equated to a salary of 1 at age } x; \text{ which is the assurance of an annuity of 1 per annum on retirement before the pension age.}$$

$$\frac{{}^r a R_x^s}{D_x^s} = \text{the present value at age } x \text{ of the assurance of an annuity, on retirement before the pension age, equivalent to the total salary received from age } x \text{ until retirement equated to 1 of salary at age } x.$$

(II) BENEFITS BASED ON LAST SALARY.

*Return of Last Premiums.*

$${}^d C_x^s = {}^d C_x \times s_x$$

$${}^w C_x^s = {}^w C_x \times s_x$$

$${}^r C_x^s = {}^r C_x \times s_x$$

$$\frac{d}{65}M_x^{ls} = {}^dC_x^s + {}^dC_{x+1}^s + \dots + {}^dC_{64}^s$$

$$\frac{w}{65}M_x^{ls} = {}^wC_x^s + {}^wC_{x+1}^s + \dots + {}^wC_{64}^s$$

$$\frac{r}{65}M_x^{ls} = {}^rC_x^s + {}^rC_{x+1}^s + \dots + {}^rC_{64}^s$$

so that

$$\left. \begin{aligned} \frac{d}{D_x^s}M_x^{ls} &= \\ \frac{w}{D_x^s}M_x^{ls} &= \\ \frac{r}{D_x^s}M_x^{ls} &= \end{aligned} \right\} \begin{array}{l} \text{The present value, at age } x, \text{ of} \\ \text{the assurance of the return} \\ \text{of the last year's full salary,} \\ \text{equated to a salary of 1 at} \\ \text{age } x, \text{ receivable in the} \\ \text{year of} \end{array} \left\{ \begin{array}{l} \text{death.} \\ \text{withdrawal.} \\ \text{early retirement.} \end{array} \right.$$

### *Pension on Early Retirement.*

$${}^{ra}C_x^{ls} = {}^rC_x^s \times (a'_{x+.5} + .5) = {}^rC_x \times s_x \times (a'_{x+.5} + .5)$$

$$\frac{ra}{65}M_x^{ls} = {}^{ra}C_x^{ls} + {}^{ra}C_{x+1}^{ls} + \dots + {}^{ra}C_{64}^{ls}$$

$$\frac{ra}{65}R_x^{ls} = \frac{ra}{65}M_x^{ls} + \frac{ra}{65}M_{x+1}^{ls} + \dots + \frac{ra}{65}M_{64}^{ls}$$

so that

$$\frac{ra}{D_x}M_x^{ls} = \text{the present value, at age } x, \text{ of the assurance of an} \\ \text{annuity at the end of the year of early retirement,} \\ \text{of the amount of the full last year's salary} \\ \text{receivable in the year of retirement.}$$

$$\frac{ra}{D_x^s}M_x^{ls} = \text{the same as above equated to 1 of salary at age } x.$$

$$\frac{ra}{D_x}R_x^{ls} = \text{the present value, at age } x, \text{ of the assurance of an} \\ \text{annuity at the end of the year of early retirement,} \\ \text{of the amount of the full last year's salary} \\ \text{receivable in the year of retirement, multiplied by} \\ \text{the number of years between age } x \text{ and the age at} \\ \text{the end of the year of retirement; that is, a} \\ \text{pension of the last salary for every year of service.}$$

$$\frac{ra}{D_x^s}R_x^{ls} = \text{the same, equated to salary of 1 at age } x.$$

## APPENDIX II.

## LIST OF CORRECTIONS AND ALTERATIONS.

Vol. xxxvi.

Page 220.—In the foot-note, for  ${}_{64}^d\mathbb{N}_x$  read  ${}_{65}^d\mathbb{N}_x$ .

Page 221.—In all the formulas in Problem IIIA, for  $a'_{x+1}$  read  $(a'_{x+.5} + .5)$ .

Page 223.—In the first formula in Problem VIA, for  $-P({}^dR_x + {}^vR_x)$  read  $+P({}^dR_x + {}^vR_x)$ .

Page 224.—In the second line in Problem IXA, for “benefits in Problem VIIA” read “benefits in Problem VIIIA.”

Page 227.—In Problem XIIIA, 11th line, for  ${}^vR_r$  read  ${}^vR_x$ ; and in all the formulas for  $({}^r\mathbb{N}_x - {}^r\mathbb{N}_{x+15} - 15{}^rD_{x+15})$  read  $({}^r\mathbb{N}_x - {}^r\mathbb{N}_{x+16} - a_{(16)}{}^rD_{x+15})$ .

Page 233.—In the denominator of the two formulas at the bottom of the page, for  ${}^vD_x^s$  read  $D_x^s$ .

Page 235.—In the formula at bottom of page, for—

$({}^dM_x^s + {}^vM_x^s + {}^rM_x^s)$  read  $({}^dM_x^{ls} + {}^vM_x^{ls} + {}^rM_x^{ls})$   
and refer to vol. xxxvii, page 193.

Pages 238–240.—In Problems XB and XIB, for  $a'_{x+1}$  read  $(a'_{x+.5} + .5)$  throughout.

Page 241.—In 4 places; for  $({}^r\mathbb{N}_x^s - {}^r\mathbb{N}_{x+15}^s - as_{15}{}^rD_{x+15})$  read  $({}^r\mathbb{N}_x^s - {}^r\mathbb{N}_{x+16}^s - (as)_{(16)}{}^rD_{x+15})$ .

Page 245.—Cancel “Section VI” and refer to vol. xxxvii, page 195.

Page 248.—Cancel whole page and refer to vol. xxxvii, page 196.

Page 269.—Cancel whole page and refer to vol. xxxvii, page 207.

Page 274.—Cancel whole page and refer to vol. xxxvii, page 207.

Page 275.—Heading to column 7, for  ${}^vR^s \div D$  read  ${}^vR_x^s \div D_x^s$ .

Page 276.—Cancel columns 6, 7, and 8, and refer to vol. xxxvii, page 208.



# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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*Vaccination and the Act of 1898.* By ARTHUR FRANCIS BURRIDGE, *Actuary and Secretary of the Equity and Law Life Assurance Society, and a Vice-President of the Institute of Actuaries.*

[Read before the Institute, 28 April 1902.]

### I.—HISTORICAL.

OUR *Journal* contains singularly little information on Vaccination—a brief remark by S. H. Ward, M.D., in 1860 (*J.I.A.*, viii, 339); some paragraphs in a general paper by H. W. Porter, in the same year (*J.I.A.*, ix, 151); and a paper by Dr. Sprague in 1877 (*J.I.A.*, xx, 216), are the only references to the subject that I have been able to trace.

Dr. Sprague asked the question, “Does vaccination afford any protection against small-pox”? He examined the statistics put forward by medical men in support of vaccination, and also gave a full account of a report by Dr. Keller, an opponent of vaccination, on the sickness from small-pox among the employees of the Austrian State Railway Company in the year 1873.

Although Dr. Sprague had no reason to doubt the trustworthy character of Keller’s statistics, yet he arrived at the conclusion that the figures did not justify Keller’s inference that vaccination had proved useless.

A strong light has been more recently thrown upon Keller’s statistics in a work by Herr Körösi, Director of the Municipal Statistical Bureau of Buda Pesth, and Honorary Fellow of the

Royal Statistical Society.\* This book was brought to my notice by Dr. E. J. Edwardes, a well-known writer on vaccination, author of "Vaccination and Small-pox."† Dr. Edwardes, dealing with Keller's figures, writes as follows:—

" . . . . . the sanitary medical men of Vienna, the Stadtphysicat, went over the very same ground as Keller, and brought out quite opposite results, *i.e.*, they found that the evidence told strongly in favour of vaccination. Again, Keller's results are not based on sound statistical principles, for he deduces percentages of deaths from one actual death, or two or three deaths—a ridiculous procedure, which has been aptly called by the anti-vaccinists themselves the 'percentage swindle.'

" But worse remains behind. Herr Kőrösi, in 1887, went over the whole ground again, and found that *Keller directly falsified the returns* originally supplied to him by the Railway surgeons. Kőrösi obtained from eight surgeons copies of their original returns; *all eight had been altered by Keller* so as to tell against vaccination. Keller's bias was well known in Vienna.

" Dr. Pichler, of Steiersdorf, had fortunately sent a copy of his return in 1873 to the Comitätsbehörde of Lugos; in his case Keller had altered the figures '68' to '38.' At the International Medical Congress in Washington, 1887, a Special Committee was appointed to examine and report on Keller's statistics. The Committee reported—and the signatures include that of Dr. A. B. Arnold, President of the Section of General Medicine—that all the returns submitted to them were without exception falsified in such a manner as to raise the mortality from small-pox amongst the vaccinated, while that of the unvaccinated was lessened."

I have referred to Herr Kőrösi's book, and verified Dr. Edwardes' quotations therefrom. In the debate on Compulsory Vaccination in the House of Commons, on 13 May 1893, Sir Walter Foster, M.P. (then Parliamentary Secretary of the Local Government Board), called attention to Kőrösi's exposure of Keller and to the Report of the Special Committee.

\* Kritik der Vaccinations—Statistik und Neue Beiträge zur Frage des Impfschutzes. Josef Kőrösi: Berlin, 1889.

† Vaccination and Small-pox. E. J. Edwardes, M.D. Lond. J. and A. Churchill, 1892.

History of  
Vaccination.

As it would appear desirable that some account of the history of vaccination should be found in the *Journal*, I have recorded the main facts as follows:—

#### NOTES ON EARLY HISTORY OF VACCINATION AND LEGISLATION.

Inoculation for  
Small-pox.

The practice of inoculation for small-pox, that is, the artificial introduction of the virus into the system by the insertion of fluid from a variolous pustule\* into wounds of the skin made for the purpose, was first definitely brought to general notice in England by a letter from Lady Mary Wortley Montagu, in 1717. It had long been practised in Turkey and other Eastern countries, and its utility was well known, before its introduction into England, both in the south of Wales and in the Highlands; but the first clearly recorded case in England is that of the daughter of Lady Mary Wortley Montagu (whose son had some time before been inoculated at Constantinople), inoculated by Maitland in London, in April 1721. It was very slowly adopted in this country, and it was not until it had been practised on six criminals (whose liberty was promised to them if they recovered, which they fortunately did) that it was generally received.

In 1746 an Inoculation and Small-pox Hospital was started in London, and in the latter half of the eighteenth century the practice of inoculation seems to have been very general, more especially among the more wealthy, though the incidence of the inoculated varied largely in different localities.

3 and 4 Vict.  
c. 29.

After the introduction of vaccination, the practice of inoculation fell into disuse, and was rendered a penal offence by the first Vaccination Act, in 1840.

Vaccination.

That infection with cow-pox would give immunity from small-pox had been a popular belief among country people long before the apprenticeship of Jenner; by the medical profession, however, the belief was either ignored or looked upon as a popular error.

The idea that it might be possible to propagate the cow-pox, and with it the security from small-pox, first appears to have struck Jenner about the beginning of the last quarter of the eighteenth century, but it was not until 1796 that he made his first decisive experiment. On the 14 May 1796, a boy named Phipps, eight years of age, was successfully vaccinated with

\* Or, to be precise, from the "vesicle," as it would now in its earlier stage be called.



lymph taken from the hand of a dairymaid, who had been infected with the disease by milking cows suffering from cow-pox. On the following 1 July, Phipps was subjected to what is known as the "variola test", that is to say, he was inoculated for small-pox without sensible effect, and was thus shown to be insusceptible to small-pox. Some months afterwards the boy was again inoculated, as before, without effect.

As the result of this and other similar experiments, Jenner published in 1798 his first memoir, "An Inquiry into the causes and effects of the Variolæ Vaccinæ."

In January 1799, cow-pox was discovered in a dairy in the Gray's Inn Lane, and became the subject of experiments on a large scale by Dr. Woodville and Dr. Pearson at the Small-pox Hospital. Seven persons at the hospital were originally vaccinated with matter taken direct from one of the Gray's Inn Lane cows, and others from that taken from the hands of a dairymaid.

From these cases was established the stock distributed at home and abroad, and known as "Woodville's lymph"; this, also, was the beginning of the more general practice of vaccination, since Jenner's original stock had come to an end. But although Jenner received supplies of Woodville's lymph for his own use and for purposes of distribution, he also obtained and distributed lymph obtained from other sources—notably from a farm in Kentish Town. In July 1800, 33 of the most eminent physicians and 40 distinguished surgeons of London signed a declaration of adhesion to the new doctrine.

In 1802, a Committee of the House of Commons, after gathering evidence from experienced witnesses, reported that vaccine inoculation was preventive in its effect, and introduced a milder disorder than inoculated small-pox, which was not capable of being communicated by contagion.

In 1807, the Royal College of Physicians presented a report to the House of Commons, the result of an extensive enquiry, to a similar effect; as an outcome of their enquiry the College strongly recommended the practice of vaccination. In this year was founded, by Royal Warrant, the National Vaccine Establishment. The data for determining to what extent vaccination was practised in the first quarter of the 19th century are too uncertain to allow of any value being attached to the result; it is certain, however, that the amount was very considerable, as such isolated records as the following show:—Of over 8,000 children under



12 years of age admitted into the Royal Metropolitan Infirmary between October 1820 and April 1822, 41 per-cent had been vaccinated and 6 per-cent inoculated.

In 1824, of 8,112 inhabitants of Cambridge, who had been born since the introduction of vaccination, 48·7 per-cent had been vaccinated.

The year 1837 is of interest, as in that year the present system of registration of deaths commenced in England, so that more exact statistics of mortality from small-pox are thereafter available.

3 and 4 Vict.  
c. 29. Prior to the Victorian era no statute was passed dealing with the subject of vaccination. The year 1840 is the starting-point of vaccination legislation; on the 23 July of that year was passed "An Act to extend the practice of Vaccination." This Act, by directing the Guardians and Overseers of every parish in England and Wales to contract with their medical officers for the vaccination of all persons resident in such parishes, provided for the extension of gratuitous vaccination to persons of every class.

Inoculation of the small-pox was declared by the Act to be an offence liable to summary conviction.

The Poor Law Commissioners were empowered to carry the Act of 1840 into operation. Reporting in August 1841, they state that of the 583 Boards of Guardians in England and Wales, 533 had entered into vaccination contracts up to the end of April. Some of the unions had delayed taking these steps from a feeling of misapprehension, as to whether the Act authorized the payment of the vaccinators out of the poor rate; and, in some cases, it had been erroneously considered that the acceptance of the benefit provided by the Legislature would deprive the person whose child might thus be vaccinated of the parliamentary franchise (the Act of 1841 described below was intended to remove these doubts).

The Commissioners thus describe the arrangements for vaccination in the various unions:—

"The union is divided into districts, and in each district vaccination stations have been appointed, so numerous and near to each other that few persons will have to walk more than two miles to the station, and ordinarily the distance will be much less. The vaccinator for the district attends at each station periodically on a fixed day and hour, and attends at the same station on the eighth

day (the day week) afterwards, to inspect the cases which he has vaccinated at such first attendance; and, having ascertained the result of the vaccination, gives to the party, if it be successful, a certificate to that effect."

4 and 5 Vict. c. 24. On 21 June 1841, was passed "An Act to amend an Act to extend the practice of Vaccination."

The Act provided that the expenses incident to the execution of the Act of 1840 were to be paid out of the rates. It further declared that persons vaccinated under the Act should not be considered to be in receipt of parochial relief, or be, on account of such vaccination, deprived of any privilege or subject to any disability. It will be seen that by these Acts of 1840 and 1841, vaccination was entirely voluntary; no compulsion was brought to bear on parents or others. Further, as the amount of the remuneration to be received was to depend on the number of persons who, *not having been previously successfully vaccinated*, shall be successfully vaccinated, it would appear that revaccination was not contemplated.

16 and 17 Vict. c. 100. By the Act of 20 August 1853, "An Act further "to extend and make compulsory the Practice of "Vaccination", infant vaccination was first made compulsory in England and Wales. The parents of every child born after 1 August 1853, were required, under a penalty not exceeding 20s., to present such child for vaccination within three months of birth.

The Public Vaccinator was empowered:—

- (1) To postpone the vaccination if the child were not in a fit state.
- (2) To give a certificate of insusceptibility if he was of opinion that a child vaccinated by him was insusceptible of the vaccine disease.

24 and 25 Vict. c. 59. The Vaccination Amendment Act, 1861, empowered the Guardians or Overseers to appoint persons to compel obedience to and enforce penalties imposed upon defaulters by the previous Acts.

30 and 31 Vict. c. 84. On the 12 August 1867, was passed "An Act to "consolidate and amend the laws relating to "Vaccination."

The Act, which came into force as from 1 January 1868,

practically continued the provisions of the previous Acts, at the same time providing for details of organization. Revaccination was specifically encouraged and paid for at the rate of two-thirds of that for primary vaccination.

34 and 35 Vict. This was an Act passed 21 August 1871, entitled  
c. 98.

“An Act to amend the Vaccination Act of 1867.” In the meantime a Select Committee had been appointed to enquire into the working of the 1867 Act, and the Act of 1871 was introduced by its chairman. The principal alteration in the existing law effected by the Act was that, whereas under the principal Act the appointment of an officer to enforce its provisions was optional, it was now made obligatory.

Since 1871, the superintendence and direction of the work of vaccination has been carried on by the Local Government Board.

By Royal Warrant, bearing date 29 May 1889, a Royal Commission was appointed to enquire into the whole subject of vaccination, and its beneficial or injurious effect, and as to the advisability of making alterations in the then existing Legislature and its enforcement.

The enquiry of the Commission extended over several years. They held 136 meetings, and examined 187 witnesses, and caused several important investigations to be conducted for their assistance. Their final Report was dated August 1896, two of the Commissioners dissenting therefrom.

61 and 62 Vict. The Act that followed was confessedly of a somewhat  
c. 49.

tentative character, framed with the object of removing, as far as possible, the objections of those opposed to compulsory vaccination.

Its provisions include :—

1. The extension of the period for primary vaccination to six months from birth.
2. The exemption from the penalty for non-vaccination, if the parent, within four months of the birth of the child, satisfies two justices, or a stipendiary or metropolitan police magistrate, that he conscientiously believes that the vaccination would be prejudicial to the health of the child.
3. The limitation in case of non-compliance with the Act to one conviction and penalty for the same offence;

and the treatment of the person so convicted as a first-class misdemeanant.

4. An extension of the powers vested in the Local Government Board.

The passing of this Act closed, temporarily at all events, the period of compulsory vaccination which had existed since 1853.

Chronological  
Table of  
Vaccination  
Legislation  
in various  
Countries.

In the following statement is shown the general trend of legislation in this country consequent upon the abolition of inoculation in 1840, and of legislation in some of the Colonies, and in foreign countries:—



*Countries in which Vaccination is enforced by legislation.*

Name of Country	Voluntary gratuitous Vaccination introduced	Vaccination of Infants made compulsory [A]; efficiently carried out [B]; modified in [C]			Gratuitous Revaccination introduced	Revaccination made compulsory
England and Wales	1840	[A] 1853	[B] 1871	[C] 1898	1867	...
Scotland . . .	1848		[A] 1863		...	...
Ireland . . .	1840		[A] 1863		1878	...
Sweden . . .	...		[A] 1816		...	[Army] 1849
Prussia . . .	...	[Army only] 1834		[A] 1874	...	1874
Italy . . .	...		[A] 1888		...	[School age] 1888
Denmark . . .	...	[On admission to school and to confirmation]		[A] 1871	...	...
*						
The Netherlands .	...	[On admission to school]		1873	...	...
Roumania . . .	...		...		...	1893
Hungary. . .	...		...		...	1887
Württemberg. . .	...		1818		...	...
Schleswig & Holstein	...	[On admission to school and confirmation]		1811	...	...
Norway . . .	...		[A] 1811		...	...
Bavaria [first in Europe]	...		[A] 1807		...	...
Kurhessen [Hesse].	...		[A] 1815		...	...
Nassau . . .	...		[A] 1818		...	...
Hanover . . .	...		[A] 1821		...	...
Baden . . .	...		[A] 1815		...	...

\* NOTE.—For some of the dates in the second section of this table, I am indebted to Dr. Edwardes.

*Countries not enforcing Compulsory Vaccination by Legislation.*

Switzerland. Five Cantons in 1895 abolished compulsory vaccination, namely, Bern, Solothurn, Graubunden, Aargau, and Heuenberg. Five Cantons in 1883 abolished compulsory vaccination, namely, Zurich, Lucerne, Basle (City),\* 1882, St. Gallen, and Thurgau.

\* In the year 1885, the mortality from small-pox in Basle was 105·6 per 100,000 population.

Belgium.	No law yet.
Russia.	Orders exist, but no organized system.
Austria.	Vaccination and revaccination, 1886; compulsory in Army.
France.	Vaccination before admission to school, compulsory by regulations; thorough vaccination began in 1888.
Spain.	Orders exist, but no organized system.

*Notes on Colonial Legislation.*

**New South Wales.** Small-pox is practically unknown in New South Wales, there not having been any deaths from it in 1899, and only three during the last ten years. Vaccination is not compulsory, and is only resorted to in times of scare. It is easy to discover from the returns of the Government vaccinators the years when the community was threatened by the disease, as at such times the number of persons submitting themselves to vaccination largely increased.

**Victoria.** Parents are required by law to have their children vaccinated. Only 26 deaths from small-pox have occurred since 1853, and none in the last 13 years.

**Queensland.** Vaccination is regulated by Public Health Act, 1900, which provides that the Commissioner, with the approval of the Governor, may appoint a place or places for the performance of vaccination in any area in which the provisions of the Act are in force.

In any such area, compulsory vaccination within six months of birth is enforced under a penalty not exceeding £5, unless the parent makes a statutory declaration of conscientious objection.

**Western Australia.** Vaccination is compulsory. Small-pox is practically unknown in the State, only one outbreak having recently occurred, and that was rapidly stamped out.

**Tasmania.** Vaccination Act, 1898:—Every medical practitioner shall, unless he objects, be appointed a vaccinator, who shall vaccinate gratuitously all children and others applying to him.

Every child to be vaccinated within 12 months of birth. In default, parent liable to penalty not exceeding £5 unless he forwards to Registrar a statutory declaration, made before a Justice of the Peace, that he conscientiously believes that vaccination will be permanently detrimental to the health of the child.

Medical practitioners can obtain a supply of calf lymph on application to the Secretary of the Central Board of Health.

**New Zealand.** Vaccination is now regulated by Public Health Act, 1900, which provides for compulsory vaccination within one year of birth, under penalty of 40s. to parent, unless he can give a statutory declaration of conscientious objection.

**Canada.** Vaccination is dealt with by Provincial legislation in the Dominion, and, consequently, the Boards of Health in the different Provinces are responsible for the regulations appertaining to it.

**Cape of Good Hope.** Vaccination is regulated by Public Health Act, 1897, which requires vaccination to take place within three months of the birth of a child. No conscience clause.

**Natal.** Vaccination compulsory. No conscience clause.

Comparative  
influence of  
Small-pox on  
total mortality  
for London in  
the last two  
centuries.

Before attempting to analyze the influence of vaccination in this and other countries, it may be well to trace, in a general manner, the relative influence of small-pox on the rate of mortality.

In the absence of trustworthy data of populations, this can be best done by a comparison of the deaths from small-pox with those occurring from all causes. The best available source of information for this purpose is found in the Bills of Mortality for London.

While the lessening influence of small-pox on the general mortality is so marked as to be unmistakable, yet too much weight should not be attached to a comparison of this nature into which two variable factors enter, namely, the small-pox deaths, and the deaths from all causes.

The figures which form the basis of the curve were prepared by Dr. Guy down to 1881,\* and completed by the present writer from the Registrar-General's Annual Returns.

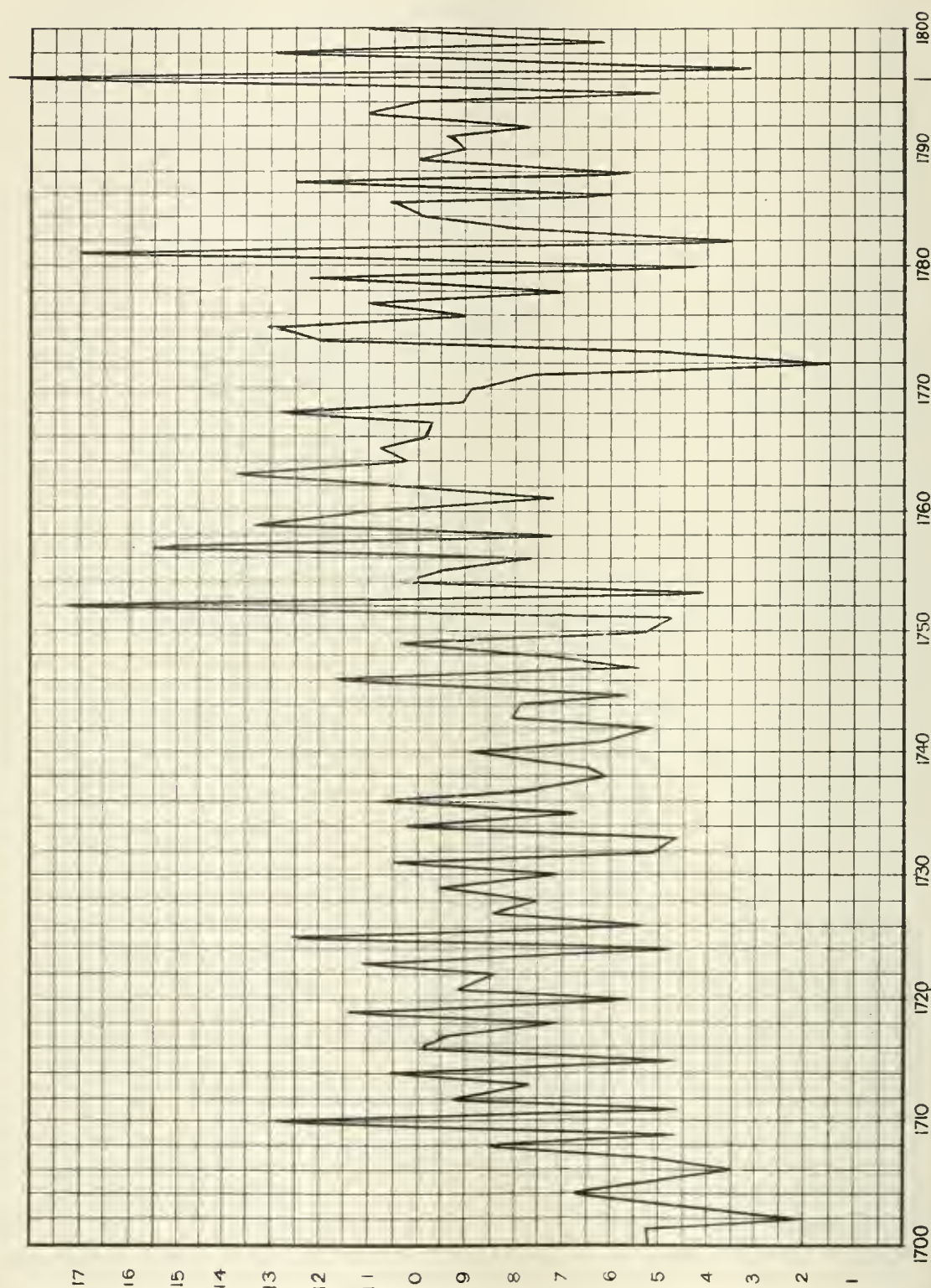
The diagrams were drawn by Mr. A. C. Thorne, F.I.A., who has also rendered much other valuable assistance in the preparation of this paper.

\* Two Hundred and Fifty Years of Small-pox in London. (*J.R.S.S.*, xlv, 399).

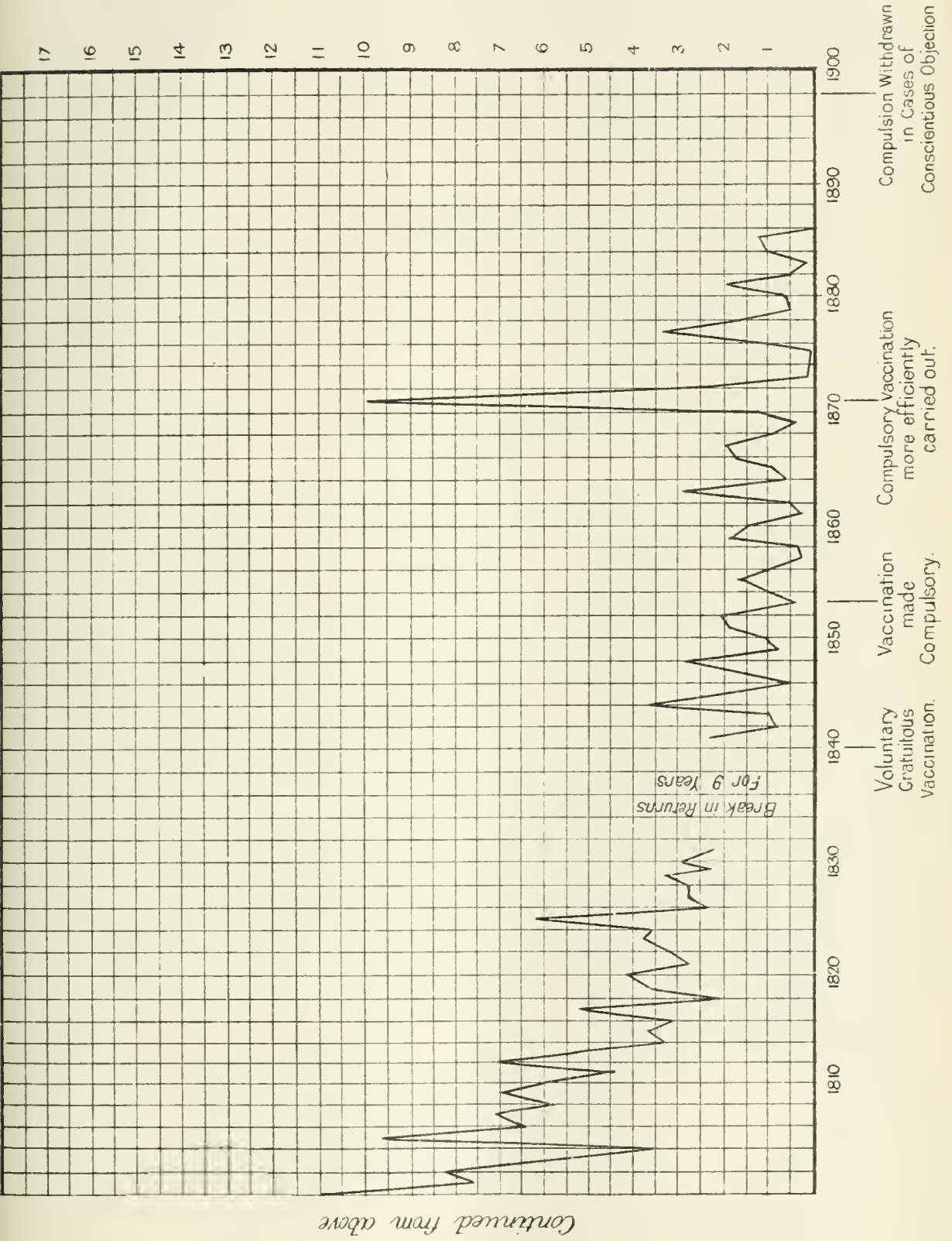


Diagram 1.

PERCENTAGE OF DEATHS FROM SMALLPOX TO DEATHS FROM ALL CAUSES IN LONDON FROM 1700 TO 1800.







Consideration of the curve shows, in a very marked way, the reduced weight of small-pox in the general mortality in the 19th, as contrasted with that in the 18th century.

The tendency of the curve is seen to be, on the whole, an upward one in the 18th century, reaching its maximum point in the year 1796, and then falling away rapidly. After 1805 no altitude at all approaching that normal in the 18th century occurred until the epidemic of 1871.

A well-marked feature is the sharp drop that would seem invariably to follow a year of epidemic.

From 1886 onwards the small-pox mortality in London was so slight as not to affect the total mortality, and the curve in the diagram, consequently, for these years becomes identical with the base line.

## II.—SMALL-POX STATISTICS AS INDICATING THE EFFECT OF VACCINATION.

Difficulty of  
obtaining  
conclusive  
Proof.

The character of the available statistics renders it difficult to show the precise effect of vaccination in individual cases, because we cannot divide the population into vaccinated and unvaccinated, nor can we trace the history, as regards vaccination, of persons attacked by or dying from small-pox. In the cases of death, even if it could be traced that vaccination had been performed, no proof is obtainable of the length of time that had elapsed. Information of this kind is only forthcoming in regard to isolated portions of the community, and, in order to obtain a broad view of the question, we must compare the small-pox mortality in different countries side by side with their vaccination laws.

Comparison of  
Small-pox  
Mortality in  
Countries  
having varying  
degree of  
Vaccination  
Legislation.

Such a comparison may be obtained by an inspection of the following diagrams, showing the numbers dying from small-pox year by year out of 100,000 living:—

COUNTRIES IN WHICH VACCINATION IS COMPULSORY.

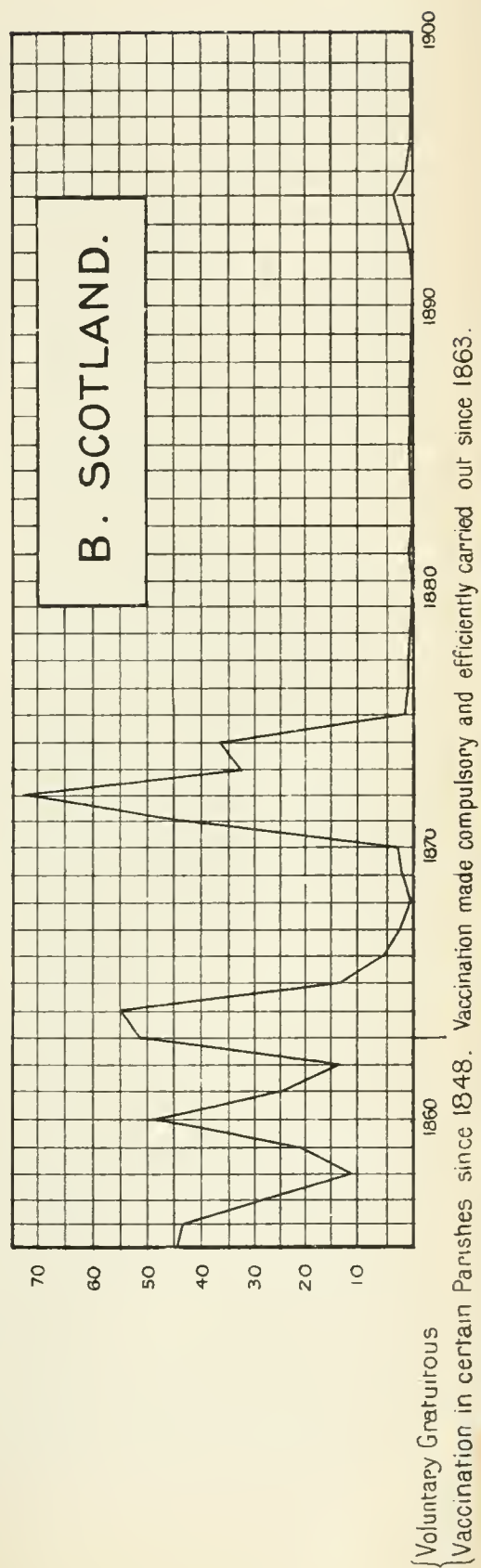
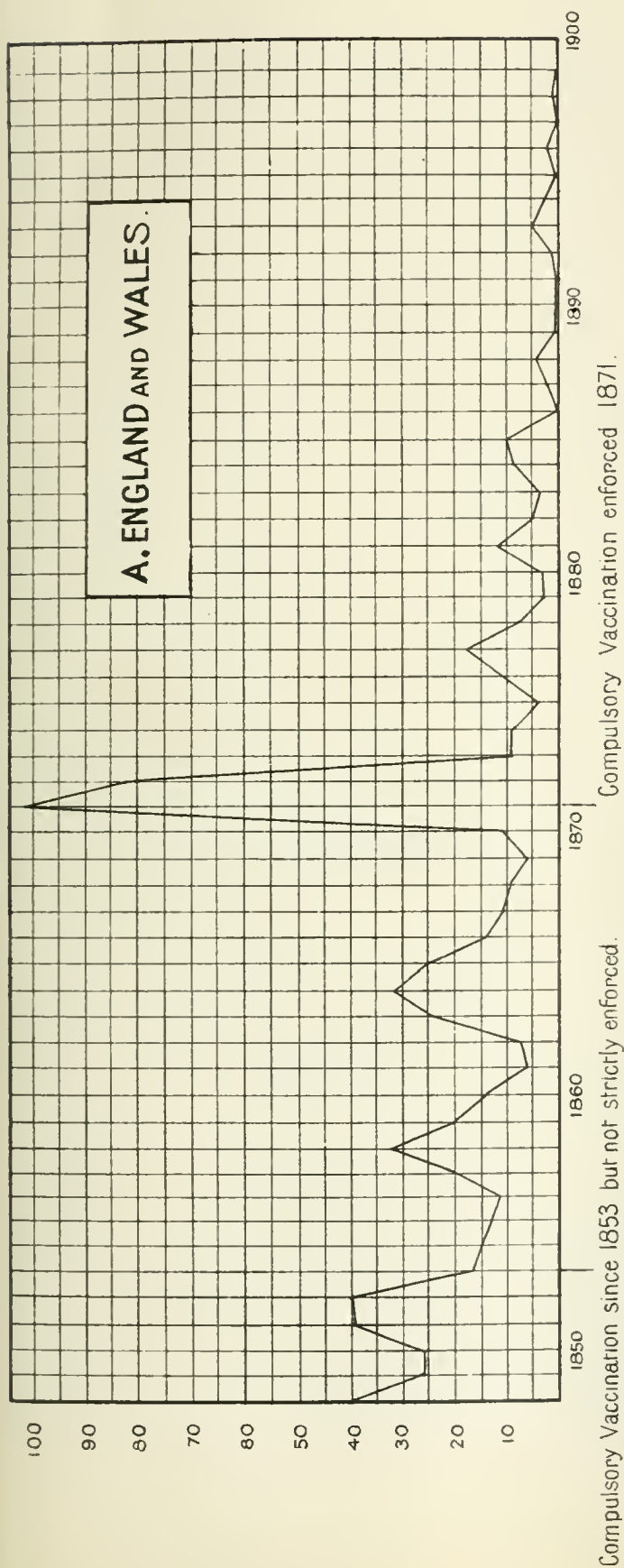
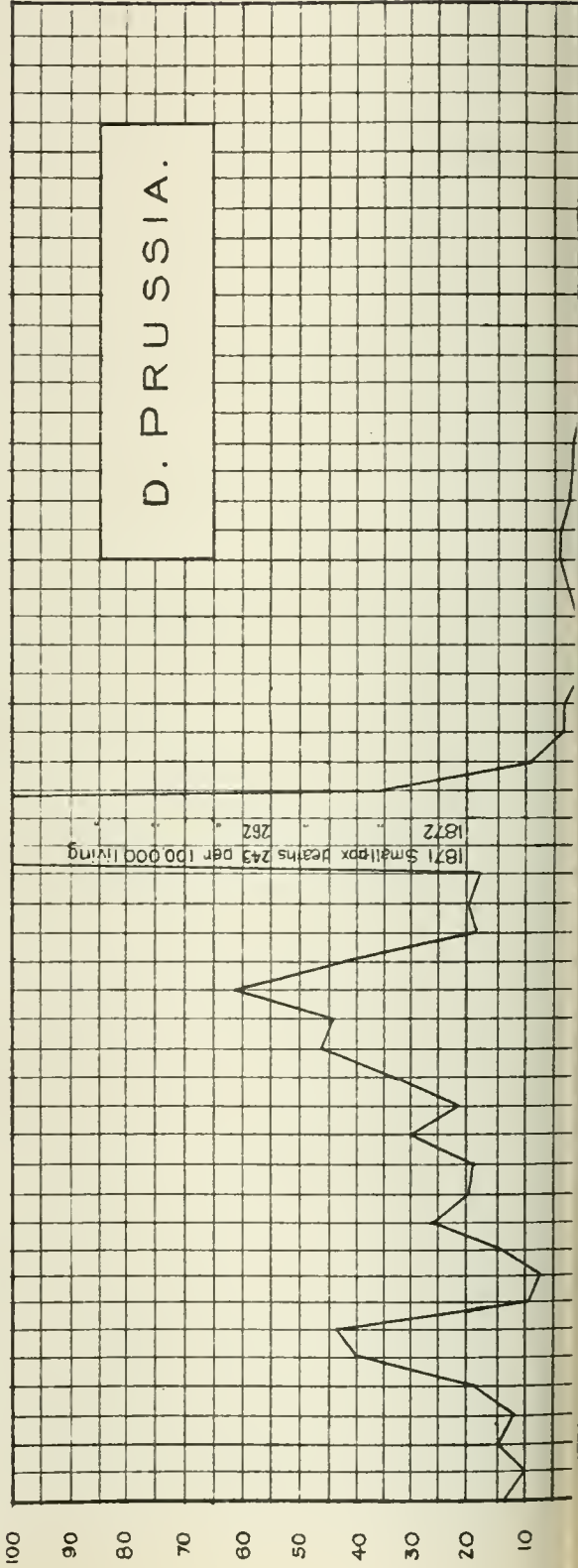
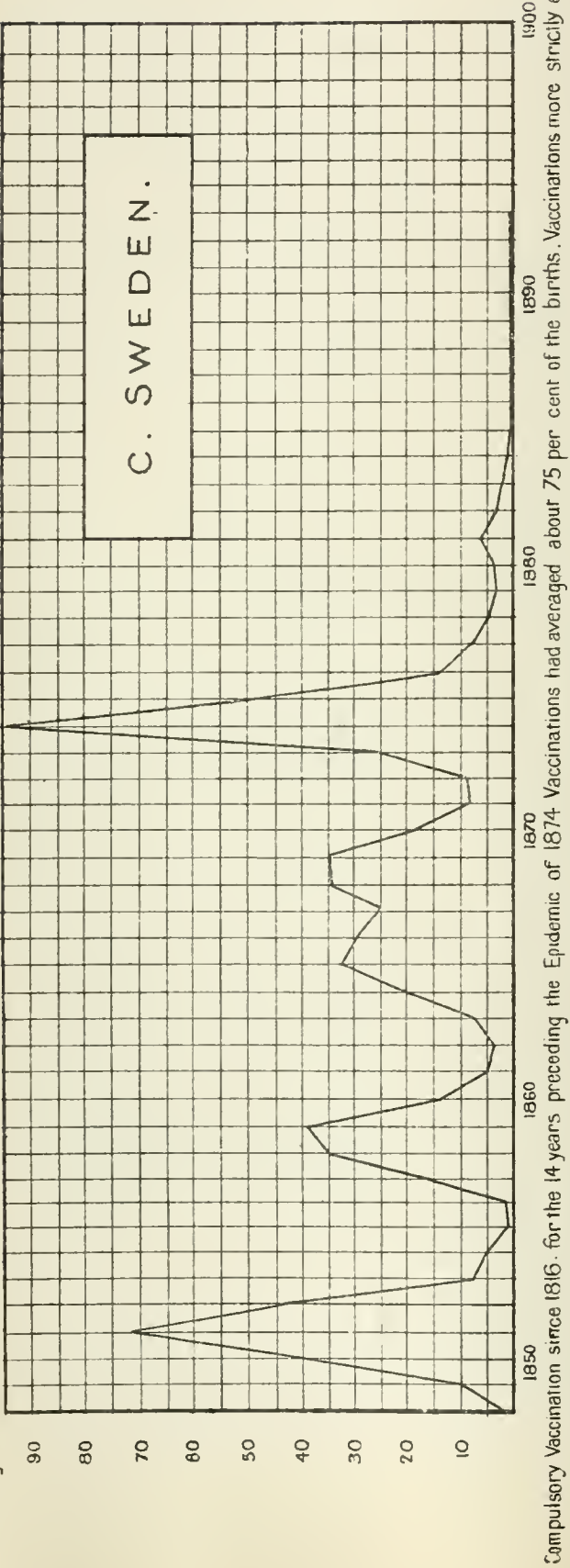
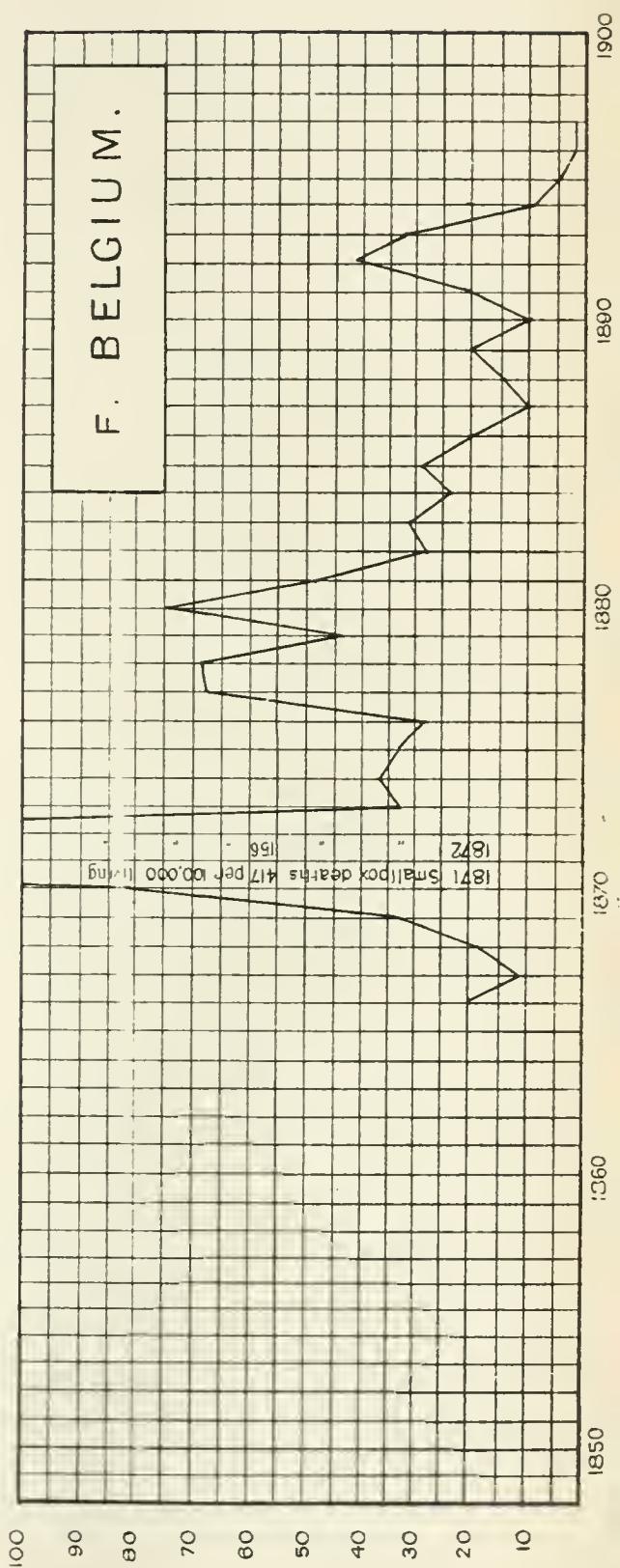
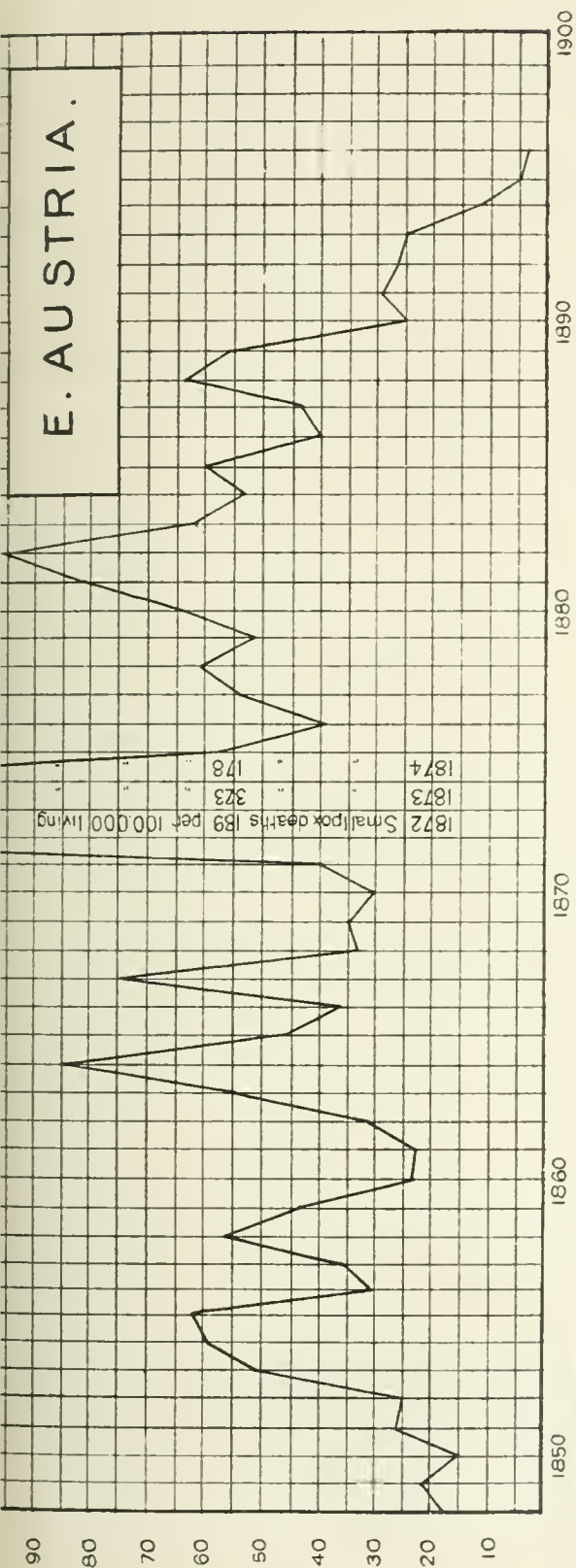


Diagram II continued







In the case of Prussia the percentages of those dying from small-pox since 1874 are so small that it becomes difficult to read them off from the diagram; the actual numbers per 100,000 living are, therefore, given below:—

*Prussia.*

Year	Number Dying out of 100,000 Living	Year	Number Dying out of 100,000 Living
1875	3·6	1887	0·5
1876	3·1	1888	0·3
1877	0·3	1889	0·5
1878	0·7	1890	0·1
1879	1·3	1891	0·1
1880	2·6	1892	0·3
1881	3·6	1893	0·4
1882	3·6	1894	0·3
1883	2·0	1895	0·08
1884	1·4	1896	0·02
1885	1·4	1897	0·02
1886	0·5	1898	0·04

The first four diagrams relate to countries in which compulsory vaccination has been enforced, namely, England and Wales, Scotland, Sweden, and Prussia. There is a certain similarity in these four curves which show:—

1st.—A relatively high but fluctuating mortality up to 1870.

2ndly.—A sharp rise during the years immediately succeeding, caused by the epidemic which spread over Europe, with the result, already commented on, of a fall to a rate below any previous normal one; and

3rdly.—A mortality rate for the last quarter of a century which, in the light of previous records, may be regarded as insignificant, and which has in fact, in the cases of Scotland, Sweden, and Prussia, become practically extinct.

Austria and Belgium are instances of countries in which compulsory vaccination is not enforced by legislation, and the diagrams (E and F) are instructive. Contrasting Austria (E) with Prussia (D), somewhat similar features are observable up to 1874, the curve for Austria being on the whole slightly the higher of the two. After 1874, however, the curves become very divergent; instead of the practical disappearance shown by the

curve for Prussia, Austrian mortality soon re-assumes its former influence.

The curve for Austria (E) again reaches a high point in 1882, and it is not until 1894 that the mortality falls below 25 per 100,000 living. Since 1891 the vaccination of children has been more sharply supervised, and, by the help of a control (inspection) of the school children, made fairly general. (*Ziemlich allgemein durchgefuhrt worden.*)\*

We have hitherto dwelt upon the mortality rates of general populations at aggregated ages in various countries, and shown that the rates among those populations have varied directly according to the stringency of vaccination legislation.

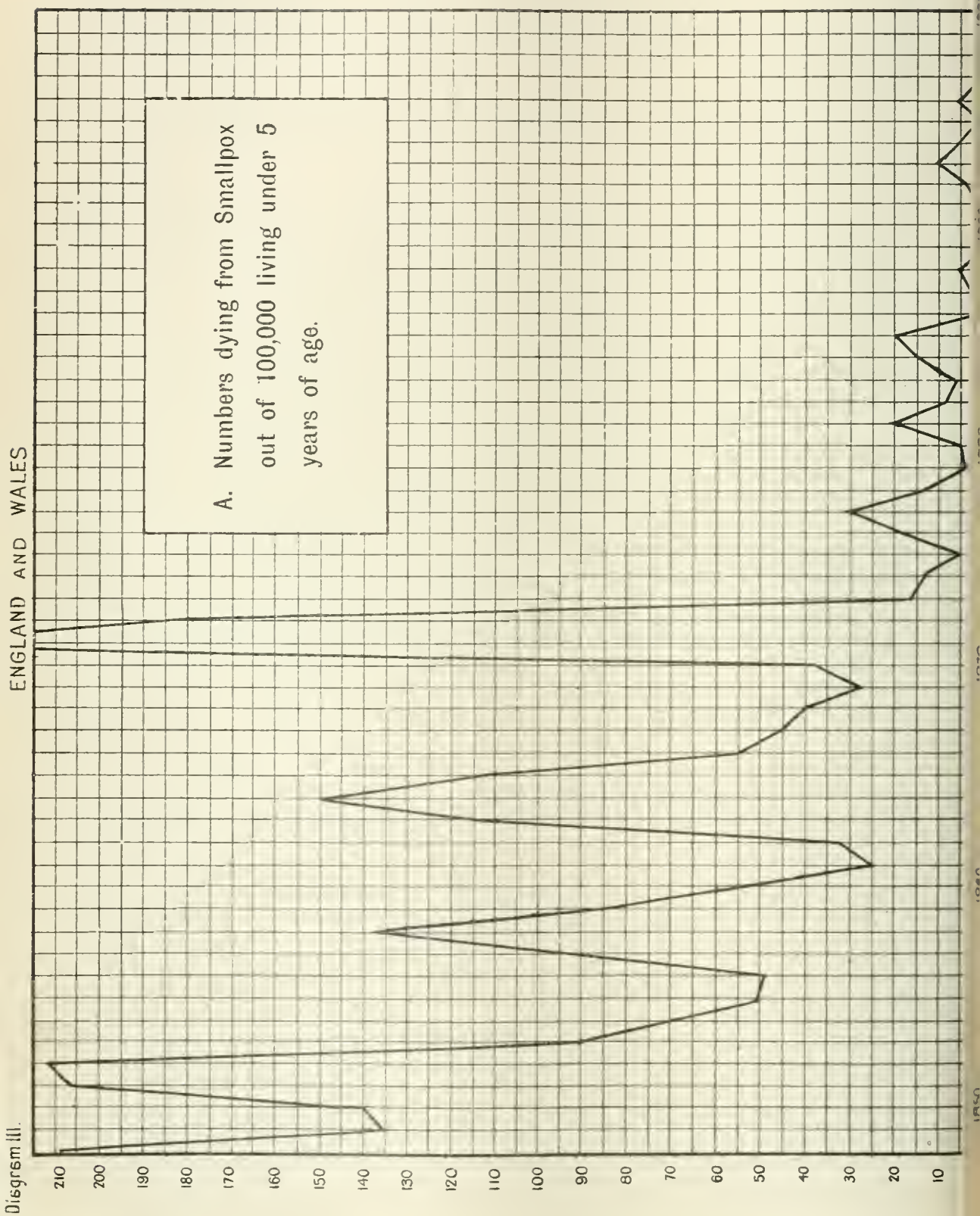
Analysis  
according to  
Age-incidence. The foregoing method, however, by no means exhausts the deductions from the figures, and a different point of view is obtained by an examination of the age-incidence of the mortality.

This has been done for England and Wales by Dr. Newsholme from the figures in the final Report of the Royal Commission; but the following diagrams have been independently drawn and brought up to date from the Registrar-General's Returns.

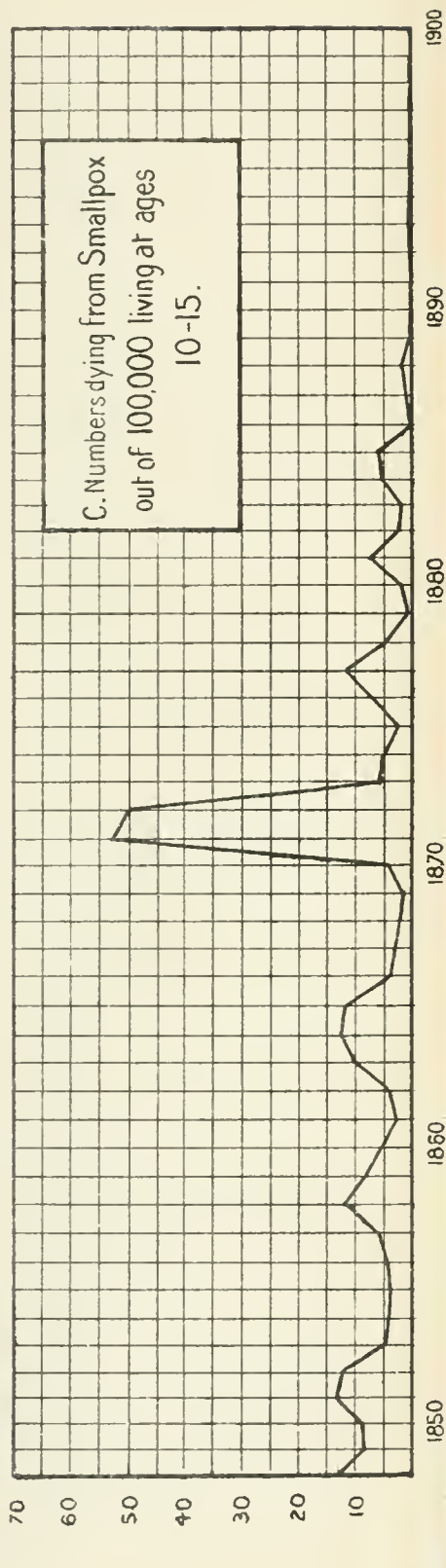
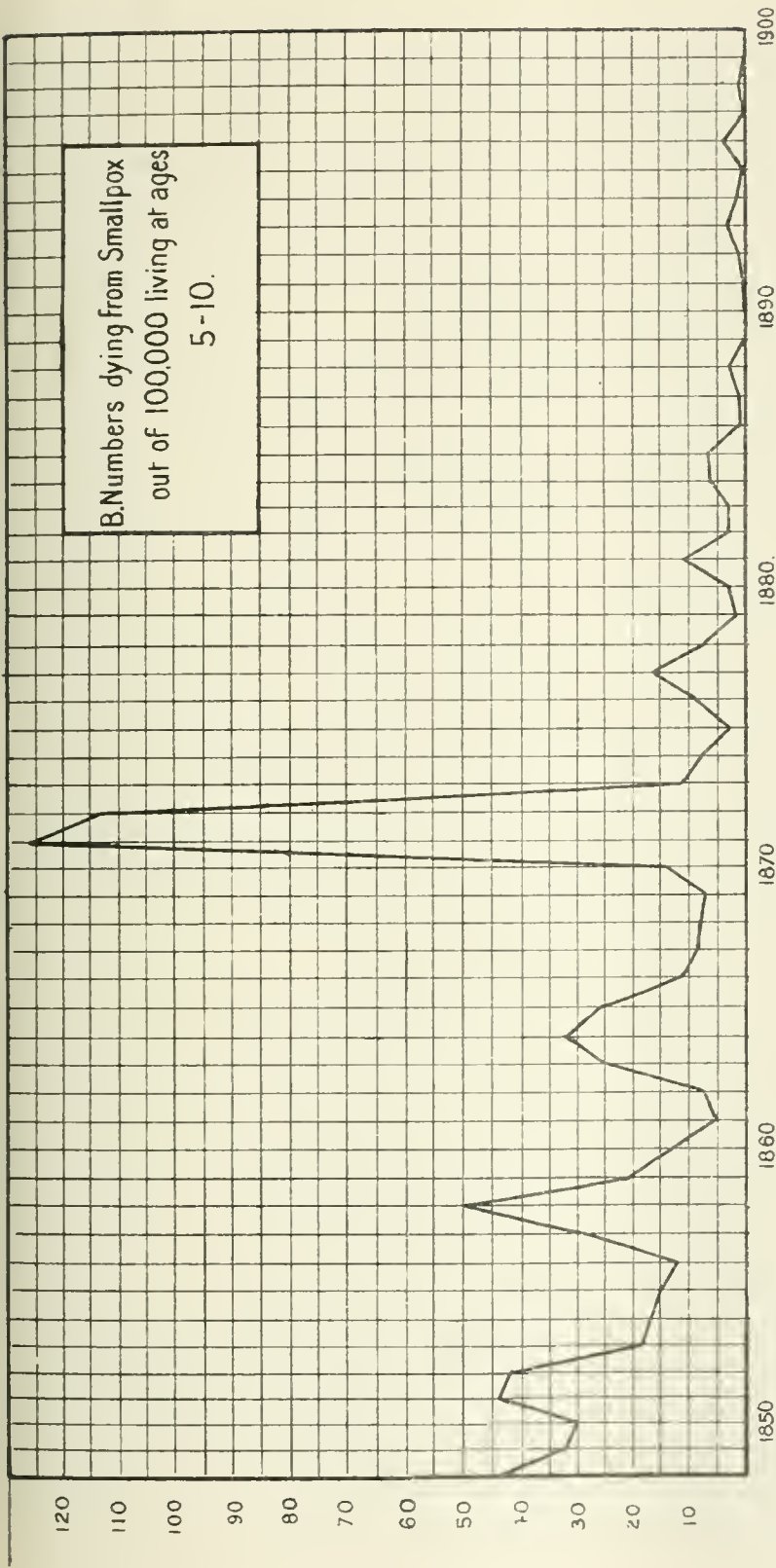
The diagrams are an analysis of the mortality from small-pox as affecting the population of England and Wales, divided into age-groups.

\* Dr. Kubler's Geschichte. Der Pocken und der Impfung.









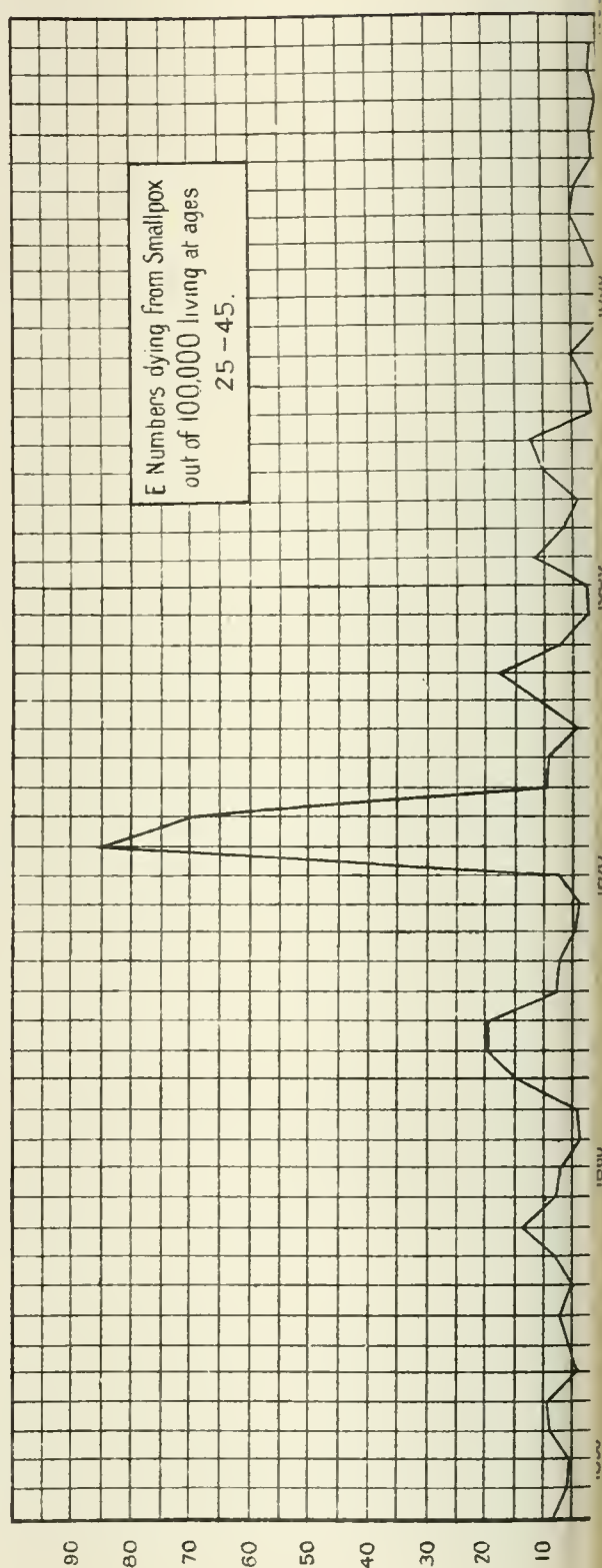
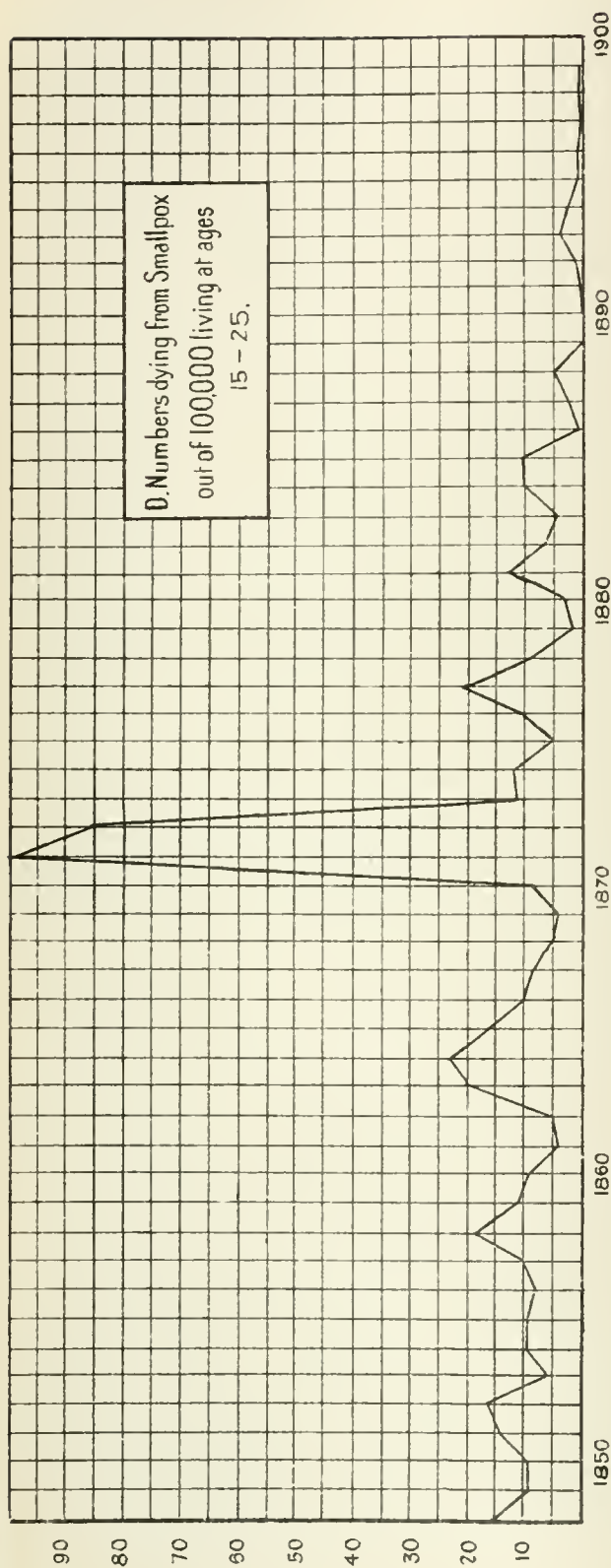
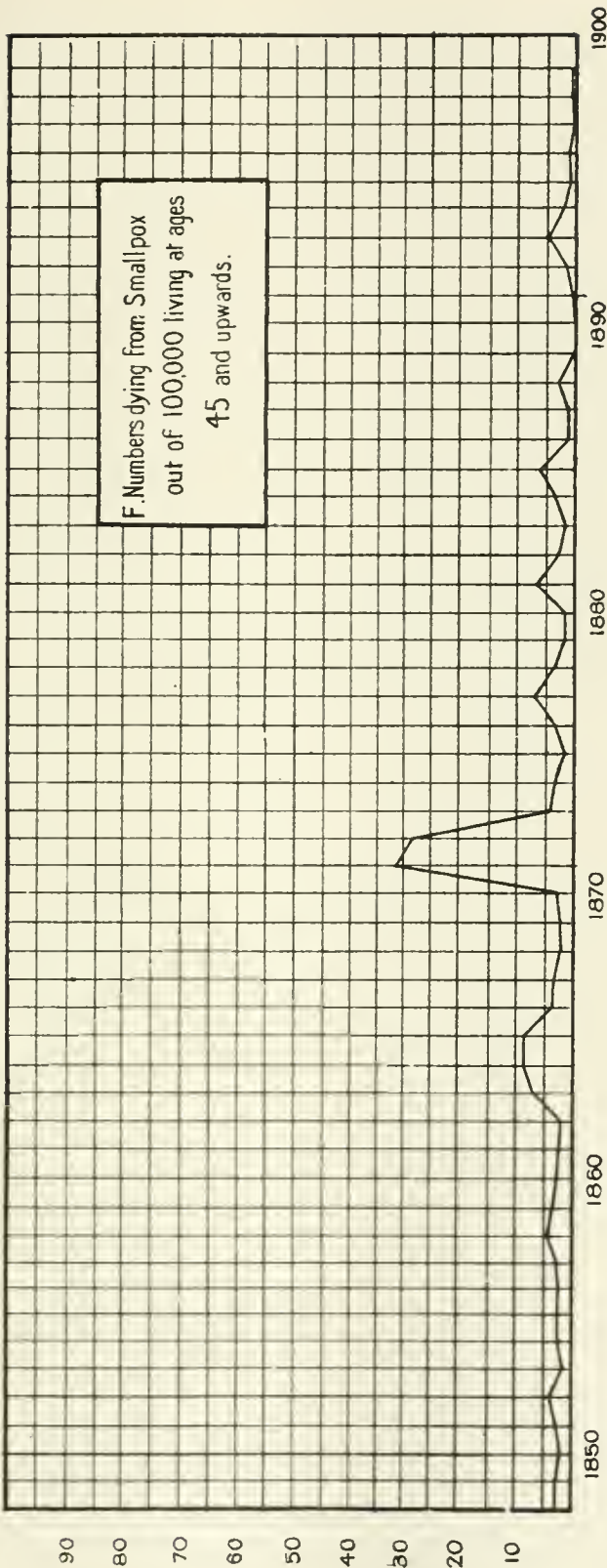


Diagram III. continued.



The curve for the period 0-5 years reproduces, in a more pronounced manner, the downward inclination already commented on, although the regularity of the curve is interrupted in this and all other age-periods by the epidemic of 1870-71.

As we pass along the periods of life, the decline in the curve loses its acuteness, until, for the oldest age-group, the tendency of the mortality is to produce a curve parallel to the base line.

It would, therefore, appear that the immunity from small-pox has mainly affected children, the intensity of such relief varying inversely to the age.\*

Improved  
sanitation  
does not alone  
account for  
immunity from  
Small-pox.

It has been suggested that improved sanitation has accounted for the diminution in small-pox mortality; but, if this were the case, we should expect to find an equal improvement in the death rates from other febrile diseases, *e.g.*, measles, diphtheria, and scarlet fever.

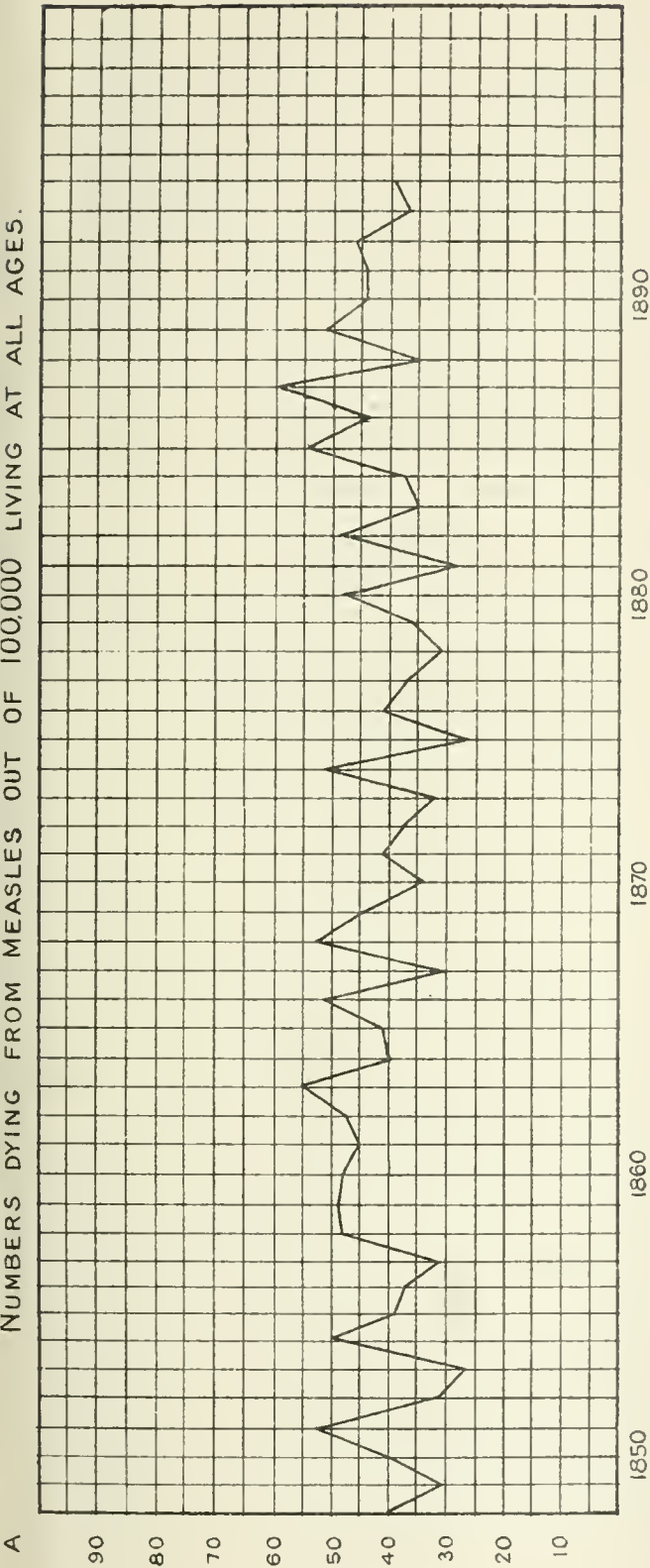
Trend of  
Mortality from  
other Zymotic  
Diseases.

The following curves show the course of these three diseases, which should be compared with the diagram on page 259 for small-pox.

\* This analysis bears out the opinion expressed by the late Sir George Buchanan, as long ago as 1884, that vaccination has only a protective influence for a limited time, and that its general effect has been to reduce the number of cases of small-pox in early life.



NUMBERS DYING FROM MEASLES OUT OF 100,000 LIVING AT ALL AGES.



NUMBERS DYING FROM DIPHTHERIA OUT OF 100,000 LIVING AT ALL AGES.

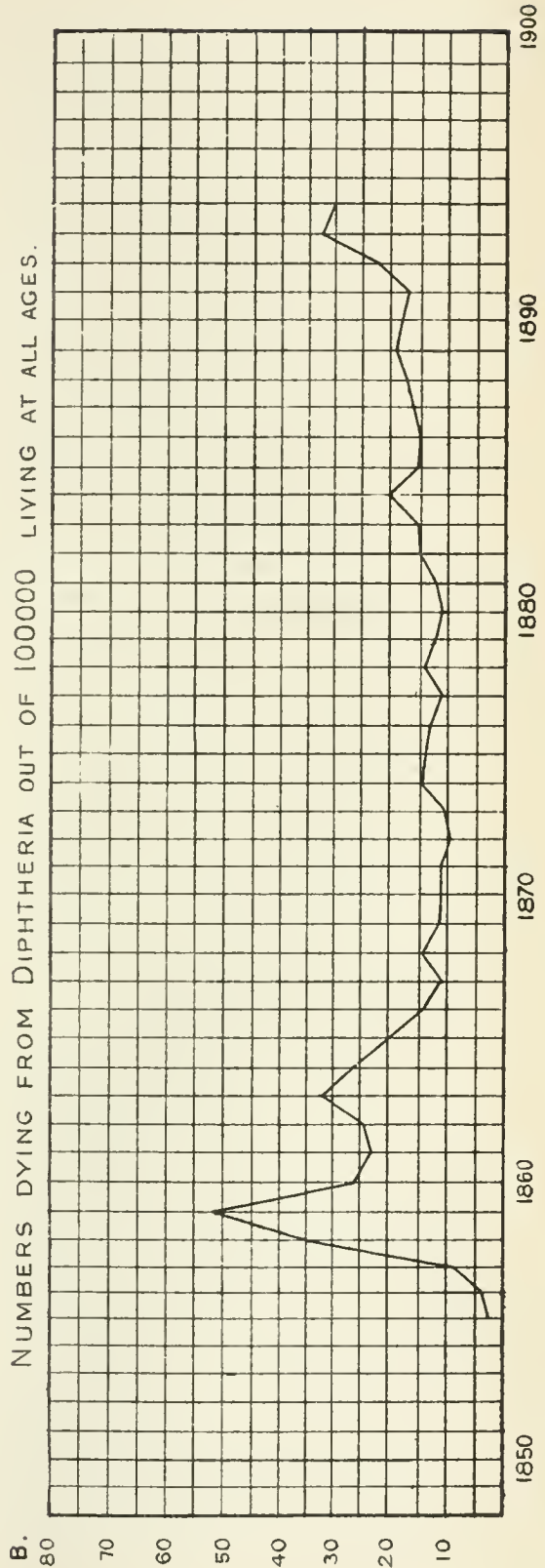
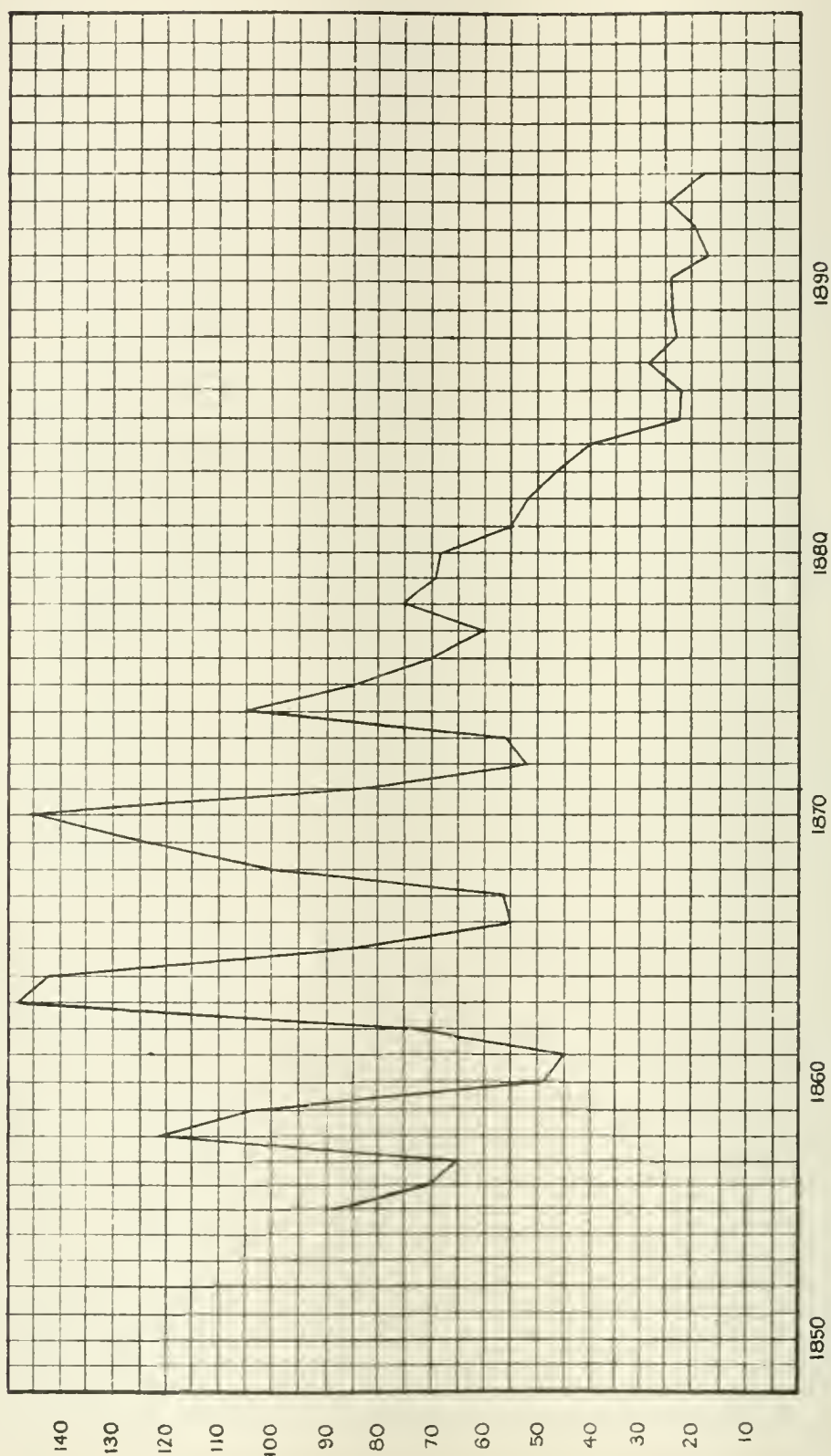


Diagram IV.

## ENGLAND AND WALES.

C. NUMBERS DYING FROM SCARLET FEVER OUT OF 100,000 LIVING AT ALL AGES.



The figures on which these curves are based are given in the final Report.

This comparison would show that there must be some other cause, in addition to sanitation, which has favourably affected small-pox mortality.

Summary of  
conclusions to  
be derived from  
foregoing  
analyses.

The conclusions arrived at from the investigation up to the present point may be summarized as follows:—

1. That the mortality from small-pox in the various countries under review would seem to have followed the vaccination legislation, and that a marked difference appears since the epidemic of 1870-74 as between those countries which enforce and those which do not enforce vaccination; and, further, that in the only example in which re-vaccination is compulsory, namely, Prussia, small-pox has been practically eradicated.
2. As regards England and Wales: The last 50 years may be divided into two periods, namely, the period 1848-70, during which vaccination was compulsory, though not strictly enforced; and the period 1871 onwards, in which vaccination was strictly enforced. The mortality in the first of these periods fluctuated considerably, but showed on the whole a slightly downward inclination. The second period opened with an epidemic, and the figures for 1871-72 were abnormal. The mortality in the following year fell to a low point, thereafter fluctuated within much narrower limits, and remained as a factor of relatively little weight.
3. That the effect of the decline in mortality has principally affected children under 10, the influence being less and less felt as the higher age-groups are reached, until, for ages above 45, the mortality from small-pox remains at the present day as heavy as it was 50 years ago.
4. That sanitary legislation alone does not sufficiently account for the decrease in small-pox mortality.

The next step in the investigation has been an examination into the statistics of special communities and classes in which the vaccination condition is ascertainable.

*Sheffield Epidemic.*

Of all the epidemics of small-pox, that of Sheffield provides the best means of determining directly the influence exercised by vaccination upon the extent and intensity of the disease ; because, for that borough, we have, for the period under review, not only the attacks and mortality among the vaccinated and unvaccinated, respectively, divided into quinquennial age-groups, but, also,—the quite as important factor—the corresponding numbers, vaccinated and unvaccinated, of the population exposed to risk in those age-groups.

To attain this end a “vaccination census” was taken, under the superintendence of Dr. Barry, the Medical Departmental Inspector of the Local Government Board, that is to say, a house-to-house visitation of the borough was made, and particulars obtained for each inmate in the following form:—

Name	Age	Vaccinated	When	Revaccinated	When	Had Small-pox	When

By this means Dr. Barry built up the figures of vaccinated and unvaccinated populations as given in columns (2) and (3) of the following table. In the examination of these results by the Royal Commission, however, it was pointed out that as the Census was taken *during* the Epidemic, a certain number of persons previously unvaccinated would have been vaccinated, and included, therefore, as such in the returns, unduly reducing the unvaccinated “exposed to risk.”

Subsequently, therefore, Dr. Barry made the following analysis of the returns of certain streets, with a view to determining the extent to which the true incidence of the exposures had been disturbed, as between the two classes, by the method adopted in making up the returns.



Name of Street	Number of persons returned as vaccinated	Number of persons returned as vaccinated, who had been first vaccinated during the Epidemic
Court 7, Smithfield . . . . .	17	0
Court 11, Allen Street . . . . .	44	0
Hollis Croft . . . . .	346	4
Collier's Row . . . . .	142	0
Court 1, Rockingham Street . . . . .	31	0
Newcastle Street . . . . .	206	3
Court 11, Bailey Street. . . . .	30	0
Court 1, Bailey Lane . . . . .	68	1
Sheldon Street . . . . .	436	9
	1,320	17

The original returns for the above streets gave 1,320 vaccinated, and 44 unvaccinated; the following analysis shows that a more correct statement of the respective exposures would have been 1,303 vaccinated, and 61 unvaccinated. In other words, the "unvaccinated" should have been larger by 38·6 per-cent (not 28 per-cent, as stated by Dr. Barry, and adopted as a correction by the Royal Commission). In column (5) of the following table, Dr. Barry's "unvaccinated population" (column 3) has been increased by 38·6 per-cent, the numbers corresponding to the increase being transferred out of the "vaccinated population" (column 2); the adjusted vaccinated population appears in column (4). The remaining columns explain themselves.

Age Periods	Population Enumerated		Population adjusted for Vaccinations during the Epidemic		Cases of Small-pox		Deaths from Small-pox		Attacks per 100,000 Living		Deaths per 100,000 Living	
	Vaccinated	Unvaccinated	Vaccinated	Unvaccinated	Vaccinated	Unvaccinated	Vaccinated	Unvaccinated	Vaccinated	Unvaccinated	Vaccinated	Unvaccinated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Under 5 years	33,393	1,981	32,628	2,746	121	128	1	66	371	4,661	3·1	2404·5
5 and under 10	34,843	278	34,736	385	232	100	5	34	669	25,974	14·4	8831·2
10 „ 15	32,965	235	32,874	326	629	91	11	32	1,913	27,914	33·5	9816·0
15 „ 20	27,111	282	27,002	391	979	84	19	53	3,626	21,514	70·4	13555·0
20 „ 25	23,795	416	23,634	577	765	76	36	45	3,240	13,173	152·4	7799·0
25 „ 30	20,993	468	20,812	649	502	22	33	16	2,412	3,390	158·6	2467·5
30 „ 35	19,475	319	19,352	442	350	18	28	11	1,810	4,072	144·7	2488·7
35 „ 40	16,626	338	16,495	469	200	13	26	10	1,212	2,772	157·6	2132·5
40 „ 45	15,001	295	14,887	409	153	7	16	3	1,028	1,712	107·5	733·5
45 „ 50	12,017	239	11,925	331	86	7	5	3	721	2,116	42·0	906·5
50 „ 55	9,903	251	9,806	348	58	1	11	...	591	287	112·2	...
55 „ 60	6,432	190	6,359	263	23	1	...	...	362	380	...	...
60 and upwards	12,587	396	12,434	549	29	2	9	1	233	364	72·5	182·1
Age unknown	3,256	27	3,246	37	24	2	...	...	739	...	61·6	...
All Ages	268,397	5,717	266,190	7,922	4,151	552	200	274	1,559	6,968	75·1	3478·7

[Upper Curve: Rate of Attack from Smallpox among the Unvaccinated per 100,000 living at each Age.

A { Shaded Portion " " Mortality

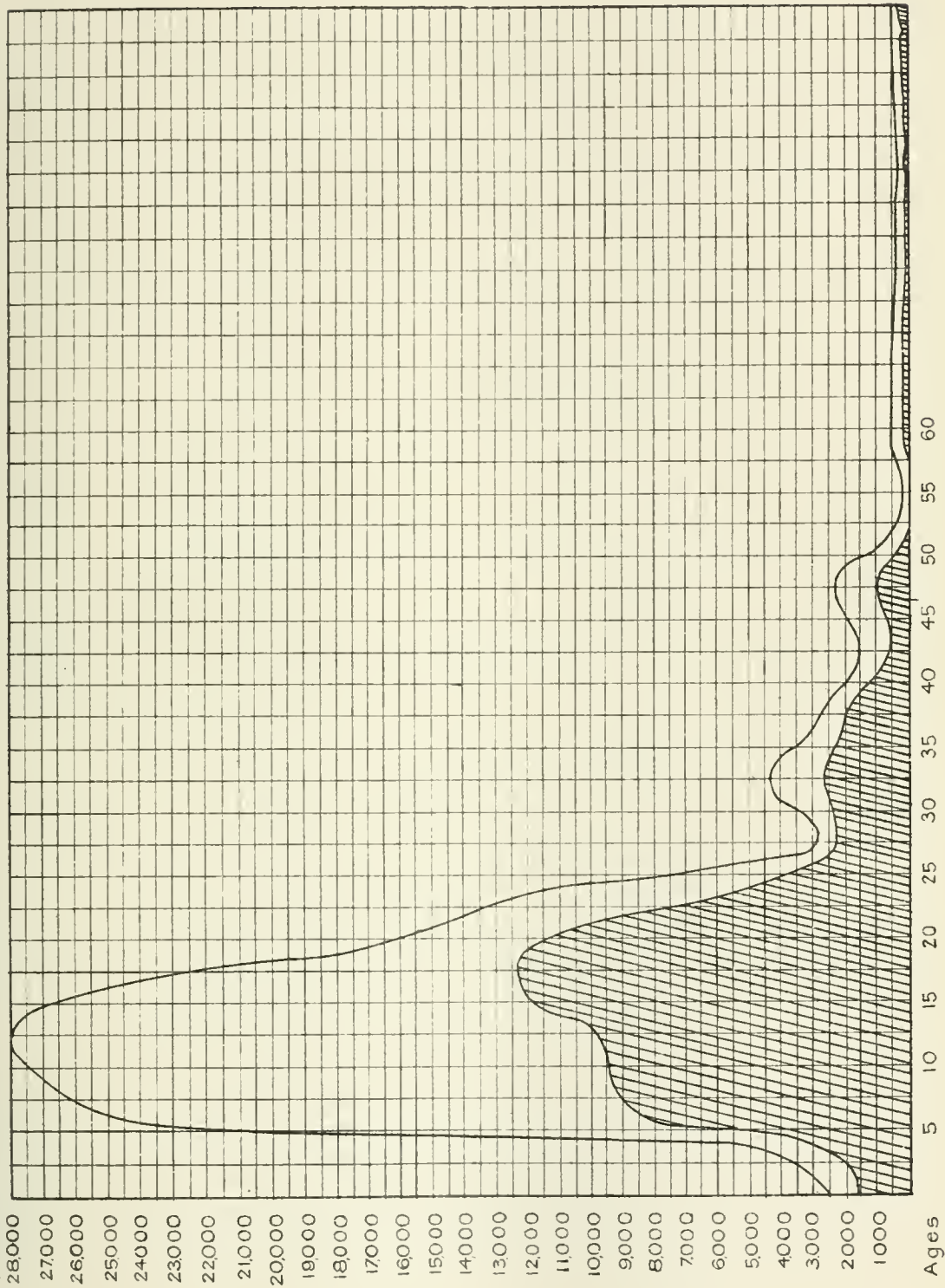
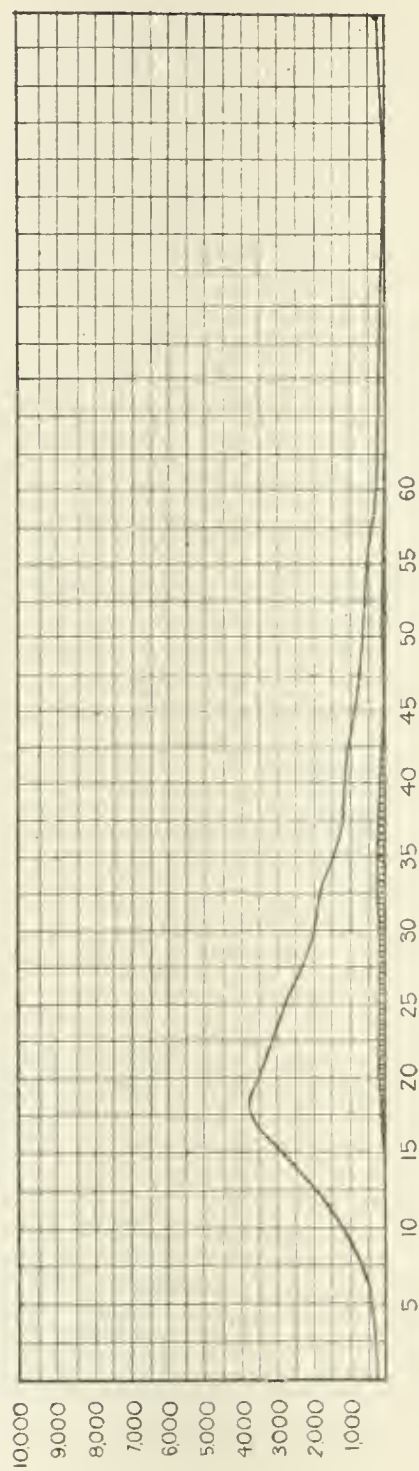


Diagram V continued

## SMALLPOX EPIDEMIC AT SHEFFIELD 1887-1888.

B { Upper Curve: Rate of Attack from Smallpox among the Vaccinated per 100,000, living at each Age.  
 Shaded Portion " Mortality





The most striking features that are brought out by these figures, and by the curves that illustrate them, are:—

1. The very large immunity from attack enjoyed by the vaccinated class, as compared with the unvaccinated.
2. The gradual approximation of the rates among the two classes as the groups at the higher ages are reached.
3. The very much smaller percentage of fatal attacks among the vaccinated classes who had contracted the disease than among the unvaccinated.

The following percentages may help to bring out the first two points:—

Ages	Percentages borne by attack rate among Vaccinated to that among Unvaccinated
Under 10 . . .	3·4
10-20 . . .	11·2
20-30 . . .	34·1
30-40 . . .	44·2
40 and upwards . . .	60·4

while the third feature is brought into prominence by the following comparison:—

Ages	PERCENTAGE OF DEATHS FROM SMALL-POX TO NUMBER OF ATTACKS.	
	Vaccinated	Unvaccinated
Under 10 . . .	1·7	36·7
10-20 . . .	1·9	47·3
20-30 . . .	5·5	70·0
30-40 . . .	10·0	67·5
40 and upwards . . .	11·4	37·5

Further  
illustrations  
drawn from  
Epidemics.

The evidence given by the statistics of those brought under observation only by the fact that they have been attacked by the disease will not alone be entirely satisfactory. The cumulative evidence, however, of records such as the following must carry some weight:

\* In Bavaria, in the epidemic of 1871, of 28,081 persons

\* Klinger: The Small-pox epidemic of the year 1871, and Vaccination in Bavaria, Nuremberg, 1873.

attacked by small-pox (of whom 906 were under one year of age, 4,128 were aged between 1 and 20, and 23,047 were aged more than 20), 1,251 were known to be unvaccinated, and the rest had been vaccinated.

Of the 26,830 vaccinated, 3,619, or 13·5 per-cent, died.

Of the 1,251 unvaccinated, 752, or 60·1 per-cent, died.

In the Small-pox Hospitals of Berlin, Munster, and Posen, of those attacked by small-pox the percentage of deaths was five times as great among the unvaccinated as among the vaccinated.

\* In the district of Oppeln (Silesia), in 1871-1875, of 28,163 vaccinated persons attacked, the deaths were 3,391, or 12 per-cent; of 2,862 unvaccinated persons attacked, the deaths were 1,329, or 46·4 per-cent.

### *Experience of Armies.*

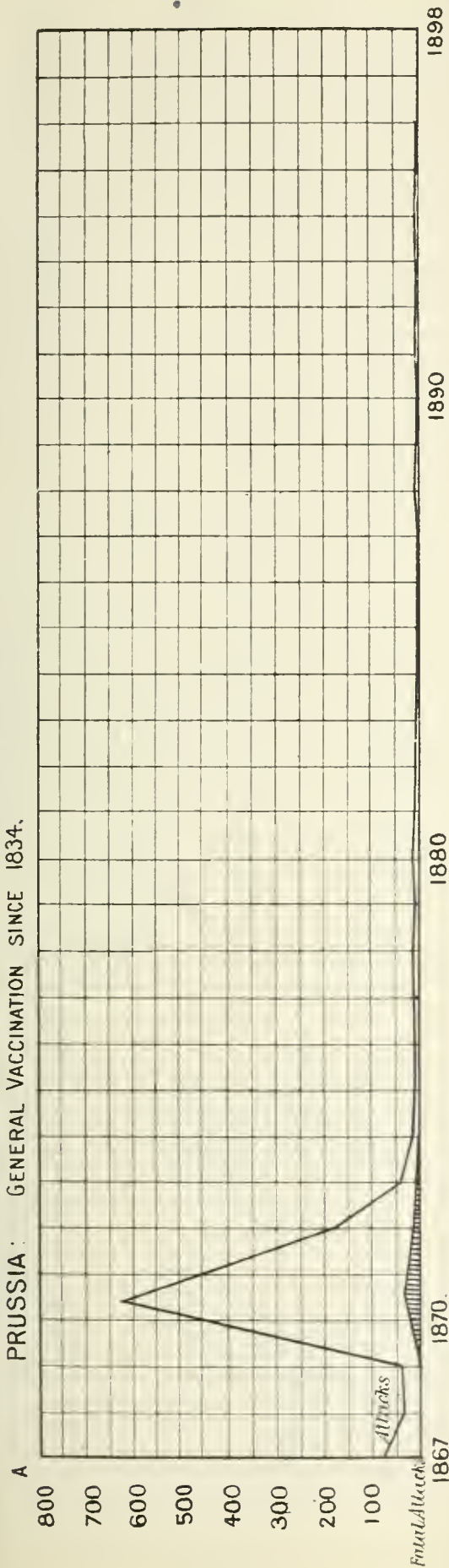
The experience of armies forms the largest available material to be derived from communities of which the individuals are under constant supervision, and whose vaccination condition is a more or less known factor. The armies of Prussia, France, and Austria offer, as communities in which the degree of vaccination has been very different, suitable ground of comparison, both as between one another and at different periods of their own history. In Prussia, general vaccination on entering the Army has been strictly enforced since 1834; in France, although repeatedly ordered, it has only been thoroughly carried out since 1888, and in the Austrian Army there was no vaccination necessary, prior to 1886.

The curves representing the attack and mortality ratios are given in the following diagrams; as, however, in the case of Prussia since 1873, and France and Austria from much more recent dates, the curves approximate closely to the base line,† the actual numerical ratios may, perhaps, provide a better means of comparison, and they are given, therefore, in the subjoined table:—

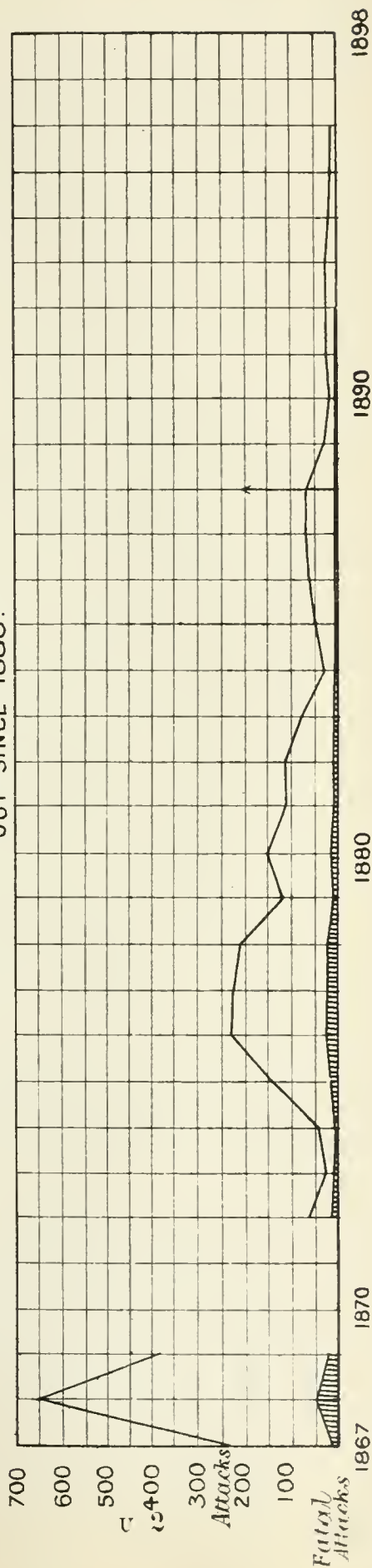
\* Pistor: Report on Public Health in Oppeln, for the years 1871-1875. Vaccination in Oppeln, 1876.

† The mortality curve for the Prussian Army, with the exception of the year 1884, in which there was one death from small-pox, has been identical with the base line since 1873.

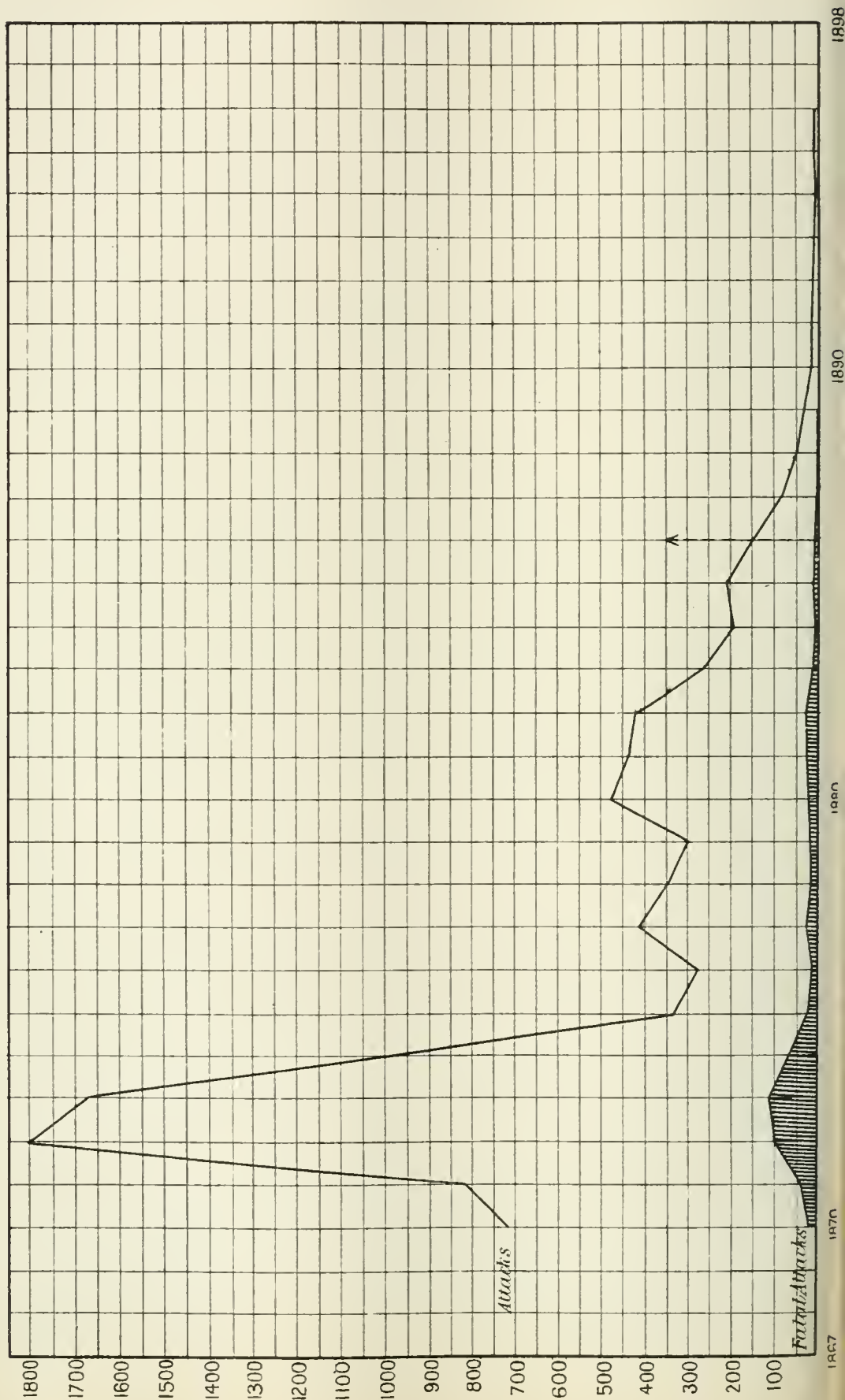
ATTACKS AND DEATHS FROM SMALLPOX IN DIFFERENT ARMIES  
PER 100,000 MEN  
GENERAL VACCINATION SINCE 1834.



**B. FRANCE :** GENERAL VACCINATION REPEATEDLY ORDERED, BUT ONLY THOROUGHLY CARRIED OUT SINCE 1888.



C. AUSTRIA: NO VACCINATION IN EARLIER YEARS, GENERAL VACCINATION SINCE 1886.





*Numbers Attacked by and Dying from Small-pox out of  
100,000 Men.*

Year	PRUSSIAN ARMY		FRENCH ARMY		AUSTRIAN ARMY	
	Attacks	Fatal Attacks	Attacks	Fatal Attacks	Attacks	Fatal Attacks
1867	74.2	0.8	231.1	18.2	...	...
1868	38.7	0.4	650.2	42.8	...	...
1869	43.4	0.4	382.8	22.8	...	...
1870	1260.3	57.8	...	...	718.3	18.5
1871			...	...	815.8	40.1
1872	175.0	5.4	60.0	10.7	1798.4	101.4
1873	41.6	3.0	27.5	4.0	1657.5	108.9
1874	9.6	.0	39.7	3.3	1002.8	66.9
1875	7.0	.0	141.8	17.8	336.5	21.5
1876	6.6	.0	230.5	28.2	274.7	10.4
1877	4.1	.0	222.2	19.6	412.0	25.5
1878	5.6	.0	213.1	20.1	344.0	15.4
1879	2.6	.0	115.6	8.9	303.8	22.7
1880	8.5	.0	153.6	14.9	475.3	25.2
1881	5.5	.0	111.2	7.9	434.2	29.1
1882	6.0	.0	118.8	9.1	423.0	27.7
1883	2.6	.0	81.4	3.3	261.9	16.3
1884	1.8	0.3	36.4	3.3	194.6	7.7
1885	1.8	.0	47.4	1.3	210.6	12.5
1886	1.8	.0	61.1	3.6	140.1	7.9
1887	1.2	.0	68.0	3.9	79.3	3.0
1888	5.2	.0	68.0	2.8	50.4	1.8
1889	1.7	.0	36.2	3.8	34.8	2.1
1890	1.4	.0	18.8	0.8	17.3	0.4
1891	6.2	.0	20.1	0.6	17.9	0.7
1892	5.0	.0	22.3	0.2	15.9	0.0
1893	2.4	.0	25.1	0.8	11.4	0.3
1894	5.3	.0	17.8	1.3	10.1	0.3
1895	2.7	.0	11.2	1.1	10.9	0.7
1896	2.5	.0	9.9	0.5	5.9	0.3

Leaving out of account the years in which the effect of the great epidemic was apparent, and dividing the twenty-two years, 1875-1896 inclusive, into two periods, the mean annual ratios per 100,000 men are:—

		Period 1875-1885 Inclusive	Period 1886-1896 Inclusive
Prussian Army	{ Attacks . . .	4.7	3.2
	{ Fatal Attacks	0.0	0.0
French Army	{ Attacks . . .	133.6	32.6
	{ Fatal Attacks	11.3	1.8
Austrian Army	{ Attacks . . .	333.7	35.8
	{ Fatal Attacks	19.5	1.6

Or, again, if the six years 1881 to 1886 inclusive be taken, during which vaccination was not carried out to any extent in the French Army, and was not required at all in the Austrian Army, and compared with the six years 1891-1896 inclusive, during which vaccination was compulsory and was strictly enforced in both Armies, it will be seen that, while in the earlier period the rates of attack by small-pox in the French and Austrian Armies were respectively 23 and 85 times as heavy as in the Prussian Army, in the later period the attack rate in the French Army had fallen to four times, and that in the Austrian Army to three times, that of the Prussian Army rate.

### *Revaccination.*

Incidental reference has been made to the protection conferred by revaccination, as in the case of Prussia since 1874, and the Prussian Army since 1834.

The following example of a well-vaccinated community side by side with one not compulsorily vaccinated is given.

In the Prussian Army for the years 1834-1869, the mortality from small-pox was only 40 per-cent of that of the civilian population between ages 15 and 50, the actual ratios being :—

Prussian Army	...	...	...	1·4 per 100,000 men
Civilians (ages 15-50)	...	...	...	3·6 per 100,000 population

Small-pox  
Hospital  
Nurses'  
Experience.      An illustration of the immunity given by revaccination is quoted by Dr. Newsholme from the experience of nurses employed in Small-pox Hospitals.\*

“ . . . . in the experience of the Staff of the Metropolitan Asylums Board Hospitals, in the six years 1890-1895, out of a Staff varying from 1,160 to 2,514, the number who contracted scarlet fever, diphtheria, or typhoid fever during each year varied from 4·0 to 7·3 per-cent of the total staff; while out of a Staff varying from 64 to 320 the percentage attacked by small-pox was nil, except in 1892, when it was 1·4, and in 1893, when it was 1·9.”

A further striking example, derived from the present epidemic, of the protection afforded by revaccination is furnished by J. Harley Brooks, M.D., Medical Superintendent of the Mile End Infirmary.

The Nursing Staff on duty at the Mile End Infirmary consisted of 40 nurses and 2 matrons; of these, 29 nurses and the 2 matrons had been revaccinated at periods ranging from

\* The Elements of Vital Statistics.—Arthur Newsholme, M.D. Lond. Swan, Sonnenschein & Co., Ltd., 1899.

2 days to 2 years prior to the outbreak in the institution, and 4 nurses were revaccinated immediately after the outbreak. None of these 35 persons contracted small-pox. The remaining 7 nurses contracted small-pox, and of these 3 had not been revaccinated; the other 4 were revaccinated within the incubation period, as is shown in the following statement:—

	Date of Revaccination	Date of Removal
1 nurse . . . . .	January 24	January 30
1 nurse . . . . .	January 29	January 31
2 nurses . . . . .	January 29	February 1

These four were, therefore, virtually not revaccinated.

### *Deductions from foregoing Results.*

We have now examined the evidence, as derived by comparison between countries with and without stringent legislation, respecting vaccination; also between groups of persons at different intervals of time since vaccination. We have, further, investigated the comparative influence of an epidemic upon vaccinated and unvaccinated classes, and, finally, the influence of a stringent revaccination upon communities of which the individuals are under strict observation.

The conclusions arrived at point, irresistibly, to the value of vaccination as an agent in reducing both the rate of attack and the virulence of attack of small-pox. They, also, indicate the necessity of renewing the protection afforded by vaccination in infancy at a later period of life as appearing, for instance, by comparison of the diagrams (Diagram III, A and F), where it is shown that while the mortality in infancy has been reduced to  $\frac{1}{40}$ th of its former rate, the mortality for ages 45 and upwards has not been diminished.

The virtue of revaccination is shown in the case of Prussia, where, since 1885, not one person in 100,000, and since 1894, not one in a million, has died of small-pox.

### III.—NATURE AND EXTENT OF VACCINATION AS AT PRESENT CARRIED OUT.

**Efficient and inefficient Vaccination.** In the foregoing investigation no account has been taken of the character of the operation of vaccination; for, it must be remembered, the mere act of vaccination gives no protection unless it be efficient, and the operation of vaccination has not been legally defined. An examination of the number of marks affords some guide to the

degree of protection, the evidence, upon the whole, pointing to the conclusion that the greater the number of marks the greater the protection. The following figures are derived from an examination of hospital cases.

The percentages for the years 1836-1851 were derived from observations of 3,094 cases of post-vaccinal small-pox, and those for years 1852-1867 from an experience of 10,661 cases.

Cases of Small-pox classified according to the Vaccination Marks borne by each Patient respectively	Percentage dying of Persons attacked 1836-1851	Percentage dying of Persons attacked 1852-1867
Having one vaccination cicatrix . .	7·6	13·8
„ two „ cicatrices. .	4·3	7·7
„ three „ „ . .	1·8	3·0
„ four „ „ . .	·7	·9

Similar results were obtained from the observations of 6,839 cases, as follows :—

No. of Marks	Cases	No. of Deaths	Per-cent
1	1,357	85	6·2
2	1,971	115	5·8
3	1,997	75	3·7
4	1,514	34	2·2

These figures, while illustrating the argument, are not, in my opinion, conclusive, for the reason that the persons classified according to marks may have been differently distributed as to ages; but the consistency between the two sets of figures and the general uniformity of their progression, would be difficult to explain on other grounds than as a result of the degree of vaccination.

If the conclusions up to this point be accepted, namely, the necessity for vaccination universally applied and efficiently carried out, it remains to enquire to what extent the operation of the law has given effect to this requirement.

**Degree of Primary Vaccination.** Vaccination, as we have seen, was made compulsory by the Act of 1853, but its universal practice was not strictly enforced until the Act of 1871, when it was rendered obligatory on the part of the Guardians to appoint vaccination officers to enforce the operation of the law.

For many years subsequently to this Act, the number of infants unaccounted for in the year of birth was about 5 per-cent only; the remaining 95 per-cent having either been reported as :—



- (a) Successfully vaccinated.  
 (b) Insusceptible of vaccination.  
 (c) Having had small-pox.  
 (d) Having died unvaccinated.  
 or (e) Having received certificate of postponement.

Since 1883, however, the percentage of infants unaccounted for has been steadily on the increase, the actual percentages from 1872 to the present time being as follows:—

*Children not finally accounted for by Vaccination (including cases postponed), shown as percentages of the Total Births for each year from 1872 to 1899.*

1872 . . .	5.1	1885 . . .	5.8
1873 . . .	4.8	1886 . . .	6.4
1874 . . .	4.8	1887 . . .	7.1
1875 . . .	4.7	1888 . . .	8.5
1876 . . .	4.3	1889 . . .	9.9
1877 . . .	4.5	1890 . . .	11.3
1878 . . .	4.7	1891 . . .	13.4
1879 . . .	5.0	1892 . . .	14.9
1880 . . .	4.9	1893 . . .	16.1
1881 . . .	4.5	1894 . . .	19.2
1882 . . .	4.8	1895 . . .	20.5
1883 . . .	5.1	1896 . . .	22.9
1884 . . .	5.5		

	Percentage excused by Conscientious Objection	Total
1897 . . .	3.4 . . .	22.7 . . . 26.1
1898 . . .	5.1 . . .	21.5 . . . 26.6
1899 . . .	3.6 . . .	17.2 . . . 20.8

It is probable that some portion of the increase may have been due to the fact that the general question of vaccination was under consideration by a Royal Commission during the seven years 1889-1896.

From the births for 1897 onwards, a large number of children have been removed from the column "not finally accounted for", by the fact that certificates of conscientious objection had been obtained in respect of them; for the purpose of deciding the thoroughness of vaccination in each year, these children, however, should be included, and the result appears in the table.

The actual number of children excused by conscientious objection were as follows:—

In respect of the births of 1897 . . .	31,687
"          "          1898 . . .	47,423*
"          "          1899 . . .	33,573

\* The total number of certificates of conscientious objection received during 1898 amounted to 203,413. Most of these related to children born and registered before 1898, and were granted under section 2, sub-section 2, of the Act, which gave to all parents with unvaccinated children, no matter when they were born, the option of obtaining certificates of conscientious objection within four months from the passing of the Act.

In Scotland, the corresponding figures of children not finally accounted for from 1864 to the present time, have always been smaller than in England and Wales; the proportion until the year 1898 never having risen to 5 per-cent, the percentage since then having been:—

In respect of births of 1898	.	.	.	.	5·5 per-cent
„ „ 1899	.	.	.	.	5·2 „

In fact, there has never been any considerable opposition in Scotland to the practice of vaccination.

The Act of 1898 provides that unless the parent, or other person having the custody of the child, satisfies, within four months from the birth of the child, two justices, or a stipendiary or metropolitan or police magistrate, in petty sessions, that he conscientiously believes that vaccination would be prejudicial to the health of the child, and within seven days thereafter delivers to the vaccination officer for the district a certificate by such justices or magistrate of such conscientious objection, and obtain a certificate of exemption, the child must be vaccinated within six months from the birth; and, as provided under previous Acts, unless the operation be postponed under the certificate of the public vaccinator, the parent, or other person, is liable to be proceeded against summarily.

After allowing for the children excused by conscientious objections, there yet remains a large percentage of children not finally accounted for; and as the administration of the vaccination laws is in the hands of Boards of Guardians, who in many cases have shown themselves to be antagonistic to vaccination, the responsibility for deficient vaccination must be, to some extent, attributed to them.

On this point the Royal Commission reported as follows:—  
 “We were anxious to learn to what extent the Guardians in  
 “England and Wales had ceased to put the law requiring the  
 “vaccination of children in force. We, accordingly, made  
 “enquiry of the Guardians throughout the country. Answers  
 “were received from 620 out of the 648 unions. We found that  
 “the law was not being enforced in 122 out of these 620 districts;  
 “in 46 out of the 122, however, the Guardians based their action  
 “upon the fact that a Royal Commission had been appointed to  
 “enquire into the subject of vaccination and had not yet  
 “reported.”

An indication of the local feeling as regards vaccination is given by the following statement, in which are tabulated the

unions in England and Wales in which, in respect of the births for the year 1899, the cases not finally accounted for are upwards of 20 per-cent.\*

<b>Bedford.</b>		<b>Herts.</b>		<b>Northampton.</b>		<b>Wilts.</b>	
Bedford . . . . .	62·9	St. Albans . . . . .	29·3	Brackley . . . . .	23·9	Bradford-on-Avon . . . . .	44·4
Luton . . . . .	43·4	Heimel Hempstead . . . . .	26·9	Brixworth . . . . .	37·9	Calne . . . . .	29·9
<b>Bucks.</b>		<b>Hunts.</b>		Daventry . . . . .	25·0	Chippenham . . . . .	25·6
Reading . . . . .	62·0	Nil.		Kettering . . . . .	62·1	Malmesbury . . . . .	21·0
<b>Bucks.</b>		<b>Kent.</b>		Northampton . . . . .	60·7	Trowbridge and	
Nil.		Gravesend and Milton . . . . .	35·6	Wellington . . . . .	61·0	Melksham . . . . .	67·3
<b>Cambridge.</b>		Maidstone . . . . .	22·5	<b>Northumberland.</b>		Westbury . . . . .	30·5
Nil.		<b>Lancaster.</b>		Castle Ward . . . . .	30·2	<b>Worcester.</b>	
<b>Chester.</b>		Burnley . . . . .	28·7	Morpeth . . . . .	41·4	King's Norton . . . . .	23·2
Nil.		Chorlton . . . . .	23·4	<b>Nottingham.</b>		<b>York, E. R.</b>	
<b>Cornwall.</b>		Fylde, The . . . . .	20·2	Basford . . . . .	23·9	Kingston-upon-Hull . . . . .	26·6
Falmouth . . . . .	31·2	Rochdale . . . . .	20·7	Mansfield . . . . .	32·5	<b>York, N. R.</b>	
Redruth . . . . .	29·9	<b>Leicester.</b>		Newark . . . . .	29·8	Scarborough . . . . .	21·8
Stratton . . . . .	23·3	Ashby-de-la-Zouch . . . . .	24·0	Nottingham . . . . .	24·4	<b>York, W. R.</b>	
Truro . . . . .	21·9	Barrow-on-Sour . . . . .	33·0	<b>Oxford.</b>		Bradford . . . . .	24·9
<b>Cumberland.</b>		Blaby . . . . .	27·3	Nil.		Dewsbury . . . . .	34·8
Cockermouth . . . . .	35·2	Hinckley . . . . .	48·4	<b>Rutland.</b>		Halifax . . . . .	63·1
<b>Derby.</b>		Leicester . . . . .	75·2	Nil.		Keighley . . . . .	24·8
Belper . . . . .	20·9	Loughborough . . . . .	22·0	<b>Salop.</b>		Skipton . . . . .	21·0
Chesterfield . . . . .	28·3	Market Harborough . . . . .	53·3	Wellington . . . . .	22·9	<b>Anglesey.</b>	
Derby . . . . .	66·7	<b>Lincoln.</b>		<b>Somerset.</b>		Nil.	
Shardlow . . . . .	32·9	Bourne . . . . .	23·7	Axbridge . . . . .	22·2	<b>Brecknock.</b>	
<b>Devon.</b>		<b>London.</b>		Bath . . . . .	21·5	Crickhowell . . . . .	22·0
Plymouth . . . . .	27·0	Bethnal Green . . . . .	47·8	Bridgwater . . . . .	24·8	<b>Cardigan.</b>	
Devonport . . . . .	28·7	Camberwell . . . . .	28·6	<b>Southampton.</b>		Nil.	
<b>Essex.</b>		George, St., in the		Christchurch . . . . .	24·4	<b>Carmarthen.</b>	
Weymouth . . . . .	29·3	East . . . . .	34·9	<b>Stafford.</b>		Nil.	
<b>Gloucester.</b>		Giles, St., and St.		Burton-on-Trent . . . . .	29·8	<b>Carnarvon.</b>	
Bristol . . . . .	32·2	George . . . . .	32·3	Walsall . . . . .	29·4	Nil.	
Cheltenham . . . . .	23·0	Hackney . . . . .	37·7	W. Bromwich . . . . .	32·3	<b>Denbigh.</b>	
Gloucester . . . . .	34·6	Holborn . . . . .	30·8	Wolverhampton . . . . .	23·3	Nil.	
Stroud . . . . .	27·7	Islington . . . . .	28·2	<b>Suffolk.</b>		<b>Flint.</b>	
Newkesbury . . . . .	43·7	Lambeth . . . . .	24·0	Ipswich . . . . .	25·7	Nil.	
Wheatenhurst . . . . .	52·4	Lewisham . . . . .	20·6	<b>Surrey.</b>		<b>Glamorgan.</b>	
<b>Hereford.</b>		Marylebone . . . . .	20·2	Croydon . . . . .	39·4	Nil.	
Nil.		Mill End Old Town . . . . .	69·4	Kingston . . . . .	20·9	<b>Merioneth.</b>	
<b>Herts.</b>		Olave, St. . . . .	22·9	<b>Sussex.</b>		Nil.	
<b>Hunts.</b>		Pancras, St. . . . .	35·2	Brighton . . . . .	21·3	<b>Montgomery.</b>	
<b>Kent.</b>		Poplar . . . . .	49·6	Eastbourne . . . . .	52·8	Nil.	
<b>Lancaster.</b>		Southwark . . . . .	29·6	E. Grinstead . . . . .	27·5	<b>Pembroke.</b>	
<b>Leicester.</b>		Shoreditch . . . . .	52·2	Lewes . . . . .	27·0	Nil.	
<b>Lincoln.</b>		Stepney . . . . .	46·6	<b>Warwick.</b>		<b>Radnor.</b>	
<b>London.</b>		Wandsworth and		Atherstone . . . . .	36·2	Rhayader . . . . .	45·7
<b>Middlesex.</b>		Clapham . . . . .	23·4	Coventry . . . . .	30·5		
Edmonton . . . . .	27·1	<b>Middlesex.</b>		Meriden . . . . .	24·7		
<b>Monmouth.</b>		Edmonton . . . . .	27·1	Nuneaton . . . . .	44·9		
Nil.		<b>Norfolk.</b>		<b>Westmoreland.</b>			
<b>Norfolk.</b>		King's Lynn . . . . .	59·9	Nil.			

\* The recent decision in the case of Moore v. Keyte is of considerable importance. This was an appeal from a conviction by the Magistrates of Leicester, under the Vaccination Acts, the Appellant having been prosecuted by the Vaccination Officer, as such, against the orders of the Leicester Guardians, and fined 1s. and costs, for neglecting to have his child vaccinated. The Lord Chief Justice, in giving judgment, said that there was no ground for saying that the consent of the Guardians was made a condition precedent to the prosecution. The Vaccination Officer was bound to obey the order of the Local Government Board, and any order of the Guardians, interfering with that order, would not be a legal order that he would be bound to obey.



The aim of the Royal Commissioners was "so to frame "and to administer the law that opposition to vaccination "should not spread to other districts, and that it should cease "or diminish in those parts of the country where it at present "prevails."

The Act which followed was a tentative one. It is operative only until 31 December 1903, and the near approach of its termination raises the important question of future legislation. Possibly, however, the Act may not be modified at the end of its five years, but its operation continued by including it in the schedule of the "Annual Expiring Laws Continuance Act." The working of the Act should, in any event, be thoroughly examined in the near future by another Royal Commission, or by a Departmental Committee, or some other efficient form of enquiry.

It has been shown in the course of this paper that by the means of a thorough primary vaccination, in conjunction with compulsory revaccination at school age and on joining the Army, small-pox may be practically exterminated, and immunity from epidemics given to a community thus protected.

With this example, and with the experience of the present epidemic in London, the nation should, surely, not rest satisfied until primary vaccination shall have become the universal practice and revaccination rendered compulsory.

#### IV.—SUGGESTIONS FOR INCREASING THE POPULARITY OF VACCINATION AND FOR ADDING TO ITS EFFICIENCY.

The conscientious objector, as has been shown in the table on page 285, is only responsible for about one-fifth of the infants annually evading vaccination, and, as it is believed that the indulgence conceded by the Act of 1898 removed a certain amount of opposition to the practice of vaccination, it must be assumed as probable that compulsion, with its penalties as existing before the Act, will not be re-introduced.

What, then, are the measures to be taken to secure the vaccination of the 20 per-cent of the children annually born in England and Wales, whose non-vaccination is not accountable for upon any excusable ground, and which, probably, arises, to a large extent, from mere indifference, an indifference



only, as it appears, capable of being overcome in times of epidemic?\*

1. It has been suggested that if the control were removed from local authority and placed in the hands of a central body, such as the Local Government Board, the action would be violently resisted, with the result that less, rather than more, vaccination would ensue; but a transfer from guardians to some other local authority, *e.g.*, Borough, Urban or Rural District Councils, would not be attended with this objection. Against this latter proposition, however, it is argued that the question, *primâ facie*, is likely to be dealt with from a broader point of view by a body which draws its members from a wider area, than by a body whose members represent a small area, and that as there are only some 640 Boards of Guardians, as against some 1,710 Urban and Rural District Councils, the proposed change would not be the means of better enforced vaccination.† Probably, the question would receive more thorough attention at the hands of County Councils than in the case of either of the bodies above mentioned. This is a matter, however, which has been much discussed, and should receive the careful consideration of any Committee that might be appointed prior to future legislation.
2. The national importance of vaccination would appear to make it a fitting subject for a subsidy, and if an adequate fee were allowed to every medical man, and not only, as at present, to those specially appointed, for each case of successful vaccination performed by him, it is probable that there would be a large

\* I have no means of estimating the numbers that have been vaccinated or revaccinated during the present epidemic. Our local papers, in February 1902, mentioned the "rush" to be vaccinated; the people outside the house of the public vaccinator being on some evenings of such proportions as to put one in mind of the pit crowds outside some of the more popular London theatres.

† The present feeling, on the question of vaccination, obtaining among the various Urban District Councils throughout the country is shown in the response obtained by the Beckenham Council to their circular asking for resolutions in favour of a repeal of the Conscientious Objector Clauses of the Vaccination Act of 1898. As a result, 94 Councils passed similar resolutions, 46 decided to take no action, 73 sent a formal acknowledgment, and 936 made no reply.

extension of the practice. This would, no doubt, entail an inspection of results by Government officials, and it should be an object of an enquiry whether the consequent expense could reasonably be undertaken by the Government.

3. The operation of vaccination has never been defined by the Legislature, and any future Act should contain a statement of the maximum number and area of cicatrices.
4. Hitherto, lymph from the Government vaccine establishment has only been supplied to Government vaccinators, but it would seem advisable that all medical men should be able to obtain it. If, in the present position of the Government laboratories, the output would be insufficient to meet such a demand, this department should be enlarged sufficiently to enable the very general desire to obtain lymph guaranteed by the Government to be complied with. Such lymph need not, necessarily, be supplied gratis, but might be sold at the same cost as that produced from trade sources.

It would be undesirable to discourage, entirely, the manufacture of lymph by private enterprise, because the demand is occasionally, as in times of epidemics, excessive and greater than the Government might be able to meet, having regard to the fact that the lymph, as pointed out by Dr. Monckton Copeman, retains full activity for only a limited period. But, as the preparation and testing of glycerinated calf lymph is a matter requiring great skill and care, it should be necessary that the lymph manufactured by private persons should be either under the supervision of, or be bacteriologically\* tested by, Government officials. This would prevent the possibility of inert lymph being supplied to medical men, of which instances have recently been given in the Medical Press.†

The processes employed in the purification and preservation of Government lymph, for which

\* If practicable, a clinical test, also, would be advisable.

† *British Medical Journal*, 2 November 1901, page 1380.

Dr. Monckton Copeman is responsible, have been described by him at length in his Milroy Lectures, delivered before the Royal College of Physicians in 1898, and, more recently, in an address to the Royal Medical and Chirurgical Society in December 1901.

5. The future Act should include the enforcement of revaccination, for which, as is shown in the instance of Germany, the school age would offer the suitable opportunity.

#### APPENDIX.

##### *Practice of British Life Assurance Offices.*

For the purpose of this paper information was collected as to the present practice of Life Assurance Offices with regard to the question of vaccination.

The replies of the Offices may be summarized as follows:—

##### *Practice as regards Enquiries made to Proposer.*

- 60 enquire "Has the life been vaccinated?"
- 8 enquire date of last vaccination.
- 1 enquires now, "Has the life been vaccinated?" but intends to ask date of last vaccination in future.
- 2 do not make any enquiry *re* vaccination.

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##### *Practice of Office where the Life proposed is Unvaccinated.*

- 7 do not make any extra charge.
- 23\* charge extra. Extra varying from 2s. 6d. per-cent to 10s. per-cent, or life rated up from two to ten years.
- 13 insert condition, declaring policy to be null and void in the event of death from variola.
- 3 insert condition, returning only surrender-value in the event of death from variola.
- 9 charge extra, or insert condition declaring policy to be null and void in the event of death from variola.
- 1 charges extra, or inserts condition returning only surrender-value in the event of death from variola.
- 5 charge extra, or insert condition returning only premiums paid in the event of death from variola.
- 11 decline case if unvaccinated.
- 1 has no fixed practice.

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None of the nine Industrial Offices, who furnished information, make any enquiry or charge an extra premium.

\* 1 Office charges extra if life has not been revaccinated.

*Practice of American Offices.*

It was a somewhat general custom in America, up to a recent date, to require either evidence of vaccination within a reasonable period, or revaccination, in default of which a clause would be inserted relieving the Office of liability in case of death from small-pox.

The larger Companies have, apparently, discontinued action of this kind, in view of the general custom of vaccination, and the, consequently, slight danger of small-pox even among unvaccinated persons.

Mr. Emory McClintock writes :—

“The impression prevails among our medical men that deaths from small-pox among those who hold policies not carrying that risk are very rare, they being only a small fraction of a community in which there are but few deaths from that disease. The health authorities of the several States have broad powers, and when small-pox shows itself require vaccination in the locality. Your ‘conscience clause’ would not be approved here.”

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Sir Henry M. Stanley, G.C.B., contributes some interesting facts as the result of his experience among the natives employed in his African Expeditions.

2, RICHMOND TERRACE,  
WHITEHALL, S.W.  
20 November 1901.

DEAR SIR,

My faith in vaccination has been confirmed by my experiences in five African expeditions.

In 1871-72, I was too inexperienced in African travel to provide against the pest of small-pox, which generally decimated native caravans. As a result, I lost 23 men out of 196 by small-pox.

In 1874-77, I had over 350 men. I had them all vaccinated before starting. The loss by small-pox during three years was five men.

In 1879-84 my first contingent numbered 68 men. They were all vaccinated by myself. During the three years I lost none. Second contingent of 250 men were also vaccinated. There was no loss from small-pox during their term of service. Third contingent of 226 men, brought by Lieutenant Valcke, were not vaccinated, and when they reached the Cape from Zanzibar, the ship they were in was put in quarantine while at Cape Town. When she reached the Congo a month later, 22 men had died, and over 50 were on the sick list from the disease. The worst feature connected with this contingent was that only 79 men were ever reported fit for duty.

In 1887-89, 620 men and boys belonged to this expedition. During the first year there was no case of variola in the expedition.



In the second year, 130 were detached for separate duty, in doing which they became mixed up with a body of over 330 natives, a few of whom had already been attacked with small-pox.

Compelled to travel together for nearly 600 miles, through a hostile country, there were no means of isolating those afflicted from the other people. At the muster made 22 December 1888, it was found the journey's casualties were 106, of whom nearly one-fifth had died from small-pox.

Out of the 620 vaccinated in February 1887, one had become delirious from a confluent case of small-pox, and committed suicide by jumping into a cataract. Two had slightly contracted the disease, and for a week were relieved from duty. At the end of that week they resumed their place in the marching column, and appeared none the worse.

Yours faithfully,

HENRY M. STANLEY.

A. F. BURRIDGE, Esq.

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LIST OF BOOKS, &c., REFERRED TO FOR THE PURPOSES  
OF THIS PAPER.

Evidence taken by and Reports by Royal Commission. Vols. I-VI.  
1889-1893.

Final Report of Royal Commission. 1896.

Annual Reports of Medical Officer of the Local Government Board.

Annual Reports of Registrar-General :—

- I. England and Wales.
- II. Scotland.

Records supplied by the Agents-General for the following Colonies :—

New South Wales.	New Zealand.
Victoria.	Canada.
Queensland.	Cape of Good Hope.
Western Australia.	Natal.
Tasmania.	

Kritik der Vaccinations-Statistik und Neue Beiträge zur Frage des Impfschutzes. Josef Kőrösi. Berlin, 1889.

Statistical Year-Book of the German Empire (22nd). Berlin, 1901.

Blattern und Schutzpockenimpfung. Berlin, 1901.

Medizinalstatistische Mittheilungen aus dem Kaiserlichen Gesundheitsamte.  
Berlin, 1901.

Der Pocken und der Impfung. Dr. Kubler.

La vaccinazione nell'esercito e l' "Antivaccinismo." Dott. Ridolpo Livi.  
Roma, 1899.

Report on an epidemic of Small-pox at Sheffield during 1887-1888.  
Dr. Barry.

Vaccination and Small-pox statistically considered. E. J. Edwardes, M.D., M.R.C.P. Lond. (From the *Practitioner* for May 1896.)

Vaccination and Small-pox in England and other Countries. Edward J. Edwardes, M.D. Lond.

Encyclopædia Britannica. Article—Vaccination.

Two hundred and fifty years of Small-pox in London, together with a supplement relating to England and Wales. William A. Guy, M.B. Cantab., F.R.C.P., F.R.S. (*J.R.S.S.*, xlv, 399-437.)

The Small-pox Epidemic, as affected by the state of War and Peace. William A. Guy, M.B. Cantab., F.R.C.P., F.R.S. (*J.R.S.S.*, xlv, 577-587.)

English Vaccination and Small-pox Statistics. Noel A. Humphreys. (*J.R.S.S.*, lx, 503-551.)

Statistics of Small-pox and Vaccination. Alfred Milnes, M.A. Lond. (*J.R.S.S.*, lx, 552-612.)

Vaccination. S. Monckton Copeman, M.A., M.D. Cantab., F.R.C.P.

Modern Methods of Vaccination and their Scientific Basis. S. Monckton Copeman, M.A., M.D. Cantab., F.R.C.P. (Paper read before the Royal Medical and Chirurgical Society in December 1901.)

An enquiry into the prevalence of Small-pox in Kilmarnock in the last century. J. McVail, M.D.

The Report of the Royal Commission on Vaccination. A Review of the Dissentients' Statements. J. McVail, M.D.

Vaccination Vindicated. J. McVail, M.D.

Vital Statistics. A. Newsholme, M.D. Lond.

Small-pox in London. J. F. J. Sykes, M.D.

*The Edinburgh Review*, April 1899. Article—The History and Effects of Vaccination.

*Times* Correspondence. January 1899, October and November 1901. Mrs. Garrett Anderson, M.D., and others.

*British Medical Journal*.

*Lancet*.

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## DISCUSSION.

THE PRESIDENT (Mr. C. D. Higham) said that when he asked Mr. Burridge to prepare a paper on vaccination, it was not with any idea that actuaries needed convincing of its importance: but a quarter of a century had elapsed since the matter had been before them, and it seemed that, with an epidemic in their midst, it was possible that the Institute, within whose purview any question of life and death came, might possibly render some public service if it could in any way re-state the arguments, or make some new presentment of the facts and figures, so that, if possible, they might confirm the faith of any who were wavering, or, better still, convert

from the error of their ways some who were now in the opposite camp, but who yet had kept an open mind, and might be led to see how great a protection could be obtained at a small cost, and very little expenditure of time and trouble. He thought the members would agree that the author had put his case very well, and done his work excellently, especially in the charts which appealed to the eye. He was afraid the discussion was likely to be one-sided, and that not altogether by their fault. They would have liked to have asked one or two representative anti-vaccinationists, but it was felt that they could not decently invite a man as a guest and then, metaphorically, knock him down; and although he had hoped that some leading anti-vaccinationist, if there were such, might have applied for a card of invitation, he had been disappointed. Nevertheless, if their deliberations were one-sided, he hoped they would endeavour to make them practical. The thing to be decided was, could the case be stated so cogently as to compel the attention of that great, inert, sluggish mass of people, who would not take the trouble to think or do, and who went on placidly running risks to themselves and to those dear to them, rather than submit to a trifling inconvenience? He would like to know if there was any reason why the fee paid by the authorities for vaccination should be conferred exclusively on a public vaccinator, because it appeared to him that if every medical man who efficiently performed vaccination on a child were to have the fee, the active support would be obtained of a large body of medical men of what might be called the lower rank, if there were any lower rank, in that noble profession. Another suggestion which he had heard was, he thought, worthy of attention, namely, whether the present system of penalties could not be supplemented by a system of rewards; and before they worried the man who did not have his child vaccinated, could they not remunerate him who did. In other words, was there any reason why some small fee should not be paid to the man or woman who brought his or her child to be vaccinated soon after birth? And it did seem that if their laws were to be altered in that way, something would be done to engage the attention of the numbers who at present would not trouble about vaccination. There was the expense, but there would be economy on the coercive side; and, after all, there were worse ways of spending public moneys than in saving life and preserving health to the community.

Mr. J. DOUGLAS WATSON, in opening the discussion, said that probably no one who had not been into the subject closely, realized how difficult it was. With regard to the paper, apart from the title, he did not find anywhere any very clear definition of the scope of the investigation, but he thought before one dealt with a subject statistically it was necessary to go somewhat into the theory which one was asked to support. As far as he could gather, Jenner's attention was originally drawn to the subject owing to a popular belief that inoculation with cow-pox had the effect of protecting people from small-pox. After making investigations on the subject, Jenner explained in his historic paper how he had elaborated the process of vaccination, and made the following remarks: "What renders the cow-pox virus so extremely singular is that the person who has been thus affected is for ever after secure from the infection



of the small-pox ; neither exposure to the variolus effluvia nor the insertion of the matter into the skin producing the malady." For some time it was considered that cow-pox was an independent disease to small-pox, and he had found that rather a difficulty. The result of investigations since Jenner's time, however, made it clear that cow-pox was probably derived from small-pox in the human being, and that by passing the small-pox virus through a certain number of cows, the so-called cow-pox was eventually produced. If that were so, it did not require a very great knowledge of recent medical science to see an analogy between the process of vaccination and the process of inoculating with anti-toxins for diphtheria, hydrophobia, and other kindred diseases, which had been so successful in recent years. Moreover, if that were admitted, it explained also why vaccination had singularly failed in one of the claims that was originally made for it, namely, that its effect was permanent. Leaving the original claims made by Jenner on behalf of vaccination, and the subsequent intermediate claims which have been made from time to time, he thought they might state the present claims on behalf of vaccination in the following way: (1) That efficient vaccination gave complete immunity from attack for a certain indefinite period, probably seven or ten years; (2) that the state of complete immunity might be reinstated by subsequent re-vaccination from time to time; and (3) that even after the above complete immunity had worn out, the vaccinated person was left in such a state that if he then contracted small-pox it was of a milder type. Those were the three problems actuaries had to attack practically from the statistical point of view, and when one went into the matter he thought it would be quite evident that the statistics available were not very clear, and did not give such definite information as was desirable. Just as he thought the general vaccination case had been weakened by the original claims for it not having been borne out by the facts, so, in papers like that of the author's, it was possibly a logical mistake to give statistics which did not actually prove the points it was desired to make. A handle must not be given to the anti-vaccinationist, and the author had been very careful in that respect. For instance, in introducing his first diagram on page 256, he drew attention to the fallacy involved in comparing the deaths from any one disease with the total deaths. But there was a further difficulty about the diagram, which was not referred to. Supposing it was said that the reason for the general character of the curve there shown was that inoculation became fashionable and then died out, he did not see how that argument was to be encountered. It seemed to him that these and similar considerations rendered the first diagram, as a contribution to the particular subject, rather inconclusive. Turning to the second section of the paper, headed "Small-pox statistics as indicating the effect of vaccination", it would be seen that the author again drew attention to the difficulty of finding suitable statistics which gave conclusive proof. After carefully considering diagrams 2 and 3, he found great difficulty in satisfying himself that they showed satisfactory evidence as to the efficiency or otherwise of vaccination. In the first place, in any investigation in which it was sought to show that one out of



many possible causes was the efficient cause, the difficulty of isolating that one cause was well known, and in a case of the kind under discussion, to do so was almost impossible. For instance, if it was argued that in spite of occasional epidemics of small-pox the general decrease in the disease was due to improved sanitation, or to increased attention to the evils of overcrowding, or to the recognition of the importance of immediate and complete isolation of small-pox and other cases of infectious disease, it was difficult, from the diagrams alone, to show that those various causes had not been the effective agents as distinct from vaccination. It must be admitted that vast improvements in those various precautions had taken place. For instance, Mr. Porter, in a paper written in 1860, and referred to by Mr. Burridge, said:—"The necessity is now, happily, beginning to be recognized for providing special carriages for the conveyance to the hospitals of small-pox and fever patients. Too many are still conveyed in the common street cabs, by which course these diseases are no doubt much disseminated." That paragraph indicated a state of things which was very different from that now existing under the régime of the Metropolitan Asylums Board. In that connection he could not quite agree with the deductions made in regard to the diagrams as to other so-called zymotic diseases. It seemed to him that effect had not been given in the three diagrams to the questions of the relative age distribution of fatal attack. If they remembered Dr. Sprague's general conclusion, that statistics in which the age distribution was not given effect to must be rejected, he thought there were great difficulties in admitting that the diagrams in the paper as to the other zymotic diseases conclusively showed the effect or otherwise of vaccination. Turning from these general diagrams, the author next considered those in which the condition with regard to vaccination of the section of the community dealt with was known. To make such an investigation satisfactory, he thought they required to have more detailed statistics than were as a rule available. From a statistical point of view, if one was going to deal with the question in that way, one must theoretically have the means of dividing the population accurately into a vaccinated and an unvaccinated class; not only that, but the vaccinated population must be homogeneous, with regard to age distribution, class distribution, occupation, sanitary condition, geographical position, and also as regards time. Not only must the vaccinated population be homogeneous in those various respects, but the unvaccinated population must also be similarly homogeneous. In practice these theoretical requirements were not even approximately met by the available material. But he took it that the author had carefully considered these difficulties in regard to the Sheffield population, and he had, no doubt, also considered the peculiar difficulty of that census, in that there was a 38 per-cent adjustment which had been applied irrespective of age, which might introduce a very serious error. If, making allowance for all those difficulties, they were still satisfied that the deductions drawn from the curve were fair, it must be admitted that it showed very strongly the beneficial effect of vaccination. The difficulty he had mentioned about time seemed to him to apply peculiarly to an army. A further point must, moreover, always be considered, namely, that in dealing with the question there must be equality in "liability to

infection." If one population was subject to the infection of small-pox and was compared with another that was not, he did not think very much proof was obtained from showing that there was much small-pox in the one and very little in the other. That must be particularly borne in mind in dealing with armies, where proper isolation was available, and now-a-days always employed. The next section of the paper dealt with special classes, such as hospital nurses and attendants, and introduced quite the most satisfactory figures. The ages and sexes were distributed homogeneously. The vaccination condition was accurately known, and it might be taken that every unit tabulated was practically equally subject to infection. Indeed, he failed to see any loophole by which one could escape from the conclusion that these statistics proved conclusively the beneficial effects of vaccination. Passing on to the final recommendations, he had gathered from the paper that a return to compulsory vaccination was not contemplated. As a believer in vaccination, he could not help thinking that any return to compulsion would be a mistake, and would only have the effect of producing a large "army of martyrs." He thought it would be well if some means could be devised to reach the large class of indifferent persons of whom the President had spoken. If it were possible to make it more troublesome and uncomfortable for a man to remain unvaccinated than to be vaccinated it would be very valuable. The suggestion that every doctor should be in a position to vaccinate gratuitously was a very practical and useful one, but he did not see how Government inspection of the vaccinated cases could be undertaken under such conditions. In conclusion, he would be very glad to hear the opinions of the more experienced members of the Institute on the purely life assurance aspects of the question. There was no doubt that, if in the past the offices had adopted a uniform practice in connection with the subject, their records would contain a very valuable storehouse of facts available for statistics, but, apart from the value of any such statistics which might be collected in the future by the offices, he thought any enquiry on the matter unnecessary, and that the actual risk of death from small-pox amongst the classes they insured, whether unvaccinated, vaccinated, or re-vaccinated, was so small that it might be taken with other similar risks.

Dr. SYMES THOMPSON said the medical view of the question was a very comprehensive one, which it was exceedingly difficult to deal with in a few moments. The paper was of great value, as it contained material for the medical profession when they desired to issue statistics in favour of vaccination. But the paper was still more calculated to impress those who had an open mind upon the matter; in fact, he did not doubt that the paper was calculated to convince all except those who were incapable of being convinced. The evidence was necessarily difficult, and the subject was necessarily confused, and they might have to wait a long time before obtaining conclusive evidence. But the present evidence was quite sufficient to act upon, and quite sufficient to overcome the arguments on the other side. On such a matter one could scarcely expect to have evidence which would satisfy all statisticians. As had been clearly shown, there were elements of difficulty which of necessity entered into the construction of the tables, and made it quite



impossible to avoid dangers from error. With regard to the historical part of the paper, as Gresham Professor he had had to deal with the subject over and over again, and it struck him that the historical point had been exceedingly well and forcibly put by the author. It was true there was increased mortality from chest disease at the time of the introduction of vaccination, and it had been maintained by the opponents of vaccination that such mortality was due to the introduction of vaccination. Inasmuch as the two things were contemporaneous, it was very natural to infer there was some relation between them. It was only necessary to remember that the stethoscope was introduced at the same time as vaccination, in order to disprove and set aside the assumption that chest disease was due to vaccination, because many diseases which had previously been classified under other heads were then, owing to the examination possible through the stethoscope, put down to chest diseases. That was an illustration which one only had to mention in order to carry with it a good deal of conviction. With reference to Jenner's statement which had been quoted, Jenner believed when he introduced vaccination that he was introducing that which was a permanent barrier to the occurrence of small-pox, but he found in his early experience after the introduction of vaccination that it would occur after the lapse of years. During the long period which had elapsed new discoveries had been made. They had discovered, for instance, that the influence diminished with time, and he thought the statistics the author had put before them were exceedingly valuable as showing that the danger of small-pox recurred after ten or twelve years of age, emphasizing more than had been done in the past the immense importance of re-vaccination. The value of such a proceeding was conclusive from the returns of the small-pox hospitals. But re-vaccination must be efficient re-vaccination. Many of the incidents which had occurred with reference to the protective power of vaccination were due to the fact that it had not been efficiently applied, but now doctors used vaccine obtained direct from the calf it was easy to recognize how difficult it was to expect efficient vaccination under the old methods. When he remembered the little bits of quill with which he used to vaccinate, he was not at all surprised that the vaccination of the past was not so efficient as they desired it to be. With reference to mortality, it must not be forgotten that, although the protective power of vaccination against small-pox might diminish as the years went on, the protective power against severe and fatal small-pox was prolonged throughout life. There were some people who were subject to small-pox again and again; probably many present had heard of the celebrated case of the poor woman who had seven attacks. These were exceptional cases. It had also been found in small-pox that a previous attack was not necessarily a preservative; in fact, not nearly so preservative as recent re-vaccination. With reference to the duty of guardians, the guardians in many parts of the country had been obstructive, with the result that vaccination had, in consequence of their action, been much less efficient than it would have been. In regard to the conscientious objector, it seemed to be a curious fact that a large number of people had conscientiously objected and had satisfied the magistrate,

and yet almost immediately after had had their children vaccinated. They felt it necessary to offer opposition, because of the conscientious feelings they held. So that, although there were a great many conscientious objectors, curiously enough there had been an increase in the numerical efficiency of vaccination since the Act. The lymph was a matter of very great importance. The Government inspectors did their work with care. He happened to know one of the Government inspectors, who examined the lymph with the very greatest precision and accuracy, and he thought the danger of introducing disease with the lymph had now been minimized; practically there was no danger at all. The addition of glycerine tended to put a stop to any other morbid microbe which might possibly have got into the solution; and it might be taken as granted that the lymph as supplied was free from any danger of introducing any other disease. With regard to the application of the matter to life assurance, he thought with the last speaker that any necessity for special action on the part of life offices was not very great, but still he thought it was for the medical men, when they came in contact with those who refused to be vaccinated, to put pressure upon them, and, without necessarily excluding them, to use private influence in getting them to see the right course of action.

Mr. G. H. RYAN said it would be the unanimous opinion of the meeting that, whether they regarded the paper in the light of its historical description, or of the arguments and statistics with which it was illustrated, or of the soundness of the conclusions enforced, it would prove a most valuable contribution to their transactions. There were, however, some omissions observable, though perhaps they were not of very serious importance. He failed, for instance, to find any reference to Dr. Farr, who thirty years ago was a well-known authority and active controversialist upon the subject, and whose name was familiar to everyone in that room. He thought, also, it would have been of the greatest interest if the author had given some statistics relating to the death rate from small-pox in Switzerland since compulsory vaccination was abolished in 1885. Then he found no reference in the paper to what legislation the United States had passed, or what statistics the United States could furnish in the matter. In the presence of the Gresham Professor of Medicine it was presumptuous for actuaries to deal with the medical side of the question, but, nevertheless, the subject fell legitimately within the study of vital statistics, in which the actuary's function and competence would not be called in question. Indeed, if one looked at the historical development of public opinion on the subject, they must confess to a feeling of surprise that the medical authority had not been more powerful in achieving the end they all had in view. In fact, it would appear that vaccination formed one of a small body of subjects in regard to which the unenlightened public resisted, if they did not actually resent, authority. He was lately reading some old numbers of the *Edinburgh Review*, and in one of the numbers, which appeared about the beginning of the century, he found a paper on vaccination. It reviewed nineteen pamphlets and documents dealing with the subject, commencing with the opinion of the College of Surgeons, to which the author had called



attention. Most of the pamphlets were favourable to vaccination, but some of them were hostile to the new theory. As the Edinburgh reviewer took the side of the College of Surgeons and the vaccinationists, he need scarcely state that the article dealt very unsparingly with those who were on the other side. In the course of the review the remark was made, "The majority, we may say all the respectable part of the medical profession, now concur in considering vaccination a safe and perfect cure of small-pox." It was scarcely worth while to refer to that article, except that it showed the standard of opinion which existed at the beginning of the century; and, bearing that in mind, it seemed strange that not until 1853 was vaccination made compulsory, that not until 1871 was it enforced with anything like rigour, and that in 1898 the clock was put back by the recognition by Act of Parliament of that political sacro-sanct, the conscientious objector. In view of these facts, he thought they must come to the conclusion that medical authority, resting alone, had not been so successful as might have been expected. He believed it was in the direction of relying more and more upon the stern and inexorable teachings of experience—in other words, upon trustworthy statistics—than upon mere authority, that the end in view would be achieved. He thought it would be folly to expect that the statistics likely to be obtained on such a subject would ever fulfil the ideal which Mr. Watson had set up. When large bodies of facts were dealt with, it must be recognized that there were inherent limitations and errors of observation in them; and, while they should not allow those defects to obscure the issue, they must, on the other hand, duly recognize them, and make suitable allowances and deductions in such conclusions as were based upon them. For example, it would be remembered that Dr. Sprague referred to the fact that any comparison between the mortality of vaccinated and unvaccinated children might be fallacious, unless it was borne in mind that the unvaccinated children would probably be of weaker physique and poorer health than the vaccinated, for the reason that an unhealthy child would often either get relief from, or a postponement of, vaccination; or, if it was vaccinated, would be more prone to an attack of disease, and have less power to overcome it. Such a condition in the problem must be borne in mind, or their conclusions would be unsound. Again, it was worth remembering that a new state of things confronted them in the vaccination question, owing to the immense aggregation of population in large centres. The world had never seen such masses of people as to-day could be found in London, Chicago, New York, and other big cities of the world, and such a fact altered and influenced the conditions of the problem. He had no doubt that points like this would be carefully borne in mind by the statisticians of the future, and he was sure that if they could carefully impress upon the public mind the teachings of their statistics, they would do a great deal to combat the ignorance and prejudice which, they all felt, constituted a real danger to the health and lives of the people.

Dr. GARRETT ANDERSON said she welcomed extremely the opportunity of hearing a discussion upon such a subject, from the point of view of it being one of the facts on which they were

anxious as much as possible to educate public opinion. The subject should be discussed as thoroughly as possible, in order that public opinion might be more enlightened upon it than it had hitherto been. The subject was a very difficult and complicated one, and it was not easy to get what had been called a statistical proof, or what she would rather call a mathematical proof. It was impossible to have a mathematical proof under the conditions in which human beings were dealt with. Although she did not wish to recommend such a thing, she had proposed on one or two occasions that their anti-vaccinationist friends should go with her to a small-pox hospital, but they never seemed likely to adopt her suggestion. If they could bring themselves to send ten unvaccinated and ten vaccinated children into a small-pox hospital, and let them stay there for three or four days, mathematical proof would be obtained, and it would be a very distressing experience, she had no doubt, for the unvaccinated children. Apart from such a group of experiments, they must be content with taking things in the lump, and must not be too fastidious as to the statistics not being exactly true in all directions. The points she was particularly interested in were as to how the Act of 1898 could be improved. The Act would very soon have to be considered, and there were several things which must be pressed upon the public with all the power they could command when the time for improving the Act came. First of all, she would very much like to see the duty of enforcing the action of the vaccination officer taken out of the hands of the guardians and put in the hands of the health authority. The burden of coping with an epidemic of small-pox fell upon the health authority, and it was only reasonable that the health authority should have the power of enforcing the action of vaccination. Dr. Hope, of Liverpool, had pointed out lately that there was incidentally a great waste of time at the most critical moment at the beginning of an epidemic, just when a few hours were of the greatest importance, before one could get patients removed, because the Officer of Health had to communicate with the Guardians. In the next place, she felt very strongly that a much better supply of lymph should be available. It was abominable that in a rich country like England trustworthy lymph was not obtainable by the ordinary medical practitioner. At the end of last year the tremendous difficulty of getting any lymph was a scandal all over London. She had no doubt that hundreds of people were said to be re-vaccinated, and as nothing happened they supposed everything was safe. They were told by Dr. Copeman that the lymph did not keep good for a week. Was it likely that a private manufacturer would keep his lymph if it went wrong after it was a week old? It was not in the least likely. She had a good deal of fault to find on that score. In Germany she believed there were 25 State institutions for the manufacture of lymph, and, considering the enormous cost of a small-pox epidemic, she thought it was most foolish on the part of the Government not to have sufficient number of State institutions for the thoroughly good manufacture of lymph, so that anybody could get it. If they chose to let doctors pay for it, well and good; but they should at any rate be able to get



trustworthy lymph. Personally, she thought re-vaccination ought to take place at early adult life. "Compulsory", perhaps, was not a very wise word, though if the public liked "obligatory" better she would alter it to that. She would make it necessary for people to be vaccinated before they went into schools or went from them, and before they went into any kind of business. Business people, such as actuaries, should insist upon it that they would not employ people who could not produce their certificate of re-vaccination. If a little trouble was taken to make that certificate a proper thing, so that every person would have to have one for almost everything connected with business life, it would help the cause of vaccination very much indeed. Mr. Ryan had spoken of the extraordinary difficulty arising from the massing of the people. Not only had those masses of people to be considered, but the very great powers of locomotion which at present existed. A hundred or two hundred years ago locomotion was nothing compared with what it was at present, but, even so long ago as then, in London 500 more people died a week from small-pox than in the present epidemic, and that in a small community one-tenth the size of the present London. She also agreed with the danger that arose from the use of cabs. She knew of five cases which went to the nearest hospital at the beginning of the epidemic on separate days, all of them by the Underground Railway or omnibus, and three out of the five had small-pox. It was a wonder there was not a huge epidemic at present ten times the size, when one considered how the people had been going about. The people were exposed to very great risk, and the sooner they woke up to the necessity of getting themselves more thoroughly protected the better.

Mr. S. G. WARNER thought a service was done by occasionally selecting such a topic as vaccination for their deliberations. The subjects usually discussed were technical in their character, and it could not but be a good thing sometimes to have one on which actuaries, as students of mortality statistics, had a right to speak, but which at the same time elicited professional opinion of another kind, and experience different from their own; so that they might confer with members of another profession, especially when that was one which was so justly held in honour, and on which they all in so many important matters relied—the medical profession. The controversy which prevailed between the vaccinator and non-vaccinator created a position difficult, unique, and unfortunate. Mr. Ryan had shown something of the strange persistence of that controversy, how after proceeding for 100 years it still existed in sufficient strength to have led a few years ago to a very important legislative concession to the non-vaccinationist view. It was quite clear that a movement of that kind could not be ignored, and must be carefully considered. The position was difficult, because they were confronted with a mass of opposition, not well reasoned in many cases, often ignorant, and complicated by social and even political considerations which had no real place in a scientific question. It was a unique position, because at present, at all events, he supposed small-pox was the only well-known prevalent epidemic disease for which a scientific antidote was recommended.

Further, the position was unfortunate. None of them could help feeling regret that sections of the community, however misguidedly and unadvisedly, should maintain upon the subject the attitude they did, an attitude which was by no means entirely confined to the uneducated, but which found champions amongst people of considerable culture and intelligence. It was clear, further, that the anti-vaccinator would not take up the position he did if he had not some deep-seated mistrust of the process itself. All their statistical controversy—the true bearing of which had been so admirably exhibited in the paper—proceeded from something which lay behind, and without which it would not exist at all—the feeling that the remedy recommended had in itself elements of injury and of evil. The objections might be roughly classified under two heads, the first of which related more especially to defective machinery, causing imperfect vaccination—a difficulty with which they might hope to cope more and more effectively as years went on. The chief objections upon that score were, if not already entirely removed, in rapid course of being so. They must admit that there had been too much ground for them in the past, but might confidently look forward to the time when they would disappear. The same thing, unfortunately, did not apply to the remaining class of objections, which were, rightly or wrongly, based upon the nature of the process of vaccination itself, and which asked whether in attempting to remove the evil the scientific process recommended might not bring other evils in its train. To that class of objections belonged the one dealt with by Dr. Symes Thompson with regard to the spread of chest disease, but there was a whole class of such objections, with which, of course, unscientific men and non-experts were entirely unable to deal, which depended upon the exact scientific nature of the process which took place—whether it either subtracted from the human frame a part of its vitality or added some foreign element; so, in either case, while securing immunity from one disease, possibly increasing liability to others. One would like to know how far the non-vaccinated were a source of danger to the vaccinated. Was the public danger which existed in the case of the non-vaccinated confined to that class itself, or were they a source of danger to the general community who had submitted to vaccination? There were other less important questions about which in the lay mind a strange confusion seemed to exist which the recent epidemic had revealed. A great many rather wild statements, dogmatically made, had been heard. For instance, one had heard that no one need trouble about being re-vaccinated after the age of 19, or of 40, or of 60; while it was within his own experience that there had been heroic candidates for re-vaccination during the recent epidemic who had attained to the age of 78. They were also told—and here he spoke feelingly—that if one submitted himself to the process and did not as a result undergo quite the requisite physical misery for the requisite number of days, one had to be vaccinated again; while, on the other hand, the assertion was made that such a result was a proof of non-susceptibility to the disease.

Dr. COPEMAN, in expressing his indebtedness to the Institute for the opportunity of being present, and his pleasure at being



enabled to listen to the interesting discussion on the author's admirable paper, said he found it rather difficult to speak on the subject, because, in the first place, in the presence of such an audience of statisticians it would obviously be absurd for him to attempt to deal with that portion of the subject; and, again, on the question more particularly dealt with by the author, in regard to suggestions for increasing the popularity of vaccination, he felt the difficulty of his position on the medical staff of the Local Government Board. It was almost impossible for him to discuss that particular point. But one or two statements had been made by previous speakers concerning which he would like to make a remark. Mrs. Anderson made a statement which he thought it would be very unwise to allow to go before the public as a definite statement of fact. He would probably be forgiven for being a little egotistical, because he was more or less responsible for the methods adopted, not only in this country but abroad, in the manufacture of the lymph distributed by the Government. Therefore, when Mrs. Anderson said that the lymph supplied by the Government would only hold good for about a week he felt bound to contradict that statement. Of course, occasionally it happened, even in the best regulated laboratories, and with the most experienced operators, that, for reasons which at present they were unable to explain, one particular batch of lymph was not particularly good, and did not give as good effects as one could wish; but, speaking of the official lymph supplied, the Government now took every opportunity of testing every lot of lymph manufactured, before it was distributed. A veterinary surgeon, for instance, invariably reported to the Government on the condition of the calf, and if anything was found suspicious after it had been slaughtered the lymph was not distributed. But supposing that all was well, the lymph was further tested on a number of children at the Government stations before any large amount was distributed for use, so that it would be quite easy to see whether the particular source was good or not, and if it were not of the full efficiency it would not be distributed. With regard to the keeping-property of the lymph, he had frequently had samples which had been manufactured in the Government laboratories for periods of three or six months, or even a year, which had after that interval of time given 100 per-cent inoculation successes when employed at Government stations. He did not mean to say that that would always be the case. What he imagined Mrs. Anderson referred to was that they issued now to all public vaccinators a little notice requesting them to use the lymph if possible within a week of receipt. That was a very different thing, of course. The reason for the notice was that it was not infrequently found that through carelessness or oversight the lymph was left in a hot room and, possibly, in the sunlight. Both high temperature and sunlight were very dangerous to the efficacy of the lymph, and consequently they did not wish to run any risks of that kind. But that did not mean that, with proper precautions, such as were observed in the Government laboratories, lymph was sent out which was useless after such a brief time. With reference to the supply running short recently, that only occurred when the first great rush of

re-vaccination came in the early part of last year, when it was found that the demand for lymph went up to between 20 and 50 times the ordinary demand. Of course, it was impossible to cope with such a sudden rush, and, as it was only likely to come on at epidemic time, the Local Government Board did not feel justified in suddenly increasing their laboratories to the enormous extent it would be necessary to render them always capable of meeting such an exceptional demand. He was happy to say that the President of the Local Government Board had already stated in the House of Commons that the laboratories were perfectly capable at the present moment of meeting any possible demand which could be made upon them, not only for primary vaccination, for which they alone could be held responsible under the existing circumstances, but also in the way of re-vaccination, which was likely to be wanted in the present or in the near future. At the present moment it was not intended to supply private practitioners, but only those who were public vaccinators; but it was not improbable that in the future the scope of the supply of lymph might be increased. There were a few other points which had been raised, namely, the question of indirect compulsion in regard to vaccination, and the question of the results in Switzerland since the Vaccination Act was done away with. He had occasion to go to Switzerland when investigating the subject of the production of vaccination lymph for the Government, and there he found that apparently vaccination had been carried out. It might be of interest to the members to know what he discovered. He asked the Minister of the Interior whether there was any compulsory law, and was told there was not, but he also heard that vaccination was very thoroughly carried out. Asking for the explanation, the Minister informed him that he had had to do with the very general system in Switzerland of free education. The Swiss were a thrifty people, and as they were able to get first-rate education without expense to the individual parents, a great majority availed themselves of it. There was no compulsory vaccination, but every child on coming to school in the first instance was required to bring with him or her a certificate of successful vaccination; consequently, although there was no compulsion, Switzerland was a well vaccinated community. There was one other personal point. The opener of the discussion had called attention to the fact that Jenner believed cow-pox and small-pox to be related, if they were not really the same disease, and he stated it was desirable to find out what were the exact facts with regard to the matter, because if it could be proved they were one and the same disease it would have great scientific value. Of course, that was so, but the matter was by no means so easy as was assumed. For the last 100 years innumerable investigators had been endeavouring to transfer small-pox to the cow or calf, certainly with some success, but on the whole with very indifferent success, as would be judged when he stated that Badcock, the well-known chemist of a century ago, carried out no less than 37 experiments before he obtained what he believed to be a successful result. But, although the proportion of success was so small, there could be no doubt about it. He might be allowed to mention as an interesting fact, that within the last few years a



method of experimentation had been devised which had definitely proved the point, because he was of opinion that it was not from a generalized form of small-pox but from the inoculated form, which was very frequent in the time of Jenner, that cow-pox originated. As Mrs. Anderson had said, of course it was impossible to arrive at a definite result in dealing with human beings, because that was undesirable; but he had attempted to prove his point by dealing with monkeys instead of human beings. He found out some years ago they were capable of contracting small-pox, and, by carrying the infection through monkeys, then transforming it to a series of cows, obtaining the lymph and transferring it to human beings—among others himself—he had demonstrated the fact in every instance that it had been possible to transfer the inoculated form of small-pox. He, therefore, felt justified in saying that, at any rate, the two diseases were most intimately related, and one was capable of being derived from the other.

Dr. NEWSHOLME congratulated Mr. Watson on the able manner in which he had presented to some extent the anti-vaccinationist cause. He quite agreed with Mr. Watson, and also with Mr. Warner, that the question was not merely a statistical one, and that one only obtained a distorted view of it if one confined himself entirely to the statistical side. The subject was also an important biological question. As Dr. Copeman had just explained, there could be no doubt that vaccination was simply small-pox modified in such a way that its visible effects were local instead of general. Remembering that fact, one could perfectly understand how vaccination, and the protection obtained from it, was not perfect and permanent, inasmuch as small-pox itself was frequently—at any rate, occasionally—repeated. He had himself known one case, which was almost unique, in which a second attack of small-pox occurred within a month after the first. If there were exceptional cases of that kind, and more frequently cases in which a second attack of small-pox occurred after a lapse of years, one could not be surprised that the protection afforded by vaccination was neither perfect nor permanent. Having said that, it was necessary also to say that it was a very great protection indeed to the majority of mankind. In every community there must be exceptions. Mr. Watson had also mentioned the importance of proper corrections for age and sex distribution, and alluded to the fact that in the author's diagram—and one of the most happy and successful points about the paper was the fact that one could see at a glance everything taught in it—the proper correction had not been made for age distribution with regard to measles and whooping cough. Fortunately he was able to supply that defect. Taking the proportional death-rate under five years of age, the proportional death-rate at all ages being given as unity, then in 1851–60 the proportional death-rate from measles under five years of age was 6·8, and 1881–90 it was 7·1; similarly scarlet fever, diphtheria, and whooping cough had not altered. Fevers had altered from 1·5 to ·8, and small-pox had altered from 4·7 to 1·8, one third of what it was formerly. So that the only instance in which, after careful investigation, there happened to have been any such transference of the mortality from one age period to another at all comparable with that of small-pox

was that of fever, including in that name typhus, typhoid, and simple continued fever. But when carefully looked into, the particular circumstance of fever was not analogous to that of small-pox. Small-pox, when fatal, was a well-defined disease, and there was no chance of mistaking it. That did not apply at all to fever, and in former times there was not the slightest doubt that a large number of cases returned as fever were really tuberculosis, and other diseases which were now relegated to their proper head, and which were then determined under the head of fever. Consequently, no single case was left in which there was a transference of mortality from one age to another, such as occurred in the case of small-pox. Was there any explanation apart from vaccination? Explanations had been given which did not explain the remarkable transformation which had taken place. One explanation which had been attempted was the longer interval that occurred between epidemics as contrasted with the past. That did not suffice. If he had time he could show that did not explain it; but even supposing that were a fallacy, it was a fallacy which was entirely absent when one came to compare the statistics of a single epidemic in the same town, the vaccinated part of the town with the unvaccinated part of the town during the same epidemic. Then he thought even Mr. Watson must be satisfied that, as nearly as possible, homogeneous groups were obtained. The Royal Commission took six towns. They sent down special investigators, who investigated the incidence of small-pox, sickness, and death according to the age, whether the patients were vaccinated or not; and the result of their investigation, so far as small-pox was concerned, was seen by the following comparative statement. Among the unvaccinated the number of deaths per hundred cases in those six towns was 35, while among the vaccinated it was 5·2. Then, dividing the two groups, vaccinated and unvaccinated, into those under ten and above ten, the fatality among the unvaccinated under ten was 30·3 per-cent, as compared with 2·3 per-cent among the vaccinated, and over ten the relative fatalities were 34·3 per-cent among the unvaccinated, compared with 5·4 among the vaccinated. These were comparable details obtained from the same population, in which, apparently, the only difference was the difference of vaccination and non-vaccination. It appeared to him there was no way out of statistics like that, except that vaccination had a life-saving property. They knew it had power to diminish the incidence of the attacks amongst the vaccinated in invaded families, as compared with the incidence among those who had not been vaccinated. A great deal had been made by statisticians of the fact that during the epidemic a scare occurred, and many people rushed to be vaccinated. Consequently, the unvaccinated group was being reduced every day, while the vaccinated group was being increased, with the result that when an investigator went down at the end of the epidemic and produced his statistics relating to it, the vaccinated group had been swollen at the expense of the unvaccinated, and consequently the unvaccinated were handicapped in comparison. That was a very plausible objection; but he ventured to think that with the statistics actuaries were familiar with in the construction of a life table, they had valuable means of checking and disproving the force of that contention. At the end of the epidemic the number



of vaccinated and unvaccinated was known, the number of cases of small-pox which had occurred week by week during the epidemic was known, and the number who were vaccinated during the epidemic was also known. By those means they could form their columns of population, deaths, and cases. He challenged any person who did not believe in vaccination to so construct a life table for the course of the epidemic, by putting the vaccination that occurred during the epidemic at any period of it he liked. He would never be able to bring the fatality of the vaccinated up to the fatality of the unvaccinated, or the proportional number of cases of small-pox in that group up to the number in the other. Obviously the best way would be for him to put the transference of the unvaccinated to the vaccinated at the end of the epidemic; but even if he did this, there would still be a remarkable difference between the fatality of the two respective groups. If the experience of ten nurses who had been successfully and recently re-vaccinated engaged in nursing small-pox cases, were compared with that of ten other nurses of the same age who had not been re-vaccinated, we should have a justifiable and convincing experiment, which he was quite certain would turn a large number of those who were now anti-vaccinators into believers in vaccination.

Mr. T. E. YOUNG, in closing the discussion, said that, though by speaking last, he enjoyed the advantage of surveying the discussion as a whole, and the further, though doubtful, advantage of freedom from criticism, yet he laboured under the graver disadvantage of finding his arguments largely anticipated, and many observations which he had laboriously manufactured already advanced by other speakers. The paper shows once more, in a marked form, the conspicuous superiority of graphical construction over tabular statements, and the large economy of mental power, with the capacity of clear and rapid decipherment of results, which can be obtained from a complete picture competent of being comprised within a single view, in place of a series of detached sketches which involve the mental toil of combination into a whole. He concentrated his attention upon the diagrams relating to England and Wales, Sweden, Prussia, Austria, and Belgium; and he proposed to apply to those figures two of the inductive canons of research, first systematized by John Stuart Mill as the organized and scientific expression of ordinary methods of judgment and assessment of valuation. Statistics, of course, in sociology especially, are never "pure" and precisely commensurable; but extensive numbers, collected under approximately similar conditions, and appertaining to the same periods of time, may be confidently accepted as comparable. He ventured to premise that the question does not seem to be a medical, but essentially a statistical, one, since there does not appear to have been discovered any physiological or chemical or other causal nexus between the introduction of the virus and the subsequent immunity. The eminent authority, Dr. Foord Caiger, has rightly and generally stated that it is an established fact that one attack of an infective disease, whether acquired experimentally or naturally, confers security for a certain period upon the animal or person against subsequent infection. True; but the proposition is simply a *fact of observation*,

and the circumstance of small-pox immunity is ranged within the compass of that induction from experience merely as a particular instance. The relation, therefore, between vaccination and freedom from attack forms what the scientists term an empirical law, or to speak more strictly, an empirical uniformity; a uniformity, that is to say, which has not yet been resolved into an ultimate law where the *cause* itself is discerned; though he conceived that the uniformity in this case is free from the limitation incidental to physical empirical laws generally, by reason of the circumstance that the existence of this special uniformity can be relied upon universally. In short, regarded from the medical point of view, we only possess—to employ an old scholastic phrase—the *ratio cognoscendi*, but unhappily have not attained to the *ratio essendi*. This being the position, he accordingly founded this ascertained relation exclusively upon the basis of carefully-scrutinized and appropriate statistics such as those which Mr. Burridge had presented. And in support of his contention that the subject under discussion is entirely a statistical one, he cited two examples of the modern use by scientific men of purely statistical methods of research in biology and physics. Professor Karl Pearson has recently emphasized the fact that the entire doctrine of evolution and heredity rests in reality and ultimately upon a statistical basis; and Professor Clerk-Maxwell, in the investigation into molecules contained in his great treatise on Heat, relinquished the strict kinetic course of enquiry, and adopted the statistical method of considering the average number of groups of molecules selected according to their velocities. For the purpose of employing Mill's canons, he compared together the diagrams for England and Wales, Sweden and Prussia, where vaccination is compulsory, and found that the relevant curves approximated. If we now scrutinize the similar curves for Austria and Belgium we discover a congruity between them, but very different from the course of the former curves. Now, apply Mill's canon of the method of Agreement to the curves for England and Wales, Sweden and Prussia, and we find that (to follow his form of expression) the instances in which the phenomenon under discussion appears (that is to say, the diminution of mortality from small-pox) possess in common—in direct relation to the phenomenon in question—the one circumstance only of the compulsory introduction of vaccination, and we may, therefore, justly conclude, in pursuance of the principle of that method, that this common circumstance (namely, the compulsory requirement of vaccination) is the cause of the phenomenon of the diminution of mortality from small-pox. And now, instituting a comparison of the curves for England and Wales, Sweden and Prussia, with the curves for Austria and Belgium—the two sets of curves being congruent respectively between themselves—we are able to apply, and with final decisiveness, the canon of the method of Difference: that is to say, that the instances in which the phenomenon (or the diminution of mortality from small-pox) occurs and the instances in which it does not appear, possess, approximately speaking, every circumstance in common except one (namely, compulsory vaccination), this one existing only in the former set of observations—so that we



are entitled again to infer, in accordance with this canon, that the circumstance in which alone the two groups of instances differ (namely, compulsory vaccination) is the cause of the phenomenon of the reduction of small-pox mortality which we are investigating. He had already stated that, in applying these canons to extensive collections of statistics, like those of nations diverse in their racial character, though descended from a common stock, in their climatic conditions, their social habits, and their sanitary and medical supervision, difficulties must necessarily intrude. This fact, however, is incidental to all large accumulations of compared statistics, and seeing that we are dealing with nations which exhibit practically the same degree of civilization, all of which act upon modern methods of sanitation, possess an approximately similar environment, and are served with admirable medical skill and knowledge, and seeing further that the same spaces of time have been adopted for the investigation, it seems indubitable that we can properly regard the compared instances as congruous, and that we may contend, with convinced accuracy, that the statistics of Mr. BurrIDGE are not merely valuable and valid in themselves, but that his inferences are consistent with those which are deducible by the application of the strictly scientific tests which he had described and employed.

On the motion of the PRESIDENT, a hearty vote of thanks to Mr. BurrIDGE for his paper was carried by acclamation.

Mr. BURRIDGE, in reply, said he felt extremely grateful to the Institute for its generous reception of his paper. It had had the effect he hoped it might have, namely, of producing a debate of great educational influence on the subject. If he did not go into details of so many of the excellent speeches to which they had listened, and if he did not answer the exceedingly interesting remarks of the opener, it was out of no disrespect to the speakers, but because of the lateness of the hour. No one could be more aware of the difficulties of the subject than he was, but in the absence of strict mathematical proof, he took those implements which were readiest to his hands; and nothing he had heard in the discussion had weakened his faith in the curves which he had submitted in the paper. On the contrary, the very eloquent and trenchant remarks to which they had listened from Mr. Young confirmed him—and he was glad to have had such confirmation—in the use to which he had put the curves. From the discussion which had taken place, he ventured to think the Institute was capable of making a very strong pronouncement on the question, and was entitled to do so; and, as that pronouncement had been re-inforced by the high medical authorities who had spoken, he hoped it would have some practical results.

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## CORRESPONDENCE.

*To the Editor of the Journal of the Institute of Actuaries.*

## VACCINATION.

SIR,—The following table, giving the comparative results of the mortality from small-pox in London at three different periods, may be of interest to the readers of Mr. Burridge's valuable paper on vaccination.

*Mortality from Small-pox in London.*

Quinquennial Periods	Number of Deaths by Bills of Mortality	Annual Average	
1799–1803	7,762	1,552	
1809–1813	5,297	1,059	
1819–1823	3,390	678	
Year	Population of London	Deaths from Small-pox Average of Quinquennium	Mortality per 100,000
1801	958,863	1,552	162
1811	1,138,815	1,059	93
1821	1,378,947	678	49

Jenner's first publication on the subject was in 1798. It may, therefore, be fairly assumed that in 1801, when the first census was taken in England, the entire population of London was unvaccinated. In 1811, the period of the next census, a portion of the population would have been vaccinated, probably a small portion only. But in the next ten years the practice materially increased, so that in 1821, the date of the succeeding census, the proportion of the vaccinated must have been much greater. For the purpose of comparing the number of deaths with the population, the average number of the deaths in quinquennial periods has been taken, as particular years are sometimes subject to epidemics of disease.

The result, it will be observed, is that the number of deaths from small-pox per 100,000 of the population in London was, in 1801, 162; in 1811, 93; and in 1821, only 49.

This simple statement furnishes, I think, conclusive evidence of the marked diminution in the mortality from small-pox by the introduction of the practice of vaccination.

Yours obedient Servant,

ARTHUR H. BAILEY.

30 May 1902.

# THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

## *Summary of the Life Assurance and Annuity Revenue Accounts.*

[Extracted from the Parliamentary Return for 1901, published in 1902.]

I N C O M E	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year . . . . .	240,688,985	20,034,162	260,723,147
Premiums . . . . .	21,795,619	9,296,047	31,091,666
Consideration for Annuities . . . . .	1,716,027	6,986	1,723,013
Interest and Dividends (less Tax)	8,910,353	676,180	9,586,533
Increase in value of Investments . . . . .	279,905	807	280,712
Fines, Fees, &c. . . . .	13,480	1,285	14,765
Capital Paid-up . . . . .	41,255	41,366	82,621
Customs Timber Measuring, &c. . . . .	3,904	...	3,904
Transfers from other Accounts . . . . .	293,233	90,493	383,726
Miscellaneous . . . . .	25,678	4,445	30,123
	273,768,439	30,151,771	303,920,210
O U T G O	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims . . . . .	16,937,676	3,866,537	20,804,213
Cash Bonuses and Reduction of Premiums . . . . .	1,068,021	960	1,068,981
Surrenders . . . . .	1,125,455	38,059	1,163,514
Annuities . . . . .	1,775,793	7,858	1,783,651
Commission . . . . .	1,166,565	2,306,842	3,473,407
Expenses of Management . . . . .	1,825,516	1,640,051	3,465,567
Bad Debts . . . . .	5,390	771	6,161
Decrease in value of Investments . . . . .	266,586	9,672	276,258
Interest on Capital and Dividends and Bonuses to Shareholders . . . . .	434,018	477,263	911,281
Transfers to other Accounts . . . . .	278,193	50,074	328,267
Miscellaneous . . . . .	8,850	2,119	10,969
Balance* at the end of the Year . . . . .	248,876,376	21,751,565	270,627,941
	273,768,439	30,151,771	303,920,210

\* This Balance includes the whole of the Life and Annuity Funds (£266,608,306), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

## Summary of the Balance Sheets (1901).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . . .	11,729,334	1,670,290	13,399,624
Life and Annuity Funds . . .	246,129,803	20,478,503	266,608,306
Fire Funds of Companies trans-acting Life Business . . .	10,846,404	...	10,846,404
Marine Funds of Companies trans-acting Life Business . . .	672,498	...	672,498
Reserve Funds . . .	4,431,505	850,000	5,281,505
Other Funds . . .	1,829,737	215,435	2,045,172
Profit and Loss Balances . . .	4,533,249	...	4,533,249
Depreciation and Investment Balances . . .	1,271,062	18,549	1,289,611
Globe Annuitants (Liverpool and London) . . .	1,102,800	...	1,102,800
Outstanding Claims . . .	4,057,060	51,361	4,108,421
Outstanding Accounts . . .	760,420	23,403	783,823
Temporary Loans . . .	404,946	7,453	412,399
	287,768,818	23,314,994	311,083,812
ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages . . .	82,697,835	2,475,946	85,173,781
Loans on Policies . . .	13,188,380	46,097	13,234,477
„ Rates . . .	25,107,981	7,977,191	33,085,172
British Government Securities . .	5,812,769	1,905,561	7,718,330
Indian and Colonial Government Securities . . .	18,968,747	362,073	19,330,820
Foreign Government Securities . .	10,449,126	415,989	10,865,115
Debentures . . .	49,912,000	2,082,781	51,994,781
Shares and Stocks . . .	35,005,576	105,633	35,111,209
Companies' own Shares . . .	630,066	...	630,066
Land and House Property and Ground Rents . . .	22,842,864	6,139,131	28,981,995
Life Interests and Reversions . .	7,460,326	2,013	7,462,339
Loans on Personal Security . . .	1,280,143	7,941	1,288,084
Agents' Balances and Outstanding Premiums . . .	5,911,027	628,131	6,539,158
Outstanding Interest . . .	2,625,969	206,052	2,832,021
Cash, Deposits, Stamps, &c. . .	5,610,347	496,716	6,107,063
Customs Timber Measuring Balances, &c. . .	1,223	...	1,223
Deficiencies, Preliminary Expenses, &c. . .	264,439	463,739	728,178
	287,768,818	23,314,994	311,083,812



INCREASE (+) or DECREASE (—) in the Chief Items of this Year's  
SUMMARY as compared with the corresponding Items for the  
previous Year.

	Ordinary Companies	Industrial Companies
INCOME.	£	£
Premiums . . . . .	+ 536,433	+ 873,470
Consideration for Annuities . . . . .	— 422,987	— 647
Interest and Dividends (less Tax) . . . . .	+ 155,237	+ 60,677
Net Result of Realization and Re-valuation of Investments . . . . .	— 194,264	— 1,699
OUTGO.		
Claims . . . . .	+ 1,056,508	+ 455,895
Annuities . . . . .	+ 61,220	+ 490
Surrenders . . . . .	+ 119,732	+ 2,687
Commission . . . . .	+ 7,350	+ 187,843
Expenses of Management . . . . .	+ 46,625	+ 160,843
LIABILITIES.		
Paid-up Capital (including sundry Share- holders' Balances) . . . . .	+ 291,196	+ 37,666
Life and Annuity Funds . . . . .	+ 7,932,972	+ 1,708,455
ASSETS.		
Mortgages (including Loans on Rates) . . . . .	+ 3,356,262	+ 803,930
Life Interests and Reversions . . . . .	+ 221,645	+ 3
Loans on Policies . . . . .	+ 763,411	+ 7,961
British Government Securities . . . . .	+ 944,795	+ 42,691
Indian and Colonial Government Securities . . . . .	+ 543,573	— 19,056
Foreign Government Securities . . . . .	+ 293,739	+ 9,456
Debentures . . . . .	+ 783,530	+ 45,861
Shares and Stocks . . . . .	+ 532,295	+ 7,467
Companies' own Shares . . . . .	— 2,164	...
Land and House Property and Ground Rents . . . . .	+ 1,409,835	+ 504,599
Loans on Personal Security . . . . .	— 126,296	+ 1,003

#### NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 96, of which 81 have been classed as Ordinary, 11 as Industrial, and 4 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately. The returns of the United Provident are included for the first time.

During the year four names have been added to the Official List of Companies, namely, the Citizens' Life Assurance Company, Limited; Empire Guarantee and Insurance Corporation, Limited; Nelson and Company, Limited; and the Profits and Income Insurance Company, Limited; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies*  
ORDINARY BUSINESS.

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances	Net
	No.	Amount	No.	Amount	No.	Amount	Amount	Amount
ASSURANCES.		£		£		£	£	£
Whole Term of Life	793,885	377,871,245	146,458	69,885,620	940,343	447,756,865	26,939,271	420,817,5
Limited number of Premiums . . .	48,012	29,513,614	10,036	4,338,970	58,048	33,852,584	1,596,528	32,256,0
Endowments . . .	841,897	407,384,859	156,494	74,224,590	998,391	481,609,449	28,535,799	453,073,6
Endowment Assur- ances . . .	1,780	389,569	19,680	4,943,318	21,460	5,332,887	62,000	5,270,8
Joint Lives . . .	851,485	142,003,128	56,408	17,654,587	907,893	159,657,715	2,447,884	157,209,8
Last Survivor . . .	16,416	3,347,075	2,743	1,040,179	19,159	4,387,254	272,196	4,115,0
Contingent . . .	877	718,484	1,106	1,466,821	1,983	2,185,305	374,447	1,810,8
Issue . . .	37	70,363	4,091	6,121,060	4,128	6,191,423	1,508,200	4,683,2
Miscellaneous . . .	11	25,250	1,369	5,064,524	1,380	5,089,774	1,447,131	3,642,6
	2,472	1,368,633	7,992	10,137,850	10,464	11,506,483	2,048,391	9,458,0
	1,714,975	555,307,361	249,883	120,652,929	1,964,858	675,960,290	36,696,048	639,264,2
ANNUITIES.								
Immediate . . .	...	...	...	...	33,129	1,736,726	54,237	1,682,4
Deferred . . .	...	...	...	...	12,516	358,533	23,689	334,8
	...	...	...	...	45,645	2,095,259	77,926	2,017,3

## INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances	Net
	No.	Amount	No.	Amount	No.	Amount	Amount	Amount
ASSURANCES.						£	£	£
Whole Term of Life	...	...	...	...	18,067,991	175,029,996	1,180	175,028,8
Limited number of Premiums . . .	...	...	...	...	1,362	8,504	...	8,50
Endowments . . .	...	...	...	...	18,069,353	175,038,500	1,180	175,037,3
Endowment Assur- ances . . .	...	...	...	...	1,294,046	11,187,540	...	11,187,5
Joint Lives . . .	...	...	...	...	279,536	3,151,488	88	3,151,40
Contingent . . .	...	...	...	...	362,186	5,797,741	...	5,797,7
Miscellaneous . . .	...	...	...	...	4	1,910	400	1,51
	...	...	...	...	2	2,100	1,000	1,10
	...	...	...	...	20,005,127	195,179,279	2,668	195,176,61
ANNUITIES.								
Immediate . . .	...	...	...	...	63	1,949	...	1,94
Deferred . . .	...	...	...	...	7	178	...	17
	...	...	...	...	70	2,127	...	2,12

The above figures are based on Returns deposited, for the most part, during the last five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of one Company, namely, the Customs Fund, the amount of business at a more recent date has been included. The figures of the Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

## THE INSTITUTE OF ACTUARIES.

## THE BRITISH CENSUS OF 1901.

It is desirable that the *Journal* should contain some account of the communications which passed between the President (Mr. Higham) and the Registrar-General (Mr. MacLeod) as to the results of the recent Census; but so much of the exchange of opinion has been in the nature of friendly conversation that a detailed statement is impracticable.

The President invited the Registrar-General to hear Mr. Ryan's paper on Census Reform\* read, but exceptional pressure of business in connection with the Census returns prevented the Institute from having that pleasure, and the following letter was then written:—

19 December 1901.

DEAR SIR,—It was very gratifying to the Institute of Actuaries to learn from your kindly letter to me of the 26th ult. that the modifications in the system and procedure of census-taking that we think so important are receiving your personal attention.

Some of the desired changes will obviously require parliamentary sanction, but there are others which, so far as we are able to see, might be made with respect to the Census lately taken if they should fortunately commend themselves to your judgment. I would mention for instance the making public what the enumerated population is at each separate age, the explanation of the method of graduation adopted, and the publishing of the results in a form, if possible, that will be followed in due course by our Colonies and India, that statisticians may have less difficulty than at present in comparing one part of the world with another.

Possibly you would be willing to learn more fully, either from a deputation or in writing, what we are so anxious to see done. If so, I would with pleasure call on you, bringing a couple of other Fellows of the Institute with me, or address to you a communication setting out our views at greater length. But if the responsibilities of your position permit you to make these alterations, the gratitude is certain not only of those whom I have the honour to represent, but of many others who desire to see this country lead the way in this as in other matters.

I am, DEAR SIR,

Yours faithfully,

(Signed) C. D. HIGHAM.

REGINALD MACLEOD, Esq., &c.

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\* *J.I.A.*, xxxvi, 329.



This was very courteously acknowledged by the Registrar-General, with a request for information in writing, it being mentioned that it was too late to abstract ages in greater detail; and the annexed letter was accordingly sent, and the reply and rejoinder are appended:—

11 *January* 1902.

DEAR SIR,—I have delayed to thank you for your letter of the 20th ult. because I desired to confer with some of my colleagues as to its purport.

The item of chief importance referred to in mine of the 19th was the publication of figures representing the enumerated population at individual years of age, and if your statement that it is too late to concede this point must be regarded as final, I cannot forbear from saying that the decision would be a source of genuine disappointment to the actuarial profession as well as statisticians. Moreover, in such case, it would hardly be opportune to urge upon your further consideration the other matters, for they could well remain in abeyance until the larger issue received attention.

But since the difficulty as to abstracting from the present returns the figures for individual ages probably arises from a question of expense—and I should judge, I hope not rashly, of no very great expense—I do venture to bespeak your kind reconsideration of this matter, and to assure you of the grateful interest with which my profession, and, I am sure, all students of economics, would receive an intimation from you that you could see your way to adopt the suggestion I have ventured to press on your notice.

I am, DEAR SIR,

Yours very faithfully,

(Signed) C. D. HIGHAM.

REGINALD MACLEOD, Esq., C.B., &c.

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20 *February* 1902.

DEAR SIR,—Your letter of the 11th ultimo intimated that you thought the conference at one time suggested would serve no useful purpose. I therefore did not send any immediate reply. It may be well, however, to inform you that it is intended to publish, with the Census Report for England and Wales, a table giving figures representing—to use your own words—the enumerated population at individual years of age, but these figures will be calculated from the results abstracted in quinquennial periods, and will be distributed to each year by interpolation.

For the reasons why it is held this gives a nearer approach to accuracy than a presentation of the numbers arrived at by actual abstraction, I may refer you to the Report in vol. iv of the Census Returns of 1891.

Considerably more than half the abstraction in regard to ages for England and Wales has already been completed, and I could not—even if it were desirable—have the work re-done. The matter is not

only one of cost, but also of accuracy, since extreme sub-division tends undoubtedly to error. If you have any observations to make as to the method of graduation to be pursued, I need hardly assure you they shall be most favourably considered.

I remain,

Very faithfully yours,

REGINALD MACLEOD.

C. D. HIGHAM, Esq.

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25 *March* 1902.

DEAR SIR,—I am greatly obliged by your letter of the 20th ult. which has been carefully considered by my colleagues and myself, but the figures you propose to publish will hardly represent the population as enumerated, but rather form an adjusted table showing what are believed to be the facts—or in other words, if you will allow me to say so, the numbers as they ought to have come out rather than as they have.

It would be a greivous disappointment to the Institute of Actuaries, and I believe to others also, if the present Census were to pass by without any concession to what seems to be a legitimate appeal for further information, and I am constrained to ask if it is possible for you to see me on the subject, as I am loth to close this correspondence until everything has been done to further the object we have in view.

I am, SIR,

Yours faithfully,

(Signed) C. D. HIGHAM.

REGINALD MACLEOD, Esq., C.B., &c.

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A personal conference then took place between the Registrar-General, with whom were Mr. Noel Humphreys and Dr. Tatham, and the President, and, while it appeared to be impracticable to give the schedules at individual ages on the present occasion (though in India it had been possible, and for that country the graduation methods had been fully explained), it was gratifying to find a ready inclination to consider the views of the Institute, especially as to a quinquennial Census, a permanent department for that purpose, and the adoption of the card system; and the Registrar-General has been good enough to promise to publish in his Report the figures for each age in respect of five selected districts of varying character, each with a population of about 100,000. He proposes also to insert a note of caution as to the impossibility of trusting to the ages having been accurately returned—an opinion with which actuaries would not desire to

quarrel—and the profession and the public will thus have before them valuable and interesting data for comparison with the graduated figures and other useful purposes.

The Institute's Census Committee, meanwhile, will await an opportunity of raising the questions again when the proper time arrives.

## EXAMINATIONS OF THE INSTITUTE, APRIL 1902.

### EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

*Examiner*—PROF. S. L. LONEY, M.A.

*Supervisors*—MESSRS. B. A. BERRY, B.A., and S. G. WARNER.

#### *First Paper.*

1. In a race of a quarter of a mile A allows B 15 yards and C 40 yards start, and they all reach the goal together. How much start can B allow C when B has to run the whole distance?

2. The proprietor of a boarding school, having already 30 pupils, finds that an addition of 5 increases his gross yearly expenditure by £300, but diminishes the average cost per head by £1. What did his expenses originally amount to?

3. Find the H.C.F. of

$$x^4 - 16x^3 + 93x^2 - 234x + 216 \text{ and } 2x^3 - 24x^2 + 93x - 117.$$

Obtain the square root of  $\frac{11}{2} - 2\sqrt{6}$ .

4. Solve the equations—

$$(1) \quad \begin{aligned} bx - ay &= b^2, \\ (a - b)x + by &= a^2; \end{aligned}$$

$$(2) \quad x^2 - x + \sqrt{2x^2 - 3x + 5} = 15 + \frac{x}{2}.$$

5. Find the sum of  $n$  terms of an arithmetic progression whose first term is  $a$  and whose common difference is  $d$ .

Insert 10 arithmetic means between 8 and  $-5$ .

6. Find the number of permutations of  $n$  things taken  $r$  at a time.

10 similar presents are to be distributed amongst 6 children so that each child receives at least one present; in how many ways can the distribution be made?



7. Write down the general term [*i.e.*, the  $(r+1)$ th] in the expansion of  $(a+x)^n$  by the Binomial Theorem.

Obtain, in their simplest forms, the coefficients of  $x^9$  in  $\left(2 + \frac{3}{2}x\right)^{17}$  and of  $x^{2r+1}$  in the expansion of  $\left(x - \frac{1}{x}\right)^{2n+1}$ .

8. Define a logarithm, and prove that  $\log_a x = \log_b x \times \log_a b$ .

With the help of the tables, find the value of

$$\sqrt[5]{\frac{(78.39)^4 \div \sqrt{143.5}}{1.235 \div (.00037)^{\frac{1}{3}}}}$$

9. Define Probability, and show how it is mathematically measured.

A pack of 52 playing cards is dealt out at random (with the cards face downwards) into 4 sets of 13 cards each. The top card of one of these sets is lifted and found to be the ace of hearts. Show that the chance that the king, queen, and knave of hearts are in this same set is about  $\frac{1}{9.5}$ .

10. A has a dice-box containing 4 dice, with the faces numbered 1 to 6, and B has a dice-box containing 3 dice similarly numbered. They throw in succession, A commencing, and the one that first throws 10 receives the stake of £1. Find the values of their respective expectations.

11. Find the value of  $\Delta^n u_x$ , where  $u_x$  is equal to

$$x(x+1)(x+2) \dots (x+m-1),$$

and show that the  $n$ th difference of a rational integral algebraic function of the  $n$ th degree in  $x$  is constant.

12. When  $m$  consecutive equidistant values of a function  $u_x$  are given, find an approximate general expression for it.

Find a rational integral function of  $x$  which for the values  $x=2, 3, 4, 5$  shall have the respective values 2, 14, 40, 86.

### *Second Paper.*

13. A man pays insurance on his life amounting to one-tenth of his income; after paying income-tax at the rate of 1s. 2d. in the £ on the remainder of his income he has £1,066. 8s. 9d. left; what was his gross income?

14. Find the condition that the expression  $ax^2 + bx + c$  may retain a constant sign for all values of  $x$ .

If  $a$  be positive and less than 2, show that the equation

$$(x+3)(1-ax) - (a+1)(x+3) = 2$$

has no real roots.

15. If  $A$  vary as  $B$  when  $C$  is constant, and if it vary as  $C$  when  $B$  is constant, show that it varies as  $BC$  when both  $B$  and  $C$  vary.

Three circular coins of radii  $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$  of an inch, and of thicknesses  $\frac{1}{20}$ ,  $\frac{1}{10}$ , and  $\frac{3}{20}$  of an inch respectively, are melted together and formed into a circular coin  $\frac{1}{8}$  of an inch in thickness. Assuming that the areas of circles vary as the squares of their radii, find, to three places of decimals of an inch, the radius of this single coin.

16. If  $a, b, c, d$  are in G.P., prove that

$$(a+b+c+d)^2 = (a+b)^2 + (c+d)^2 + 2(b+c)^2.$$

Sum, to  $n$  terms, the series

$$x + 2x^2 + 3x^3 + 4x^4 + \dots$$

17. Find the sum of the squares of the terms of the series  $1, 4, 7, \dots, 3n-2$ .

18. Assuming the truth of the Binomial Theorem for a positive integral index, prove its truth, with a certain restriction, for any index.

Show that the sums of the two infinite series—

$$1 + \frac{3}{8} + \frac{3.9}{8.16} + \frac{3.9.15}{8.16.24} + \frac{3.9.15.21}{8.16.24.32} + \dots$$

and

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

are equal.

19. Prove that  $\log_a m^n = n \times \log_a m$ .

A wine seller has 40 gallons of wine; as soon as he has sold half a gallon he mixes with the remainder half a gallon of water; how often can he repeat this process before the amount of wine in the mixture is less than half of the whole?

20. Assuming the truth of the Exponential Theorem, prove that

$$\frac{1}{2} \log_e \frac{1+x}{1-x} = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \text{ad inf.}$$

Find the value of  $\log_e \frac{101}{99}$  to 9 places of decimals, and show how the value of  $\log_{10} \frac{101}{99}$  could be obtained.

21. Show that the product of any number of positive quantities, whose sum is given, is greatest when the quantities are all equal.

22. Explain what is meant by Inverse Probability.

A bag contains 8 balls, each of which is equally likely to be white or black; three balls are drawn and turn out to be two white and one black; these three balls are then removed, and another ball is drawn; show that it is an even chance that this is a white ball.

23. Prove the formula—

$$u_n = u_0 + n \cdot \Delta u_0 + \frac{n(n-1)}{1 \cdot 2} \cdot \Delta^2 u_0 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \Delta^3 u_0 + \dots + \Delta^n u_0$$

Given  $u_0 = 89,685$ ,  $u_1 = 88,994$ ,  $u_2 = 88,294$ , and  $u_3 = 87,585$ , find the value of  $u_9$ .

24. Find the integral of an expression of the form—

$$\frac{1}{u_x \cdot u_{x+1} \cdot u_{x+2} \cdot \dots \cdot u_{x+m-1}},$$

where  $u_x = ax + b$ .

Sum to  $n$  terms the series  $\frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \frac{1}{4 \cdot 6} + \dots$

### EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

*Examiners*—MESSRS. J. E. FAULKS, B.A., W. HUTTON, O. KENTISH,  
and A. W. TARN.

#### *First Paper.*

1. Obtain, from first principles, the system of equations—

$$S = P(1+i)^n = P\left(1 + \frac{j}{m}\right)^{mn} = Pe^{n\delta}$$

and explain the meaning of each of the symbols occurring therein. ✓

Give the corresponding equations if the operation of discounting instead of that of accumulating be considered, and show by a verbal explanation that, for the same effective rate of interest, the forces of interest and discount are equal.

2. Find an expression for the present value of an annuity-certain where the reproductive rate differs from the remunerative rate, and show how the successive payments of such an annuity are divisible between interest and principal.

Discuss the terms upon which such a contract may be allowed to be terminated during its currency when such termination is at the desire of (a) the person paying (b) the person receiving the annuity. ✓

3. A life assurance office has on its books at the beginning of a year  $m$  policies of assurance on the lives of as many persons each aged  $x$ . Find expressions for the most probable number of deaths and the expected deaths.

How would you approximate to the “expected claims” in the cases of:—

- (a) Joint life policies,
- (b) Policies on the survivor of two lives,
- (c) Term policies.



4. Explain clearly the difference between  $\frac{T_0}{l_0}$  and  $\frac{2Y_0}{T_0}$ . What does  $\frac{T_x - T_{x+n}}{l_x}$  represent?

A community otherwise stationary is subject for  $n$  years to an annual increase from immigration at age  $x$  to the extent of 10 per-cent of the number who attain that age. Show how to ascertain the effect of this immigration upon the average age at death at the end of the  $n$  years. What practical consideration would vitiate your result?

5. Express, in terms of the D and N columns and the rate of discount, the annual premiums for

- (i) an endowment assurance to mature in  $n$  years,
- (ii) a whole-life assurance, premiums limited to  $n$  payments.

Subtract the second from the first and give a verbal interpretation of the result.

Prove by general reasoning that  $P_x = v - \frac{a_x}{1 + a_x}$ .

6. How would you calculate a premium conversion table where  $\bar{a}_x$  is the argument and  $\bar{A}_x$  the result?

What would be the result of entering such tables calculated for single and annual premiums with  $\bar{a}_n$ ; and what does  $\frac{1}{\bar{a}_x} - \delta$  represent?

7. Find an expression for the value of an annuity of 1 payable during the joint lives of  $(x)$  and  $(y)$ , and for  $t$  years longer should the survivor live so long.

8. Obtain a formula for the office annual premium for an endowment assurance policy on  $(x)$  to mature in 20 years, the premium to be based on select tables and to provide for a compound reversionary bonus of  $p$  per-cent per annum declared quinquennially, with interim bonuses at the same rate after the first five years, and the loading to provide for an initial commission of  $k$  per-cent on the sum assured spread over the whole term, a constant of  $l$  per-cent on the sum assured and a percentage of  $m$  on the gross premium.

9. At the commencement of a certain year a company has on its books  $l_x$  persons who have been assured for 1 each for  $n$  years, and who will be subject throughout the year to a special rate of mortality  $q_x + w_x$ , and in respect of the claims which occur amongst them the company undertakes to pay only the reserve values at the end of the year. On the assumption that the extra mortality will cease at the end of the year and that it will not prejudicially affect the lives remaining assured, state under what conditions the company will make a profit from the arrangement, and find an expression for the amount of such profit. (Assume the premiums due at the beginning of the year.)

10. From the returns of two censuses taken at an interval of 10 years—which give the numbers living for quinquennial groups of ages 0–5, 5–10, &c.,—and a register of the deaths for each of the 10 years, explain how you would proceed to form a mortality table. State what you consider to be essential requirements as regards data for a mortality experience, and in what respects objection may be taken to the employment of population returns for that purpose.

11. Give formulas, for purposes both of calculation and verification, for the construction of Tables of  ${}_nV_x$ .

Show how this function can be readily tabulated by means of the Arithmometer.

12. Find the differential coefficients of  $x^n$  and  $\log_e x$ , and the integrals of  $e^x$  and  $a^x$ .

Demonstrate the formula

$$(IB) = -(1+i) \frac{dB}{di},$$

and apply it to find the present value of an ordinary increasing annuity for  $n$  years certain.

### *Second Paper.*

[“*A Short Collection of Actuarial Tables*” will be supplied, and should be used in answering Questions Nos. 13, 14, 15, 19, 21 and 22.]

13. A borrower desirous of raising £10,000 upon security of a depreciating nature is offered the money from different sources on the following terms respectively, namely—

- (a) Interest 4 per-cent, the loan to be repayable by equal annual instalments over 20 years;
- (b) An annuity of £743. 19s. 8d. to be paid for 20 years to repay the loan with interest;
- (c) Interest  $3\frac{3}{4}$  per-cent, a sinking-fund policy to mature at the end of 20 years to be effected for the amount of the loan in an office which calculates its rates of premium on a net  $2\frac{1}{2}$  per-cent basis.

Compare fully the three offers from the borrower's point of view.

14. A foreign corporation issues a loan of £390,000 4 per-cent bonds, repayable by annual drawings as follows—

£10,000 at the end of 5 years			
11,000	„	„	6 „
12,000	„	„	7 „

and so on, till the whole is repaid. The issue price being  $94\frac{1}{2}$  per-cent, what rate of interest is paid by the corporation?

15. Write down formulas for all the possible combinations of the probability of dying in or surviving a year among three lives, and prove the truth of your answer.

The probability that two persons aged respectively 20 and 40 will not both be alive at the end of 20 years is .38823. Out of 96,223 persons alive at age 20, 6,358 die before they attain age 30. Find  ${}_{30}q_{30}$ .

16. Give an algebraical proof that—

$$a_x = \sum v^n (1+i)^n \frac{(1+i)^n - 1}{i} {}_{n-1}q_x - 1$$

17. State what assumptions are made with regard to the sum assured in the respective symbols  $A_x$ ,  $A_x^{(m)}$ , and  $\bar{A}_x$ , and with regard to the annuity in the symbols  $a_x$ ,  $a_x^{(m)}$ , and  $\bar{a}_x$ ; and show how the former symbols can be expressed in terms of the corresponding annuities.

18. Show that in a mortality table which follows Makeham's law the value of an annuity on  $m$  joint lives may be expressed as the value of an annuity on a single life at a different rate of interest.

19. A proposal for assurance on a life aged  $x$  has been accepted by an office at the premium applicable to age  $x+m$ . The proposer wishes, however, to pay the normal premium at age  $x$ , and to take a policy subject to a debt of  $\kappa$  if he should die in the first year,  $\kappa\left(1 - \frac{1}{n}\right)$  if he should die in the second year, and so on, until the  $n$ th year, after which the full sum assured would become payable. Obtain a formula for  $\kappa$ , using net premiums and ignoring any question of loading.

Having given  $x=32$ ,  $m=3$ ,  $n=30$ , find  $\kappa$  at 3 per-cent interest.

20. Show how  $\pi_x$  may be transformed into

$$vq_{x+n}(1 - {}_nV_x) + vp_{x+n}({}_{n+1}V_x - {}_nV_x) - (1-v){}_nV_x,$$

and give a verbal interpretation of the result.

21. Under a certain "Reversible Premium" scheme, an office undertakes to grant whole-life assurances under which, after  $n$  years, the company pays the premiums to the assured instead of receiving them from him, and this continues until the life drops, when the sum assured becomes payable. If  $n=20$ , find the office annual premium at age 40 by the Text-Book Table at 3 per-cent, allowing for a loading of 15 per-cent of the net premium and 5s. per-cent on the sum assured.



22. Find, without using commutation columns, an expression for the annual premium for an assurance to be payable only in the event of both of two lives, now aged  $x$  and  $y$  respectively, dying within  $n$  years.

How would you in practice determine the office annual premium for an assurance to be payable only if the survivor of (25) and (30) dies within five years?

23. Deduce Lagrange's Interpolation Formula.

Having given  $a_{65,65,65} =$  at 3 per-cent, 4.254,  
 „  $3\frac{1}{2}$  „ 4.167,  
 „ 4 „ 4.084,  
 „ 5 „ 3.925,

find the value at  $4\frac{1}{2}$  per-cent.

24. Explain verbally how, by means of approximate summation formulas, the integral calculus may be applied to find the values of isolated assurance and annuity benefits.

Write down the integrals corresponding to

$$\bar{A}_{xyz}^2 \quad \bar{A}_{x:yz}^1 \quad \bar{a}_{x|yz}$$

and draw up a schedule showing in detail how you would calculate the value of the first of these by one of G. F. Hardy's formulas of summation.

### EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

*Examiners*—MESSRS. H. W. ANDRAS, F. T. M. BYERS, W. P. PHELPS, M.A.,  
and E. A. RUSHER.

#### *First Paper.*

1. Show how a Mortality Table may be constructed from Census returns. How would you deal with years 0 to 10?

2. Give a description of the Graphic method of graduation. What objections have been made to it, and under what circumstances would you consider it a suitable method to employ? How would you apply it to the graduation of a Mortality Table constructed from a limited experience?

3. What do you consider the most suitable method to employ in graduating "Select Life" Tables of Mortality, and how would you proceed?

What modification in the formula would be necessary if it were desired to graduate according to Makeham's law?

4. In what respects do the rates of Sickness exhibited in Mr. Sutton's Friendly Societies' Experience Table, Males 1876-80, differ from the rates shown in the Manchester Unity 1866-70 Rural, Town, and City Table?

5. Mr. Woolhouse's formula for graduation may be expressed:

$$\begin{aligned} V_x = & \cdot 2u_x + \cdot 192(u_{x-1} + u_{x+1}) + \cdot 168(u_{x-2} + u_{x+2}) \\ & + \cdot 056(u_{x-3} + u_{x+3}) + \cdot 024(u_{x-4} + u_{x+4}) \\ & - \cdot 016(u_{x-6} + u_{x+6}) - \cdot 024(u_{x-7} + u_{x+7}) \end{aligned}$$

What do you consider the advantages and disadvantages of his method?

Prove that if the third differences of the original data are constant the formula reproduces the original data.

6. Give a brief history of English Life Table No. 5, and state its distinctive features as compared with those of English Life Table No. 3.

7. What general deductions may be made from the unadjusted data of the "Institute and Faculty" Mortality Experience of Assured Lives?

What, in your opinion, would be the general effect of substituting the  $O^M$  Table of Mortality for the  $H^M$  Table of Mortality in calculating the reserves of an ordinary Life Assurance Company at 3 per-cent interest? Give reasons.

8. Describe the re-insurance method of valuation proposed by Mr. J. A. Higham, and state any feature of note in the reserve made by that method.

9. Describe Dr. Sprague's method of distribution of surplus, and its characteristics. What modifications of the plan have been suggested, and what are their advantages?

10. A Company values its Endowment Assurances in groups, all policies in each group having the same unexpired term of years to run before maturing; the premiums on a large number of these policies cease some years before the Sum Assured is payable, and it is considered undesirable to treat them separately. The groups are valued as if in all cases the premiums were payable throughout the term.

Suggest a simple means of adjustment which would allow for the inclusion of these limited payment cases in the groups.

11. An office proposes to issue Whole-Life Assurance policies under which the premiums are to be returned in respect of any period of disablement from disease or accident after the first two months of such disablement. How would you calculate the premium for such a benefit, and what data would you employ?

What modification would you suggest should be made for the period of old age?

12. How would you calculate tables of premiums for Whole-Life Assurances, Endowment Assurances, and Children's Endowments, in : (1) an Industrial Assurance Company ; (2) a Friendly Society ?

What are your views as to taking into account rates of lapsing ?

13. What various methods have been adopted in practice for valuing the Stock Exchange Securities in the balance-sheet of a Life Office, and what are the advantages and disadvantages of each ? Which method would you adopt, and for what reason ?

How would you deal with "wasting" securities ?

*Second Paper.*

14. A Life Assurance Company values quinquennially all Life Assurance Contracts on the  $H^M$  3 per-cent basis for the purpose of ascertaining the gross surplus, and at each valuation carries forward a moiety of the resulting gross surplus to the next valuation

What general effects will this have on the amount of the divisible surplus from time to time ?

15. A fund nominally of £10,000, producing £360 per annum, is settled as regards the income to three lives aged respectively 30, 35, and 40, with benefit of survivorship, the absolute reversion being to a fourth person. The four persons have agreed to bring the fund, which is represented by Stock at 90, into possession : indicate how you would apportion their respective shares.

16. How would you calculate the following scales of office premiums for an Ordinary Life Assurance Company, and what tables of mortality, rates of interest, and methods of loading, would you adopt ?

(a) Deferred Assurances for Children, commencing after age 21, all premiums paid being returned in case of death before that age.

(b) Children's Endowments payable at 21, with return of premiums paid at previous death, all premiums to cease at the death of the parent.

17. Upon what principles would you make a valuation of the liabilities of a Company A with a view to its transfer to a Company B, the policyholders in Company A to participate in future in bonuses declared by Company B ?

18. How would you ascertain the gain or loss from mortality in an Annuity Society ?

19. How would you calculate the following ?

(a) Single premium for a Reversionary Annuity to a married woman, to commence at the death of her husband, both lives being resident in the West Indies.

(b) The annual premium for a Contingent Assurance on  $x$  against  $y$ ,  $x$  having just been accepted for a Whole-Life Assurance at an extra premium of 10s. per-cent for residence and occupation.



20. A, a male, aged 21, is entitled to an estate in fee simple of the value of £250,000 if he survive B, a female, aged 60, and C, aged 70, provided that the latter, a bachelor, die without leaving issue. The estate is charged with certain annuities, namely, £1,000 per annum to each of D, a female, aged 40, and E, a male, aged 50, during their joint lives, and £1,500 per annum for the life of the survivor. How would you find the market value of A's interest?

21. Describe Mr. G. F. Hardy's method of ascertaining the profit or loss from mortality in a Life Assurance Company. What do you consider the principal objections to it, and how would you propose to overcome them in practice?

22. State the formulæ you would use in a net premium valuation for the net premiums, reversions, and annuities in valuing a large number of Whole-Life Assurance and Endowment Assurance Contracts issued at true half-yearly and quarterly premiums.

23. What are the essential differences between the "Pure" and the "Gross" premium methods of valuation as employed at the present time? Under what circumstances would you consider the latter the preferable method to adopt?

24. A trust-fund invested in first-class Securities of the value of £21,000 is to be divided on the death of a gentleman (A) and his wife, both aged 55 and in good health, among all the children of A by his present or any future wife, who shall survive him or attain 21 in his lifetime: A has three children all now living.

Give your opinion of the market value of the reversion of the eldest son, aged 25, and state what proportion you would recommend to be advanced on its security, by way of mortgage, and at what rate of interest.

25. A, aged 40, is entitled as tenant-in-tail in remainder to an estate of the value of upwards of £50,000, expectant on the death of the survivor of B, aged 65, and C, aged 60, who have successive life interests. A desires to borrow £3,000, but is unable to keep down interest. B is moribund and incapable of joining in the transaction. C is willing to assist A to raise the money on the most economical terms, and for this purpose will covenant for the payment of premiums and interest after the death of B, assigning her reversionary life interest as security for her covenant. It is proposed that A should give a charge, redeemable within three years, on repayment of capital and premiums, with compound interest. It is also suggested that the necessary assurances, which are to be free from all restrictions as to residence and occupation, should be partly immediate, under reduced premiums for the first three years, and partly deferred, and that a single premium should be paid for the privilege of deferring part of the risk.

State on what terms you would recommend the advance to be made, and investigate formulæ for ascertaining the amount of the ultimate annuity, redemption money, and policies required?

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW  
(PART IV).

*Examiners*—MESSRS. L. F. HOVIL, W. HUGHES, G. MARKS, and  
G. TODD, M.A.

*First Paper.*

5 1. Describe briefly the form of a mortgage. What courses are open to a mortgagee of real property in the event of failure on the part of the mortgagor to pay interest?

Discuss the power of a first mortgagee to compound interest on his security as against a subsequent mortgagee.

0 2. An office is asked to settle a claim without retaining possession of the deeds of title. Explain the present legal position in respect of an "acknowledgment of right to production" and an "undertaking for safe custody" of the deeds.

0 3. Discuss the legal position and responsibilities, under the Life Assurance Companies Acts, of life assurance offices, mutual and proprietary, in relation to Sinking Fund policies.

5 4. What considerations would guide you in advising whether a projected amalgamation between two insurance offices is necessary or desirable, and what enquiries would you make in order to ascertain whether or not such amalgamation is feasible?

5 5. What proof of title would you require before paying a claim to the representatives of a person who died domiciled elsewhere than in the United Kingdom?

Do the provisions of the Colonial Probate Act affect the titles to policies of the representatives of persons who die domiciled in British possessions? Give reasons for your answer.

6. What particulars would you require if you were asked to advise as to the establishment of a staff pension fund to give the following benefits, in exchange for a contribution of 5 per-cent of the salaries?

8 (a) A Pension on the attainment of age 65, such pension being based on the average salary throughout service.

(b) Return of all contributions on death or retirement through ill-health before 65.

(c) Return of half contributions on withdrawal before 65.

How would you ascertain what pension would be provided by the suggested contribution?

0 7. Describe the methods you would adopt, and the Ledger Accounts you would open, to deal conveniently with investments and interest in the books of a Life Office, referring to any entries necessary at the close of a financial year.

8. Enumerate the points to be taken into account in considering the advisability of extending the operations of a Life Office to a Colony or Foreign Country.

9. What is the legal relation between banker and customer?

What is meant by a special endorsement of a cheque payable "to order", and what is its legal effect?

10. What are the advantages and disadvantages attaching to the investment of funds in purchase, or on mortgage of freehold and leasehold ground rents? What points must be specially taken into consideration in selecting them for investment?

11. The Directors of a Life Office which has been established about 20 years have under consideration a proposal to make all ordinary life policies payable at age 80 or at previous death. Draft a report on the subject for the consideration of the Directors.

12. Discuss the merits of the following securities as investments for a Life Assurance Company:

- (a) British Government Funds.
- (b) American Railroad Securities.
- (c) Colonial Government Stock or Bonds.
- (d) British Railway Ordinary and Debenture Stock.
- (e) Stock of Gas and Water Companies.
- (f) Municipal Stock.

### *Second Paper.*

13. Define and distinguish between joint tenants, tenants in common, and coparceners. How is a partition of a joint tenancy of real property effected?

14. Define the powers of leasing exercisable (a) by the life tenant of settled property and (b) by the mortgagee in possession of a life interest.

15. What special covenants should be inserted in (a) a mortgage of a policy of life assurance, (b) an assignment of a life policy to trustees of a settlement?

16. Draft a form of declaration to be signed by a proposer for life assurance, and a form for notifying acceptance of the risk. Discuss the intention and legal effect of your wording.

State the effect of the judgment of the Court of Appeal in the case of *Roberts v. The Security Company*.

17. What considerations would influence you in determining the bases on which a Friendly Society granting the usual benefits should be valued?

How would you deal with the surplus or deficiency shown by the valuation?



5 18. The Directors of an old-established manufacturing Company employing about 500 men, wish to provide pensions of 15s. per week at age 65 to all present and future workmen who on attaining that age have been in their service 20 years or more. The Directors are willing to contribute £1,000 a year for this purpose.

The only data available are the present ages and length of service of the men now in the Company's employment.

You are asked to advise whether the proposed contribution of £1,000 per annum would be sufficient for the purpose contemplated, and also as to the best means of applying it in order to effect that purpose. How would you proceed?

5 19. Enumerate and distinguish between the various classes of Friendly and Benefit Societies. What advantages, if any, attach to the Affiliated Orders as compared with other classes?

3 20. What system would you adopt to verify the income from (a) premiums, (b) interest and dividends of a Life Assurance Company? Draft the headings of the books you would require.

3 21. What has been the effect upon the finances of the Indian Government of the fall in the gold price of silver which began in 1873?

What measures has the Government taken to place the currency of India upon a firm basis, and what has been the effect of those measures?

8 22. What is the "Consolidated Fund", and what was its origin? Why are the Funded and Unfunded Debts so called, and of what items do they respectively consist?

7 23. Draft a short report to the Directors of a Life Assurance Office, the funds of which are large and rapidly increasing, outlining a financial policy and suggesting directions in which new investments should be sought in future.

24. Explain the statements and references contained in the following sentences, with any comments which occur to you.

8 "Money was not quite so abundant to-day, but the discount market was still easy with a further decline in quotations, a reduction in the Bank rate to-morrow being expected in some quarters. The increasing demand for bar gold, however, the price of which has risen to-day to 77s. 10 $\frac{1}{4}$ d., as well as the continued low level of the Paris cheque, which remains at gold point, and the decline in other Continental exchanges, have not, perhaps been sufficiently considered by those who are inclined to take bills freely at present rates. Should the Bank decide to maintain the present official minimum to-morrow, it will not be very long before the income-tax payments begin to reduce the market supply of money to an extent which would probably bring the open market rate close up to that of the Bank."—(*Times*.)

## PROCEEDINGS OF THE INSTITUTE.—SESSION 1901-1902.

*First Ordinary Meeting, 25 November 1901.*

The first ordinary meeting of the Session 1901-1902 was held at the Hall of the Institute, on the 25th day of November 1901.

The President (Mr. C. D. HIGHAM) in the Chair.

A paper entitled "The Case for Census Reform", by Mr. G. H. Ryan, was read by the author.

The following gentlemen took part in the discussion:—Messrs. A. F. Burrige, J. H. Barnes, A. H. Bailey, G. King, G. F. Hardy, Dr. Dudfield (a visitor), and the President.

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*Second Ordinary Meeting, 30 December 1901.*

The President (Mr. C. D. HIGHAM) in the Chair.

Messrs. Walter Denham, F.F.A., and A. Graham Donald, F.F.A., were duly elected Associates, and Mons. G. H. Adan was also duly elected a Corresponding Member of the Institute.

A paper entitled "An Enquiry into the Methods and General Principles to be adopted in calculating the Rates of Contribution, or Percentage Deductions from Salary, in the case of Superannuation Funds and Pension Funds generally", by Mr. James McGowan, was read in abstract by Mr. Woods (Hon. Secretary).

The following gentlemen took part in the discussion:—Messrs. E. C. Thomas, T. G. Ackland, S. J. H. W. Allin, H. W. Manly, G. King, and T. E. Young.

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*Third Ordinary Meeting, 27 January 1902.*

The President (Mr. C. D. HIGHAM) in the Chair.

The Essay on "The Actuarial Aspects of Recent Legislation, in the United Kingdom and Other Countries, on the subject of Compensation to Workmen for Accidents", to which the first and only prize was awarded from the Brown Prize Fund in the year 1900, was read in abstract by the author, Mr. John Nicoll.

The following gentlemen took part in the discussion:—Messrs. L. Stahlschmidt, F. B. Wyatt, A. Hewat, H. E. W. Lutt, A. W. Watson, S. G. Warner, and W. Hughes; also Messrs. R. T. Thompson and C. H. Green (visitors), and the President.

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*Fourth Ordinary Meeting, 24 February 1902.*

The President (Mr. C. D. HIGHAM) in the Chair.

A paper entitled "Some Notes on the Net Premium Method of Valuation, as affected by recent Tendencies and Developments", by Mr. S. G. Warner, was read by the author.

The following gentlemen took part in the discussion:—Messrs. B. A. Berry, H. W. Manly, J. Chisholm, A. R. Barrand, T. E. Young, R. Todhunter, J. B. Gillison, A. Levine, A. G. Hemming, and G. King.

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*Fifth Ordinary Meeting, 24 March 1902.*

The President (Mr. C. D. HIGHAM) in the Chair.

Mr. H. F. Falconer, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "The British Offices Life Tables, 1893: an Investigation of the Rates of Mortality in different Classes of the Assurance Experience, and of the resulting Net Premiums and Policy Reserves", by Mr. T. G. Ackland, was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. L. F. Hovil, H. E. Nightingale, G. F. Hardy, G. King, A. Levine, J. Chatham, A. Moorhouse, and H. W. Manly.

*Sixth Ordinary Meeting, 28 April 1902.*

The President (Mr. C. D. HIGHAM) in the Chair.

A paper entitled "Vaccination and the Act of 1898", by Mr. A. F. Burridge, was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. J. D. Watson, G. H. Ryan, S. G. Warner, and T. E. Young; also Drs. Symes Thompson, Garrett Anderson, Copeman, and Newsholme (visitors).

*The Fifty-fifth Annual General Meeting, 2 June 1902.*

The President (Mr. C. D. HIGHAM) in the Chair.

The proceedings at the Annual General Meeting will be found on page 342.

REPORT, 1901-1902.

The Council have the pleasure to report to the members upon the progress of the Institute during the session of 1901-1902, the fifty-fourth year of its existence.

There has been an *increase* of 24 in the number of members, as compared with the previous year. 119 candidates have been admitted as Probationers, and 53 as Students conditionally on their passing Part I of the Examination. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of members was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

On 31 March	1896, 788,
"	1897, 826,
"	1898, 860,
"	1899, 834,
"	1900, 822,
"	1901, 818,
"	1902, 842.



The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

*Schedule of Membership, 31 March 1902.*

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1901 .	1	204	271	324	18	818
ii. Withdrawals by						
(1) Death . . .	...	3	1	1	2	42
(2) Resignation . . .	...	1	2	12	...	
(3) Default in pay- ment of Sub- scriptions . . .	...	...	3	17	...	
	1	200	265	294	16	776
iii. Additions to Membership						
(1) By Election . . .	...	...	4	...	9	66
(2) By Order of Council	...	...	...	53	...	
(3) By Re-instatement	...	...	...	...	...	
	1	200	269	347	25	842
iv. Transfers						
(1) By Examination:						
<i>from Associates</i>	...	...	14	...	...	...
<i>to Fellows</i> . . .	...	14	...	...	...	...
	1	214	255	347	25	842
(2) By Examination:						
<i>from Students</i>	...	...	...	2	...	...
<i>to Fellows</i> . . .	...	2	...	...	...	...
	1	216	255	345	25	842
(3) By Examination:						
<i>from Students</i>	...	...	...	13	...	...
<i>to Associates</i> . . .	...	...	13	...	...	...
v. Number of Members in each class on 31 March 1902 .	1	216	268	332	25	842

The Council have, with great regret, to report the loss by death of one Fellow, Mr. F. Marchbank; two Associates, Mr. J. P. Bourne and Mr. C. Povah; one Student, Mr. W. Lamb Daughtrey, Jun.; and one Corresponding Member, Mons. E. A. Badon-Pascal.

The Annual Subscriptions, together with admission and other fees, amounted to £1,781. 17s., showing a slight increase as compared with those of the previous year. The total Income for the year was £2,389. 7s. 6d., and the total Expenditure £2,407. 15s. 7d. The Revenue Account and Balance Sheet are given herewith (p. 339). The slight diminution in the amount of Funds is more than accounted for by the donation to the Queen Victoria Memorial Fund, irrespective of other exceptional payments.

The stock in hand of the Institute publications on 31 March was as follows :

No. of Copies	Description of Work
11,747 . . . .	Parts of <i>Journal</i> .
490 . . . .	Index to Vols. 1 to 10.
1,001 . . . .	„ to Vols. 21 to 30.
1,463 . . . .	<i>Text-Book</i> , Part I (New Edition).
720 . . . .	Government Joint-Life Annuity Tables.
798 . . . .	Select Life Tables.
662 . . . .	A Short Collection of Actuarial Tables.
281 . . . .	Messenger Prize Essay (Friendly Societies).
19 <i>in cloth</i> }	{ Lectures on Finance and Law (Clare and
3,045 <i>in paper</i> }	
1,718 . . . .	Lectures on the Companies Acts (A. C. Clauson).
1,717 . . . .	Lectures on the Law of Mortgage (W. G. Hayter).
42 . . . .	Logarithm Cards.
886 . . . .	Transactions of the Second International Congress of Actuaries.

The following papers were submitted at the sessional meetings of the Institute, namely :

- 25 *November* 1901.—“The Case for Census Reform”—Mr. G. H. Ryan.
- 30 *December* 1901.—“An Enquiry into the Methods and General Principles to be adopted in calculating the Rates of Contribution, or Percentage Deductions from Salary, in the case of Superannuation Funds and Pension Funds generally”—Mr. James McGowan.
- 27 *January* 1902.—“The Actuarial Aspects of Recent Legislation, in the United Kingdom and other Countries, on the subject of Compensation to Workmen for Accidents”—being the Essay by Mr. John Nicoll, to which the First and only Prize was awarded from the Brown Prize Fund in the Year 1900.
- 24 *February* 1902.—“Some Notes on the Net Premium Method of Valuation, as affected by recent Tendencies and Developments”—Mr. S. G. Warner.
- 24 *March* 1902.—“The British Offices’ Life Tables, 1893: an Investigation of the Rates of Mortality in Different Classes of the Assurance Experience, and of the resulting Net Premiums and Policy-Reserves”—Mr. T. G. Ackland.
- 28 *April* 1902.—“Vaccination and the Act of 1898”—Mr. A. F. Burridge.

For the Examinations held in the United Kingdom and the Colonies on 18, 19, 21, and 22 April last, 250 entries were received, namely :

109	for Part	I.
85	„	II.
37	„	III.
19	„	IV.

The results of the Examinations will be duly announced.\* The Council warmly acknowledge the valuable services of the Honorary Examiners and Supervisors.

His Royal Highness the President of the Royal Commissioners of the Patriotic Fund having asked for a valuation as at 31 December last of certain of their Funds, a further interim Report has been made by the Council; and a special valuation of the Transvaal War Fund, which was also requested, is almost ready.

There was recently published in the *Journal* the correspondence which took place when the Council were consulted by the Government of India as to the statistics of Indian Government Establishments.

Suggestions made on behalf of the Institute for improvement in the methods of collecting and using the data of a census in this country are receiving the attention of the Registrar-General.

It will be observed that the Institute has endeavoured to fulfil the duty to the public thrown upon it by its Royal Charter. It has also, by the courteous vigilance of its Fellows and friends across the Atlantic, hindered an unwarrantable appropriation of its name in Canada, and it is hoped that representations made through Lord Strathcona and Mount Royal, the High Commissioner for the Dominion in London, will prevent any renewal of the attempt.

As regards the Mortality Investigation, jointly undertaken by the Institute of Actuaries and the Faculty of Actuaries in Scotland, the Council have to report that good progress, under the excellent supervision of Mr. Ackland, has been made during the year with the computation of monetary values based upon the graduated Tables. The monetary tables deduced from the  $O^M$  and  $O^{M(5)}$  Experience have been computed in London and have been printed, and the volume is now almost ready for publication. The *Select* mortality tables for Whole-Life Participating Assurances on Male Lives have been graduated with great skill by Mr. G. F. Hardy, and monetary tables based upon this graduation are now in course of preparation in London. The Monetary Tables for Single and Joint Lives, based upon the Life Annuity experience for male and female lives, which have been computed in Edinburgh under the able direction of Mr. James Chatham, are now completed, and are about to be placed in the printer's hands with a view to early publication.

The laborious tasks so skilfully executed by Mr. Todhunter and Mr. King in connection with the Text-Book were acknowledged last year, and the publication of their works has now enabled members to form for themselves a true estimate of their great value.

Mr. Todhunter having resigned his office as Tutor for Part II of the Examinations, the Council have appointed Mr. A. Levine, M.A., to succeed him, and Mr. H. M. Trouncer, B.A., has been appointed Tutor for Part I in the place of Mr. Levine.

The Council much regret to have to report that no essay has been received in connection with the Messenger Prizes offered in March 1901 for essays on Endowment Assurances.

\* These results for the United Kingdom and the Colonies, combined, are given on page 340.



Amount of Funds at the beginning of the year—	£	s.	d.	Journal—	£	s.	d.
General Fund . . . . .	7,606	16	4	Printing of Nos. 201, 202, 203, and 204 . . . . .	481	18	4
Messenger Legacy Fund . . . . .	348	15	2	Clerical assistance . . . . .	45	0	0
Brown Prize Fund . . . . .	231	12	1				
Subscriptions—							
Fellows . . . . .	592	4	0	Text-Book, Part I, New Edition—			
Associates . . . . .	494	11	0	Printing, Binding, and Editorial Expenses . . . . .			
Students . . . . .	326	11	0	Library—binding and purchases . . . . .	35	0	5
Probationers . . . . .	61	8	6	Expenditure on account of Card Indexes . . . . .	150	15	9
Application Fees—							
Associates . . . . .	6	6	0	Meetings . . . . .	185	16	2
Students . . . . .	39	18	0	Examinations charges . . . . .	68	7	5
Probationers . . . . .	36	4	6	Lecturers for classes in Parts I and II . . . . .	70	14	5
Examination Fees collected during the year . . . . .	82	8	6	Lectures on Law of Mortgage . . . . .	225	15	0
Class Fees . . . . .	224	14	0	Legal charges . . . . .	108	7	0
Sales of Publications—				Office Expenditure—			
Journal . . . . .	146	18	0	Rent . . . . .	275	0	0
Text-Book, Part I (New edition) . . . . .	92	3	6	Salaries . . . . .	346	15	4
Text-Book, Part II (1st edition) . . . . .	57	7	6	House expenses . . . . .	65	16	5
Text-Book, Part II (2nd edition) . . . . .	2	0	0	Corporation Duty . . . . .	10	13	8
Government Annuity Tables . . . . .	6	10	4	Fire Insurance . . . . .	14	2	3
Select Life Tables . . . . .	11	19	9	Stationery and Printing . . . . .	112	8	1
Short Collection of Actuarial Tables . . . . .	4	8	5	Postage and Telegrams . . . . .	35	6	1
Hardy's Friendly Societies . . . . .	1	3	0	Furniture . . . . .	4	3	0
Legal and Financial Lectures . . . . .	14	18	4	Sundries . . . . .	4	13	4
Logarithm Cards . . . . .	1	16	7				
Transactions of Second International Congress . . . . .	36	6	0	Donation to Queen Victoria Memorial Fund . . . . .	868	18	2
Dividends and Interest (less tax)—				Expenditure on Valuations for the Commissioners of the	52	10	0
General Fund . . . . .	217	8	5	Patriotic Fund . . . . .			
Messenger Legacy Fund . . . . .	7	0	0	Interest on Loan from Bankers . . . . .	51	19	9
Brown Prize Fund . . . . .	7	10	8	Amount of Funds at the end of the year, as per Balance Sheet . . . . .	7	3	4
				Examined and found correct, 15 April 1902.			
				A. MACKAY,			
				A. M. LEVEAUX,			
				H. E. WILSON,			
				Auditors.			
					£10,749	16	1

## Balance Sheet, 31 March 1902.

LIABILITIES.	£	s.	d.	ASSETS.	£	s.	d.
General Fund . . . . .	7,573	17	7	Natal 3 per-cent Inscribed Stock (£3,000), cost . . . . .	2,846	6	0
Messenger Legacy Fund . . . . .	233	9	2	Metropolitan Railway 4 per-cent Debenture Stock (£1,050), cost . . . . .	1,185	11	3
Accumulated Dividends . . . . .	122	6	0	Great Eastern Railway 4 per-cent Debenture Stock (£1,000), cost . . . . .	1,303	14	9
Brown Prize Fund . . . . .	200	0	0	Great Northern Railway Preferred Ordinary Stock (£1,000), cost . . . . .	1,142	11	9
Accumulated Dividends . . . . .	39	2	9	Great Western Railway 4½ per-cent Debenture Stock (£800), cost . . . . .	1,258	12	5
				Advance to the Joint Mortality Experience Committee . . . . .	250	0	0
				Outstanding Subscriptions . . . . .	91	7	0
				Cash on Current Account . . . . .	90	12	4
				[The Institute also possesses certain copyrights			
				and stocks of publications (see p. 337).]			
				Examined and found correct, 15 April 1902.			
				A. MACKAY,			
				A. M. LEVEAUX,			
				Auditors.			
					£8,168	15	6

The Card Index to the *Journal* has been found most useful, and a Card Index to the Library has also been prepared and placed in the Hall of the Institute for the use of Members.

A Book-Plate has been designed and engraved by Mr. C. W. Sherborn with much success.

The Library now contains a scrap-book for cuttings from newspapers and other ephemeral literature, and a subscription has been paid to one of the institutions which supply such extracts, though they will also be thankfully received from any quarter. The information thus registered may often be of present use, and is likely to be of exceeding interest to future generations of actuaries.

Members of the Institute will be glad to notice in the Hall the bust of Dr. Sprague, which has been presented by the subscribers to a fund raised to perpetuate his memory. The number of portraits of distinguished mathematicians of old time possessed by the Institute is growing.

With a view to the general convenience of Members, arrangements have been made for an installation of the telephone by the General Post Office.

Mr. A. G. Wiggins, who for ten years has occupied the post of Assistant Secretary, having expressed his desire to retire from that position, the Council have granted him a small pension in recognition of his faithful services; and have appointed as his successor Mr. S. H. Jarvis, whose satisfactory work for ten years in the office of the Institute is well known.

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### EXAMINATIONS, 1902.

Examinations were held on the 18th, 19th, 21st, and 22nd of April, 1902, in the United Kingdom and the Colonies, at London, Edinburgh, Dublin, Sydney, Melbourne, Adelaide, Wellington, Montreal, Toronto, and Ottawa, with the following results, the names in each class being arranged in alphabetical order:—

#### PART I.

One hundred and nine candidates sent in their names, of whom one hundred and three presented themselves, and sixty-seven passed, namely:—

#### *Class I:*

Askwith, T. N.  
Coates, F. G.  
Deck, J. G.  
Gould, W. H.  
Houston, C. C.  
Humphreys, C. A.

Kemper, J. M. de B.  
Langstaff, J. M.  
McGee, C. H.  
Muckle, C. P.  
Shrubsole, S. S.  
Sugars, R. M.

*Class II:*

Chandler, F. J.  
 Clinton, L. E.  
 Edwards, T. B.  
 Ellis, R. G.  
 Farrow, A. E.  
 Gleave, C. S.  
 Gordon, W. H.  
 Gorham, E. A.  
 Gysin, H.  
 Hammant, F. C.  
 Hodge, C. W.  
 Hughes, C.  
 Hugill, H.  
 Humphry, E. W.

Jackson, H. M.  
 Macfarlane, J. A.  
 McKellar, J. A.  
 Maudling, R. G.  
 Moore, H. F.  
 Mullin, A.  
 Nicholls, A. W.  
 Phillips, T. H. B.  
 Proud, C. W.  
 Reynell, G. C.  
 Robertson, A. C.  
 Sare, T. H.  
 Tully, A. P. T.  
 Wellisch, F.

*Class III:*

Beresford, C.  
 Biden, N. F.  
 Bowerman, J. P.  
 Brook, F. A.  
 Capon, G. W.  
 Collins, W. E.  
 Corbett, A. G.  
 Denmark, R. J.  
 Edwards, E. S.  
 Godsill, R. C.  
 Hardy, R. H.  
 Humphreys, J. A.  
 Jones, E. F.

Lancaster, C. H.  
 Lang, F. J.  
 Macorquodale, F. D.  
 Morton, F.  
 Paton, H. A.  
 Smith, S. W.  
 Smither, H. B.  
 Strong, W. B.  
 Sturt, H. R.  
 Sudell, A. H.  
 Thwaites, F. G.  
 Tope, M. W.  
 White, W. C.

Woodhouse, D. A.

## PART II.

Eighty-five candidates sent in their names, of whom seventy-five presented themselves, and twenty-one passed, namely:—

*Class I:*

None.

*Class II:*

Carter, N. J.  
 Grant, M. D.

Gray, R. A.  
 Kilgour, D. E.  
 Milligan, C. L.

*Class III:*

Dawson, M. M.  
 Denmead, J. C.  
 Doust-Smith, E. C.  
 Downes, S. C.  
 Fitzgerald, C. R.  
 Gaines, John M.  
 Hooper, G. D.  
 Hunter, R. G.

Pigrome, G. D.  
 Robinson, H. T. K.  
 Spurgeon, E. F.  
 Stamp, H. E.  
 Traversi, A. T.  
 Wandless, J. R.  
 Wares, H. W.  
 Weatherill, C.



## PART III.

Thirty-seven candidates sent in their names, all of whom presented themselves, and twelve passed, all in the third class, namely :—

Adlard, S.	†May, B.
Bacon, J.	†Richmond, G. W.
†Coutts, C. R. V.	Weatherill, H.
Falconer, H. F.	Wood, A. B.
†Gillies, G.	†*Workman, W. A.
†Kenchington, C. W.	Worthington, W.

## PART IV.

Nineteen candidates sent in their names, of whom eighteen presented themselves, and nine passed, namely :—

*Class I:*

None.

*Class II:*

†Anderson, T. F.	McArthur, H. de C.
†Appleton, F.	†*Workman, W. A.

*Class III:*

Rae, J.	†Tinner, T.
†Strong, W. R.	†Woolmer, A. H.
Young, A. S.	

Those marked (†) have now completed the examination for the Class of Fellow.  
(\*) Mr. Workman passed in both parts of the Final Examination.

## PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Monday, 2 June, the President, Mr. Chas. D. Higham, in the chair.

The Report of the Council (given on p. 335) having been read,

The PRESIDENT, in moving the adoption of the report and accounts, remarked that the papers read at the sessional meetings had covered a broad area, and had been varied and interesting. In connection with these meetings, he wished to mention that, in future, members who desired to have a proof of the paper sent to them should communicate their desire every year to the Assistant Secretary. They were anxious that those who wished should have them, because it promoted discussion, and it also saved time in reading the papers, but they did not want to keep on sending them where they were not wanted. With regard to their public duties, they had been very busy during the year with the Patriotic Commissioners' Funds. They had valued most of them again, and certain new Funds. They were just now dealing with the Transvaal War Fund, and that meant taking out a new marriage rate, but it was satisfactory to find their efforts so warmly appreciated and so kindly acknowledged in the messages that were sent to them. He mentioned that the Indian Government had asked their

opinion as to some statistics: whether, first, they were likely to be of any value to the public, and next, what, if so, would be the best way of setting about producing them. That, he ventured to think, was an enlightened method of procedure on the part of a public department, in that they took expert advice before spending public money to know whether the task was worth doing and how to do it. A report had been sent out, and was under consideration. With regard to the Census of their own country, they had been in communication with the Registrar-General, with whom he had had more than one pleasant conference. While it seemed that it was impracticable on the present occasion to get their principal wishes fulfilled, the Registrar-General had been very courteous in making certain concessions to their desires, and they had had the satisfaction of finding that there was a good deal of agreement between themselves and him and his principal officials, though naturally there was Parliament to be thought of, and they could not get all they wanted at present. But they would begin to agitate again at the proper time, and it was gratifying to have established friendly relations with an important Government department. Then came the *magnum opus* of the Faculty and themselves. He had in his hands what he might call the O<sup>M</sup> "Hardy." It did not contain all the functions given by Mr. R. P. Hardy in his book of H<sup>M</sup> Tables, but it had certain others instead, and more might be published later, while those it did contain were arranged very much in his method, and it would be observed that the matter as to the O<sup>M(5)</sup> Tables was on pale green paper to prevent mistakes. As soon as this type was free, the annuities, which were all complete, would be produced. They had been done in Scotland, thanks to Mr. Meikle's and Mr. Chatham's assiduity; and following these the select values were well on their way, and would come later. With that last they hoped to include a clear explanation of all their plans and methods, and of those magnificent graduation curves for which they were indebted to the genius of Mr. G. F. Hardy. He had said before that no words could express what they owed to Mr. Ackland; and they had the willing services of Mr. R. P. Hardy, the chairman of the committee, to keep them all together. Mr. Todhunter now retired from his position as lecturer, and they hoped to welcome him on the Council. Mr. Levine succeeded him, and Mr. Trouncer would follow Mr. Levine, and he hoped equally successfully. He had been asked to mention that it was very possible they might somewhat vary these classes. Hitherto it had been the practice for many a young man to read the whole of the subjects in a year, and at once present himself for examination, and he was afraid very often fail; and then read the whole again during the second year, and go up once more, with generally a better result. It would perhaps be arranged that there should be two classes—one of a more elementary character, and the other for the more advanced subjects, so that in the ordinary way a man would generally get well grounded in the first year, and carry on his studies in the second, and only make one effort, and that successful, for his examination. Or the thing might be arranged in some other way by lengthening the period of study in the year, but they wanted to do what they could to help the younger men to get thoroughly prepared. Whether they would ever be able to give any help in the way of classes for parts three and four of the

examination he did not know, but it was not forgotten, and if it ever became possible they would do their best. He was very sorry to say that they did not have a single essay sent in for the Messenger prizes. It was thought the endowment assurance subject would have been popular, but it might be that intending candidates had been waiting for the new endowment assurance mortality. Still, it was a disappointment, and for the last time he would appeal to those who had only lately passed their examinations not to get rusty, but to keep their hands in by competing for these prizes. The card indices for the library and the *Journal* had been finished, and an immense work had been completed by Mr. Marks, for which they thanked him, as well as for his heavy labours as librarian. He did not think gentlemen always remembered the amount of quiet work that went on there, and what with receiving new books, looking through catalogues of second-hand sales, watching Parliamentary publications, perpetual correspondence, and one thing and another, they were very greatly indebted to their librarian. Then the mention of the *Journal* reminded him of another of the officers who was less known to the public, namely, the Editor of the *Journal*. To Mr. King they owed much. Some of them got a little relief in the holiday months, but the *Journal* went on always, and it required a good deal of tact, mathematical knowledge, literary ability, and unceasing patience, to bring out the numbers quarter by quarter. Mr. King was spared one trouble which had been experienced by previous editors—he had no lack of matter. There was a time when they occasionally used to wonder how they would get the *Journal* full, but now they had almost more matter than they knew what to do with. Besides the January and April numbers, they had published a Double Number in between; and he believed Mr. King had now more than enough for the rest of the year. He thought the time was coming when they would have to publish four numbers for a volume instead of six, and have one volume each year. The book-plate would soon be ready, and when they saw it, he thought they would say that Mr. Sherborn had done his work well, and that it was pretty. They had adopted the motto of *certum ex incertis* which had come from one of their Fellows, Mr. Newton. Finally, they would be very soon losing Mr. Wiggins. They never had a better Secretary. But now, with advancing years, he claimed his rest, and they had granted him a small pension, not a pension worthy of his deserts, for with their income they could not do everything they wished, but they hoped that for many years he might enjoy his retirement. He had little doubt from his personal observation during these two years, and before, that Mr. Jarvis, who had been appointed to succeed Mr. Wiggins, would be a thoroughly worthy successor of the good men who had been before him.

Mr. F. B. WYATT, in seconding the motion, said he would like to make a few general remarks as to the work which did not appear altogether from the report. The report was a statement of hard facts, and did not give a full idea of the work that had been done. Speaking from experience on the Council, the work done during the last year was, if anything, greater than in any year that had gone before. In particular, he would point out that the scope of usefulness of the Institute had very greatly increased, and he thought he might say without fear of contradiction that the position of the profession at the present moment was better than ever it had been.



Out of the six papers that were read before the Institute during the past session, three could not be said to be on essentially actuarial subjects, but they were all subjects of great importance to the nation, on which those in authority on such matters were always glad to have actuarial advice. Possibly the most important of them was the one which dealt with workmen's compensation. That, he thought, in the course of time, would become essentially an actuarial subject. There was only one other point he would touch upon, namely, that, unfortunately, during last year there had been no lectures. These lectures in the past had been very successful, and, as a retiring member of the Council, he ventured to hope that in the coming session some other lectures would be given. He would mention the subjects of political economy, and the fiscal arrangements of this and other countries, as possible heads for lectures. It appeared to him that, though they were subjects on which the actuary had a considerable amount of knowledge, much additional knowledge might certainly be gained from expert lecturers.

The motion was unanimously agreed to.

The PRESIDENT, before the ballot was taken for the new Council, wished to call attention to the fact that they were in the unusual position of having three vacancies among the Vice-Presidents. Mr. Wyatt had to go because no one could be re-elected more than three times; and they were also losing Mr. G. F. Hardy. Then Mr. Chisholm was unfortunately leaving the Council, for he was retiring from the active exercise of his profession, and thought it right to resign his position on the Council, as well as that of Vice-President.

The following gentlemen, nominated by the Council, were balloted for and elected as officers for the ensuing year:

*President.*

WILLIAM HUGHES.

*Vice-Presidents.*

RALPH PRICE HARDY.

ARTHUR FRANCIS BURRIDGE.

THOMAS G. C. BROWNE.

GEORGE KING.

*Council.*

\*THOMAS GANS ACKLAND.

HENRY WALSINGHAM ANDRAS.

ARTHUR DIGBY BESANT, B.A.

\*JAMES BLAKEY.

THOMAS G. C. BROWNE.

ARTHUR FRANCIS BURRIDGE.

JAMES CHATHAM.

HENRY COCKBURN.

\*GEORGE STEPHEN CRISFORD.

STANLEY DAY.

GEORGE FRANCIS HARDY.

\*RALPH PRICE HARDY.

AUGUSTUS HENDRIKS.

CHARLES DANIEL HIGHAM.

LOUIS FREDERICK HOVIL.

WILLIAM HUGHES.

GEORGE KING.

GEORGE JAMES LIDSTONE.

GEORGE MACRITCHIE LOW.

HENRY WILLIAM MANLY.

GEOFFREY MARKS.

HARRY ETHELSTAN NIGHTINGALE.

FREDERICK SCHOOLING.

JOHN BELL TENNANT.

GEORGE TODD, M.A.

\*RALPH TODHUNTER, M.A.

SAMUEL GEORGE WARNER.

\*ALFRED WILLIAM WATSON.

ERNEST WOODS.

THOMAS EMLEY YOUNG, B.A.

\* Not Members of the last Council.

*Treasurer.*

HENRY COCKBURN.

*Honorary Secretaries.*

FREDERICK SCHOOLING.

| GEORGE TODD, M.A.

The PRESIDENT wished Mr. Hughes a happy and prosperous reign.

Mr. WILLIAM HUGHES, in tendering thanks on behalf of the newly-elected Council, said they might be assured that the Council as a body, and each member in particular, would devote every energy to the promotion of the interests of the Institute. Perhaps he might be permitted to add a word or two of special thanks in recognition of the honour they had done him in electing him as their President. He was very deeply conscious of the honour of being elected to fill a chair which had been occupied by such a long line of illustrious predecessors, but he felt that there were also great responsibilities. He hoped, however, that he would be able to fulfil the duties of the office, if not, it might be, so well as his predecessors, at least acceptably to uphold the dignity of the Institute, to aid in its councils, and to extend its usefulness.

On the motion of Mr. BARTON, seconded by Mr. WEATHERILL, Messrs. H. E. Wilson, A. M. Leveaux, and Bernard Woods were elected auditors for the ensuing year.

Mr. R. P. HARDY, in proposing a vote of thanks to the President, the Vice-Presidents, the Council, the Officers, the Examiners, and Supervisors, for their services during the past session, said that, looking at the work as a whole, there was one thing abundantly clear, and that was that, both in the ordinary and in the special administration of the Institute, its credit had suffered no diminution at the hands of its officers; but, on the contrary, it had been enhanced and carried still further forward by, amongst other things, their freely imparted technical knowledge in response to the demands of the public; and lastly, but by no means least, by the unselfish devotion of those upon whom the administration had devolved. He would specially draw their attention to the very high compliment paid to the Institute, as a chartered and scientific body, in making it *amicus curiæ* in respect of the financial position of those several funds administered by the Patriotic Commissioners. It was well known that for some years past considerable friction had ensued between the naval and military services and the Commissioners as a body. This Institute's report will entirely remove that tension. The Institute's action in respect to the Indian and the Census questions should not be overlooked in their review of the year's working. They were probably aware that he had just returned from a lengthened stay amongst some of their distant brethren. To them the proceedings of this Institute were of supreme interest; they followed closely all movements here, and were equally up-to-date on all points of theory and practice. In Sydney alone, there was a considerable body, having their own local Institute, with papers and discussions of the highest merit; they would, since their studies had taught them an independent line of thought, bye-and-bye found a school of their own, one dealing with their own special problems, but still in indissoluble connection with their

*alma mater.* The Institute had been the means of supplying the Australian public with a technically-educated class drawn from the ranks of her own citizens, capable of safeguarding all the important interests of life assurance. These Australian *alumni* were all nurtured upon the solid food of Part II of the *Text-Book*; all had mastered that admirable summary they owed to Mr. King's wide and accurate professional skill, and to the public spirit and discernment of the Institute as its projector.

Mr. M. N. ADLER, thought it was in the fitness of things that one who like himself had been connected with the Institute for some forty-five years, should have a word to say in support of the resolution. He would advert to the President's success in placing members of the Institute on a proper footing in connection with the authorities at Somerset House; it was at the Actuaries' Club where the matter with regard to the unfriendly spirit with which the Somerset House people used to regard valuations and opinions given in by members of the Institute was first mooted, and he was delighted to find that the President took the matter up with energy, and successfully had the position of Actuaries recognized by the department. It appeared, then, that the past two years, which had seen the expansion of the country to the utmost, had been concomitant with the expansion of the Institute. They hoped that the new President would have like success in the career upon which he was about to enter. The office of Examiner in former days was a light undertaking, when but ten or fifteen candidates came forward to be examined, but nowadays it was a different matter altogether. He was sure they earnestly appreciated the work of Examiners.

The resolution was carried with acclamation.

The PRESIDENT said they did well to associate in the vote conjointly with the President the Officers and those by whom he was surrounded, without whom it would be impossible for him to get along, and he was the more glad to say this because it had occasionally happened that when some piece of work had been done to their liking most of the glory went to the President, whereas the work had been more equally shared. But there were not only the Officers and Council, there were the unofficial members, and many of the younger members, who all lent a hand when asked. They were a united body, and he had never called for help in vain. He was grateful above all because they had given him the opportunity of rendering service to the Institute they loved so well.

Mr. JAMES CHISHOLM, moved a vote of thanks to the auditors (Messrs. A. Mackay, H. E. Wilson, and A. M. Leveaux) for their services during the past year. They might be sure that when they had experts of the standing of the gentlemen he had named there was nothing to be found fault with, either in the matter or the form of the accounts.

The resolution, seconded by Mr. A. H. Bailey, was carried unanimously.

Mr. LEVEAUX briefly responded on behalf of the Auditors.

Mr. R. P. HARDY, said he had been asked by the brethren in Sydney to tender to the brethren in London their fraternal greetings. He hoped the message would be received here with the warmth and with the intention that dictated it.

The PRESIDENT said they one and all reciprocated in the warmest possible manner the kindly messages conveyed from Australia.



*Additions to the Library.*

The following works have been added to the Library since the publication of the *Journal* for October 1901:

*By whom presented  
(when not purchased).*

Accountants, Institute of Chartered, in England and Wales. List of Members, 1902.	<i>The Institute.</i>
Actuarial Society of America. Transactions of the	<i>The Society.</i>
Actuaries, Faculty of List of Members, 1902. Transactions of the	} <i>The Faculty.</i>
American Mathematical Society. Transactions of the	
American Statistical Association. Transactions of the	<i>The Association.</i>
Australian Mutual Provident Society. Fifty-third Annual Report, 1902.	<i>The A.M.P. Society.</i>
Austria. Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für das Königreich Böhmen in Prag. 1900. Die Privaten Versicherungsunternehmungen in den im Reichsrathe vertretenen Königreichen und Ländern im Jahre, 1899. Mittheilungen des Verbandes der österr. und ungar. Versicherungs-Techniker. 1901-2.	} <i>The Austrian Government.</i>
Belgium. Bulletin du Comité des Congrès Internationaux d'Actuaries. No. 5. 1901.	
Bulletin de l'Association des Actuaires Belges.	
Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Épargne et de Retraite. 1901.	
Bertillon (Dr. Jacques). Cours Élémentaire de Statistique conforme au pro- gramme arrêté par le Conseil supérieur de Statistique pour l'examen d'admission dans diverses administrations publiques. Paris, 1895.	<i>G. H. Ryan.</i>
Birmingham, Insurance Institute of Transactions, 1900-1901.	<i>The Institute.</i>
Bourne's Handy Assurance Manual, 1902.	<i>The Publishers.</i>
British Offices Life Tables, 1893. Tables deduced from the graduated experience of Whole-Life Participating Assurances on Male Lives (Aggregate Tables). 1902.	} <i>The Joint Mortality Committee.</i>
Chartered Accountants' Students' Society of London. Transactions of the 1892.	

*By whom presented  
(when not purchased).*

**Coghlan (T. A.).**

Child-birth in New South Wales: A Study in Statistics.  
Sydney, 1900.

*R. P. Hardy.*

Statistics of the Six States of Australia and New  
Zealand. 1861-1900.

The Wealth and Progress of New South Wales.  
1900-1901.

*The Author.*

**Davies (Griffith).**

Report and Valuation for the Bengal Military Fund.  
1844.

*A. Smither.*

**De Magellan (J. H.).**

Collection de Différens Traités sur des Instrumens  
d'Astronomie, Physique, etc. 1780.

*M. N. Adler.*

**De Morgan (Sophia E.).**

"Threescore Years and Ten." Reminiscences of the  
late, to which are added letters to and from her  
husband, the late Augustus De Morgan, and  
others. Edited by Mary A. De Morgan. 1895.

*Purchased.*

**Denmark.**

Statistics of Life Assurance Companies in Denmark.  
1900.

*Dr. J. Münthe Brün.*

**De Parcioux (M.).**

Essay on the Probabilities of the Duration of Human  
Life (With Tables and additional Essay). Trans-  
lated from the French. MS. 1746, 1760.

*Purchased.*

**Dougharty (H.).**

How to insure against everything.

*The Author.*

**Dizler (Carl).**

Sofort beginnende Leibrenten mit Rückvergütung der  
Baareinlagen abzüglich der bereits bezogenen  
Renten. 1901.

*The Author.*

**Duguid (Charles).**

How to read the Money Article. 3rd edition, 1901.

*Purchased.*

**Elderton (W. P.).**

Tables for testing the goodness of fit of Theory to  
Observation. 1901.

*The Author.*

**Elsner (Dr. A. F.).**

Assecuranz Almanach, 1902.

*The Publisher.*

**Emminghaus (Dr. A.).**

Mittheilungen aus der Geschäfts- und Sterblichkeits-  
Statistik der Lebensversicherungsbank für  
Deutschland zu Gotha für die Fünfzig Jahre von  
1829 bis 1878. 1880.

*The Gotha  
Life Office.*

**Encyclopædia Britannica.**

The Part for November, 1856, containing articles on  
Insurance by "W.T.T." and Interest by "J.R.M."  
8th edition, 1856.

*J. Chisholm.*

**Engineers, Institution of Civil**

List of Members, 1902.

*The Institution.*

*By whom presented  
(when not purchased).*

**Euler (Leonard).**

Elements of Algebra. Translated from the French ; with the Notes of Mons. Bernoulli, etc., and the Additions of Mons. De la Grange. 5th edition, by the Rev. John Hewlett, 1840. To which is prefixed a memoir of the life and character of Euler, by Francis Horner.

*J. Chisholm.*

**Farrell (J.).**

Industrial Life Assurance. 1902.

*The Author.*

**Fenn (Charles).**

A Compendium of the English and Foreign Funds. 16th edition, by S. F. Van Oss. 1898.

*Purchased.*

**France.**

Bulletin de l'Institut des Actuaire Français.

*L'Institut.*

**General Reversionary and Investment Company's Act, 1851.**

*D. A. Bumsted.*

**Germany.**

Deutscher Verein für Versicherungs-Wissenschaft. Rules, 1899. List of Members, 1901.

Zeitschrift für die gesammte Versicherungs-Wissenschaft.

*The Society.*

Zeitschrift des Königlich Preussischen Statistischen Bureaus. 1901, 1902.

*The German Government.*

Zustand und Fortschritte der deutschen Lebensversicherungs-Anstalten im Jahre 1901.

*The Publisher.*

**Giffen (Robert).**

The Growth of Capital. 1889.

*Purchased.*

**Goodeve (—).**

Modern Law of Personal Property. 3rd edition, 1899. Revised and partly re-written by J. H. Williams, LL.B., and W. M. Crowdy, B.A.

*Purchased.*

**Hall (Rev. T. G.).**

A Treatise on the Differential and Integral Calculus, and the Calculus of Variations. 4th edition, 1846.

*J. Chisholm.*

**Hewat (A.).**

An Investigation of the Marriage and Mortality Experience of a Scottish Ministers' Widows' and Orphans' Fund, with relative Statistical and Monetary Tables. 1902.

*The Author.*

**Holland.**

Archief voor de Verzekerings-Wetenschap, 1901-1902.

*The Society.*

Jaarboekje van de Vereeniging voor Levensverzekering. 1895, 1902.

*The Algemeene Maatschappij.*

Mededeelingen der Vereeniging voor Levensverzekering. 1902.

*The Society.*

Mémoires pour servir a l'histoire des Assurances sur la vie et des rentes viagères aux Pays-Bas. Collected and published by the "Algemeene Maatschappij van Levensverzekering en Lijfrente." Amsterdam, 1898.

*A. G. Wiggins.*



*By whom presented  
(when not purchased).*

"Insurance Register," 1901, 1902.

*C. & E. Layton.*

Italy.

Studi Sulle Pensioni e Loro Riordinamento Proposta di un nuovo regolamento. (Comune di Bologna.) 1901.

*G. Toja.*

Ivory (James), A.M.

On the Attractions of Homogeneous Ellipsoids. (From the *Philosophical Transactions*.) 1809.

*M. N. Adler.*

"Journal of the Federation of Insurance Institutes of Great Britain and Ireland."

Vols. III. and IV. 1900-1901.

*Purchased.*

Kersseboom (W.).

Essays in Political Arithmetic, contained in Three Treatises. Translated from the Dutch. MS. 1748.

*Purchased.*

King (G.).

On Life Contingencies, including Life Annuities and Assurances. (Institute of Actuaries' Text-Book, Part II) 2nd edition. 1902.

Lacroix (S. F.).

The Elements of Algebra. Translated from the French by W. H. Spiller. 1835.

*J. Chisholm.*

Laplace (P. S. de).

Supplément à la Théorie Analytique des Probabilités. N.D.

*A. Smither.*

Lawson (John B. D.).

A Synopsis of all the Data for the Construction of Triangles, from which Geometrical Solutions have hitherto been in print. 1773.

*M. N. Adler.*

Lees (George).

Elements of Arithmetic, Algebra, and Geometry. 1826.

Leslie (Sir John).

Elements of Geometry and Plane Trigonometry, with an Appendix. 3rd edition. 1817.

The Philosophy of Arithmetic; exhibiting a progressive view of the Theory and Practice of Calculation. 1817.

*J. Chisholm.*

Levasseur (E.).

La Population Française. Histoire de la Population avant 1789 et Démographie de la France comparée à celle des autres nations aux XIXe siècle, précédée d'une introduction sur la Statistique. 3 Vols. Paris. 1889-1892.

*Purchased.*

Manchester, Insurance Institute of

Report, 1900-1901.

*The Institute.*

Martin (S. G.).

The Principal Causes of Mortality in Queensland. 1901.

*The Author.*

Moinet (Dr. Francis W.).

Guide to Medical Examination for Life Assurance. 1876.

*J. Chisholm.*

*By whom presented  
(when not purchased).*

**Morgan (William).**

On the Method of Determining, from the Real Probabilities of Life, the value of a contingent Reversion in which three lives are involved in the Survivorship. (From the *Philosophical Transactions*.) 1789.

*M. N. Adler.*

**Muirhead (Dr. Claud).**

Report on the Causes of Death among the Assured in the Scottish Widows' Fund and Life Assurance Society, from 1874 to 1894 inclusive.

*The Scottish  
Widows' Fund.*

**Neild (Lieut.-Col. J. C.), M.P.**

Report on Old Age Pensions, Charitable relief, and State Insurance in England and on the Continent of Europe. Sydney, 1898.

*R. P. Hardy.*

**Neison (F. G. P.), F.L.S.**

Report on the Bengal Military Fund. 1847.

*A. Smither.*

**New South Wales, Insurance Institute of**  
Transactions of the

*The Institute.*

**Newton (Sir Isaac).**

Arithmetica Universalis; sive de compositione et resolutione Arithmetica Liber. 1707.

*J. Chisholm.*

**New Zealand, Insurance Institute of**  
Report of Proceedings, 1901.

*The Institute.*

**Nikitin (Basil), M.A., and Sonvoroff (Proctor), M.A.**

Elements of Plane and Spherical Trigonometry. Translated from the Russian by the Authors. 1786.

*M. N. Adler.*

**Owen (Dr. S. Holgate).**

Report on the Health of the Moss Side District. 1901.

*The Author.*

**Ozanam (M.).**

Cours de Mathematique. Vol. I. New edition, Paris, 1697.

*J. Chisholm.*

**Packer (A.).**

Monetary Frauds and Extortions. 1878.

**Parliamentary Papers.**

Bills.

Employers' Liability Extension.  
Friendly Societies Disqualification Removal.  
Infant Life Protection Act (1897) Amendment.  
Old Age Pensions (5).  
Police Superannuation (Scotland).  
Poor Law Officers' Superannuation (Scotland).  
Prevention of Corruption (4).  
Vaccination.  
Workmen's Compensation Act (1897) Amendment (3).

*Purchased.*

Census, 1901, England and Wales.

Preliminary Report and Tables of the Population and Houses.

Census, 1901, Ireland.

Preliminary Report and Tables of the Population and Houses.

*By whom presented  
(when not purchased).*

**Parliamentary Papers—(continued).**

Census, 1901, Scotland.

Preliminary Report and Tables of the Population  
and Houses.

Charitable Agencies for Relief of Widows and Orphans  
of Soldiers and Sailors. Report from Joint Select  
Committee of the House of Lords and the House of  
Commons. With Minutes and Evidence. 1901.

*Purchased.*

Colonies.

Canada.

Report of the Superintendent of Insurance for  
the years 1898, 1899, and 1900.

*The Government  
Insurance Dept.*

New South Wales.

Census, 1901.

Part I. Ages of the People.

Summaries of the

Correspondence respecting Friendly Societies'  
Further Amendment Bill. 1901.

*The Government  
of N. S. W.*

Statistical Register, 1900.

Another copy.

*R. P. Hardy.*

Vital Statistics. Report for 1900.

*The Government  
of N. S. W.*

New Zealand.

Friendly Societies. Twenty-fourth Annual  
Report of the Registrar, 1901.

Official Year-Book, 1901.

Statistics of the Colony for the year 1900.

*The Government  
of N. Z.*

South Australia.

Annual Report of the Government Geologist,  
1894.

*The Public  
Actuary of S. A.*

Victoria.

Friendly Societies.

Eleventh Report of the Registrar for  
the Year 1900.

Twenty-third Annual Report of the  
Actuary for the Year 1900.

*The Government  
of Victoria.*

Statistical Abstract for the Years 1893 to  
1898, and for 1899 and previous Years.

Victorian Year-Book, 1895-8.

*R. P. Hardy.*

Education.

School Teachers' Deferred Annuity Funds, England  
and Scotland. Accounts for 1900, together  
with the Reports of the Comptroller and the  
Auditor-General thereon. 1901.

*Purchased.*

School Teachers' Superannuation. Report of  
Committee. 1894.

*J. Chisholm.*

School Teachers' Superannuation (Isle of Man)  
Rules. 1901.

*Purchased.*

Emigration and Immigration.

Statistics for 1900.

Friendly Societies.

Building Societies. Sixth Annual Report, Parts  
I and II. 1900.

Chief Registrar's Reports—

For 1899. Parts B and C.

„ 1900. „ A, B, C, and D.

Guide Book of the Registry Office, 1901.

*Purchased.*

Government Insurance and Annuities.

Accounts, 1900 and 1901.

India (East).

Financial Statement for 1901-1902.



*By whom presented  
(when not purchased).***Parliamentary Papers—(continued).**

Life Assurance Companies.  
Return, 1901.

*The Board of Trade.*

Local Taxation. England and Wales.

Final Report of the Royal Commission, 1901.

National Debt.

Annuities. Accounts to 5 Jan. 1901 and 5 Jan. 1902.

Military Savings Banks. Accounts to 5 Jan. 1901 and 5 Jan. 1902.

Return showing Liabilities and Assets at the close of each financial year, from 1835-6 to 1900-1, both inclusive.

Savings Banks and Friendly Societies. Accounts to 20 Nov. 1900 and 20 Nov. 1901.

Sinking Funds. Account showing amount received from the Exchequer in the year ended 31 March, 1901.

Patriotic Fund.

(1) Account of the Net Moneys paid over to the Commissioners in respect of the Transvaal War Fund. }

(2) Schedule of Beneficiaries. 1901. }

Thirty-ninth Annual Report of the Royal Commissioners. 1901.

Police Superannuation (Scotland) Bill.

Reports from the Select Committee, together with Minutes of Evidence, &c. 1901.

*Purchased.*

Public Health.

Vaccination, &c.

Twenty-ninth Annual Report of the Local Government Board, containing the Report of the Medical Officer for 1899-1900.

Thirty-fifth, Thirty-sixth, and Thirty-seventh Annual Reports of the Registrar-General for Scotland. 1899-1901.

Registrar-General. England.

Sixty-second and Sixty-third Annual Reports on Births, Deaths, and Marriages. 1899, 1900.

Registrar-General. Ireland.

Thirty-seventh detailed Annual Report on Births, Deaths, and Marriages for 1900.

Registrar-General. Scotland.

Forty-fifth, Forty-sixth and Forty-seventh Annual Reports on Births, Deaths, and Marriages. 1899-1901.

Forty-fifth detailed Annual Report on Births, Deaths and Marriages. (Abstracts of 1899.) 1901.

Savings Banks and Friendly Societies.

Accounts for 1900.

**Periodicals.**

Accountants' Magazine.

*Purchased.*

Australasian Insurance and Banking Record. 8 Vols. 1893-1900.

Australasian Review. 1900.

Institute of Bankers' Journal,

*R. P. Hardy.**R. P. Hardy.**The Institute.*

*By whom presented  
(when not purchased).*

Periodicals—(*continued*).

Insurance Record.  
London Mathematical Society's Journal.  
Post Magazine.  
Post Magazine Almanac.  
Royal Statistical Society's Journal.

*The Editor.*  
*The Society.*  
*The Editor.*  
*The Editor.*  
*The Society.*

Poore (Dr. G. V.).

The Influence on Life Assurance of a History of } *L. A. Medical*  
Appendicitis and Insanity. } *Officers' Ass'n.*

Potts (Robert).

Euclid's Elements of Geometry. The first six books,  
chiefly from the text of Dr. Simson, with  
explanatory notes. 1850.

Ritchie (Rev. Wm.).

Principles of the Differential and Integral Calculus,  
familiarily illustrated, and applied to a variety of  
useful purposes. 2nd edition, by J. A. Spencer,  
1847.

*J. Chisholm.*

Sang (E.).

Arithmetic. Elementary and Higher. 2 vols. 1856,  
1857.  
Essays on Life Assurance. 2nd edition, 1840.

School Board for London.

Report of the School Management Committee for the  
year ended at Lady Day, 1894.

Schouten (P.).

Grondbeginselen der Levensverzekerings-wiskunde.  
1901.

*The Author.*

Scientific and Learned Societies of Great Britain and  
Ireland.

Official Year-Book, 1901.

*Purchased.*

Smee (Dr. A. H.).

Tables and Diagrams illustrating Comparative Rates of  
Mortality from various causes and in various  
occupations. 1901.

*G. S. Odling.*

Spitzer (Simon).

Tabellen für die Zinses-Zinsen und Renten-Rechnung. }  
1897. }

*Purchased.*

Streeter (T. E.).

The Elements of the Theory of Probabilities. 1902.

*The Author.*

Switzerland.

Rapport du Bureau Fédéral des Assurances sur les }  
Enterprises Privées en Matière d'Assurances en }  
Suisse. 1900.

*The Swiss  
Government.*

Tacquet (L. A.).

The Elements of Euclid; with select theorems out of  
Archimedes. 3rd edition, by W. Whiston. 1714.

*J. Chisholm.*

Tate (William).

The Modern Cambist; forming a manual of Foreign  
Exchanges. 7th edition, 1852.

*A. Smither.*

*By whom presented  
(when not purchased).***Thomson (James).**

An Introduction to the Differential and Integral Calculus; with an Appendix illustrative of the Theory of Curves. 1831.

**Thomson (W. T.).**

"Insurance or Assurance." (As written for the Encyclopædia Britannica. 8th edition, 1856. Proof sheets.)

*J. Chisholm.***Todhunter (I.).**

Algebra, with numerous examples. 4th edition, 1866.

**United States of America.**

Official Publications—

New York State.

Fortieth, Forty-first, Forty-second, and Forty-third Annual Reports of the Superintendent of Insurance for the years 1898, 1899, 1900, and 1901.

*The Government  
Insurance Dept.*

Report and Proceedings of the Joint Committee of the Senate and Assembly appointed to investigate Trusts. 1897.

*Purchased.*

Wisconsin.

Annual Report of the Commissioner of Insurance, 1902.

*W. H. Bloomer.***Victoria, Insurance Institute of**

Proceedings, 1901.

*The Institute.***Wallace (William).**

New Series for the Quadrature of the Conic Sections, and the Computation of Logarithms. 1808.

*M. N. Adler.***Ward (J.).**

Clavis Usuræ; or a key to Interest, both Simple and Compound. 1710.

*J. Chisholm.***Whitworth (W. A.).**

Choice and Chance, 5th edition, 1901.

*Purchased.***Williams (Joshua).**

Principles of the Law of Personal Property; intended for the use of Students in Conveyancing. 15th edition, by T. Cyprian Williams. 1900.

*Purchased.***Yorkshire, Insurance Institute of**

Proceedings, 1900-1901.

*The Institute.*

The following additional copies of works already in the Library have also been added:—



*By whom presented  
(when not purchased).*

Ackland (T. G.) and Hardy (G. F.).

Graduated Exercises and Examples; with Solutions. 1889.

Actuarial Society of America.

Papers and Transactions, 1895-1900.

Actuaries, Institute of

Discussions. Sessions 1865-66 and 1866-67.

Life Tables, 1872.

Mortality Experience of Life Assurance Companies, 1869.

Ansell (C.).

A Treatise on Friendly Societies. 1835.

Ansell (C.), Jr.

Statistics of Families in the Upper and Professional Classes. 1874.

*J. Chisholm.*

Bowser (W. A.).

Friendly Societies' Valuation and other Tables, deduced from the mortality and sickness experience of the I.O.F.M.U. Friendly Society, during the years 1866-70. 1896.

Carment (D.).

Tables of Endowment Assurance Policy Values, &c., according to H<sup>M</sup> Mortality. 1884.

Chisholm (David).

A new Method of constructing a table of the Probabilities of Survivorship between two lives, &c., &c. 1852.

Chisholm (James).

Tables for finding the values of Policies of all durations, according to any Table of Mortality or any rate of interest, with other useful Tables. 1885.

*A. G. Wiggins.*

Coghlan (T. A.).

Statistical Account of the Seven Colonies of Australasia. 1899-1900.

Wealth and Progress of New South Wales. 1898-9.

*R. P. Hardy.*

De Morgan (A.).

The Differential and Integral Calculus, 1842.

Elements of Algebra, preliminary to the Differential Calculus. Second edition, 1837.

An Essay on Probabilities, and on their application to Life Contingencies and Insurance Offices. 1838.

Farr (Dr. W.).

Vital Statistics. A Memorial Volume of Selections from his Writings, edited by N. A. Humphreys. With Biography. 1885.

*J. Chisholm.*

France.

Traité Complet de l'examen Médical dans les Assurances sur la vie. Paris, 1887.

Galloway (T.).

A Treatise on Probability. 1839.

*A. Smither.*

*By whom presented  
(when not purchased).*

**Gompertz (Benjamin).**

The Application of a Method of Differences to the species of Series whose sums are obtained by Mr. Landen by the help of impossible quantities. (From the *Philosophical Transactions*.) 1806.

*M. N. Adler.*

**Gray (Peter).**

The Construction of Survivorship Assurance Tables. 1852.

Tables and Formulæ for the Computation of Life Contingencies. 1870.

**Hardy (R. P.).**

Valuation Tables, based on the Institute of Actuaries Mortality Experience ( $H^M$ ) Table, at 3,  $3\frac{1}{2}$ , 4, and  $4\frac{1}{2}$  per cent. 1873.

**Hardy (Peter).**

Some Considerations in the Theories of Combinations, Probabilities, and Life Contingencies. 1852.

**Hewat (A.).**

Widows' and Pension Funds. 1896.

*J. Chisholm.*

**International Actuarial Congresses.**

Transactions of the First (1896) and of the Second (1898).

**Jellicoe (C.).**

The conditions which give rise to Surplus in Life Assurance Companies, &c., &c. 1852.

**Jones (David).**

On the Value of Annuities and Reversionary Payments, with numerous Tables. 2 vols. 1844.

**"Journal of the Institute of Actuaries."**

Vols. I to XXXV, both inclusive. Index to vols. I to X; and Index to vols. I to XX, Charter and Bye-laws, and List of Members, 1888, in one volume.

**Karup (Prof. J.).**

Die Finanzlage der Gothaischen Staatsdiener-Wittwen-Societät, am 31 December 1890.

*G. King.*

**King (G.).**

Institute of Actuaries' Text-Book, Part II. Life Contingencies, including Life Annuities and Assurances. 1887.

*J. Chisholm.*

**Lacroix (S. F.).**

An Elementary Treatise on the Differential and Integral Calculus (with an Appendix and Notes). 1816.

Traité Élémentaire du Calcul des Probabilités. 1822.

Traité Élémentaire du Calcul des Probabilités. 1833.

*J. Chisholm.*

*A. Smither.*

**Laplace (P. S.).**

Théorie Analytique des Probabilités. 1812.

*A. Smither.*

**Lisle (G.).**

Accounting in Theory and Practice. 1900.

*Purchased.*

*By whom presented  
(when not purchased).*

M'Candlish (J. M.).

The Future Interest of Money, considered chiefly as an element in Life Assurance Calculations. 1866.

Macdonald (W. R.).

Notes on the Theory of Logarithms. 1885.

Meech (Levi W.).

System and Tables of Life Insurance (Thirty American Offices' Experience).

*J. Chisholm.*

Meikle (J.).

An Analysis of the Profits of Life Assurance. 1865.

The Rationale of Life Assurance Premiums. 1861.

The Relation of Tables of Mortality to Values of Policies. 1871.

Milne (J.).

A Treatise on the Valuation of Annuities and Assurances on Lives and Survivorships. 2 vols. 1815.

*A. Smither.*

Morgan (William).

On the Method of Determining, from the Real Probabilities of Life, the value of a contingent Reversion in which three lives are involved in the Survivorship. (From the *Philosophical Transactions*.) 1791, 1794, and 1799.

On the Probabilities of Survivorships between two persons of any given ages, and the method of determining the values of Reversions depending on those survivorships. (From the *Philosophical Transactions*.) 1788.

*M. N. Adler.*

Neison (F. G. P.), F.L.S.

Contributions to Vital Statistics. 3rd edition, 1857.

*J. Chisholm.*

Parliamentary Papers.

Colonies.

New South Wales.

Vital Statistics for 1898 and previous years.

Victorian Year-Book, Vol. II. 1893.

Education.

School Board for London (Superannuation) Bill. Report 1891.

*R. P. Hardy.*

Quetelet (A.).

Lettres sur la Théorie des Probabilités. Brux. 1846.

The same. Translated by O. G. Downes. London. 1849.

*J. Chisholm.*

Robertson (A.).

The Nature, Value, and Disputability of Life Assurance Policies, considered in a letter to Lord Campbell. 1860.

Sang (E.).

Address to the Actuarial Society of Edinburgh. 1868.

Essays on Life Assurance. 1852.



*By whom presented  
(when not purchased).*

**Parliamentary Papers—(continued).**

**Spens (W.).**

Is there a materially greater risk in the assurance of a select life of from 40 to 45, than of a select life of from 20 to 25, for one year? 1853.

**Sprague (Dr. T. B.).**

A Treatise on Life Insurance Accounts. 1874.

**Sutton (W.).**

Institute of Actuaries' Text-Book, Part I. Interest (including Annuities-Certain). 1882.

**Thoman (F.).**

Tables de Logarithmes à 27 Decimales pour les calculs de précision. 1867.

**Thomson (W. T.).**

Notes on the Pecuniary Interests of Heirs of Entail. 1849.

**Todhunter (R.).**

Institute of Actuaries' Text-Book, Part I. Interest (including Annuities-Certain). New edition. 1901.

**Turnbull (A. H.).**

Capital, Money, Interest, and Investment. 1870.

**Wolford (G.), Editor.**

General and Public Statute Laws of the several States of the United States, relating to Fire, Marine, Life, &c. Insurance Corporations, &c. 1870.

*J. Chisholm.*

# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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*Opening Address by the President, WILLIAM HUGHES, Esq.*

[Delivered 24 November 1902.]

THE attempt to fulfil the duty which custom imposes upon the President of the Institute, of delivering an address at the opening of the session, has fully convinced me of the truth of an observation made by one of my predecessors, that it would be likely to impose a heavy burden on the future occupants of this chair. In addressing an audience which includes so many who are my superiors in learning and experience, I feel myself in danger, on the one hand of presumption, and on the other of uttering mere commonplaces, or of repeating in feebler phrase what has been already said by better men than myself. I hope to escape the first of these dangers by avoiding the discussion of any purely scientific or technical subject, and, though I fear I may not be quite successful in escaping the second, I venture to offer some observations on the aims and objects of the Institute which seem to me to be appropriate to the stage in its history at which we have arrived.

The end of the 53rd year of the Institute's existence may seem to be rather late for a statement of its objects; but the course of events during half-a-century, and the circumstances of the time, have naturally modified our views and broadened our

ideas of the functions which the Institute is capable of fulfilling beyond those that were entertained by the founders.

Not that these were by any means narrow or restricted. The first paragraph of the original Constitution announced that it was founded for the purpose of elevating the attainments and status of all who are engaged in occupations connected with the pursuits of an Actuary, and embraces as its peculiar province of enquiry all monetary questions involving a consideration of the separate or combined effects of Interest and Probability. This statement, which reappears in the preamble to the Royal Charter of Incorporation, granted to the Institute in 1884, is of a very wide and general character, and the means to be employed to elevate the attainments and status of Actuaries are stated in the same document in almost equally brief and general terms. The collection and arrangement of data, the construction of tables, the dissemination of practical and theoretical information by means of papers published in its Transactions, and the institution of examinations for students, are stated as the principal means to be adopted in furthering the objects of the Institute. In addition to the objects enumerated in these formal statements, the original promoters were desirous of obtaining for the profession a more definite and acknowledged status in the eyes of the public, and the question of applying for a Charter of Incorporation was discussed at a very early stage. Circumstances, however, which need not now be particularly referred to, prevented any active steps being taken in this direction, and it was not until the Institute had been in existence for thirty-six years that the Charter was obtained.

One of the objects of the Institute which seems to have been very prominently in the minds of its founders and early members was the suppression of unsound assurance companies. Fifty years ago, and indeed for some time afterwards, many life assurance companies were in existence which were founded on erroneous principles and conducted on unsound methods. A writer in the first volume of our *Journal*, commenting upon the then recently-founded Institute, expressed the hope that it would be the means of enabling the profession not only to "give the proper direction to honest and legitimate enterprise but would also place in its hands the means of exposing and extinguishing such as are of an unsound and delusive description." This object may be said to have been completely accomplished. The Life Assurance Companies Act of 1870, though not formally



the work of the Institute as a body, was drafted under the advice of one of its most eminent members. The passing of this Act, and the dissemination by the Institute of sound and accurate knowledge, has resulted in the almost complete disappearance of unsound assurance companies, and the highly satisfactory condition of life assurance in this country at the present time may fairly be attributed to the influence of the Institute.

It is not necessary, however, to pursue any further its past history. The main facts of its career are tolerably well known to most of us, and have been very fully set forth in some of the inaugural addresses by preceding Presidents. For half a century it has been employed in laboriously acquiring and systematising the knowledge of its subjects and in elaborately instructing and training its students with a view to fitting them for professional work; but it has, as yet, done but little in the way of spreading actuarial knowledge beyond professional circles. The public in general know as little of the Institute of Actuaries as they do of the locality in which it is situated. To most people, probably, Staple Inn is principally known by the description in Dickens' *Edwin Drood* as a little nook behind the most ancient part of Holborn, containing a little hall with a little lantern in its roof, to what purposes devoted they know not.

My immediate predecessor in this Chair, in his address two years ago, made the remark that "it may be a small matter that "so many have no idea of the functions of an Actuary and that we "find ourselves sometimes confused with strange company in the "popular mind." I am inclined to think this is not after all so small a matter. It is much to be desired that the public should have a more definite idea of what is meant by the word Actuary, and should be more fully informed as to the aims and functions of the Institute. It is a matter of common knowledge among ourselves that the popular notions on this subject are extremely vague, and even presumably well-informed writers on economical and financial subjects frequently display extraordinary ignorance when they venture to touch upon actuarial subjects.

It is true that the subject of life assurance is being daily pressed upon public attention by means of advertisements, prospectuses, canvassing-leaflets, and pamphlets, and by the oral arguments of agents anxious to do business, but the public nevertheless remains with the vaguest and most nebulous notions on the subject. The only question in which they seem to take any real interest is "which Office gives the biggest bonus and the

most liberal surrender value", but with an apparently satisfactory answer to this question the average man ceases to take any further interest. The principles upon which the science of life assurance rests are perhaps dimly perceived by some, but to the greater part of the world, even to business men of acuteness and intelligence they are often a hopeless mystery. By many the Actuary is regarded as merely a higher kind of accountant or book-keeper to a Life Assurance Company, though by others he sometimes seems to be looked upon as a sort of financial juggler, who, by the aid of the mysterious power of compound interest, can perform incredible feats of monetary magic.

The appearance, however, of the Theory and Practice of Life Assurance as one of the subjects which may be taken by students of the University of London for the B.Sc. degree is an indication that an intelligent interest in actuarial matters is no longer to be confined to professional students. The London School of Economics has announced that, with a view to meeting the needs of those who wish to take advantage of the new facilities offered by the University, they have arranged for two courses of lectures on life assurance subjects. These lectures are to be delivered by two of our most distinguished members, and will doubtless be attended not only by students of this Institute, but also by a large number of others who have no definite intention of becoming professional Actuaries, but are taking up the subject simply with a view to their degree. The amount of knowledge which the University will require for their examination in the subject does not at present appear, but it will probably fall considerably short of that which the Institute requires for its Fellowship. At all events, there is every reason to think that the inclusion of the subject in the University curriculum will speedily attract the attention of schoolmasters, who will seek to introduce it into their school course.

In order to do this, a text-book much more elementary than the Institute *Text-Book* will inevitably be required. I venture to suggest for your consideration whether such a manual might not be prepared and issued under the direct sanction of the Institute. I by no means suggest what is known as a popular book, nor a mere condensation of the Institute *Text-Book*; neither should its preparation be confided to anybody who himself has no more than an elementary knowledge of the subject. I do not think that the preparation of such a work would be beneath the dignity of even the most learned member of this Institute, and we may remember,

as a precedent, that a Lord Chancellor did not think it beneath his dignity to write an elementary treatise for the information of laymen on so intricate a subject as the Law of Real Property.

It may be objected that the existence of such a book might afford undue encouragement to persons disposed to dispense with professional assistance in actuarial matters, but I think that it would rather have the contrary effect by indicating the real nature of the subject, and suggesting the intricacies which it would stop short of developing. The student of such a book would at least be disabused of the notion sometimes entertained by auctioneers' and solicitors' clerks, that a valuation of a reversionary interest can be estimated with sufficient accuracy with no other aid than the Table of Expectation of Life in Whitaker's Almanack, and an ordinary Table of Compound Interest.

Whatever may be done for the general dissemination of actuarial knowledge by the Universities or other agencies, the Institute in England and the Faculty of Actuaries in Scotland are the only schools for professional purposes; and for those who purpose practising as Actuaries, the attainment of the Fellowship of one of these bodies is and must be indispensable. Although the standard of attainments now required for the Fellowship is very considerably higher than that which was demanded in earlier days, the student of the present day is in a much better position than his predecessors of a former generation. Until the publication of the *Text-Book*, the student was compelled to acquire his knowledge principally from somewhat ancient treatises, which, valuable as they are, are not suitable for elementary text-books, and by painful search among various papers scattered in the *Journal* of the Institute. The absence of any uniform system of notation constituted another difficulty for the student. It was not until the Institute had been in existence for many years that it undertook the task of providing systematic instruction for its students. By the publication of its *Text-Books*, the settlement of a system of notation, the provision of authorized tutors, and the establishment of lectures on financial, legal, and economical subjects, the Institute has now furnished its students with an efficient educational apparatus. But at the same time it has very materially raised the standard of acquirement, and the examination papers of the present day, both in precision and range, demand a much more extensive knowledge than was



formerly thought necessary. It has been sometimes said that the standard has been unduly raised, and it may be contended that the rather large proportion of failures in some recent examinations afford some proof, or at least a presumption, that this is the case.

In connection with this question, we have to consider the object which the Institute has in view in instructing and examining its students. Broadly speaking, this object is to train them to become efficient officers of Life Assurance Companies, and other similar institutions which involve (to quote again the words of the Charter) the separate or combined effects of Interest and Probability. To this end it is essential that students should exhaustively study first principles and essential processes. But it is not enough to study merely mathematical analysis and recondite theories. It may be that at one time there was a tendency to devote too exclusive attention to this branch of enquiry, but it is now recognized that the aim of the Institute is to be not a mere academy of philosophical theory, but a complete school of insurance practice. Hence we demand from our students not only an acquaintance with mathematical theory and arithmetical processes, the collection and arrangement of data, the elaboration of formulæ, and construction of mortality and other tables, but we require also some considerable acquaintance with those branches of general finance and banking, and legal and economical subjects with which we come in daily contact as administrators of the business of life assurance.

There is, however, one department in which, as yet, we have required no evidence of knowledge on the part of our students, and which, nevertheless, is of the greatest importance for a life assurance official. It is the merest platitude to say that the prosperity of a Life Assurance Company is largely dependent upon the due selection of the risks which it undertakes. On this subject, however, not only do we require no evidence of knowledge on the part of our students, but it is one which has received but scant attention from our members generally. I am not forgetful of the fact that several contributions are to be found in our Transactions which deal with some aspects of this subject. They, however, deal principally with the mortality which has been experienced among rated-up lives, but the adequacy of the extra ratings and the principles upon which they were imposed has received but a small share of attention.

The subject of selection (by which I mean not the *effects* of



selection, but the principles upon which it should be made) is one on which it is doubtful whether we have advanced much beyond the point arrived at in 1874, when, in a discussion on a paper "On the practice of the Eagle Company with regard to lives classed as unsound", Mr. Bailey referred to it as an obscure subject on which, up to that time, very little progress had been made, and no real information had been obtained as to the proper premiums to charge.

On the question of rating-up it may be that Actuaries have been too much in the habit of implicitly accepting the recommendations of their medical examiners. In this matter the old saying "doctors differ" receives abundant illustration; one examiner, for example, will recommend an addition of three years, where another, on the same set of facts, will perhaps suggest ten.

We are absolutely dependent, of course, upon the examining doctor for the facts in each particular case. We have to consider his report upon the physical condition and medical history of the applicant, and we are further assisted by the expert opinion of the principal medical adviser of the Office on the set of facts presented by the examiner. But in order to appreciate the advice of the experts, and to determine the terms upon which the risk is to be accepted, or whether it is to be accepted at all, the Actuary must possess some amount of medical knowledge.

I venture to make the suggestion that it might be desirable to impart some elementary medical instruction to students, and, possibly, as a consequence, to include the subject in the examinations. I do not suggest, of course, that we should require them to study medicine exhaustively; but, as we demand from them some elementary knowledge of law, in order that they may intelligently appreciate the advice which is given them by their legal advisers, it would seem not unreasonable that they should be required to show some acquaintance with some of the first principles of certain departments of physiology and medicine. The lectures which have been given in this Hall on certain legal and financial subjects have been most interesting and valuable, and it is worth consideration whether they might not be supplemented by similar lectures on certain branches of medical knowledge which bear upon our more immediate pursuits.

Passing, however, from the question of requiring medical knowledge from students, and as an illustration of the kind of medical subjects which might well engage the attention of the Institute generally, I may refer to the controversy which has

recently engaged the attention of the medical world as to the nature of pulmonary consumption.

A great medical authority has stated that the mortality by consumption has been reduced by two-thirds during the last 30 years, and that there is some reasonable prospect of its total disappearance in 30 years more. It is stated that the disease, so far from being an inevitable consequence of hereditary taint, is an infectious malady that may be effectively controlled by suitable preventive measures.

If this be true, as we hope it may be, it obviously has a direct and most important bearing upon our work. But the subject is still in the stage of experiment and enquiry, and we have not yet arrived at a point which will justify a reversal, or even a substantial modification, of our practice with regard to the terms and conditions on which to grant assurances on lives with a consumptive family history.

At a recent meeting of the Sanitary Institute, Sir J. Crichton Browne made some valuable remarks which may well be quoted here by way of caution to any who may be disposed to take too optimistic a view of this subject: "That pulmonary consumption  
" is no longer to be considered as an hereditary disease I regard  
" as a somewhat dangerous doctrine. In connection with  
" tuberculosis, we have not only to consider the seed, but the  
" soil, and it is undoubtedly true there are certain kinds of human  
" soil which are peculiarly favourable for the reception of the  
" tubercle bacillus, so that there is a tuberculosis in the  
" temperament, in the nature of certain classes of people who are  
" rendered peculiarly liable to contract the disease. It is a  
" matter of common knowledge that the disease does run in  
" families, and often in families widely scattered from each other,  
" and I think we ought not to banish the idea that there is a  
" certain amount of heredity in connection with it."

It would seem that medical opinion on this subject is as yet in a state of suspense, and we have to wait the result of further enquiry, and investigation by the medical profession, mainly of a purely physiological character. But considerable light can doubtless be thrown on the subject by statistics, and in this branch of the enquiry the experience and observations of actuaries cannot fail to be of signal service, not only in furnishing facts and figures, but also in the equally important matter of examining and checking the inferences derived from them. Ten years ago Mr. Manly contributed a paper on the subject of consumptive

family history, in which he made some most valuable remarks on the common tendency to the misuse of statistics arising out of the attempt to draw general conclusions from partial or imperfect observations.

Not only in medical matters, but on many other social and economical subjects, the training of an Actuary enables him to detect and expose fallacies in popular conclusions from supposed facts, and to correct popular impressions. Take, for example, the exaggerated ideas sometimes expressed of the effect which the improved conditions of life in modern times have had upon its average duration. It is of course true that the wonderful advances which have been made in medical science, the increased attention which has been given during the last half century to sanitation and the drainage and water supply of towns, the great changes which have taken place in the habits of all but the lower classes in the matter of temperance, and the spread of general education, have led to considerable improvement in the health and longevity of the people. But I believe that the extent of this improvement is popularly thought to be much greater than it really is, and, indeed, is after all less than might have been reasonably expected. For, together with all these favourable influences, there have been others working (but less obviously) in a contrary direction. I believe I am right in saying that, with a diminution in the frequency and intensity of zymotic disease, there has been an increase in maladies of other kinds, and at all events the increased strain on brain and nerve induced by the stress of modern business life—in common phrase, the greater pace at which we live—has done much to impair the effects of improvements in sanitation and therapeutics.

I have mentioned incidentally the improvement that has apparently taken place in recent years in the matter of temperance. It is a matter of common observation that among the middle and upper classes a great change has passed over society in drinking habits. The practice of total abstinence from alcoholic drinks has largely increased, and is increasing, thanks very largely to the efforts of ardent temperance advocates. Among the arguments which have been used in this propaganda, one of the most powerful and most frequently urged has been the asserted influence of total abstinence upon the duration of life, and in support of this argument it is not unusual to cite the experience of those Assurance Societies which distinguish between abstainers and non-abstainers. I do not



think, however, that any Actuary of these Societies has ever made any contribution to our proceedings giving particulars of the experience of so-called Temperance Sections.

It is much to be desired that we should have detailed information on a matter so important, not only because it is of the highest interest to us as Actuaries, but because a most beneficent moral influence might be expected to follow a rigidly scientific demonstration that total abstinence does actually prolong life. At present we are asked to place reliance upon certain results, but it would obviously be far more convincing if we could examine minutely the data and the processes by which these results are arrived at. We have had more than one paper demonstrating the high rate of mortality which prevails among licensed victuallers and other dealers in intoxicating liquors, a matter on which scientific proof was almost superfluous, and it is to be hoped that we may be favoured by contributions proving with equally minute and scientific accuracy the truth of the oft-repeated assertions as to the effect of total abstinence upon the duration of life, assertions which I venture to say rest at present upon somewhat uncertain ground.

Another subject on which we are all at present more or less in the dark, and on which there is no doubt a good deal of popular misconception, is that of the risks of warfare. When the South African war first broke out the newspapers were full of letters on the hardships arising out of the demand for increased premium for assurances on the lives of soldiers, and it was hinted, or even broadly asserted, that the Offices were taking undue advantage of the circumstances to call for extra premiums far in excess of the necessities of the case. The conclusion of the war leaves us in possession of a mass of facts and figures on which to form an opinion as to how far, if at all, there was any foundation for these complaints, or whether on the other hand the premiums demanded were not in fact altogether inadequate. On this subject I am glad to be able to announce that a member of this Institute is preparing a paper which will almost immediately be submitted, dealing very exhaustively with the statistics of mortality by disease, as well as by what are by the War Office euphemistically called casualties, among the troops engaged in South Africa. I believe that no serious contribution to our knowledge on the subject of the risks of warfare has been made since those by Mr. Ackland and Dr. Smee in 1890. There is a paper by Mr. M'Lauchlan, in 1898, on the Mortality of the

British Army and Navy—but the specific topic of risks of warfare is only briefly and incidentally referred to. During the twelve years which have since elapsed the conditions of warfare have undergone considerable changes; the nature of the weapons employed, and more especially the nature of military methods and tactics have greatly altered, and it will be of the greatest interest to see how far the results of the late war confirm or modify the assumptions and conclusions of the authors of the papers to which I have referred.

Somewhat similar to this question of the proper extra premium for the risks of war, is the more general question of the rates to be charged for residence in tropical and other unhealthy climates. When such an extra premium is asked for from a policyholder it is no uncommon thing for him to attempt to persuade us that the particular place to which he is going is really one of the healthiest spots in the world, and he may even produce facts and figures in support of his assertion. It is usually easy to detect the fallacy in such arguments, but I think it must be admitted that it is always difficult and often impossible to determine with any accuracy the true rates which should be required for the risk. The subject has from time to time received careful attention, but, with the exception of Mr. Sprague's paper on West African Mortality, and a note by Mr. Hart, most of the papers dealing with questions of this kind which appear in our Transactions are of somewhat remote date. Since most of those papers were written many parts of the world then almost or altogether unknown have become peopled by Englishmen, and populous cities have sprung up in what were till lately desert places. At the same time the conditions of travel and foreign residence have undergone many changes. The medical profession has directed special attention to the causes and treatment of malaria and other tropical diseases, and it is probable that residence in the tropics has become less dangerous to Europeans than formerly. Whether this is really the case, and to what extent, can only be determined by careful examination and analysis of statistics. Possibly a sufficient mass of facts and figures has not yet been accumulated for an exhaustive treatment of the subject, but it is one on which all available information should be examined without delay in view of the probability that a large number of Englishmen may at no distant date take up a temporary or permanent abode in Southern or Central Africa. The fact that it is now a very common practice to grant what are

known as whole-world policies may make the question of extra premiums a comparatively small matter, but this very fact makes the question of the extra risk incurred by foreign residence of all the more real importance.

The question of vital statistics, not only in tropical climates but in all other parts of the world beyond the limits of the United Kingdom, is becoming of increasing interest and importance with the rapidly increasing frequency of travel and the spread of the Empire. In the collection and investigation of facts in connection with enquiries of this kind we may be materially aided by the researches of our members in the Colonies. It is very gratifying to find that so great an interest is taken in the Institute by its now numerous members in Australia, Canada, and other parts of Britain beyond the seas, who have already contributed several valuable and interesting papers to the pages of our *Journal*. The increasing number of Colonial students who submit themselves for examination is additional evidence of the great interest taken in actuarial subjects in the remoter parts of the empire. The examiners report that the work of the Colonial students is generally of a high degree of excellence, and we may hope at no distant time to receive from them, as they pass beyond the student stage, many a communication throwing new light on old subjects, as well as valuable additions to our knowledge on topics which have not hitherto engaged our attention.

Although by the nature of the case it can be but rarely that we can be favoured by the actual presence of Colonial members to read papers in this Hall, we may hope that they will with increasing frequency send communications to be read for them. Even if they do not send us full-sized contributions for such formal reading and discussion, short notes for insertion in the *Journal* will be gladly received, and, although I believe the Editor is already somewhat perplexed by the increasing bulk of the *Journal*, I am sure he will contrive to find room for such communications.

It has been sometimes asserted—though not, I think, by any of its members—that the Institute has exhausted all the subjects which are its proper subjects of enquiry, that nothing remains for it to do but to retrace old and familiar ground, and to produce new variations upon well-worn themes. Even if this were true, there remains much that has yet to be discovered upon the old and familiar ground, and the variations on the well-worn themes are capable of infinite variety. The march of events, and the ever-changing circumstances of the times, are continually



bringing us into positions from which we view the oldest and most familiar objects in entirely new aspects, and it becomes necessary to review our opinions, and sometimes even to reject as fallacious old arguments which at one time seemed unquestionably sound. In illustration of this, I may remind you of the lively interest excited last session by Mr. Warner's paper on "Net Premium Valuations", a subject on which thirty years ago it was very probably thought that the last word had been said. Yet this paper gave rise to perhaps the most animated debate which we have had for a very considerable time, and gave occasion to the expression of opinions which would at one time have been considered absolutely heretical. We are in no danger of exhausting subjects for enquiry and discussion, even if we confine ourselves to topics immediately connected with the business of life assurance. But although the circumstance that most of us are actively engaged in the work of administering the affairs of Life Assurance Companies has no doubt the effect of confining our attention, if not quite exclusively, at all events mainly, to subjects immediately connected with life assurance, we cannot overlook the fact that actuarial principles are applicable to many subjects which lie altogether beyond the sphere of insurance work.

In ordinary commerce, for example, the merchant estimates the probability of the rise and fall of prices, the abundance, or scarcity, of supply, and the varying pressure of demand. In most of such cases, no doubt, ordinary experience and sagacity afford a sufficient guide, but there are some which might well be made the subject of enquiry and analysis to which they have not yet been adequately subjected.

It would not, I think, be travelling beyond the province of this Institute to investigate some of these problems in which the laws of probabilities, other than those involving the duration of human life, are involved. Many of these problems would, no doubt, involve the consideration of elements of extreme complexity, depending perhaps upon such continually altering and shifting conditions as to defy complete analysis, but yet presenting an amount of regularity sufficient to enable us to arrive at approximate solutions of great practical value. Many years ago, Mr. Brown mentioned, as examples of this kind of question, the law of the recurrence of commercial crises, questions connected with Fire and Marine Insurance, and the regularity of the action of the human will. To these may be added questions connected

with the rate of growth of the population, the distribution of wealth, and the incidence of taxation. Indeed, there is hardly any department of Political Economy which does not present problems of a kind which demand the application of actuarial principles for their solution.

One of these problems which seems to demand particular attention is that of the Death Duties, and I think that anyone who would make an exhaustive examination of their probable ultimate effects, if the present law remains unaltered, would be doing a great public service.

Another question is that of old age pensions, of which we have heard so much for some years past, and of which we are certain to hear a great deal more. The pressure of public opinion will almost certainly compel the Government, sooner or later, to deal with the question in some form or other, and whatever form the proposed measures may take, they will be sure to be of a kind to require careful study by the members of our profession. It is hardly likely that any attempt will be made to carry into effect the extreme proposals which at one time found favour with certain ardent philanthropists, who fixed their attention solely upon the desired ends with entire disregard of the cost at which only they could be obtained. But it is very probable that schemes will yet be proposed and pressed, less obviously impracticable but equally unsound, based upon imperfect or erroneous statistics and supported by illogical and fallacious arguments. It will be the duty of Actuaries to keep a careful watch upon any proposals that may be made and to use every possible means to direct public attention to errors in premises and fallacies in arguments, and especially to the probabilities of the consequences which may be expected to follow the adoption of any particular schemes that may be suggested.

Is it too much to hope that before any Bill dealing with the subject receives the active support of Government it may be submitted for the formal consideration of this Institute?

Independently, however, of legislative enactment, some progress towards the solution of the old age pensions problem is being made by the noticeably increasing popularity of endowment assurances.

The original idea of life assurance was that of protection against loss by death, especially in the case of those persons whose families were dependent upon their personal earnings. In early days, and indeed until comparatively recently, all the arguments

in favour of assurance were directed to the necessity of making provision for widows and children, and stress was laid upon the duty of every head of a family to deny himself and make a sacrifice of part of his present income so as to secure a provision for those he should leave behind him. Every agent used to be armed with bundles of leaflets enforcing this argument in which the phrasology of religious tracts was largely adopted, and even texts of Scripture were freely employed as aids in the procuration of business. I am glad to observe that this very objectionable form of advertisement appears to be falling into disuse. The appeal is now not so much to self-sacrifice as to selfishness. Life assurance is now very generally represented as only incidentally a family provision and mainly as an excellent investment, and the presentation of this aspect of the case is found to be a much more powerful means of securing new business than the old-fashioned appeal on behalf of the bereaved family.

In some quarters, I believe, the growing popularity of endowment assurances is viewed with distrust and dislike. It is said that it is not life assurance, that it is at variance with the purposes for which the companies exist, and is converting them into mere savings banks or investment societies. Even if there be some truth in this, it is nevertheless certain that the offer of endowment assurances has been the means of inducing a large number of parents, who would never have effected ordinary assurances, not only to provide for their young children against the risk of orphanage, but also to secure for themselves a provision for their old age when the children are grown up and provided for. Moreover, these contracts, as compared with whole-life policies, possess the advantage of providing against the converse contingency of prolonged life, when not only does the necessity for family provision usually disappear, but it may happen that the continued payment of premiums becomes a heavy burden, and the total amount paid may at last considerably exceed the amount to be received.

The only valid objection which there would appear to be to endowment assurances is that the policyholder is made to contribute, perhaps unknowingly, to a fund in which it is possible he may never participate. In other words, he is a contributor to something in the nature of a tontine, a form of investment which appears to be regarded by some people as inherently immoral. But this objection would equally apply to deferred annuities, except that in this case the true nature of the investment is



at once apparent to the contributor, a fact which no doubt largely accounts for the general unpopularity of this class of transaction. But the essential practical difference appears to be that, whereas in the case of deferred annuities a certain number of the contributors will never receive any pecuniary return for their money, the grantee of an endowment assurance must in any event receive a return exceeding (and, in the event of early death, very considerably exceeding) the amount of his investment. It is true that the assurance part of the benefit has been secured by only a part of the contribution, the rest enuring for the benefit of the survivors, but the investment is made in most cases mainly with a view to securing provision for old age, and with the hope and expectation of surviving the term. For my own part, I look upon the popularity of endowment assurances, if not made payable at too early an age, with every satisfaction. Life Assurance Companies are thus becoming not only a means of providing indemnity, but also an agency for the promotion of thrift, especially in the case of persons of small income, who are thus provided with a means of investment for small sums which they would otherwise probably fail to save. I have had the opportunity of watching the growth of this class of investment somewhat closely, and I have been much interested to observe evidence of the existence of habits of thrift among the lower-middle, and artizan classes. These classes are often represented as extravagant and improvident, and we are sometimes asked to believe that they spend the greater part of their surplus income upon beer and betting. Although no doubt a deplorably large part of the earnings of the people is actually wasted in this and other undesirable ways, it is also true that a very much larger proportion of the working classes than is generally believed is in the habitual practice of provident habits, and they are themselves taking a no inconsiderable step towards the solution of the old age pension problem.

The rapid increase in recent years of the total amount of the funds of Life Assurance Companies makes the question of their proper investment of correspondingly increasing importance. The Board of Trade Returns show that in 1891 these funds amounted to £217,500,000, in 1896 the figure was £263,800,000, and at the date of the last return in the present year the total appears as £311,000,000. These figures, however, do not of course fully indicate the total amount for which investments have to be found. Old mortgages are continually being paid off,

in many cases to be replaced at rates more favourable to the borrower, and large amounts of capital are being returned in the shape of instalments of terminable annuities. In former times when the amounts seeking investment were comparatively small, and securities of the highest class were readily obtainable at 4 per-cent, or even higher rates, the question of investment naturally attracted but little attention. But with the vast sums that have now to be placed, the comparative scarcity of old-fashioned securities, and the marked tendency to a decrease in the rate of interest, attention is being directed to the possibility of finding new directions for investment.

For the moment a wave of lower prices, and consequently higher yield, has, to some extent, diverted attention from this subject, but there is every reason to regard this condition as merely temporary, and the downward trend in the rate of interest on first-class securities will re-appear. In the discussion which followed Mr. Barrand's paper on the question of the suitability of commercial and trading securities for the investment of life assurance funds, it appeared that the desirability of extending the area of investment was fully realized, although his particular suggestions did not meet with very general acceptance. It has always been a cardinal doctrine with the managers of assurance companies that the absolute safety of the principal is paramount, and that, although the realization of as high a rate of interest as possible is all-important, it must stand only in the second place. But as the realizable rate on what are called gilt-edged securities seems to be permanently diminished, the question of widening the area of investment can no longer be disregarded. It would be most interesting and valuable if we could ascertain or estimate with any degree of accuracy what would be the effect of investing a large number of the small amounts in what may be called speculative securities, and of carrying the amount of interest realized, in excess of a certain minimum, to a guarantee or insurance fund, itself to be invested in first-class securities to be used to make good losses of capital or deficiencies of revenue as they occur. I by no means advocate a trial of the experiment in actual practice, but it might be useful if anybody possessed of the necessary patience and perseverance would make the experiment on paper by selecting a number of such securities, and following their fortunes to see what the result would be.

In connection with the question of investments, I may refer to a subject on which it would be well if some consensus of

opinion could be arrived at, namely, the proper mode of estimating the value of marketable securities, and the figure at which they should appear in a company's balance-sheet. To write down to the market quotations on the day of closing the accounts those securities which were purchased at a higher price, and at the same time to retain at the purchase price those which have appreciated, may be defended on the score of safety, but it is obviously devoid of any other principle, and must result, in the course of time, in seriously under-valuing the assets. When, as happened on the last day of the last century, market values are unusually and considerably lowered, the inconvenience of such a proceeding becomes apparent. A paragraph in the recently-issued Report of the Select Committee of the House of Commons on Savings Bank Funds may be quoted in illustration. They say: "Although the law requires an annual valuation of the capital assets on the basis of the market value of the day, they cannot but regard it as somewhat misleading. In 1897, when Consols stood at 113, a similar valuation showed a surplus to the Trustees' Savings Bank Fund of £4,453,000 and to the Post Office Savings Bank Fund of £12,780,000. In 1901, when Consols stood at 91 $\frac{7}{8}$  on 20 November, these figures were turned into a deficiency on the Trustee Savings Bank Fund of £2,680,000 and on the Post Office Savings Bank Fund of £5,000,000. These deficiencies have since been largely reduced by the rise in securities, but no valuation on this basis gives a correct view of the real position, for the securities belonging to the Funds are practically held as permanent investments."

The retention of the purchase price of this class of securities in the balance sheet, together with a suitable Investment Fluctuation Fund, is probably the best practical mode of dealing with the question; but other ways of meeting the difficulty have been suggested which are not undeserving of some attention. One of these, which bases the value upon a capitalization of the income, appears to me to be worthy of consideration, and, although it has been referred to somewhat disparagingly by Mr. Higham as a "curious Italian system", I think it might very well be more closely examined.

In the foregoing remarks, I have endeavoured to direct attention to practical subjects which I think should engage the attention both of the students and of the older members of the Institute. I have made no reference to any purely scientific or technical subjects, partly for the reason indicated in my opening



remarks, and partly because there is no necessity for pressing them upon your attention. There is no danger that the scientific functions of the Institute will ever be overlooked or neglected. On the contrary, I believe the opinion has been expressed that mathematical research is its only true function, and that discussion of practical questions should be only incidental and occasional. This I hold to be not only a mistaken but even a dangerous doctrine. An Actuary should above all things be a practical man, but if his education and training have been mainly or altogether technical, he is in danger of becoming a mere pedant, and though possessed of the highest scientific acquirements may yet be lacking in all the essential qualifications of a business man. The Actuary of a Life Assurance Company, if he is not also the Manager, should have a large share in the actual management and direction of its affairs. He should be much more than the mere scientific adviser of the management, without an active voice in the direction of the Company, and should not be like the engineer of an ocean steamer who reports to the captain the pressure of the steam and the number of revolutions of the screw, but has no part in the setting of the course or the navigation of the vessel. The discussion of mathematical theory, sometimes of a very abstruse and technical character must necessarily form a large part, and to many minds by far the most attractive part, of our proceedings; but if the Institute is to be, as I think it should be, a complete school of insurance, the discussion of practical subjects necessarily claims its proper share of consideration. In dealing with these practical subjects, however, it is necessary to beware of regarding them from what may be called a too practical point of view; in other words, the practical subjects should be discussed in a scientific spirit.

The practical and the theoretical can never indeed be entirely dissociated, for in the discussion of the most abstract subject its ultimate utilitarian application should never be entirely lost sight of, nor must any proposition of a practical character be adopted without careful survey of its character from a theoretical standpoint. Life Assurance is a business, and although a business of a highly technical character, must after all be conducted with a due regard to ordinary commercial principles and practices. And here may I say a passing word on the subject of advertising? In common with every other trader it is necessary and right that the Assurance Manager should display his wares to the best advantage, and call attention to them by every legitimate

means. But in so doing he should, I think, be reasonably reticent and dignified, and in the preparation of documents for public circulation should beware of a tendency which is sometimes observable to employ the meaner arts of advertisement and to adopt phrases and catch-words which are unpleasantly suggestive of the "wonderful value" and "phenomenal bargain" tickets of the lower class shop windows. The stress of competition naturally leads a manager to do all he can to make his Company attractive to the public, and he may in consequence yield too readily to popular demand for what are called liberal concessions, and may be tempted to introduce novelties of various kinds to secure public attention and patronage. Unless duly fortified by technical knowledge, so as to be able to estimate the real effect and cost of these concessions and novelties, he may involve his Office in difficulties not easy to repair. For these concessions are costly, and if under the pressure of competition surrender-values and other benefits are increased, and foreign residence and dangerous occupation risks are accepted at inadequate extra premiums, or none at all, while expenses do not diminish, and the rate of interest declines, it must happen ultimately that profits will dwindle and bonuses decrease, and, as the magnitude of the bonus is the criterion by which the public estimate the merits of an Assurance Company, the popularity which was the object of the concessions may receive a serious check.

On the other hand, the too philosophical Actuary, whose principal anxiety is to discover and adopt the most scientific methods, and to work out his arithmetical result correct to the last place of decimals, may by the neglect of, and, perhaps, contempt for, ordinary commercial considerations, allow himself to be altogether outstripped by his more practical competitors in the race for business.

We are looking forward with pleasure to the meeting of the Fourth International Congress of Actuaries which is to be held in New York in September. The fact that the place of meeting on this occasion is in an English-speaking country may induce some to attend who did not go to the Brussels and Paris meetings; and, although a voyage across the Atlantic is somewhat more formidable than a journey across the Straits of Dover, it is to be hoped that neither the greater distance nor the consequent necessity of a longer absence from home will prove a serious obstacle to the attendance at the Congress of a

goodly number of our members. Successful as the former conferences have been, there is every prospect that the forthcoming meeting will surpass its predecessors in utility and intent. Many subjects which were discussed at the Paris Congress, have since received more particular attention, and we may expect that the contributions to the forthcoming meeting may throw new light upon such subjects as provident funds, invalid and accident assurance, occupation mortality, the effect of reduction in the rate of interest, various methods of profit distribution, and other important and interesting subjects. The permanent Committee of Congress has, no doubt, profited by the experience of the former meetings to introduce certain desirable improvements in the conduct of the proceedings, and has, no doubt, duly considered the hints on this subject by Mr. Higham in his comments on the last meeting. The fact that the Congress will meet upon American soil will, no doubt, be the occasion of directing special attention to certain matters in which the methods and ideas of American Actuaries and life assurance officials differ from those which prevail in this country, and if they fail to persuade us that these matters are better dealt with by them than by ourselves, we may at least learn to better appreciate their point of view. We may be quite sure that, whatever may be our differences of opinion on matters of practice or of theory, we shall receive a welcome characteristic of the hospitable American people. To some of us, many of the principal American Actuaries are already personally known, and it will be a great pleasure to us to make the personal acquaintance of many others known to us as yet only by their writings, and to have the opportunity which will, no doubt, be afforded us, of inspecting some of the details of the numerous and vast assurance organizations whose existence and magnitude are almost daily impressed upon us by their activities in competition with ourselves.

Addressing myself now, according to custom on these occasions, to the students and younger members of the Institute, I desire to say a word or two on matters to which I have already referred in the earlier part of this address. I can well believe that some of you have heard with something like dismay the suggestion of the possible addition of the subjects of medicine and physiology to our already voluminous syllabus. Although the suggestion has been made, I doubt whether it could be carried into effect without the sacrifice of some other subject, and



it is not easy to see how this could be done. But, whether medical matters be ultimately included in the subjects for examination or not, it is quite certain that some knowledge of this kind is essential for the due performance of an Actuary's work, and it must be acquired, even if it is not studied for examination purposes. And so with many another department of knowledge. It is of course impossible that every kind of knowledge required for the conduct of life can ever be included in an examination syllabus. And it is equally true that success, and even brilliant success, in any set of examinations is no guarantee (though it may afford a strong presumption) of general efficiency. Success in life does not inevitably follow success in the examination room, for the latter can be achieved by close attention to the subjects within the necessarily limited bounds of the syllabus, to the absolute exclusion of all that lies beyond it. It is common enough to find young men preparing for examinations who steadily decline to read anything that, as they express it, "does not pay," with the result that they emerge from the examination room, like Dickens' Sol Gills, "chock full of science," but almost absolutely ignorant of everything beyond the technicalities of their profession; and what is worse, ignorant of their ignorance, and with an exaggerated estimate of their own abilities. This devoted attention to text-books, and the highly concentrated form in which knowledge is now obtainable for examination purposes, may indeed defeat its own end, and perhaps the large proportion of failures to which I have referred may be partly the result of attempting to take in too much at once in too short a time, and with a mind and intellect ill adapted to receive it, through the neglect of broader and more general intellectual training.

I have referred to the disabilities under which earlier students laboured, as compared with those of the present day, but those very disabilities may have been actually advantageous in so far as they compelled attention to the broad study of principles, and made it necessary for them to think out for themselves many matters which are now presented to the modern student cut and dried, ready to be stored, if possible, in the memory, to be produced in the same dry form in the examination room, but which fail to obtain effectual entrance into the intelligence. I have heard of a schoolboy who succeeded in the prodigious feat of learning by heart the whole of the first book

of Euclid, without having any idea of its meaning; and I have seen answers to some of our own examination questions which have raised a suspicion of a similar process of assimilation of pages of the *Text-Book*. Many of the failures of our examinees arise from hurry and imperfect preparation, and too mechanical and exclusive attention to the special subjects in the syllabus; and it would be well if a longer time and slower pace were adopted, so as to allow of opportunity for the training of the intellect by general reading and the study of subjects not directly connected with the examination.

To some of you it may perhaps appear little short of ridiculous that I should recommend as a preparation for our examinations that a student should take up the study of history or general literature, but I am quite sure that the deliberate study of some apparently remote subject, not under the forced draught with which the technical subjects are studied, but deliberately, quietly, and with no examination end in view, would so invigorate and brace the mental powers that the technical subjects would be more easily and completely mastered. I would especially recommend the study of the great masters of English prose, as of the greatest practical utility, not only by way of storing the mind with useful information, but as inducing clearness of thought and lucidity of expression, qualities often conspicuously absent from business documents, and not invariably found even in contributions to our own *Transactions*.

I am pleased to be able to make an announcement which I am sure will be very welcome to students. After a rather long delay, occasioned by unforeseen circumstances, the new building adjoining this hall is at length approaching completion, and I hope that in a few weeks we shall enter upon the occupation of an additional large room which will be available for classes and as a reading room, and will also enable us to hold the examinations next April and henceforward under much more favourable conditions of air and space than have hitherto prevailed.

It is very satisfactory that the increasing number of our students and the growing popularity of our science should have made this widening of our space necessary. It is only a few years since this ancient Hall, after having been devoted for two centuries to the service of students of the law, was in danger of demolition as a useless anachronism. Just as its fate seemed inevitable, it fortunately became the seat of this Institute, and

entered upon a new career of usefulness. Its ancient kitchen, in which no doubt many a banquet for the legal worthies of Staple Inn has been prepared, was transformed into a class-room for actuarial students, who have now become far too numerous for its inconvenient and narrow bounds. For their better accommodation the new and spacious room has now been prepared, but the Hall itself will still be the place of our general meetings and discussions, and, as the knowledge of actuarial subjects becomes more generally diffused, will achieve a renown far exceeding any that ever attached to it as one of the Inns of Chancery.

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*The Use of Quadrature Formulæ and other Methods of Approximation for the Calculation of Survivorship Benefits. By JAMES BUCHANAN, M.A., F.I.A., F.F.A., of the Scottish Widows' Fund Life Assurance Society.*

THE great value of quadrature formulæ for the calculation of survivorship and other benefits involving three or more lives has been already illustrated in the pages of the *Journal*. From certain remarks made by Mr. G. F. Hardy and Mr. George King in the discussion which followed the reading of Mr. King's paper (*J.I.A.*, xxvi, pp. 297, 301), I inferred that it would be of interest to examine the errors involved in these formulæ, and to compare the results yielded by them and by other methods of approximation, such as the substitution of an equivalent life. Mr. Sheppard has shown (*Proceedings of the London Mathematical Society*, xxxii, p. 258) how most of the best-known quadrature formulæ can be readily derived from the Maclaurin Summation Theorem, with expressions for the errors in terms of differential coefficients. In a recent paper (*Proceedings of the London Mathematical Society*, xxxiv, p. 335), I have obtained similar formulæ by direct integration from Everett's Interpolation Theorem, with expressions for the errors in terms of central differences.

Denoting by  $p$  the distance of the ordinate  $u_p$  in front of  $u_0$ , and by  $q$  its distance behind  $u_1$ , so that  $p+q=1$ , Professor Everett writes (*J.I.A.*, xxxv, p. 452),



$$u_p = \left[ q + \frac{q(q^2-1)}{3!} \delta_2 + \frac{q(q^2-1)(q^2-4)}{5!} \delta_4 + \dots \right] u_0 \\ + \left[ p + \frac{p(p^2-1)}{3!} \delta_2 + \frac{p(p^2-1)(p^2-4)}{5!} \delta_4 + \dots \right] u_1$$

where  $\delta_2, \delta_4 \dots$  are the even central differences.\*

If  $a, b, c \dots$  be the factors, including unity, of  $n$ , I showed that

$$A = \int_0^{nh} u_x dx = A_a + \epsilon_a$$

with similar equations involving  $b, c \dots$ ; where

$$A_a = ah \left[ \frac{1}{2} u_0 + u_{ah} + u_{2ah} + \dots + \frac{1}{2} u_{nh} \right]$$

and

$$\epsilon_a = -\frac{a^2 h}{720} \left[ 6\delta_1 - (a^2 + 10)\delta_3 + \left( \frac{a^4}{42} + \frac{a^2}{4} + 2 \right) \delta_5 \right. \\ \left. - \left( \frac{a^6}{1680} + \frac{a^4}{126} + \frac{7a^2}{120} + \frac{3}{7} \right) \delta_7 + \dots \right] (u_{nh} - u_0)$$

$\delta_1, \delta_3, \dots$  being the odd central differences of the series of functions  $\dots u_{-h}, u_0, u_h \dots$ . Hence

$$(p + q + r + \dots)A = pA_a + qA_b + rA_c + \dots + p\epsilon_a + p\epsilon_b + p\epsilon_c + \dots$$

This equation is exact. The expression

$$\frac{pA_a + qA_b + rA_c + \dots}{p + q + r + \dots}$$

gives an approximate value of the integral, while

$$\frac{p\epsilon_a + q\epsilon_b + r\epsilon_c + \dots}{p + q + r + \dots}$$

represents the error.

\* The central difference notation adopted here is different from that employed in the paper quoted above. There I used two operators,  $\delta$  and  $\mu$  (the latter a new one due to Mr. Sheppard, cf. *Proceedings of the London Mathematical Society*, vol. xxx, pp. 428, 460), defined by the relations

$$\delta u_0 = u_{\frac{1}{2}} - u_{-\frac{1}{2}} \\ \mu u_0 = \frac{1}{2}(u_{\frac{1}{2}} + u_{-\frac{1}{2}}).$$

This notation possesses certain advantages over that used above, and it would have been in some ways preferable to employ it again. But  $\mu$  has to actuaries a well-defined meaning, and it would have been very inconvenient to use it in two senses in the same paper; the suffix notation has accordingly been adopted.

If  $p, q, r \dots$  be chosen to satisfy the equations

$$pa^2 + qb^2 + rc^2 + \dots = 0$$

$$pa^4 + qb^4 + rc^4 + \dots = 0$$

$$\dots \dots \dots$$

the coefficients of  $\delta_1, \delta_3, \delta_5, \dots$  in the expression for the error can be made to vanish, and we get approximate formulæ which are true to third, fifth,  $\dots$  differences.

In what follows  $n$  is taken equal to 6, and the letter  $h$  is for convenience omitted from the suffix of  $u$ . If we take two factors,  $a$  and  $b$ , and put  $a=1, b=2$ , we get, since  $pa^2 + qb^2 = 0$ ,  $p : q :: 4 : -1$ , and

$$A = \frac{1}{3} [4A_1 - A_2] = \frac{h}{3} [u_0 + u_6 + 4(u_1 + u_3 + u_5) + 2(u_2 + u_4)] \quad \dots (i)$$

while the error is readily found to be

$$-\frac{h}{180} \left[ \delta_3 - \frac{31}{84} \delta_5 + \frac{557}{5040} \delta_7 - \dots \right] (u_6 - u_0)$$

This is Simpson's Rule.

If we put  $a=1 : b=3$ , then  $p : q :: 9 : -1$ , and

$$A = \frac{1}{8} [9A_1 - A_3] = \frac{3h}{8} [u_0 + u_6 + 3(u_1 + u_2 + u_4 + u_5) + 2u_3] \quad \dots (ii)$$

which is Simpson's Second Rule, with an error

$$-\frac{h}{80} \left[ \delta_3 - \frac{41}{84} \delta_5 + \frac{967}{5040} \delta_7 - \dots \right] (u_6 - u_0)$$

Taking three factors  $a, b, c$  and  $p, q, r$ , to satisfy the equations

$$pa^2 + qb^2 + rc^2 = 0$$

$$pa^4 + qb^4 + rc^4 = 0,$$

we get for  $a=1 : b=2 : c=3, p : q : r :: 15 : -6 : 1$ , and

$$\begin{aligned} A &= \frac{1}{10} [15A_1 - 6A_2 + A_3] \\ &= \frac{3h}{10} [u_0 + u_2 + u_4 + u_6 + 5(u_1 + u_5) + 6u_3] \quad \dots (iii) \end{aligned}$$

which is Weddle's Rule, with an error

$$-\frac{h}{840} \left[ \delta_5 - \frac{41}{60} \delta_7 + \dots \right] (u_6 - u_0),$$

while with four factors,  $a=1 : b=2 : c=3 : d=6$ , we should get

$$A = \frac{1}{840} [1296A_1 - 567A_2 + 112A_3 - A_6]$$

$$= \frac{h}{140} [41(u_0 + u_6) + 216(u_1 + u_5) + 27(u_2 + u_4) + 272u_3] \quad \text{. . . (iv)}$$

with an error

$$- \frac{3h}{2800} [\delta_7 + \dots] (u_6 - u_0).$$

This formula was given by Woolhouse (*J.I.A.*, xi, p. 313), but it appears to have been known to Simpson (cf. *British Association Report*, 1880, pp. 336-7).

Other formulæ can readily be obtained by elimination. Thus, the elimination of  $u_2$  and  $u_4$  between Simpson's two Rules gives

$$A = \frac{1}{11} [18A_1 - 9A_2 + 2A_3] = \frac{3h}{11} [u_0 + u_6 + 6(u_1 + u_5) + 8u_3] \quad \text{. . . (v)}$$

with an error

$$+ \frac{h}{220} \left[ \delta_3 - \frac{71}{84} \delta_5 + \frac{2197}{5040} \delta_7 - \dots \right] (u_6 - u_0)$$

while the elimination of the same two terms between (iii) and (iv) gives

$$A = \frac{1}{300} [486A_1 - 243A_2 + 58A_3 - A_6]$$

$$= \frac{h}{50} [14(u_0 + u_6) + 81(u_1 + u_5) + 110u_3] \quad \text{. . . (vi)}$$

which is Mr. G. F. Hardy's well-known formula, with an error

$$+ h \left[ \frac{3}{1400} \delta_5 - \frac{1}{224} \delta_7 + \dots \right] (u_6 - u_0)$$

Of the above, some err in excess and others in defect of the true value, and by combining them in various ways we can get others in which the coefficients of the leading differences in the expression for the error are very much reduced. Thus the mean of (i) and (v) gives

$$A = \frac{1}{33} [49A_1 - 19A_2 + 3A_3]$$

$$= \frac{h}{33} [10(u_0 + u_6) + 49(u_1 + u_5) + 11(u_2 + u_4) + 58u_3] \quad \text{. . . (vii)}$$



with an error

$$-\frac{h}{1980} \left[ \delta_3 + \frac{149}{84} \delta_5 + \frac{6823}{5040} \delta_7 - \dots \right] (u_6 - u_0)$$

while, if we double (iii) and add it to (vi), we get

$$\begin{aligned} A &= \frac{1}{900} [1386A_1 - 603A_2 + 118A_3 - A_6] \\ &= \frac{h}{150} [44(u_0 + u_6) + 231(u_1 + u_5) + 30(u_2 + u_4) + 290u_3] \dots \text{(viii)} \end{aligned}$$

with an error

$$-\frac{h}{12600} \left[ \delta_5 + \frac{143}{12} \delta_7 + \dots \right] (u_6 - u_0).$$

Many other formulæ of this kind are given in the papers already referred to.

In all the preceding formulæ the earlier differences are got rid of at the expense of increasing the coefficients of the later ones, and it is well known that in practical applications the differences generally run with great irregularity. They often change sign, and after first decreasing numerically they not infrequently increase rapidly in proceeding to the higher orders; so that a formula which is true to third differences only may give better results than one which is true to the fifth or higher orders. This is illustrated by the following examples.

Using six intervals with ordinates  $u_0, u_1, \dots, u_6$ , I computed the values of the integrals—

$$\int_0^1 \frac{dx}{1+x} = \log_e 2 \quad \dots \dots \dots (a)$$

and

$$\int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4} \quad \dots \dots \dots (b)$$

The true values to seven places of decimals are .6931472 and .7853982, and the errors in the computed values are as follows:

Formula	Order of Approximation	Number of Terms	Error in (a) $\times 10^7$	Error in (b) $\times 10^7$
(i)	3rd differences	7	+ 226	- 3
(ii)	„	7	+ 482	- 23
(v)	„	5	- 146	+ 28
(vii)	„	7	+ 40	+ 12
(iii)	5th differences	7	+ 22	+ 14
(vi)	„	5	- 15	- 179
(viii)	„	7	+ 10	- 50
(iv)	7th differences	7	+ 9	- 55

This table shows that in (a) the formulæ which are true to the higher orders give the best results, while in (b) the third difference formulæ are more accurate. If we examine the differences we find that in the former case the central differences of  $u_0$  decrease numerically up to the fifth, after which they increase and finally become infinite, since  $u_{-1} = \infty$ ; the increase in those of  $u_1$  begins much later. In the latter, the odd central differences of  $u_0$  are all zero, while those of  $u_1$  begin to increase numerically after the third. A preliminary examination of the differences will often guide us as to which set of formulæ will give the best results; but when once the values of the ordinates are known, that of the integral can be found by several formulæ with very little additional trouble, and as some err in excess and others in defect, we shall generally get very close limits within which the true value lies.

These formulæ, however, depend on selected values of the function, and it is evident that, in applying them to the calculation of life assurance benefits, any irregularities of the mortality table will be reflected in the final result; so that with a badly-graduated table the value may be considerably distorted. This will be more clearly recognized if we consider the geometrical meaning of these formulæ, which is extremely simple. All the above formulæ give an approximation to the area of a curve by dividing it into six compartments of equal width, and by joining the ends of the dividing ordinates so as to get subsidiary trapezoids, whose areas can be readily determined. Thus, in the preceding analysis,  $A_1$  is the area of the trapezoid got by joining the end of each ordinate to the next;  $A_2$  that got by joining the ends of alternate ordinates, and so on; and these formulæ show how, by taking proper multiples of these areas, that of the curve can be obtained. It is easy to illustrate geometrically what the effect of an error in the length of one of these ordinates will be, and the amount of the deviation will depend on the coefficient by which the ordinate is multiplied.

To test the effect of such irregularities, I calculated, at 3 per-cent interest according to the Carlisle Table, the values of an assurance of 1 payable in the event of a young life dying first, second, and third of self and two older lives.

The values of these functions are

$$\bar{A}_{xyz}^1 = \int v^t \cdot {}_t p_{xyz} \cdot \mu_{x+t} \cdot dt$$

$$\bar{A}_{xyz}^2 = \int v^t \cdot {}_t p_x ({}_t p_y + {}_t p_z - 2{}_t p_{yz}) \mu_{x+t} \cdot dt$$

$$\bar{A}_{xyz}^3 = \int v^t \cdot {}_t p_x (1 - {}_t p_y)(1 - {}_t p_z) \cdot \mu_{x+t} \cdot dt$$

where

$$\mu_x = - \frac{1}{l_x} \cdot \frac{dl_x}{dx}$$

and, if  $x < y < z$ , the limits of integration are respectively 0 and  $\omega - z$ , 0 and  $\omega - y$ , 0 and  $\omega - x$ ,  $\omega$  being the limiting age of the mortality table. The sum of these functions ought to give us  $\bar{A}_x$ , and, by comparing the results with the known value, we get an estimate of the extent of the error.

Theoretically, we ought to use a value of  $\mu$  which is true to the same order of differences as the formula we are using, but in practice it is essential to have a prepared table of values. If we stop at third differences, we have

$$\mu_x = \frac{7(d_{x-1} + d_x) - (d_{x-2} + d_{x+1})}{12l_x} \quad . \quad . \quad . \quad (1)$$

and, with a well-graduated table, such as that in the *Institute Text-Book*, this gives results which are practically identical with those derived from Makeham's Law, by which the table has been graduated. But in the Carlisle Table the  $d$  column runs with great irregularity, and Mr. G. F. Hardy held that better results would be got by writing

$$\mu_x = \frac{l_{x-5} - l_{x+5}}{10l_x} \quad . \quad . \quad . \quad . \quad (2)$$

As it seemed desirable to ascertain the effect of this smoothing of the table, both values of  $\mu$  were employed in calculating the above functions, and the amounts of the aggregate error are shown in the following tables.



TABLE I.—  $\mu_x = \frac{7(d_{x-1} + d_x) - (d_{x-2} + d_{x+1})}{12l_x}$ .

CARLISLE 3 %	$x=30$	ERROR IN COMPUTED VALUE OF $\bar{A}_x$				
Formula	$y$	$z=40$	50	60	70	80
(i)	40	+·00243	−·00293	−·00377	+·00031	+·00066
	50	...	−·01068	−·01190	−·00869	−·00733
	60	...	...	−·01235	−·00856	−·00574
	70	...	...	...	+·00031	+·00648
	80	...	...	...	...	+·01378
(iii)	40	+·00921	+·00344	−·00018	+·00651	+·00947
	50	...	−·00704	−·01118	−·00662	−·00270
	60	...	...	−·01481	−·01062	−·00539
	70	...	...	...	+·00169	+·01029
	80	...	...	...	...	+·02080
(v)	40	+·01478	+·00867	+·00280	+·01160	+·01668
	50	...	−·00413	−·01059	−·00494	+·00107
	60	...	...	−·01683	−·01227	−·00512
	70	...	...	...	+·00282	+·01341
	80	...	...	...	...	+·02655
(vi)	40	+·01576	+·00974	+·00377	+·01229	+·01707
	50	...	−·00302	−·00959	−·00423	+·00149
	60	...	...	−·01580	−·01160	−·00469
	70	...	...	...	+·00339	+·01360
	80	...	...	...	...	+·02645

TABLE II.—  $\mu_x = \frac{l_{x-5} - l_{x+5}}{10l_x}$ .

CARLISLE 3 %	$x=30$	ERROR IN COMPUTED VALUE OF $\bar{A}_x$				
Formula	$y$	$z=40$	50	60	70	80
(i)	40	−·00035	−·00019	−·00285	+·00060	+·00080
	50	...	−·00159	−·00470	−·00211	−·00013
	60	...	...	−·00855	−·00596	−·00351
	70	...	...	...	−·00013	+·00511
	80	...	...	...	...	+·01058
(iii)	40	+·00228	+·00478	−·00091	+·00392	+·00648
	50	...	+·00470	−·00156	+·00134	+·00594
	60	...	...	−·00909	−·00690	−·00229
	70	...	...	...	+·00083	+·00833
	80	...	...	...	...	+·01669
(v)	40	+·00437	+·00884	+·00067	+·00662	+·01114
	50	...	+·00982	+·00107	+·00419	+·01075
	60	...	...	−·00953	−·00767	−·00128
	70	...	...	...	+·00164	+·01098
	80	...	...	...	...	+·02169
(vi)	40	+·00520	+·00977	+·00153	+·00719	+·01143
	50	...	+·01087	+·00202	+·00486	+·01114
	60	...	...	−·00855	−·00699	−·00087
	70	...	...	...	+·00218	+·02116
	80	...	...	...	...	+·02161

Formulæ (i), (iii), (v), and (vi), were selected because they have easy numerical coefficients; but the errors involved in the other formulæ can be readily derived from these. Thus, if  $e_1$  represent the error in formula (i),  $e_3$  that in (iii), and so on, we have

$$\left. \begin{aligned} 14e_4 &= 9e_3 + 5e_6 \\ 2e_7 &= e_1 + e_5 \\ 3e_8 &= 2e_3 + e_6 \end{aligned} \right\}.$$

Mr. King has given a modification of (vi) by duplicating it and choosing the intervals so that  $u_{7h'}$  falls just about the end of the mortality table; and he has claimed, as the result of much practical experience, that this modification gives more accurate results. The above analysis shows that this should be the case, and that the gain in accuracy is mainly due to the larger number of intervals employed. The error in (vi) is

$$h \left[ \frac{3}{1400} \delta_5 - \frac{1}{224} \delta_7 + \dots \right] (u_{6h} - u_0),$$

while, if the formula be duplicated, the error will be

$$h' \left[ \frac{3}{1400} \delta'_5 - \frac{1}{224} \delta'_7 + \dots \right] (u_{12h'} - u_0).$$

Now, as  $u_{7h'}$  falls about the end of the mortality table, we have approximately  $\omega - x = 6h = 7h'$ , and the differences in the latter case are also slightly diminished. This formula, however, involves an entirely different set of ordinates from those given above; and one of the merits of this method of approximation is that, when the values of the ordinates are known, that of the function can be found by several formulæ with little additional trouble; so that, with a well-graduated table, we shall generally get limits of the error as well as an approximation to the true value. In applying the columnar method of arranging the work, I found it simpler to get the values of the ordinates and make the multiplications by the coefficients without the use of logarithms.

It appears that Table II gives rather better results than Table I; but in both cases the errors run with great irregularity, and are often large in amount. It was pointed out by Messrs. King and Hardy, in their paper on the application of Makeham's Law to the graduation of mortality Tables (*J.I.A.*,

xxii, p. 191), that, even after the Carlisle Table had twice undergone a preliminary graduation by Woolhouse's method, certain of the functions exhibited a well-defined series of undulations, which they attributed to the way in which the graphic method of graduation was applied by Milne. It seems pretty clear that, if the selected ordinates fall at these points of maximum or minimum, the effect may be to produce errors of considerable magnitude.

TABLE III.

$x$	$\mu_x$ by Formula (1) (1)	$\mu_x$ by Formula (2) (2)	$\mu_x$ adjusted by Makeham's Law (3)
20	·00707	·00691	·00674
30	·01010	·00916	·00942
40	·01251	·01251	·01137
50	·01355	·01487	·01618
60	·03150	·02896	·02804
70	·05133	·05594	·05730
80	·12801	·12907	·12945
90	·26467	·29225	·30742

Further, as will be seen from the above table, the values of  $\mu$  given in columns (1) and (2) often differ widely, and frequently cross one another; and, as  $\mu$  is a small quantity, any small irregularity will considerably affect the result. We might get a more accurate representation of the force of mortality by tracing on cross-ruled paper curves to represent the values of  $\mu$  as given in columns (1) and (2), and then using these as guiding curves to draw another running smoothly between them; but this would be equivalent to adjusting one set of values, while others are left untouched.

The Carlisle Table, however, has been graduated by Makeham's Law by Messrs. King and Hardy, and the values of  $\mu$  according to this adjustment are placed in column (3) for comparison. The values of  $\bar{A}_{xyz}^1, \bar{A}_{xyz}^2, \bar{A}_{xyz}^3$  were accordingly re-calculated according to their adjusted table (*J.I.A.*, xxii, p. 221), and the errors in the computed values of  $\bar{A}_x$  are given below:—



TABLE IV.—  $\mu_x$  adjusted by Makeham's Law.

CARLISLE 3% adjusted	$x=30$	ERROR IN COMPUTED VALUE OF $\bar{A}_x$				
Formula	$y$	$z=40$	50	60	70	80
(i)	40	−.00573	−.00313	−.00419	−.00412	−.00385
	50	...	−.00093	−.00226	−.00245	−.00132
	60	...	...	−.00418	−.00419	−.00161
	70	...	...	...	−.00234	+ .00196
	80	...	...	...	...	+ .00652
(iii)	40	−.00950	−.00307	−.00591	−.00677	−.00389
	50	...	+ .00239	−.00113	−.00286	+ .00077
	60	...	...	−.00569	−.00803	−.00321
	70	...	...	...	−.00718	−.00056
	80	...	...	...	...	+ .00703
(v)	40	−.01260	−.00303	−.00732	−.00893	−.00394
	50	...	+ .00510	−.00020	−.00317	+ .00249
	60	...	...	−.00694	−.01118	−.00454
	70	...	...	...	−.01115	−.00262
	80	...	...	...	...	+ .00744
(vi)	40	−.01169	−.00197	−.00637	−.00827	−.00358
	50	...	+ .00631	+ .00089	−.00237	+ .00302
	60	...	...	−.00578	−.01036	−.00400
	70	...	...	...	−.01049	−.00231
	80	...	...	...	...	+ .00750

The agreement between the various formulæ is much closer; but the preceding analysis leads us to expect that some should err in excess and others in defect, and the magnitude of the deviations, coupled with the fact that with one or two exceptions the sign is persistently negative, led me to examine whether the errors are not largely due to some other cause than the inaccuracy of the formulæ themselves. To test this, I calculated for decennial ages the values of the simple functions  $\bar{a}_x$  and  $\bar{A}_x$ , and the results are placed in Table V.

TABLE V.

<i>x</i>	$\bar{a}_x$ computed by Formula				True Value
	(i)	(iii)	(v)	(vi)	
30	20·148	20·166	20·180	20·184	20·141
40	17·689	17·701	17·711	17·711	17·686
50	14·644	14·642	14·641	14·635	14·638
60	11·167	11·163	11·160	11·155	11·165
70	7·659	7·658	7·658	7·655	7·660
80	4·634	4·634	4·635	4·636	4·635

<i>x</i>	$\bar{A}_x$ computed by Formula				True Value
	(i)	(iii)	(v)	(vi)	
30	·40147	·39995	·39872	·39952	·40466
40	·47544	·47545	·47530	·47614	·47722
50	·56804	·57195	·57515	·57601	·56734
60	·67056	·67240	·67390	·67425	·66994
70	·77394	·77421	·77443	·77428	·77358
80	·86324	·86313	·86304	·86229	·86299

While the annuity-values are in very close agreement with the truth (cf. Table S, *J.I.A.*, xxii, p. 222), those of the reversion are too small for the younger ages and too large for certain of the older ones. Now  $\bar{A}_x$ , expressed as an integral, differs from  $\bar{a}_x$  only in the introduction of the factor  $\mu$  under the sign of integration. This suggests that the values of  $\mu$  (Table R, *J.I.A.*, xxii, p. 221) are too low for the younger ages and too high for the older ones, and a reference to Table III given above seems to support this view. If this be correct, it would account for the magnitude of the deviations in Table IV, and for the persistency of sign. It will be noticed that the results derived from different formulæ agree very well with one another, except in the case of the reversions for ages 50 and 60. Speaking of the Carlisle Table, Mr. King pointed out (*J.I.A.*, xxii, p. 125) that there was a “very remarkable change in the mortality just at the point of “the Table where Dr. Farr had found it necessary to bring in “new constants in constructing the English Life Table No. 1, “and the Healthy English Table. At about age 55 new “constants had to be introduced, and with the Carlisle Table it “was the same.” Possibly this break in the Table may account for the irregularity at these ages.

It appears that, in applying these formulæ to the calculation

of life assurance benefits, the error may be regarded as made up of two parts,

(i) a part due to the inaccuracy of the formulæ themselves, and

(ii) a part due to the irregularities of the mortality table.

With a well-graduated table it will be seen that different formulæ give practically identical results; but errors of the second kind are of more importance, for it is impossible to predict their extent, or whether they will be in excess or defect. In particular, any small errors in the value of  $\mu$  are likely to affect the result considerably, and these are relatively of most importance at the younger ages where  $\mu$  is itself a small quantity.

To illustrate the effect of using a table where these irregularities are as far as possible eliminated, I have given in the comparative tables at the end of this paper the values of certain functions derived by various formulæ from the adjusted table. In most cases the agreement is remarkably close, and where the divergence is at all marked, it can be largely eliminated by using formulæ (iv), (vii) and (viii). The values according to these formulæ are not tabulated, as they can be written down without trouble if we remember that

$$14 \text{ (iv)} = 9 \text{ (iii)} + 5 \text{ (vi)}$$

$$2 \text{ (vii)} = \text{ (i)} + \text{ (iv)}$$

$$3 \text{ (viii)} = 2 \text{ (iii)} + \text{ (vi)}$$

One advantage of using the adjusted Carlisle Table is that it enables us to make a comparison with values obtained from methods based on Makeham's Law. Thus, if Makeham's Law hold, we can in a joint-life annuity substitute for any number of lives of different ages an equal number of lives of the same age. Also we know that

$$\bar{A}_{xyz}^1 = \frac{(\mu_x + \lambda s) \bar{A}_{xyz} - (\mu_y + \mu_z - 2\mu_x) \lambda s \cdot \bar{a}_{xyz}}{\mu_x + \mu_y + \mu_z + 3\lambda s} \quad \text{. . . (ix)}$$

where  $\lambda s$  is the Napierian logarithm of  $s$ , and

$$\bar{A}_{x:yz}^1 = \bar{A}_{xy}^1 + \bar{A}_{xz}^1 - \bar{A}_{xyz}^1 \quad \text{. . . . . (x)}$$

Values obtained by these methods are added for comparison, and it will be seen that they differ very slightly from those derived from quadrature formulæ.

We know that if Simpson's rule of equivalent lives is true, Gompertz' Law must necessarily hold good, so that it would not be correct to apply the rule to a table which has been adjusted by Makeham's Law. Besides, the rule assumes the existence of a



prepared table of two life annuities for all ages, and these are tabulated only for equal and for decennial ages. Accordingly, the rule was applied to the original Carlisle Table. Though the annuity-values of the original and adjusted tables do not differ very widely at any point (*J.I.A.*, xxii, pp. 208-9), we can scarcely compare results which are derived from differently adjusted data. The values derived from the original table by means of formula (i) with the value of  $\mu$  given by (2), are therefore added for comparison. It appears that the substitution of a single equivalent life gives results which agree very fairly with the others, but a repeated application of the rule might give values which are wide of the truth. Methods based on Makeham's Law, where available, give much more accurate results; and it appears that with a well-graduated table quadrature formulæ may be applied to the calculation of complicated benefits with some confidence. It should be added that the reversions given in the tables are continuous functions, those obtained by the substitution of a single equivalent life being multiplied by  $(1+i)^{\frac{1}{2}}$  so as to make them comparable with the others. The annuity-values are the ordinary annuities, so that the effect of the different methods of approximation on the values of annual premiums can easily be seen by inspection.

In a recent paper (*Transactions of the Faculty of Actuaries*, I, p. 49), Mr. A. E. Sprague has given a method for finding the force of mortality for select tables; but even if the values of  $\mu$  be accepted as correct, it seems that the application of quadrature formulæ in the ordinary way may introduce an error of some importance. All central-difference interpolation formulæ assume that the curve is continuous on each side of a point; and we have seen that all the preceding formulæ are derivable by direct integration from Everett's Interpolation Theorem. During the first few years after selection, the rate of mortality changes very quickly, and follows a different law from that which holds among lives who have been assured for a longer period, so that there is generally a discontinuity where the Select Table is joined on to the  $H^{M(5)}$ . If we remember the geometrical meaning of these formulæ, the effect of such a discontinuity will be more apparent, and the error thereby introduced will probably depend, to some extent, on the position of the first selected ordinate relative to the point where the discontinuity occurs. Probably satisfactory results might be obtained by integrating over the two portions of the curve separately, and applying a separate formula to each; but the question seems to require some investigation.

Annuity payable yearly during the Joint Existence  
of Three Lives  $x, y, z$

Carlisle 3 %

$a_{xyz}$

$x=30$

Method of Approximation	$y$	$z=40$	50	60	70	80
Simpson's Rule of Equivalent Lives	40	11·928	10·725	8·483	6·117	3·925
	50	...	9·900	8·065	5·958	3·880
	60	...	...	6·814	5·261	3·545
	70	...	...	...	4·344	3·106
	80	...	...	...	...	2·395
Milne's Modification	40	11·696	10·708	8·473	6·087	3·919
	50	...	9·878	8·051	5·930	3·878
	60	...	...	6·795	5·256	3·545
	70	...	...	...	4·335	3·088
	80	...	...	...	...	2·393
Formula (i)	40	11·845	10·725	8·446	6·106	3·915
	50	...	9·983	8·067	5·955	3·873
	60	...	...	6·807	5·250	3·542
	70	...	...	...	4·319	3·111
	80	...	...	...	...	2·396

Carlisle 3 % (adjusted by Makeham's Law)

$x=30$

Method of Approximation	$y$	$z=40$	50	60	70	80
Formula (i)	40	11·959	10·663	8·677	6·217	3·786
	50	...	9·716	8·115	5·954	3·690
	60	...	...	7·047	5·399	3·474
	70	...	...	...	4·407	3·033
	80	...	...	...	...	2·300
Formula (iii)	40	11·935	10·643	8·670	6·215	3·785
	50	...	9·698	8·108	5·953	3·690
	60	...	...	7·047	5·398	3·473
	70	...	...	...	4·409	3·030
	80	...	...	...	...	2·283
Formula (v)	40	11·916	10·627	8·664	6·214	3·785
	50	...	9·684	8·102	5·952	3·689
	60	...	...	7·046	5·398	3·472
	70	...	...	...	4·410	3·028
	80	...	...	...	...	2·269
Formula (vi)	40	11·915	10·626	8·662	6·213	3·788
	50	...	9·683	8·101	5·951	3·692
	60	...	...	7·048	5·398	3·477
	70	...	...	...	4·415	3·035
	80	...	...	...	...	2·282
Substitution of equal No. of Lives of same Age	40	11·959	10·661	8·676	6·220	3·787
	50	..	9·714	8·115	5·958	3·692
	60	...	...	7·049	5·400	3·476
	70	...	...	...	4·411	3·032
	80	...	...	...	...	2·290

*Annuity payable yearly while x is Alive with the  
Survivor of y and z*

Carlisle 3 %

$a_{xyz}$

$x=30$

Method of Approximation	$y$	$z=40$	50	60	70	80
Simpson's Rule of Equivalent Lives	40	16.970	16.275	15.495	14.994	14.692
	50	...	15.202	14.015	13.255	12.838
	60	...	...	12.244	10.930	10.152
	70	...	...	...	8.980	7.724
	80	...	...	...	...	5.941
Milne's Modification	40	17.202	16.292	15.505	15.024	14.698
	50	...	15.224	14.029	13.283	12.841
	60	...	...	12.263	10.935	10.152
	70	...	...	...	8.989	7.742
	80	...	...	...	...	5.943
Formula (i)	40	17.138	16.351	15.564	15.019	14.676
	50	...	15.219	14.047	13.284	12.851
	60	...	...	12.248	10.906	10.128
	70	...	...	...	8.988	7.727
	80	...	...	...	...	5.921

Carlisle 3 % (adjusted by Makeham's Law)

$x=30$

Method of Approximation	$y$	$z=40$	50	60	70	80
Formula (i)	40	17.180	16.374	15.622	15.082	14.715
	50	...	15.226	14.079	13.250	12.719
	60	...	...	12.424	11.061	10.228
	70	...	...	...	9.048	7.673
	80	...	...	...	...	5.664
Formula (iii)	40	17.203	16.368	15.608	15.115	14.732
	50	...	15.221	14.041	13.266	12.736
	60	...	...	12.414	11.049	10.259
	70	...	...	...	9.045	7.688
	80	...	...	...	...	5.682
Formula (v)	40	17.221	16.363	15.597	15.142	14.746
	50	...	15.217	14.010	13.278	12.750
	60	...	...	12.405	11.040	10.284
	70	...	...	...	9.042	7.700
	80	...	...	...	...	5.696
Formula (vi)	40	17.220	16.357	15.588	15.135	14.743
	50	...	15.209	13.996	13.267	12.743
	60	...	...	12.398	11.026	10.274
	70	...	...	...	9.033	7.690
	80	...	...	...	...	5.686
Substitution of equal No. of Lives of same Age	40	17.157	16.363	15.618	15.067	14.757
	50	...	15.218	14.087	13.237	12.760
	60	...	...	12.423	11.065	10.246
	70	...	...	...	9.047	7.683
	80	...	...	...	...	5.682



Single Premium for Assurance of 1 payable if  $x$  predecease  
 $y$  and  $z$  (Assurance payable at Death)

Carlisle 3 %.

$\overline{A}^1_{xyz}$

$x=30$

Method of Approximation	$y$	$z=40$	50	60	70	80
Simpson's Rule of Equivalent Lives	40	·15626	·13639	·10314	·07262	·04646
	50	...	·12393	·09737	·07069	·04596
	60	...	...	·08119	·06220	·04226
	70	...	...	...	·05123	·03751
	80	...	...	...	...	·02944
Milne's Modification	40	·15221	·13615	·10301	·07226	·04640
	50	...	·12362	·09720	·07034	·04594
	60	...	...	·08095	·06214	·04225
	70	...	...	...	·05113	·03732
	80	...	...	...	...	·02941
Formula (i)	40	·16116	·13722	·10171	·07132	·04540
	50	...	·12414	·09608	·06924	·04486
	60	...	...	·07983	·06075	·04113
	70	...	...	...	·04964	·03622
	80	...	...	...	...	·02838

Carlisle 3 % (adjusted by Makeham's Law)

$x=30$

Method of Approximation	$y$	$z=40$	50	60	70	80
Formula (i)	40	·15670	·13228	·10170	·07007	·04269
	50	...	·11684	·09402	·06697	·04167
	60	...	...	·08020	·06055	·03936
	70	...	...	...	·04942	·03471
	80	...	...	...	...	·02709
Formula (iii)	40	·15694	·13223	·10166	·07006	·04269
	50	...	·11648	·09393	·06696	·04167
	60	...	...	·08010	·06053	·03936
	70	...	...	...	·04943	·03469
	80	...	...	...	...	·02696
Formula (v)	40	·15713	·13219	·10163	·07006	·04269
	50	...	·11618	·09386	·06696	·04167
	60	...	...	·08002	·06052	·03936
	70	...	...	...	·04944	·03468
	80	...	...	...	...	·02685
Formula (vi)	40	·15716	·13217	·10160	·07005	·04270
	50	...	·11613	·09382	·06694	·04169
	60	...	...	·08000	·06051	·03939
	70	...	...	...	·04946	·03473
	80	...	...	...	...	·02697
Formula (ix)	40	·15631	·13209	·10159	·07013	·04266
	50	...	·11680	·09394	·06698	·04167
	60	...	...	·08017	·06054	·03937
	70	...	...	...	·04944	·03469
	80	...	...	...	...	·02706

*Single Premium for Assurance of 1 payable if  $x$  predecease the survivor of  $y$  and  $z$  (Assurance payable at Death)*

Carlisle 3 %

 $\bar{A}_{xyz}$  $x=30$ 

Method of Approximation	$y$	$z=40$	50	60	70	80
Simpson's Rule of Equivalent Lives	40	·27238	·24560	·22961	·22100	·21709
	50	...	·21141	·18870	·17628	·17090
	60	...	...	·15565	·13550	·12538
	70	...	...	...	·10732	·09098
	80	...	...	...	...	·06897
Milne's Modification	40	·27645	·24584	·22974	·22135	·21715
	50	...	·21172	·18890	·17661	·17092
	60	...	...	·15591	·13557	·12539
	70	...	...	...	·10742	·09117
	80	...	...	...	...	·06900
Formula (i)	40	·27401	·24966	·23279	·22443	·21992
	50	...	·21329	·18944	·17741	·17177
	60	...	...	·15314	·13340	·12347
	70	...	...	...	·10644	·09022
	80	...	...	...	...	·06794

Carlisle 3 % (adjusted by Makeham's Law)

 $x=30$ 

Method of Approximation	$y$	$z=40$	50	60	70	80
Formula (i)	40	·26674	·24229	·22593	·21777	·21337
	50	...	·20837	·18457	·17176	·16538
	60	...	...	·15202	·13157	·12144
	70	...	...	...	·10266	·08608
	80	...	...	...	...	·06246
Formula (iii)	40	·26799	·24366	·22628	·21877	·21431
	50	...	·20950	·18465	·17225	·16607
	60	...	...	·15219	·13143	·12181
	70	...	...	...	·10263	·08618
	80	...	...	...	...	·06259
Formula (v)	40	·26901	·24478	·22657	·21960	·21507
	50	...	·21043	·18471	·17265	·16663
	60	...	...	·15233	·13130	·12212
	70	...	...	...	·10262	·08627
	80	...	...	...	...	·06270
Formula (vi)	40	·26939	·24502	·22669	·21973	·21526
	50	...	·21056	·18468	·17260	·16664
	60	...	...	·15231	·13119	·12204
	70	...	...	...	·10256	·08618
	80	...	...	...	...	·06260
Formula (x)	40	·26699	·24191	·22608	·21756	·21379
	50	...	·20790	·18443	·17141	·16548
	60	...	...	·15187	·13152	·12145
	70	...	...	...	·10264	·08615
	80	...	...	...	...	·06256

*The Liability of Life Assurance Companies to pay Income Tax upon Income arising from Investments in Foreign Countries.*

[Contributed, on invitation, by Messrs. Devonshire & Co., Solicitors to the Gresham Life Assurance Society, Limited.]

THE GRESHAM LIFE ASSURANCE SOCIETY, LTD.

*versus*

BISHOP (Surveyor of Taxes).

The point in this case was whether the Society was liable to be assessed in respect of interest, dividends, and rents, arising from investments in foreign countries.

The Society had large funds invested in various forms in foreign countries, in some of which countries the Society carried on the business of Life Assurance, and in others of which it carried on no business of any kind. It was admitted that only a part of the income arising from these investments was remitted to Great Britain, the rest of such income being either reinvested or remitted direct to other foreign countries for investment, or applied in payment of establishment and other expenses in the country where the interest was earned, or remitted direct to other foreign countries for the general purposes of the Society.

The contention of the Crown, as set forth in the case, was that the whole of the income on the foreign investments was constructively remitted to Great Britain, and that it was therefore assessable to Income Tax under the Rule applicable to the fourth case of Schedule D of the Income Tax Act 1842, 5 & 6 Vic. chapter 35.

Appended hereto will be found—

- (1) The Case, as stated by the Income Tax Commissioners, dated 5th May 1898.
- (2) Supplemental Statement agreed between the Solicitors for the Society and the Solicitor of Inland Revenue, dated 19th June 1899.
- (3) The Judgment of the Divisional Court in *The Universal Life Assurance Society v. Bishop*, dated 11th August 1899.
- (4) The Judgment of the Divisional Court in *The Gresham Life Assurance Society v. Bishop* of the same date.
- (5) The Judgment of the Court of Appeal, dated 22nd November 1900.
- (6) The Judgment of the House of Lords, dated 16th May 1902.

The contention of the Society was two-fold, namely—

- (1) That the income in question had not been received in Great Britain within the meaning of the Act, and
- (2) That, even if such income had been so received, it was not liable to assessment on the ground that the Society having already paid tax by way of deduction at its source on an amount of income largely in excess of the profits of the Society, it was not liable to be further assessed.



It is to be noted that this second contention challenges the decision of the Court of Appeal in the *Clerical, Medical and General Life Assurance Society v. Carter*, which was decided in the year 1889 (*J.L.A.*, xxviii, page 101).

It will be seen that the decision of the House of Lords was a unanimous one in favour of the Society upon the first of the above-mentioned contentions. It was consequently unnecessary for the House of Lords to give, and they did not, in fact, give any decision upon the second contention. The decision, therefore, of the Court of Appeal in the Clerical, Medical and General case still remains unaffected by the decision in the case now under consideration. The history of the case is somewhat peculiar, having regard to the foregoing decisions of the English Courts and to the decisions of the Scotch Courts upon the subject.

In the year 1886, the Court of Session had given a decision in the *Scottish Mortgage Company of New Mexico v. McKelvie*, reported in 24 Scotch Law Reports, page 330, which appears, *prima facie*, to be in conflict with the decision now come to.

In the year 1895, the Scottish Provident Institution was assessed for income tax on Colonial investments not received in Great Britain, but received and reinvested in the Colonies. In that case, the Court of Session decided that, where interest derived from the Colonial investments of a Society for mutual insurance was not remitted home but was reinvested abroad, it was not constructively remitted to this country so as to be chargeable with duty merely by reason of its being entered in the Society's accounts. That decision was accepted by the Revenue Authorities in Scotland and was not appealed against, and, in fact, the Standard and other Life Offices in Scotland were informed that, in view of that decision, the income upon their foreign investments, not specifically remitted home, would be free from tax.

Thus, while in Scotland, the Revenue Authorities had accepted a decision in favour of the contention of the Assurance Societies, the same Authorities in England were strenuously resisting the precisely similar contention raised by the Gresham Society in the present case. When, however, the Court of Appeal in the Gresham Society's case delivered Judgment in November 1900, drawing a distinction between the case of the Scottish Provident Institution, on the one hand, and that of the Gresham Society, on the other hand, the Inland Revenue Authorities in Scotland again raised the question in Scotland, by assessing the Standard Life Assurance Company with duty on interest arising from investments of that Society in foreign countries and in India. In that case, the Court of Session decided in favour of the Standard Company, and in the course of their Judgments the learned Judges took the opportunity of criticizing and dissenting from the views expressed by the Judges of the English Court of Appeal in the Gresham Society's case.

The Inland Revenue Authorities did not appeal to the House of Lords from the decision of the Court of Session in the Standard Company's case, preferring, apparently, to have the whole question discussed and decided in the Gresham Society's case, with the result, as has been seen, that the contention of the Crown was rejected.

In order to make the literature of the case complete, we think it well to append, in addition to the documents mentioned above,

(7) The Judgment of the Court of Session in *Forbes v. The Scottish Provident Institution*.

(8) The Judgment of the same Court in *The Standard Life Assurance Company v. Allan*.

The decision in this case has attracted a considerable amount of attention, as affecting not only Life Assurance Companies but financial and industrial companies of various kinds.

The important question as to the taxation of the income arising from the investments of Life Offices, wholly irrespective of the profits arising from their Life Assurance and Annuity business, still remains in the position in which it was left by the decision in the case of the Clerical, Medical and General Society. Thus, although it is an essential part of the business of a Life Office to invest its premium income upon securities bearing interest, and although all the calculations upon which the operations of the Society are based are founded upon the assumption that a certain rate of interest will be received from the investments so made, yet, according to the decision of the Court of Appeal, the interest so arising is to be liable to pay a tax which may seriously reduce the actual rate of interest earned, and that, whether the profits arising from the operations of the business itself are large or small. The question involves many points of great interest, and is one which may hereafter have to be finally settled by the House of Lords.

## APPENDIX (I).

### IN THE HIGH COURT OF JUSTICE,

#### QUEEN'S BENCH DIVISION.

Between—THE GRESHAM LIFE ASSURANCE SOCIETY,

LIMITED - - - - - Appellants,  
and

GEORGE HENRY BISHOP, Surveyor of Taxes Respondent.

### CASE

Stated under the Statute 43 & 44 Victoria, c. 19, s. 59, by the Commissioners for general purposes of the Income Tax for the City of London, for the opinion of the Queen's Bench Division of the High Court of Justice.

1. At a meeting of the Commissioners of Income Tax for the City of London held at the Guildhall Buildings in the said City on the 12th April 1894 the Gresham Life Assurance Society Limited appealed against the assessment by way of further charge on £143,483. 7s. 3d. made upon the Society for the year ending the 5th April 1893 in respect of profits arising from interest dividends and rents which had not been taxed at their source. For the purposes of this case the word "interest" hereinafter used includes dividends and rents. The Appellants had already been assessed to and had paid income tax on £6,516. 12s. 9d. assessed in respect of such interest dividends and rents. A copy of the further assessment appears in the Schedule hereto.

2. The Gresham Life Assurance Society Limited carries on the business of life assurance and of selling or granting annuities pursuant

to the provisions of a deed of settlement dated the 3rd July 1848 as modified by a deed dated 17th July 1893 copies of which may be referred to as part of this Case.

3. The Registered and Head Office of the Society is in London where the directors and shareholders meet and whence the affairs of the Company are managed.

#### PART 1.

4. There are certain countries in which the Society does no business of any kind but the Society has funds invested in various securities in those countries. The interest on those securities is either (a) reinvested in those countries upon securities there or (b) remitted direct to other foreign countries for investment in those countries (c) remitted to Great Britain.

5. There are certain countries in which the Society carries on business of life insurance by means of local agents or managers. The Society has funds invested in various securities in those countries. By the laws of some of those countries the Society is obliged to keep invested in securities within those countries respectively a sum to answer liabilities on its policies and other engagements in those countries respectively. No part of the money so compulsorily invested can be removed until the liability in respect of the said policies and engagements has run off. The interest on the investments whether compulsory or not is either (d) reinvested in those countries upon securities there (e) applied in establishment and other expenses in the countries where the interest is earned (f) remitted direct to other foreign countries for investment (g) remitted direct to other foreign countries for the general purposes of the Appellants or (h) remitted to Great Britain.

6. It is essential for the purposes of the Appellants as an Insurance Company that the greater portion of the premiums received by them should be invested in interest-bearing securities and that from time to time the interest accruing thereon should also be invested and the investments mentioned in paragraphs 4 and 5 are accordingly made for that purpose. The said investments are made in the course of and for the purposes of the business of the Society as an Insurance Company and the total amount of such investments is taken into account in arriving at the profits of the Society.

7. All interest capitalised abroad by reinvestment in the event of the winding-up of the Society or the discontinuance of the Society's operations in any particular country form part of the assets of the Society available for the fulfilment of the Society's obligations.

8. The Society contended before the Commissioners that under the 4th Case s. 100 of 5 & 6 Vic. c. 35 only such part of the said interest as was received in Great Britain during the year of account was assessable to tax and that the interest applied as in (a) (b) (d) (e) (f) and (g) was exempt from tax.

9. Upon the above facts it was contended on behalf of the Crown that on the facts there should be no reduction of the assessment and that there was a constructive remittance of the interest and they quoted the cases of the *Scottish Mortgage Company of New Mexico v. McKelvie* 24 Sco. L.R. 330 and *Norwich Union Fire Insurance Company v. Magee* 44 W.R. 384.

The Commissioners declined to reduce the assessment.



## PART 2.

10. The amount of the surplus funds of the Society divisible as profits is ascertained by actuarial valuation once in every three years. All the investments mentioned in paragraphs 4 & 5 are made in the course of and for the purposes of the business of the Society and the total amount of such investments is taken into account in arriving at the profits of the Society. The amount of the Society's profits for the year of account so ascertained was £17,342. 3s. The Society is willing to pay tax upon this sum and contended before the Commissioners that if the total of the interest on the Society's investments in the year of account exceeds the profits of that year the excess is not taxable and the assessment should be reduced accordingly. The Society therefore claimed to have its assessable income reduced to £17,342. 3s.

11. It was contended on behalf of the Crown that on the facts the assessment should stand and that even if the Society's profits as ascertained at the actuarial valuation were less in amount than their untaxed interest such untaxed interest is properly assessable as such without deduction and quoted the case of *Clerical and Medical &c. Society v. Carter* L.R. 22 Q.B.D. 444.

12. The Commissioners confirmed the assessment whereupon the Society expressed dissatisfaction and required a Case to be stated for the opinion of the High Court of Justice upon the above contentions which the Commissioners state and sign accordingly.

The questions for the opinion of the Court are—

1. Whether the interest described in 4 and 5 (*a*) (*b*) (*d*) (*e*) (*f*) and (*g*) respectively is liable to taxation?

If the Court should decide in favour of the Appellants with respect to any of the classes of interest described in 4 and 5 (*a*) (*b*) (*d*) (*e*) (*f*) or (*g*) the assessment is to be remitted to the Commissioners to be amended accordingly.

2. Whether the assessment is to be reduced to £17,342. 3s. the amount of profits of the Society?

If the Court should decide in favour of the Appellants on the second question the assessment is to be reduced to that amount.

SYDNEY H. WATERLOW,  
H. MORLEY,  
GEO. H. CHAMBERS,  
D. P. SELLAR,  
W. D. POWLES,  
ALBERT G. SANDEMAN,  
EDWARD T. NORRIS,  
A. H. BAKER,

*Commissioners of Taxes  
for the  
City of London.*

THOMAS HEWITT,  
*Counsel and Clerk to the Commissioners.*

*The Guildhall Buildings, E.C.  
5th May 1898.*

THE SCHEDULE ABOVE REFERRED TO.

No. of Assessment	Name of Person, Corporation, Company, or Society charged	Description of Trade, Profession, Vocation, Concern, or other Profits chargeable under Schedule D	Street or Place of Abode, Assessment, or Employment	Amount of Profits charged by First Assessment, if any	Amount of Profits per Surcharge of Inspector or Surveyor	Amount of Profits confirmed by Commissioners in addition to the Profits charged by First Assessment	Deductions						Net Income chargeable by Supplementary Assessment	Single Duty on the First Assessment	Remainder of Duty payable on the Supplementary Assessment	Amount of Treble Duties or Overplus above the Single Duty or of Penalties	Total Amount payable on Supplementary Assessment
...	Gresham Life Assurance Society (per Thomas G. Aekland, Actuary and Manager)	Profits from Interest, Dividends and Rents	St. Mildred's House, Poultry	£ s. d. 6,516 12 9	£ s. d. 150,000 0 0	£ s. d. 143,483 7 3	Amount Discharged on the ground of Exemption	Amount Allowed on the ground of Abatement	Amount Abated in respect of Life Insurance	Amount Allowed for Wear and Tear of Machinery or Plant	Total Amount of Income Abated or Discharged	£ s. d. 143,483 7 3	£ s. d. 18 3 3	£ s. d. 3,587 1 8	...	...	£ s. d. 3,587 1 8

## APPENDIX (II).

## SUPPLEMENTAL STATEMENT.

Agreed between the Solicitors for the Appellants and the Solicitor of Inland Revenue on behalf of the Respondent, in pursuance of the directions of the Queen's Bench Division (Grantham and Kennedy, J.J.) of 24th March 1899.

1. The Gresham Life Assurance Society, Limited, has not been charged to income tax under Schedule D in respect of profits for the year ended 5th April, 1893, except in respect of the interest dividend of 5 per-cent on the paid-up capital payable under clause 109 of the Deed of Settlement out of the profits of the Company. Such year is hereinafter referred to as the "year of assessment."

2. The Society has a subscribed capital of £100,000 divided into 20,000 shares of £5 each and interest is annually paid to the shareholders on the amount paid up thereon. The profits of the Society are divisible among the shareholders, and participating policyholders or otherwise applied in the manner indicated in the Deed of Settlement and the Laws and Regulations of the Company, but (except as above mentioned in paragraph 1 and afterwards mentioned in paragraph 4 hereof) no such profits were divisible or divided during the year of assessment. The said Deed Laws and Regulations form part of this Case.

3. The sole and complete management and control of all the affairs, operations and business of the Society, subject to the laws of the various countries in which the Society carries on its business, both in and out of the United Kingdom were and are alike, subject to the control of general meetings of the shareholders, vested in and exercised by the Board of Directors at the head office in London, where the meetings of the Directors and the shareholders are held, and dividends and division of profits declared and made, and dividends are payable.

4. The printed accounts (revenue accounts and balance sheets, &c.) are made up annually and shew the nature and extent of the entire business and financial operations carried on by the Society both in the United Kingdom and abroad, and as one entire and indivisible business. The profits are ascertained by actuarial valuation once in three years. Interest amounting to £1,085. 12s. was paid during the year of assessment to the shareholders at the rate of £5 per cent. per annum on the total amount of their paid-up capital. Such interest was paid out of profits as above mentioned.

5. The Official Returns\* marked *a*, *b*, *c*, and *d* made pursuant to the Life Assurance Companies Act, 1870, including the revenue accounts and balance sheets of the Society for the periods ending 30th June 1891, 31st December 1892, and 31st December 1893, and also including the valuation balance sheets, consolidated revenue accounts and statements for the three years ending 30th June 1891, and for the four and a half years ending 31st December 1895, may be referred to as forming part of this Statement.

\* It is not thought necessary to reproduce these here, as they will be found in the Returns of Life Assurance Companies to the Board of Trade.—ED. J.I.A.



6. A specimen of the form in use during the year of assessment for life insurance policies granted by the Society at their French branch (being similar to the forms of policy granted by the Society at their other branches and agencies abroad), is annexed to and forms part of this Statement and is marked *e.*\* Under these policies and contracts, the claims and annuities payable thereunder are respectively payable and actually paid at the branches and agencies abroad, and no monies are remitted abroad from the United Kingdom specifically for that purpose, but monies would be remitted abroad to such countries if and when necessary to meet payments there made.

7. The agents or managers of the agencies and branches in the countries out of the United Kingdom in which the Society carries on the business of life insurance <sup>and</sup> <sub>or</sub> of selling or granting annuities, from time to time account to the head office for all monies received and paid there by or on behalf of the Society, and render to the head office full accounts setting forth all transactions at the agencies and branches.

8. The receipts at the agencies and branches abroad include (*inter alia*) premiums received from policyholders, payments for purchase of annuities and interest or dividends arising from foreign securities or investments. The payments at these agencies and branches include (*inter alia*) payments under policies on account of claims, payments on account of annuities, policy surrender values, bonuses, commissions, management and office expenses. All receipts, payments, and balances in hand at these agencies and branches are dealt with from time to time in the manner directed by means of special or general instructions by the Board of Directors from the head office in London and are controlled by such Directors by means of such instructions, and are either invested abroad, applied towards payments abroad, or are otherwise dealt with or expended as may be required or directed by the Board of Directors in London.

9. All interest and dividends, including those the subject of the assessment appealed against, are included as money received by the Society in the revenue accounts and consolidated revenue accounts of the Society under the head of "Interest, dividends, and rents", and are taken into account in arriving at the amount of the life assurance and endowment funds and life annuity fund set out in the valuation balance sheets of the Society upon which the surpluses or profits are ascertained. The accounts are made out in the United Kingdom, and are accounts made out by the head office of the Society in the United Kingdom, and are by that head office rendered to the shareholders as accounts of all the Society's transactions and affairs which are all directed and controlled as before stated by the Directors from the head office in the United Kingdom.

10. The accounts of the Society are made out in the forms prescribed in the schedule to the Life Assurance Companies Act, 1870, and no distinction is made in the accounts of the Society with regard to receipts or expenditure, whether arising or made in the United Kingdom or abroad, but the whole receipts and expenditure at home

\* It is not thought necessary to reproduce a copy of the policy here.—Ed. *J.I.A.*

and abroad are included together in one entire account in the revenue and other accounts and valuations of the Society.

11. If the interest and dividends in question, and the premiums received abroad had not been retained abroad, or remitted from one foreign country or colony to another, the Society would have been obliged to send out from the United Kingdom to their foreign agencies and branches for the payment of claims and annuities, or discharge of other obligations, or for the payment of expenses, purposes of compulsory investment or otherwise, an amount sufficient for such purposes. By not remitting the interest, dividends, and premiums in *formâ specificâ* to the United Kingdom the Society saves the cost of the exchange, expense and inconvenience which remittances in that form would involve. The Society allege (but the Crown does not accept the allegation as being the fact) that the premiums received abroad are sufficient to pay the annuities and claims payable abroad. If this allegation is material for any purposes of this Case, then the Case is to be remitted to the Commissioners for the ascertainment by them of the actual facts.

12. In paragraph 4 of the Case stated, the foreign countries described under (*b*) include those where the Society carries on no business as well as countries where the Society carries on business, and also countries by the laws of which a compulsory investment is required as indicated in paragraph 5 of the Case.

13. In paragraph 5 of the Case stated, the establishment and other expenses described under (*e*) include payment of claims under policies and annuities. The foreign countries referred to in (*f*) and (*g*) include countries where the Society carries on business as well as countries by the laws of which a compulsory investment is required. The general purposes referred to under (*g*) include payment of claims under policies and annuities.

14. The amount of the profits of the Society, if estimated under Case 1 of Schedule D, for the year of assessment on the average of the preceding years ending 30th June 1891, would be £40,472.

15. Remittances on behalf of the Society of amounts representing interest, dividends or other moneys from one foreign country or colony to another colony or foreign country are not made via the United Kingdom, nor by means of securities (negotiable instruments or others) payable in the United Kingdom.

F. C. GORE.

DEVONSHIRE & CO.

19th June 1899.

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## APPENDIX (III).

IN THE HIGH COURT OF JUSTICE,

QUEEN'S BENCH DIVISION.

DIVISIONAL COURT.

ROYAL COURTS OF JUSTICE,

Friday, 11th August 1899.

Before—

Mr. Justice DARLING, and

Mr. Justice PHILLIMORE.

Between—THE UNIVERSAL LIFE ASSURANCE SOCIETY - Appellants,  
and

GEORGE HENRY BISHOP, Surveyor of Taxes - Respondent.

## JUDGMENT.

Mr. Justice DARLING: In this case, which was argued before my brothers Grantham and Kennedy, we have merely to read the written Judgments which they have prepared.

This is the Judgment of my brother Grantham:

In this case we have to determine whether the Company is liable to pay income tax on the sum of £14,096, received by the Society in India by way of interest on investments in that country.

The question arises under the 4th Case of the Income Tax Act, Schedule D, and the answer to it is dependent on whether or not in our Judgment it can be said that that interest has been received in Great Britain.

It is admitted that it has not been received here in *formâ specificâ* but it is alleged that it has been constructively received here, and so, under the authority of the Scottish Mortgage Company of New Mexico *v.* McKelvie, 2 Tax Cases, 165, and of the Norwich Union Fire Insurance Company *v.* Magee, 3 Tax Cases, 457, the Commissioners are entitled to charge income tax on it.

During the course of the argument, I felt very strongly that this enlargement of the meaning of the word "received" by interpreting it as being equivalent to the words "constructively received" was giving a very dangerous latitude to any canons of construction known to the law and that we ought not to apply the word "constructive" or "constructively" to the plain words of the Act unless the authorities were conclusive on the cases or the facts showed that the money was treated by the Appellants as being in reality received here and was so mixed up with the moneys received here that it was impossible to ear-mark or identify the amounts or the application of the money respectively received in England, in India, or elsewhere.

During the arguments I had not an opportunity of looking carefully into the accounts and annual report of the Universal Life Assurance Society, and I was under the impression that the interest received in India was kept separately in their accounts, but, on carefully examining them, I find that they are kept in such a way as to justify the Crown in saying that the Society has treated these



dividends just as if they had been received in England and that the facts of the case form no basis for the arguments founded on what turns out to be an imaginary state of facts.

I find the principle on which their report is framed is undoubtedly to mix all the policies, all the premiums, all the investments and all the dividends up together, wherever the policies were taken out or the premiums or dividends were received, and then, having shown what the net profits of the year were, to distribute these profits, in certain proportions, to their various shareholders and proprietors and policyholders in a fixed proportion; and, as a proof that much of these dividends has been received here, I find one of the most important subsidiary funds is the "Policyholders' Exchange Fluctuation Fund", and that £4,000 was paid to that fund to enable the Society to equalize its payments, notwithstanding the heavy losses that they might incur on the remitting of money to this country from India. Moreover, I find one paragraph, in terms, refers to these remittances by stating, under the head of "Exchange Fluctuation Reserve Fund", "Considerable remittances home from India have been made in the past year", &c., while, if we refer to the first Schedule in the Revenue Account, we find that no distinction is made in the item of interest and dividends, but they are lumped together in the sum of £48,602, and out of that sum the expenses of management are paid, apparently by the office in England for both the head and branch offices—that is, the office in India.

Under these circumstances, I think we are bound by the authority of the *Norwich Union Fire Insurance Company v. Magee*, and that our Judgment must be for the Respondent.

Mr. Justice PHILLIMORE: I will read the Judgment of my brother Kennedy in this case. It is this:

I have had an opportunity of reading the Judgment of my brother Grantham, and entirely concur in the result at which he has arrived in that Judgment in deciding against the Appellants, and I have very little to add.

I think that the facts stated in the case show that this Indian interest which is in question was not merely entered in the accounts of the Society, which by itself would be a matter of little consequence, but was retained in India, really as a matter of commercial convenience, and that but for such retention an equal sum must have been remitted to India to discharge the Society's liabilities there, and that, in reality, the amount of this Indian interest was treated by the Society as part of the divisible profits upon which (in the words of the Appellants' contention) dividends according to law had been declared and paid at the Society's Chief Office in the United Kingdom.

In these circumstances, it appears to me that there is, in regard to the Indian interest, a "constructive remittance" according to the law as applied in the *Scottish Mortgage Company of New Mexico v. McKelvie* (2 Tax Cases, 165), and as stated in the Judgment in the *Norwich Union Fire Insurance Company v. Magee*, to which I was myself a party (3 Tax Cases, 460), and that the Appellants are properly assessable in respect of it under Case 4 of Schedule D. The two Scotch cases upon which the Appellants relied, *Forbes v. the*

Scottish Widows' Fund and Life Assurance Society, and *Forbes v. the Scottish Provident Institution* (3 Tax Cases, 443), appear to me to be distinguishable upon the facts. In neither case, as I understand the report, was the interest received abroad treated in any division of profits as forming part of the divisible profits. It was simply retained and used abroad for purposes of loans and investment.

I agree that the Judgment must be for the Crown.

Mr. NEILSON: The Appeal, my Lord, will be dismissed with costs?

Mr. Justice DARLING: Yes.

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## APPENDIX (IV).

### IN THE HIGH COURT OF JUSTICE,

#### QUEEN'S BENCH DIVISION.

#### DIVISIONAL COURT.

#### ROYAL COURTS OF JUSTICE,

*Friday, 11th August 1899.*

Before—

Mr. Justice DARLING and

Mr. Justice PHILLIMORE.

Between—THE GRESHAM LIFE ASSURANCE SOCIETY,

LIMITED - - - - - Appellants,  
and

GEORGE HENRY BISHOP, Surveyor of Taxes - Respondent.

### JUDGMENT.

Mr. Justice DARLING: I will now read the Judgment of my brother Grantham in the Gresham Life Assurance Society's case. It is this: In this case practically the same question has to be determined as in the last, namely, whether dividends or interest not received in England in *formâ specificâ*, can be said to be "constructively received", and so liable to duty under the Income Tax Acts. Taking paragraph 9 of the supplemental statement as correctly stating the facts, it seems to me clear that our Judgment must be in accordance with our Judgment in the last case.

"All interest and dividends, including those the subject of the assessment appealed against, are included as money received by the Society in the revenue accounts and consolidated revenue accounts of the Society under the head of 'Interest, dividends, and rents', and are now taken into account in arriving at the amount of the life assurance and endowment funds and life annuity fund set out in the valuation balance sheets of the Society upon which the surpluses or profits are ascertained."

It seems impossible to ear-mark any particular sums as having been interest or dividends received in England, or as having been received and kept abroad. They are, in effect, treated as if they

came in some way or other into England in *specie* as well as in paper, and, though as a fact doubtless they do not all so come, yet the Society treats them all in the same way and lumps them together, so that it is impossible to do otherwise than say they are "constructively received" here, and consequently my Judgment must be, as in the last case, for the Crown.

Mr. Justice PHILLIMORE: I will now read the Judgment of my brother Kennedy. He says: "In this case I am of opinion that the Appellants' case fails. I can find no substantial distinction upon the facts between this and the preceding case, and the arguments upon the hearing were substantially the same in both cases. It is, therefore, needless for me to repeat what I have said in the Judgment in the preceding case."

Our Judgment must be for the Crown.

Mr. NEILSON: The Appeal will be dismissed, with costs, my Lord?

Mr. Justice DARLING: Yes, that is so.

## APPENDIX (V).

### IN THE COURT OF APPEAL.

ROYAL COURTS OF JUSTICE,

*Thursday, 22nd November 1900.*

Before—

The MASTER of the ROLLS.

Lord Justice COLLINS.

Lord Justice STIRLING.

Between—THE GRESHAM LIFE ASSURANCE SOCIETY,

LIMITED - - - - - Appellants,  
and

GEORGE HENRY BISHOP, Surveyor of Taxes - Respondent.

### JUDGMENT.

The MASTER of the ROLLS: This is an Appeal from the Queen's Bench Division, who gave Judgment for the Crown. The subject, that is the Gresham Life Assurance Company, appealed. The point raised is whether or not under the provisions of the Income Tax Act and the rules under Schedule (D) the Crown is entitled to levy Income Tax at whatever rate it may be during the year of assessment upon dividends received on foreign securities. That is the question. That brings me down, first of all, to the Fourth Case under Schedule (D), because we must get at the true interpretation of that in the first instance before we can deal with the matters which appear upon the facts of this case.

Now this point is rather a short one—it is simply this: The Fourth Case enacts under Schedule (D) in the Income Tax Act



“Rules for ascertaining the said last-mentioned duties in the particular cases herein mentioned.” Then there are other cases (the First Case, Second Case, Third Case) which I have not to deal with. Then the Fourth Case says: “The duty to be charged in respect of “interest arising from securities” (I will read “foreign securities”) “except such annuities, dividends, and shares as are directed to be “charged under Schedule (c) of this Act. The duty to be charged in “respect thereof shall be computed on a sum not less than a full “amount of the sums (so far as the same can be computed) which “have been or will be received in Great Britain in the current year, “without any deduction or abatement.” The question in this case is, what is the meaning of the words, “have been or will be received in Great Britain?” It seems to me there is no doubt that that is a limitation which the Legislature has put upon the otherwise general form of this Fourth Case; and to get at the Income Tax payable upon the interest or dividends, being foreign securities they must “have been or will be received in Great Britain in the current year, “without any deduction or abatement.” The question is, what is the meaning of the word “received?” Now at some time in the early part of this case the word has been used, and it has been used by some of the learned Judges, that the word “received” means “constructively” received. I did not like that at all, and I do not like it now, and if I were driven to read the words “constructively received” into this Rule I should not be delivering the Judgment I am about to deliver. There must be a receipt—an actual receipt—of the amount, but that receipt need not be in specie, it may be in account; and it seems to me that a receipt in account is just as much a receipt as a receipt in specie. That in my opinion is the true meaning of this rule; and I think that is the reading in the Scotch case, which is reported in 2 Tax Cases (McKelvie) which the Lord President (Lord Inglis), Lord Mure, and Lord Shand put upon this rule. They did not in so many words, but they came to the conclusion that in that case there had been a receipt in account of foreign dividends, and they held, that being so, that the Crown was entitled to Income Tax upon the dividends so received.

What are the facts here—the true facts? I am now dealing with Clause 11 in the Case, and now the matter has been argued I see the real meaning of that clause, which I did not at first. What are the real facts of this case? Why, that the Gresham has received dividends on foreign securities. Those dividends were undoubtedly received by their agents in foreign parts. These dividends so received they have brought into account in making up their profit and loss account. That is perfectly right. They then made up their profit and loss account as appears by Clause 4 of the Supplemental Case: “Interest “amounting to £1,085. 12s. was paid during the year of assessment “to the shareholders at the rate of £5 per-cent per annum on the “total amount of their paid-up capital. Such interest was paid out “of profits as above mentioned.” That is, they brought into account amongst other things these dividends upon these foreign securities; they brought in their other income, and on the other side they debited their expenditure, and then out of the profits so ascertained they paid the shareholders—the shareholders of the Gresham in this case;

“such interest was paid out of profits as above mentioned.” Now then the question arises upon such a state of facts as this: What is the inference to be drawn? It seems to me that the irresistible inference to be drawn from such facts as those is this—that they have received these foreign dividends in account—they have dealt with them as received in account, they have distributed them as having been received in account; and therefore when the Crown says, under Case Four of the Income Tax Act, you are liable to pay Income Tax under that Clause upon this state of facts, it would not do for the Gresham Company to say: “We have not received in the United Kingdom the foreign dividends.” Therefore it seems to me that upon this point, and that is sufficient to conclude this case upon, it is the ground-work of the decision of the learned Judges of the Queen’s Bench Division that this interest does come within the Fourth Case, and that the Crown are right in the contention they set up as to being entitled to Income Tax—they are entitled to Income Tax at the current rate upon all such foreign interest as has been brought into Profit and Loss account by this Society, and upon which that Society has paid the 5 per-cent dividend or interest. As regards Forbes’ Case, undoubtedly—and the earlier case (that is McKelvie’s Case)—though it is not a binding decision upon us, it is very much in favour of the Crown. I must say at first blush that in Forbes’ Case the Judgment of the Lord President (Lord Robertson) would be against the Crown, but on looking at the Judgment, as has been pointed out by my brother Lord Justice Collins, it seems to me that the ground-work there was that the foreign dividends were left where they were—in foreign parts—they were not dealt with, no order was given as regards them; they were not brought over and brought into account in making up the Profit and Loss Account, and no dividends had been paid out of that profit to the shareholders. It seems to me, therefore, that this case falls within the First Case, McKelvie’s Case, and not within Forbes’ Case.

There are other cases, if one wants them, which are in favour of the Crown—I am talking of Bartholomay’s Case, and of Mr. Justice Vaughan-Williams’ Judgment in the Denver Hotel Case. There is also the Judgment of my brother Wright, which is very apposite, though I am not sure that it was necessary to the decision in the Bartholomay Case, in which he says: “It appears to me that if the “Fourth Case were applicable these dividends ought in point of law to “be regarded as received in England.” There are the cases. I think myself that the Queen’s Bench Division were right in the conclusion at which they arrived, and that this Appeal must be dismissed.

There is one other word I want to say. I think the true construction of Clause 11 in this specific case was this: It was inserted for the purpose of differentiating the fact of the dividend being brought over in specie, and being brought into account. It is not intended to be stated, as I at one time thought, that these dividends were left out there, and never came here at all in account or otherwise, and I think that the line in the middle of that shows this: “By not remitting the interest *in formâ specificâ*”, that means in specie. Then comes the question which is raised in that paragraph 11. For these reasons I am of opinion that the



Crown is entitled to Judgment, and that the appeal ought to be dismissed.

Lord Justice COLLINS: I am of the same opinion. I think that, unless it is to be held that there can be no receipt in the United Kingdom unless there is a payment in specie, or *in formâ specificâ*, as it is put in what we are told is Scotch Latin, unless the payment is to be made in that way I can myself find no middle principle which would be capable of being worked unless we are to hold that a receipt in this case is shown by the fact that there has been an account taken between the parties on the basis of which the party receiving, that is, the Insurance Company, has treated the sums in question as received, and made payments on that basis. It seems to me that if that does not amount to a receipt in this country—it unquestionably amounts to a receipt in point of law—there is no middle course that we can accept, or anything short of holding that the payment must be made in specie; or, as Mr. Haldane graphically put it in the argument, that the ships must be sent from one side to the other containing the bullion.

Now it seems to me that it would be an outrage upon common sense, and upset the ordinary transactions of commerce, if we were to hold at this time of day that there can be no receipt in this country unless it is a receipt in specie. It is common knowledge that every day, to a higher and higher degree, the business of the world is transacted without any actual transfer of specie at all; and what has happened in this case seems to me to have been no more than what happens in commercial transactions every day, whether between people in one country and another, or between people in different places in the same country, namely, a settlement in account, treating the sums due from the one as a set-off against the sums due from the other. The result of that is in each case to support a plea of payment in point of law, and I think when you get a payment in point of law it assumes this,—that the transaction amounts to an actual delivery by the debtor to his creditor of the sum due. It is the business of the creditor to seek the debtor, and not the business of the debtor to seek the creditor; therefore, if you get facts that will support the payment, then I think that that carries with it the presumption that the person paid has received the money at the place wherever he is. That does seem to me to be common sense, apart from authority; but in this case we have the authority of the Court in Scotland as far back as 1886, and from that day to this, so far as I know, that case has never been questioned. And it is exactly in point, it seems to me, in the present enquiry. Exactly the same elements occurred there as occurred here. The ground on which the receipt was inferred there was that the item in question, which was said to be a receipt, had been taken into account, and, on the balance so arrived at, by taking that into account, the payment had been made.

Now Mr. Haldane distinguished that case on the ground that there was what he called an estoppel, and one Judge no doubt (the Lord President) does state a fact as influencing his Judgment, and that is that to have treated the money which the Company had in its hands here as the proceeds of the moneys raised by debentures, as



being applied in the payment of dividends, would have been to assume that they had committed an illegality, and therefore he uses that fact as an additional ground for holding that the money which was legally applicable for payment of dividends was so applied by them as money coming from the other side of the water. But it seems to me, as pointed out by the Attorney-General, that that was only one stepping-stone by which he arrived at the conclusion that the settlement in account was what it purported to be, namely, a utilization of the funds on the credit side for the purpose of meeting the funds on the debit side. It was an additional fact which made it impossible to reject the *prima facie* inference derivable from the mere fact of the two sums being brought into account in that way. The other two Judges do not rely upon that element at all; but whether it was an element in the case or not, it certainly was not an estoppel, and it could not, in my judgment, differentiate the case in any way from an ordinary case where you have items on the one side and items on the other, a balance struck and the balance applied.

Now that case was followed by another case—Forbes' Case—which was decided in the next year, and Mr. Haldane relied on that case as being inconsistent with the previous case, and therefore leaving us at large to choose which of the two we like best. Without exercising that choice, even if it were open to me, I do not think it is open to me, because it does not seem to me that there is anything inconsistent between the two decisions. I think it is quite clear that the Lord President in Forbes' Case was dealing with a case where nothing had been done on this side of the water with the funds which came into existence at the other side of the water, and I think that is the key-note to the whole position. There was no payment of dividend after a balance had been arrived at; there was no payment of dividend in that case at all as there had been in the other, and in dealing with the case he says: "There is nothing, as far as appears, done with the Colonial interests in question" (the Colonial interests being the funds as to which the question was whether they had been received in this country or not), "except to leave them where they are." Therefore no dominion had been asserted over them; nor had that dominion been asserted to the extent of applying them, and applying them in this country. Therefore it fell short of the essential element out of which the evidence of receipt is made in this case. Then I think the observations that follow are to be read by the light of that preliminary statement, which is the key to the whole position; and he goes on at the end—dealing with the Mexican Case—to say that that case "was totally different. The money there could only be "said not to have been received if money sent home by bill is not "received in this country, or if no Colonial interests are received in "the United Kingdom which do not reach it in specific form"—evidently implying that there might be ways by which Colonial interest might reach this country short of a payment in specie. It seems to me, therefore, that that case adopts the previous decision, and really says nothing in the least degree incompatible with it, and, so far as the decision itself is concerned, the point really did not arise. I think, therefore, that it being found in this case—and I

treat it as found—(as the decision has found)—that as the result of an account taken in this way a balance was arrived at, and that balance was applied to the payment of dividends, there has clearly been a receipt in this country. I do not think Mr. Haldane's point that the payments of dividend must be regarded as only provisional, leaving the rights of the parties just as though no such payments had been made, can be supported in point of fact in view of the finding which has been referred to by my Lord in paragraph 4; a finding also which is based upon an obligation imposed, as the Solicitor-General pointed out, by the deed governing the Society itself.

I think the Solicitor-General was really well justified in calling attention to the central fact here, that this is one undivided Society, and that the debts are due from the Society itself wherever incurred. There are not separate businesses in each of the different parts of the world where they carry on business. It is one business creating one debtor—the Society itself, and one person entitled to receipt—the Society itself. When you carry that in your mind and apply that to the fact that the whole account is taken together, the credit items on the one side and the debits on the other, a balance struck, and all applied by that central authority whose *locus* is the United Kingdom, I think that they cannot so apply them without going through the implication that they have received that money over which they are exercising dominion and applying it in the way of paying their debts. It is not necessary, in view of the facts which have now been clearly put before us, to deal with the subordinate contention of the Solicitor-General arising upon paragraph 11. We have here conclusive evidence that they have finally and absolutely asserted their control over the whole fund in question, and dealt with it by paying out a balance arrived at, by taking it all into consideration, and therefore it is not necessary to consider the subordinate point as to orders being given for the re-investment of these interests received abroad. I pronounce no opinion on that part of the case. I decide it on the grounds I have already stated.

LORD JUSTICE STIRLING: I am of the same opinion. The question is a very short one, namely, whether certain interest or dividends which have accrued in foreign parts to this Society can be said to be received in the United Kingdom within the meaning of the Income Tax Act. It is conceded that that interest has not been specifically received by the Company in this country, but the word "specifically" is not in the Act, and the question is whether in point of fact the interest has not been in some form or other received in the United Kingdom.

Now the facts which are relied upon for this purpose as showing that the interest has been so received are, shortly, these. The Company carries on business in a great number of countries as well as in England, but the business, as is stated in the case, is one indivisible business. The accounts, which are prepared by the Company yearly, in accordance with the requirements of the Act of 1870, embrace, without distinction, all the business transactions of the Company in whatever country they are carried on.

Under one of the articles of the deed which governs the affairs of the Company, the shareholders are entitled to be paid yearly out of the



profits a dividend of 5 per-cent upon the amount of the capital which is paid up. I cannot see anything in that article or in any others to which our attention has been called which makes such a payment provisional. No doubt the Company do not every year go through the form of having a valuation of every asset ; that is only done, as I understand it, once in every three years ; but in every intermediate year they do make up an account, as, in fact, they are obliged to do by the Statute, and they bring all the property of the Company (capital, of course, as well as income) into one account, and upon the basis of that account so arrived at they pay the dividend of 5 per-cent to the shareholders.

Now that being so, the case is brought almost in terms within the case which was decided in Scotland in the year 1886 by the First Division of the Court of Session, and it was there held that such a dealing with the receipts or interest in foreign parts constituted a receipt, or was evidence of a receipt within the country. It was at first contended, as I understood, that there could be no receipt in this country unless in some form there was a specific remittance of the money which was received abroad to this country, but that could not be maintained. In the case in Scotland this happened : There was in the hands of agents of the Company in America a certain sum received in respect of interest which had accrued there ; there was in the hands of the directors at home a fund which had arisen on capital account which in the ordinary course ought to be remitted to America and invested in securities there. Dividends were payable, and with the view to payment of the dividends the directors of the Company said to the agents in America : Retain the interest which you have got there, and apply it in satisfying our liabilities in America. On the other hand, we shall retain in this country the money which ought to go out to America, and apply that in payment of the dividend to our shareholders. It was admitted that that really amounted to a receipt in this country of American dividends. I must say that seems to me to almost decide the present case. The differences are immaterial. And as to the second case which was cited (also in Scotland) it seems to me to be quite capable of being distinguished on the ground which has been already mentioned, namely, that there merely an account had been prepared in which these dividends were introduced, but that nothing further than that had been done. So that really the matter stood on the footing of a man making an entry in his own book, but not acting upon it.

The Scotch Case was decided so long ago as 1886, and it has been followed by the Courts of this country in various cases, and has never been dissented from. I should be very sorry in that state of things to depart from the law which has thus been laid down. It seems to me the decision may be well justified and that we ought to follow it.

On these grounds I agree with the conclusions which have been arrived at by my learned brothers.

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## APPENDIX (VI).

## HOUSE OF LORDS.

*Friday, 16th May 1902.*

GRESHAM LIFE ASSURANCE SOCIETY, LIMITED - - Appellants,

AND

GEORGE HENRY BISHOP, Surveyor of Taxes - - - Respondent.

*Lords present :*

## THE LORD CHANCELLOR.

LORD MACNAGHTEN.

LORD BRAMPTON.

LORD SHAND.

LORD LINDLEY.

## JUDGMENT.

The LORD CHANCELLOR: My Lords, the question in this case seems to me to depend upon the actual words used by the Legislature and I deprecate a construction which passes by the actual words, and seeks to limit the words by what is supposed to be something equivalent to the language used by the Legislature. To put the matter shortly, the Legislature has provided that, besides the proper amount of taxation upon the balance of profits and gains by any person resident in this country, he must also pay upon the interest on any investment made in foreign countries, and that in calculating that amount, the actual amount received on such investments, no calculation or deduction is to be allowed in respect of the expenses of obtaining such investment, but the duty must be levied upon the actual amount received; but then this impost is only to be levied provided the money is received in this country.

Now here the money has not actually been received in this country. It is to be observed that the Legislature has assumed, by the distinction which it has made between the mode of ascertaining the amount payable generally upon the balance of gains and profits, and the amount taxable in respect of the interest payable upon foreign investments, that it has ear-marked that sum and made it subject to distinct and peculiar incidents. The difficulty of identifying the actual sum is no limit on the inactment. The Legislature must be supposed to have contemplated the possibility of drawing a distinction between money received in this country and money accounted for or credited in account. If it were not for the difficulty of ear-marking money I should think no one would have any doubt that the money must be received in this country to bring it within the words of the Statute. If it were not money but some commodity, say tobacco, which a trader carrying on business in London and Paris was accounting for to his London house, no one would say that though the Paris tobacco was credited in account as a set-off against some expense or something that the supposed London firm had to set off against the same claim, and that as the London firm was paid by the Paris tobacco, therefore the tobacco was liable to the import duty

on tobacco because it was taken into account in the books of the London firm.

In no way that I can give any reasonable interpretation to has the money reached this country or been received in this country. It, like the tobacco in the case suggested, has not been imported, and if the Legislature had intended that bringing it into account was to be equivalent to its being received, it would have been easy to say so. It cannot be said that the use of artificial meanings to be attached to ordinary language is either unknown or unusual in legislation; and if it was intended to make this a special subject of taxation to be taxed whenever and wherever an equivalent amount was credited or booked, or in any other way recognized as having come under the dominion of the owner in this country nothing could have been easier than to enact it in plain terms.

I decline to go beyond the words used, and I do not think this money was received in this country.

I do not think any amount of book-keeping or treatment of these assets wherever they may be will be equivalent to or the same thing as receiving the amount in this country. The words are simple, intelligible, and represent an ordinary and simple thing. I cannot think we ought to go beyond the words themselves, and I think this Judgment ought to be reversed.

Lord MACNAGHTEN: My Lords, I am also of opinion that the Judgment of the Court of Appeal cannot be supported.

The question depends upon the meaning of the rule applicable to the Fourth Case of Schedule D. To my mind the language of the rule is so plain that it is difficult, if not impossible, to add anything which would make the meaning plainer.

The Appellants are possessed of foreign securities. The duty to be charged in respect of interest arising from foreign securities is, according to the rule in question, to be computed on a sum not less than the full amount of the sums which have been, or will be, received in the United Kingdom in the current year. I do not understand what is meant by constructive receipt in such a case as this, or how any sums can be said to have been received in the United Kingdom unless they have been brought to the United Kingdom, or unless there has been a remittance "payable in the United Kingdom", to borrow the language of the rule applicable to the fifth case. The circumstance that the business of the Society is "one indivisible business", and that the Society in the statement of its affairs and in its dealings with its shareholders and creditors takes into consideration its foreign assets and liabilities, seems to me to be immaterial to the present question. As my noble and learned friend Lord Robertson, when Lord President, observed in the case of *The Scottish Provident Institution* (23 R., 322), "Every man and every company having foreign or colonial investments of course knows of the interest arising from them, takes note of it, and enters it in any statement of affairs which may require to be made up." But that, as I think and as the Lord President thought, is a very different thing from bringing the interest home—a very different thing from the receipt of the money here, either in specie or as represented by a remittance payable in this country.

The difficulty seems to have arisen from a misunderstanding or a misapplication of the Judgment in the New Mexican Case (14 Ret., 98). That was a very special case. Whether the decision was right or wrong it can have no bearing upon the question now before your Lordships. Speaking for myself, I think the decision was right. In that case, as it seems to me, in the transmission to this country of money which the Company was free to distribute, and the transmission to America by way of exchange of an equivalent amount which the Company was bound to re-invest, the Company acted as their own bankers, and did for themselves, by an entry in their books, what might have been done less conveniently and less economically by an ordinary bank or financial agent on their behalf. I think that the Appeal must be allowed.

LORD SHAND: My Lords, I am also of opinion that the Appeal should be allowed. It is true that the Appellants received the interest on their foreign securities by the hands of their agents abroad. But I think it is equally true that, as they left that interest where it was gained, it was never received in this country. When it was entered in the Company's balance sheet in order to the ascertainment of the profits of the year, it was so entered as estate which had not been received in England, but as property belonging to the Company which they acquired abroad, which had not been brought home or received here, but which was part of their foreign assets. Money or securities in that position was properly taken into account in the ascertainment of the year's profits, not because it had been received in England, but because although not so received it was part of assets of value which the Company had acquired and held abroad.

In the Scottish Case of the Investment Company of New Mexico, 14 Ret., 98, the *species facti* was different, for there the Company treated the money as received in this country, and merely saved themselves the expense of cross remittances. It appeared there that the Company was not entitled to divide the money earned abroad unless it was received as profits in this country. It was treated as so received merely to avoid the expense and inconvenience of cross remittances—money sent home and the same amount sent back by cross cheques or drafts. That was a material point in the decision of the case as showing that the money had been really received in this country.

LORD BRAMPTON: My Lords, it is conceded that no part of the money in question was ever received in the United Kingdom in specie, or in any form known to the commercial world for the transmission of money from one country or place to another. But it was argued that if not actually it was "constructively" so received in the accounts of the Society. I confess I do not like that expression, nor do I quite understand what it means. If a "constructive" receipt is the same thing as an actual receipt, I see no reason for the use of the word "constructive" at all. If it means something differing from or short of an actual receipt, then it seems to me that a constructive receipt is not recognized by the Statute, which, in using the word "received" alone, must be taken to have used it having regard to its ordinary acceptation.

The Master of the Rolls (Sir A. L. Smith) in his Judgment in the



Court of Appeal, while stating that there must be "an actual receipt of the amount", added "but that receipt need not be in specie, it may be in account", and he then proceeded to deal with the accounts of the Appellants, set forth in the Appendix, and to draw from them the inference that the Appellants had actually received and dealt with these foreign dividends in the United Kingdom, and had distributed them as having been so received. Now I am not prepared to deny that accounts may be so worded as to contain admissions justifying such an inference, but I differ with the view he took that such admissions, or anything approaching them, are to be found in the accounts before your Lordships.

These accounts were framed partly to satisfy the requirements of the Life Insurance Companies Act, 1870 (33 and 34 Vict., c. 61), that at the end of each financial year a statement of the Company's revenue account and of its balance sheet in the forms contained in the 1st and 2nd Schedules should be furnished to the Board of Trade; and partly in obedience to Articles 77 and 78 of the Society's deed of settlement, directing books to be kept in which full entries shall be made of all matters which shall properly be the subject of debt or credit account, so that the financial state of the Company may at all times appear as accurately as circumstances will permit; and further, directing balance sheets to be made up yearly and sent to every shareholder. The accounts before your Lordships profess to do no more than this, and no inference of fact can be drawn from them other than, or in addition to those stated in them.

In my opinion there is a total absence of any evidence to justify a finding that the interest in question has ever been received in the United Kingdom.

For the Crown the case of *The Scottish Mortgage Company of New Mexico v. The Commissioners of Inland Revenue* (14 Court of Session Cases, p. 98), was much relied upon. I am not satisfied with the correctness of the Judgment in that case, but assuming it to be sound, it is distinguishable from the present case, for in paragraph 13 of the printed case before the Court of Sessions there was an admission that the amount charged with the income tax had been applied in payment of interest and dividends to debenture and share holders in Glasgow. No similar admission was contained in the accounts in the case before this House.

My Lords, I am of opinion with your Lordships that the Appeal should be allowed with costs.

LORD LINDLEY: My Lords, this Appeal turns upon the answer to be given to a simple question of fact. Has a certain sum of money entered by the Gresham Society in its accounts as an asset been received in this country by the Society, or has it not? If it has, the Appeal ought to be dismissed; on the other hand, if it has not, the Appeal ought to be allowed.

First, let us consider what is meant by the receipt of a sum of money.

My Lords, I agree with the Court of Appeal that a sum of money may be received in more ways than one: *e.g.*, by the transfer of a coin or of a negotiable instrument or other document which represents and produces coin and is treated as such by business men. Even a

settlement in account may be equivalent to a receipt of a sum of money, although no money may pass; and I am not myself prepared to say that what amongst business men is equivalent to a receipt of a sum of money is not a receipt within the meaning of the Statute which your Lordships have to interpret. But to constitute a receipt of anything there must be a person to receive and a person from whom he receives, and something received by the former from the latter, and in this case that something must be a sum of money. A mere entry in an account which does not represent such a transaction, does not prove any receipt, whatever else it may be worth.

Now in this case the Gresham Company's accounts and the statements in the special case clearly establish the fact that the sum of £143,483 sought to be charged with income tax, consists of interest and dividends received abroad by the agents of the Company from persons abroad who have paid those agents. The case and accounts do not state the exact mode in which the various sums making up the total of £143,483, were paid to the agents of the Company. The payment is admitted, and the receipt of that sum by the Company, through its agents, is not in dispute.

But then comes the second question: has that sum been received in this country by the Gresham Company? The special case clearly shows that it has not in fact been remitted to this country in any way whatever. Applying the test already suggested, no one here has received that sum; the agents who received it abroad still have it abroad, or have dealt with it otherwise than by sending it to the Company here. No account even is forthcoming to show that the sum has ever been treated as remitted here so as to justify the inference that in any commercial sense the sum has been received in the United Kingdom as distinguished from other countries.

What has been done and all that has been done, is that the Gresham Company, in making up its accounts with a view to ascertain what profits it could divide in a particular year, entered on its asset side the sum of £143,483, as money received during the year. This was obviously right; for the object was not to ascertain the profit made in any particular country, but the profit made by the Company on all its transactions all over the world. The Company has paid duty on the profit so ascertained, and no question arises as to that. But when required to pay duty on the item of £143,483, on the ground that this sum is made up of interests or dividends received in the United Kingdom, the Company objects on the ground that it represents nothing of the sort. Nor does it, in truth.

The fact that the profits shown by the account have been divided amongst the shareholders of the Company does not carry the case any further. No part of the £143,483 has come over here, or been in any sense received here, and then applied in payment of dividend. Some interest or dividends received abroad have been remitted here and duty has been paid on them accordingly, but the special case shows plainly that no part of the £143,483 has been so remitted, either for the purpose of paying dividend or for any other purpose.

My Lords, it must be assumed that the language used by the Legislature in laying down the rules to be observed in the various cases contained in the Income Tax Act, 1842, was carefully chosen,

and that there was some good and sufficient reason for confining the duty on interest on foreign securities (mentioned in the fourth case falling under Schedule D) to sums which have been or will be received in Great Britain during the year for which the duty is payable. The locality of the receipt is made all important, and it is only by ignoring it, or introducing the expression "constructive receipt"—which may mean anything—that the claim of the Crown can be supported.

Schedule D in the Act of 1842 was recast in 1853, and was replaced by a new Schedule D; but the cases and rules in the Act of 1842 are applicable to the new Schedule; see sections 2 and 5 of the Income Tax Act, 1853.

My Lords, authorities have been referred to, and especially the Scotch Cases of McKelvie, Forbes and the Standard Life Assurance Company. McKelvie's Case was very peculiar. Money received by the Company's agents abroad was clearly and unmistakably treated by the Company as remitted to and received by it here, and money here was treated by the Company as remitted abroad in exchange for it. The exchange was effected by a book entry, but that entry was the business mode of carrying out cross remittances which it would have been unbusiness-like and really childish to have effected in any other way. But thinking as I do that McKelvie's Case may be properly upheld, I am not prepared to adopt it as a new starting-point for further inferences. The language of the Statute is the true starting-point in each case. Forbes' Case and the Standard Life Assurance Company's Case were both based on this sound principle, and were in my opinion both clearly rightly decided. The Court of Appeal, in my opinion, considered this case undistinguishable from McKelvie's, but I am unable so to regard it. Assuming them to be undistinguishable, it would, in my opinion, be more correct to overrule McKelvie's Case than to decide the present Appeal in favour of the Crown.

In my opinion the Appeal should be allowed.

Questions put.

That the Order appealed from be reversed.

The Contents have it.

That the Respondents do pay to the Appellants the costs both here and below.

The Contents have it.

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## APPENDIX (VII).

## OPINION

BY THE LORD PRESIDENT AT ADVISING *FORBES v. THE SCOTTISH PROVIDENT INSTITUTION, THE SCOTTISH WIDOWS' FUND, &C., SOCIETY.*

The learned Counsel for the Crown stated that they considered that the facts stated in the case were not sufficient to raise their claim under Case 1 of Schedule D. They were anxious, however, to explain that it was solely on the ground of deficient statement of the facts relating to the two Institutions which are responsible in these Appeals that they did not on the present occasion advance this claim against these mutual Societies.

The claim of the Crown was accordingly rested alternatively on Section 102 of the Income Tax Act, 1842, and the 4th Case of Schedule D.

The argument under Section 102 is admittedly novel, and it involves surprising consequences.

Case 4 of Schedule D purports to state the duty to be charged on interest arising on colonial and foreign securities, and it limits, by the terms of the Rule, the taxable amount to the sums received in the United Kingdom. This reads as if it were, and has hitherto been supposed to be, an exhaustive statement of the liability to duty of this class of securities. The present argument is that Section 102 subjects to duty the whole of such interests, disregarding the distinction drawn in Case 4 between what is yielded in the country of the security and what is received in the United Kingdom. If this view be sound, Section 102 subjects to duty a class of property, profits and gains not included in Schedules A, B, C, D, or E, to wit, interests arising from colonial or foreign securities but not received in the United Kingdom.

The answer to this argument is two-fold.

First, the Statute under which the Crown primarily claims duty from the respondent Companies is the Customs and Inland Revenue Act, 1893, and the operative Section is the fifth. Now the things on which duty is charged by that Section are the property profits and gains chargeable under Schedules A, C, D and E of the Income Tax Act, 1853, and the occupation of lands (and so on) under Schedule B of that Act. These Schedules, therefore, state the limits of the Income Tax granted by the Act of 1893. The same Section (5) goes on to provide that all such provisions contained in any Income Tax Act as were in force during the preceding year should have full force and effect "with respect to the duties of Income Tax hereby granted."

If then, and in so far as, Section 102 of the Act of 1842 subjects to tax things not covered by the lettered Schedules, it was not in force during the year 1893-4. This seems to me a sound answer to the argument for the Crown, and, if sound, is conclusive.

It would appear, however, that Section 102 never had the effect

now sought to be ascribed to it. An examination of the Act of 1842 shows that the first Section sets forth the Schedules as defining the subjects of the tax, and that the group of Sections of which Section 102 is one are merely executive or administrative directions for carrying out the Schedules and Rules. Section 102 certainly is so expressed as, in terms, to charge with duty, but the general words used are, in truth, introductory to the provisoes, and they are general because they are introductory, and therefore do not rehearse the limitations which have been already expressed.

Accordingly I am against the Crown on Section 102.

On the alternative argument in Case 4 of Schedule D, I think the facts fail the Crown. There is nothing, as far as appears, done with the colonial interests in question, except to leave them where they are. The phrase "constructive remittance" in the second query in these cases is one which, if used at all, requires to be carefully guarded. As employed in the present argument, it would practically obliterate the limitation in the rule of Case 4. Every man and every Company having foreign or colonial investments of course knows of the interest arising from them, takes note of it, and enters it in any statement of affairs which may require to be made up, but this will never make the interest "received in the United Kingdom." The New Mexican Case was totally different. The money there could only be said not to have been received if money sent home by bill is not received in this country, or if no colonial interests are received in the United Kingdom which do not reach it in specific form.

In the view which I have stated of the second query, the third query in the case relating to the Scottish Widows' Fund does not arise.

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## APPENDIX (VIII).

### OPINIONS OF THE JUDGES OF THE SECOND DIVISION OF THE COURT OF SESSION

#### ON CASE FOR

#### THE STANDARD LIFE ASSURANCE COMPANY,

*On Appeal by them as to Income Tax under "The Taxes  
Management Act, 1880."*

Lord Justice CLERK: The controversy here between the Standard Life Assurance Company and the Inland Revenue, which we are called upon to decide, arises under Schedule D, Case 4 of Section 100 of the Income Tax Act of 1842, which relates to the duty to be charged in respect of interest arising from foreign securities, which is to be computed on a sum not less than the full amount of the sums (so far as the same can be computed) which have been or will be received in Great Britain in the current year without any deduction or abatement. The whole question turns on the words "which have been or will be received in Great Britain." The facts as stated in the case are that the Appellants had in foreign

countries and in India large sums invested under the control of the Government Officials of these countries as security for the due fulfilment of the Appellants' obligations in these countries. The question relates to the receipts from these investments in certain countries, and the case states that no part of these "was remitted to " the United Kingdom *in formâ specificâ* during the year ending " 15th November 1898, the whole being retained in the countries " where they arose, and so far as not required for meeting claims and " other outgoings, invested or otherwise applied there."

These sums entered the Appellants' Revenue Account for the year submitted to the Board of Trade under the Life Assurance Companies Act, and it is the annual practice to include these interests in the Appellants' Revenue Account, and they are taken into account in arriving at the amount of profit made by the Company.

The Directors of the Appellants' Company paid all their liabilities in Great Britain out of funds in their hands in that country. No part of the proceeds of the foreign investments was applied for that purpose, or was distributed among the proprietors.

The real question in the Case is whether the words of Case 4 of Schedule D, "received in Great Britain," apply to these proceeds. The Respondent maintains that they do, they having entered the Revenue Accounts of the Appellants, and that that amounts to their having been received in account. In support of this view he appeals to the English decision in the case of the Gresham Life Assurance Society against Bishop, and to the case of the Scottish Mortgage Company of New Mexico against the Inland Revenue Commissioners. The latter of these two cases seems to me to be essentially distinguishable from the present case. In the New Mexico case the Company saved the expense of the actual transmission of money from abroad—I do not mean in specie but in the ordinary course of mercantile business—for the payment of dividends by the expedient of using capital sums raised on debenture loans in this country to pay the dividends and using the profits in the foreign country instead of the debenture money for investment abroad. In that case, plainly the interests earned in the foreign country were brought into account and applied in Great Britain. The Company could not legally divide the debenture money as profits, and the investments made abroad were truly representative of the debenture money, and the debenture money applied in Great Britain was truly an application of the profits made abroad. Therefore that money was as truly "received in Great Britain" as if it had been remitted *in formâ specificâ* or by the usual modes of transmission by which specie and transmission are avoided. But the "receiving in Great Britain" was indisputable. That case, therefore, does not in essential particulars resemble the case we are dealing with. The case of the Gresham Assurance Society has a much closer resemblance to the present. The learned Judges who decided it seem to have considered that the view they took in that case was supported in the case of *Forbes v. The Scottish Provident Institution*, decided in the First Division of this Court. I cannot agree with that view. The case of *Forbes* seems to me to be in direct conflict with the decision in the case of



Gresham, and, if rightly decided, to rule the present case. The rubric of that case, which very correctly represents the decision given, is:—

“ When interest derived from the colonial investments of  
“ a society for mutual assurance was not remitted home and  
“ was reinvested abroad, held that by being entered in the  
“ Society’s accounts it was not constructively remitted to this  
“ country so as to be chargeable with duty under Case 4 of  
“ Schedule D of the Income Tax Act of 1842.”

That seems to be an exact description of the present case.

In that case, as in this case, the sums of profit made in the foreign country were entered in the Company’s statement of affairs. The question is whether that fact alone, which, as the Lord President points out, is common to all business persons and companies having investments out of the Kingdom, constitutes receiving of the sums in Great Britain—in other words, does information to the investment holder that he has made that profit on his foreign investment *ipso facto* constitute a receiving of it here? I am of opinion that it was rightly held in the case of Forbes that it did not, and therefore that in this case, which seems to me to be practically identical with it, the decision of the Commissioners was wrong.

Lord YOUNG: I am unable to agree with your Lordship’s Judgment, for I think the Commissioners’ decision was right.

Lord TRAYNER: I think the decision of the Commissioners is wrong. The duty for which that decision finds the Appellants liable is a duty or interest arising from securities held by them in His Majesty’s dominions out of Great Britain. But such interest is only liable for duty on the amount thereof “received in Great Britain”, and it appears to me that no part of the interest on which duty is now claimed was ever received in Great Britain. The contrary is set forth in the 7th Article of the Case before us, where it is stated that no part of the interest in question was remitted to the United Kingdom, but was retained and applied in the country where it arose. I do not fail to notice that this statement is qualified by the expression that the interest was never remitted *in formâ specificâ*, and that “the foreign interest is regularly included in the Company’s annual Revenue Account as a distinct source of income, and was duly taken into account in arriving at the amount of the profit earned by the Company available for distribution by way of bonus or dividend”, &c. These facts are not inconsistent with the fact that the interest was not received in Great Britain. I concede that in order to make the foreign interest liable in duty it is not necessary that it should be remitted in “*in formâ specificâ*”, if that means was remitted in coin. Anything equivalent to money or which can be turned into money will do. Nothing of that kind occurred here. The only thing which was done was that the foreign branches reported so much interest earned and obtained, which appeared in the Appellants’ accounts as part of their assets. But that asset was kept where it was earned and got, and there applied. It was not needed for any disbursement (either in dividend or otherwise) which the Appellants had to make in Great Britain, and, in fact, was not so

applied (Article 9 of the case). In the words of the Lord President in Forbes' Case: "There is nothing, as far as appears, done with the colonial interests in question, except to leave them where they are." I cannot distinguish this case in principle from Forbes' Case, and I am prepared now to repeat the decision there given.

I respectfully dissent from the judgment given in the case of Gresham. I venture to think that there is no room for the view that the Statute contemplates or provides for any "constructive" remittance. I think there is no remittance provided for except actual remittance, not necessarily of money or coin, but of something equivalent in the market to money, and that can thus be turned into money. I cannot hold that a mere report by a foreign or colonial branch to the head office in Great Britain that so much interest has been earned and is retained is a remittance either constructive or real. In the decision in the case of Gresham a reference is made to the case of Scottish Mortgage Company of New Mexico, decided in the First Division of this Court in 1886 as being a decision "exactly in point." I think, with great deference, that that is not so. In my view the two cases are essentially different. In the New Mexico Case the facts were that the Company had received capital by debenture in this country, which they could not legally apply in payment of dividends, and which was intended for foreign investment; that at same time they had interest earned abroad which could be, and was at that time, to be applied in payment of dividends in this country. But instead of sending the debenture capital abroad for investment and receiving the foreign interest at home for payment of dividends, they directed the holders of the foreign interests to invest it, and to that extent the capital at home would be retained and applied in payment of dividends. The one was the *surrogatum* for the other. In short, what the Company did was this: they held the money in their hands to be interest, and used it as such in Great Britain, directing that an equivalent sum should be held abroad as capital and used there as such. In this way the Company did get money in this country out of foreign interest to pay their dividends, and did pay it. I think that case quite consistent with the case of Forbes. The facts are essentially different. But the facts in Forbes' Case and the present case appear to me to be the same, and the result (as in Forbes' Case) is that the foreign interests "left where they are" are not liable in duty.

LORD MONCREIFF: Although the Statute which we have to interpret was passed in 1842, there are few decisions to guide us. Two Scottish and two English cases were chiefly relied on. The earlier in date of the Scottish cases, *The Scottish Mortgage Company of New Mexico v. McKelvie*, 19th November 1886, 14 R. 98, is claimed by the Surveyor as an authority in his favour; the Appellants, on the other hand, found on the later case of *Forbes v. Scottish Provident Institution*, 17th December 1895, 23 R. 322.

The English decisions, both of which were decided within the last year, are *Gresham Life Assurance Society v. Bishop*, L.R., 1901, Q.B. 153, and *The Universal Life Assurance Society v. Bishop*, 81 L.T.R. 424. They are both adverse to the Appellants; but, however worthy of respectful consideration they may be, they



are not binding on us, and, moreover, they seem to a certain extent to have proceeded upon a misapprehension of the decisions of this Court in the cases of *The Scottish Mortgage Company of New Mexico* and *Forbes*.

We have, therefore, to interpret the Statute for ourselves with the aid of such light as those decisions and the opinions of the learned Judges who decided them afford.

The Crown's claim is made under the 4th Case, Section 100, Schedule D of the Statute of 1842. The duty to be charged on securities out of Great Britain is to be "computed on a sum not less than a full amount of the sums so far as the same can be computed *which have been or will be received in Great Britain* in the current year without any deduction or abatement." The words to be construed are "*which have been received in Great Britain*." The words "*or will be received*" do not in my opinion affect the question. They simply provide for an estimate based upon the practice and requirements of the taxpayer, this being necessary because the return has to be made before the expiry of the financial year.

Parties are agreed on the facts of this case; and amongst the facts admitted are the following:—*First*, that the whole of the interests earned abroad amounting to £136,841. 11s. 6d. were "retained in the countries where they arose, and so far as not required for meeting claims and other outgoings invested or otherwise applied there." Thus the interests in question were not remitted to or received in Great Britain during the year of assessment.

The second point (which is also a matter of admission) is that it was unnecessary to draw on the foreign receipts in question for the purpose of paying the Company's liabilities or dividends and bonuses in the United Kingdom. The Appellants had ample funds in this country which they were entitled to use for all these purposes.

In my opinion these facts are sufficient to exempt the interests in question from taxation under the 4th Case which, as I read it, requires that the interests to be taxed must either have been actually remitted to and received in Great Britain, or according to the practice and requirements of the trader, should have been remitted during the year of assessment. This view receives corroboration from the terms of the 108th Section of the Statute which prescribes the places at which profits on foreign or colonial possessions or securities (taxed under the 5th Case) which are "*imported into Great Britain*" are to be charged. Those words are used in the same sense as "*received*" in the 4th Case.

It remains to consider the grounds on which the Surveyor contends for a wider construction of the 4th Case. Towards the close of his argument the learned Solicitor-General (unless I entirely misapprehended his answers to questions from the Bench) pleaded his case as high as this, that if a trader or investor in this country to whom interest on foreign securities becomes due sends directions to his agent abroad not to remit the interest to this country, but to invest it or pay debts with it abroad, or directs the debtor to pay the money into bank abroad, or indeed gives any directions for its disposal, such interest must be held to have been "*received in Great*



Britain." I was at first surprised at the answer, because it appeared to leave no case to which the limiting words in the 4th Case could apply : but I must assume that the Crown's argument required it.

Coming more closely to the facts of this case, the Crown's claim is mainly rested on this, that the interests accrued on foreign securities are regularly entered in the Appellants' Revenue Account for the year, and go to swell the profits for the year, according to the amount of which the dividend and bonus for the year are fixed and paid. It is, therefore, argued that those interests having been brought into account and constructively applied in payment of liabilities, or in payment of dividend and bonus, must be held to have been "received" in this country. The answer is that although the interests on foreign securities are necessarily entered in the annual Revenue Account, and taken into consideration in ascertaining the amount of the profits and arranging for their division they are not in point of fact remitted to and received in Great Britain, and they are not required for the discharge of any of the Appellants' liabilities or purposes here. It appears from the 9th statement that the Company are entitled to treat and uniformly treat capital and income on precisely the same footing, and to make any payments which they are required to make out of the first and readiest monies which come into their hands. Besides, there is nothing in the case to show that the income actually received in this country was not sufficient for all payments which required to be made here. Thus in no sense were the interests in question received in Great Britain, they were not remitted and they were not required.

Reliance is placed on the fact that the supreme control and management of the Company's affairs is vested in a board at Edinburgh. I fail to see the relevancy of this consideration. The Statute assumes that the person entitled to the interest is resident in Great Britain ; his liability depends on the interest reaching him there and not upon the directions which he gives as to its application.

This brings me to *The Scottish Mortgage Co. of New Mexico v. McKelvie*, 14 R. 98.

It is impossible to read the anxiously expressed opinion of the Lord President Inglis without seeing that he regarded it as a very special case. The business of the Company was carried on by borrowing money in this country at low rates and lending it on American securities at high rates, the profit consisting of the difference. The only money which was properly applicable to payment of dividends and liabilities in this country was the interest earned abroad upon foreign securities which accordingly ought to have been regularly remitted to this country. Strictly speaking, the Company had no right to apply the money which they borrowed in this country to those purposes ; but for the sake of convenience and to avoid trouble and expense the Company, instead of ordering all the interest to be sent home, and sending out all the borrowed money raised on debenture to be invested in America, retained out of the borrowed money, a sum sufficient to pay all the working expenses in Great Britain, interest to debenture holders and depositors and a dividend, and directed an equivalent amount of the interest which would otherwise have been remitted to be retained abroad and

invested. Thus one sum was set against the other and formed a proper *surrogatum* for it. The Lord President concludes his opinion thus (page 102): "So that according to the way in which this " Company keeps its books it has really converted a sum which was " received in this country as capital, into an equivalent for the " interest upon the foreign securities, *and it represents in their " books interests upon these foreign securities.* Now in these " circumstances it appears to me quite impossible for the Company " to maintain that they have not received that interest. They have " received it in this most proper sense of the term, that it enters " their books in this country as such interest, and is paid away as " such. I am, therefore, of opinion that the duty is rightly " charged under the 4th Case, and that the deliverance of the " Commissioners ought to be affirmed."

The finding of the Commissioners, which the Court affirmed, was in these terms (page 100) "19. The Commissioners found (1) that " the profits of the Company were of the nature described in the " 3rd Clause, Schedule B, Section 2, of 16 and 17 Vic. cap. 34. " (2) That the assessment fell to be imposed on the full amount of " the sums which had been received in the United Kingdom in the " year of assessment, and that according to the rule in the 4th Case, " Section 100, of 5 and 6 Vic. cap. 35, duty was chargeable on the " profits of the Company which had been brought into account in " their books in Glasgow, in so far as such profits had been applied " to the payment of interest, dividends, debit balance, and preliminary " expenses in respect that by being so brought into account and " applied they must be held to have been received in this country " in exchange for an equivalent sum raised in this country and " invested abroad, but not upon the profits which had been carried " forward, even although such profits had been brought into account " in the books of the Company at Glasgow, in respect that they had " not yet been actually dealt with, and applied as money received in " this country."

The difference in the facts from those in the present case will at once be observed. Here it was not necessary that the interests should be remitted, and they were not dealt with as having been remitted, because there were ample funds which, according to the practice and powers of the Company, were in use legitimately, to apply to all payments to be made in Great Britain or elsewhere. In my opinion, the true explanation of the judgment in the Scottish Mortgage Company of New Mexico, is that it proceeded upon the footing of bar or estoppel. The funds raised in this country for the purpose of investments abroad could not legally be applied in payment of debts and liabilities due in this country, and therefore the Company could not be held to plead that the interest which was entered in their books as having been received had not been remitted.

The material facts in Forbes' Case, 23 R. 323, were simply these. The Directors had lent out considerable sums in Australia and elsewhere out of the United Kingdom. The interest derived from these loans in the year 1892 amounted to £90,359. 8s. 9d. That interest was wholly deposited with the Company's bankers in the country where it was collected, and not being required to meet charges



against the common fund in the United Kingdom, it was not remitted to this country *in formâ specificâ*, but in terms of the Institution's power it was lent out as opportunity offered in the name of the Corporation. It formed part of the interest entered in the Revenue Account of the Institution for the year ending 31st December 1892, as given up to the Board of Trade in terms of the Life Assurance Companies Act, 1870 (33 and 34 Vic. c. 61), Section 5. No distribution of surplus took place in the year 1892, the last septennial investigation into the affairs of the Institution having taken place in 1887, when out of a surplus amounting to £1,051,035. 8s., £350,345 had been retained, and £700,690 apportioned among the participating members.

It was held that upon these facts the case for the Crown failed. Lord President Robertson said (page 327): "On the alternative argument on Case 4 of Schedule D, I think the facts fail the Crown. There is nothing as far as appears done with the Colonial interests in question, except to leave them where they are. The phrase 'constructive remittance' in the second query in these cases is one which, if used at all, requires to be carefully guarded. As employed in the present argument it would practically obliterate the limitation in the rule of Case 4. Every man and every Company having foreign or colonial investments of course knows of the interest arising from them, takes note of it, and enters it in any statement of affairs which requires to be made up. But this will never make the interest 'received in the United Kingdom.' The New Mexican Case was totally different."

These words seem to me exactly to fit the present case. They have been interpreted as meaning merely that all that was done in Forbes' Case was that the interest entered the Corporation's accounts. But that is not so, because it appears from the statement of admitted facts, with reference to which the Lord President was speaking, that the interest was not only left abroad, but, as here, "was lent out as opportunity offered in name of the Corporation."

The only difference that can be suggested is that as there was no division of surplus during the year of assessment the foreign interest had not been taken into consideration for the purpose of fixing the amount and division of profits. But that is surely not a material fact. The fact that the taxpayer takes note of the amount of interest received abroad in regulating his expenditure (whether in payment of dividend or otherwise) cannot affect the question unless the interest either has been or should have been remitted in order to meet the expenditure.

I therefore regard the case of Forbes as an authority of our own Court in favour of the Appellants.

The facts in the case of Gresham closely resemble ours in the present case as appears from the concise statement of them in the rubric. "An Insurance Society carried on their business in the United Kingdom, and by means of local agents or managers in foreign countries. The business was entire and indivisible, and was managed by a Board of Directors in London. The Society possessed funds invested in foreign countries in which they did no business. The interests on these investments was either reinvested



“ in those countries, remitted directly to other foreign countries for investment, or remitted to London. They also possessed funds invested in foreign countries in which they carried on business. The interest on these investments was either reinvested in those countries, applied in establishment and other charges in those countries, remitted direct to other foreign countries for investment, or for the general purposes of the Society, or remitted to London. Yearly accounts were prepared on which all the interest on investment in foreign countries was included, and out of the profits shown by the accounts a dividend was paid yearly to the shareholders. The surplus funds of the Society divisible as profits were ascertained by actuarial valuation once in three years, and all the interest on investment in foreign countries was included in the triennial account.”

On these facts the Court of Appeal held that all the interest on foreign investments was received in the United Kingdom within the meaning of the 4th Case. They did so on the ground that the terms of the Statute were sufficiently satisfied by a receipt in account. All their Lordships held that the Scottish Case—The Scottish Mortgage Company of New Mexico—was an authority directly in point, and they all agreed in distinguishing the case of Forbes. Speaking of the former case, the Master of the Rolls, after stating that in his judgment the true meaning of the 4th Case was satisfied by a receipt in account, adds: “ And I think that this is the reading of it arrived at by the Judges in the Scottish Case—Scottish Mortgage Company of New Mexico *v.* Inland Revenue Commissioners. They did not put this in so many words, but they came to the conclusion in that case that there had been receipt in account of foreign dividends, and they held that being so that the Crown was entitled to income tax on the dividends so received.” Lord Justice Collins is of the same opinion, and he further states that as he reads the Lord President’s opinion the latter did not proceed on the footing that the Corporation was barred from saying that the interest had not been received.

Lord Justice Stirling is of the same opinion. For reasons which I have already stated, I think their Lordships were mistaken as to the import of the decision in that case. No doubt it was decided that in that case the cross entry in the Corporation’s books was equivalent to receipt of foreign interest; but that was solely on account of the peculiar circumstances of the case which I have described.

Then as to the decision in the case of Forbes, their Lordships all treated as proceeding on the footing that there was nothing in the case except that the interest appeared in the annual account. But here again I think their Lordships are mistaken.

On the whole matter I am of opinion that the deliverance of the Commissioners, which I have no doubt was greatly influenced by the English cases, is in this respect erroneous.

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## ACTUARIAL NOTE.

*On De Morgan's Formulas for determining the Rate of Interest yielded by an Annuity.* By SIDNEY WALLIS NEWLING, B.A.

THE formulas given by De Morgan (*J.I.A.*, vol. viii, pp. 64 *et seq.*) seem to me to be still of some mathematical, although of no practical, interest; and I therefore venture to submit the following method of obtaining expressions in close agreement with them.

(1). Given  $a$  and  $n$ , to find  $\log_{10}(1+i)$ .

$$a = \frac{1-v^n}{i} = \frac{1-e^{-n\delta}}{e^\delta-1} = ne^{-n\delta} \cdot \frac{(e^{n\delta}-1)}{n\delta} \times \frac{\delta}{e^\delta-1}$$

$$\begin{aligned} \therefore \log_e \frac{n}{a} &= n\delta + \log_e \left( 1 - \frac{n\delta}{2} + \frac{n^2\delta^2}{12} - \frac{n^4\delta^4}{720} + \&c. \right) \\ &\quad - \log_e \left( 1 - \frac{\delta}{2} + \frac{\delta^2}{12} - \frac{\delta^4}{720} + \&c. \right) \dots \end{aligned}$$

$$\text{De Morgan puts } t = \frac{1}{3} \cdot \frac{n-1}{n+1} \log_e (n/a).$$

Hence

$$\frac{3(n+1)t}{n-1} = \frac{(n+1)\delta}{2} - \frac{(n^2-1)\delta^2}{24} + \frac{(n^4-1)\delta^4}{2,880} - \frac{(n^6-1)\delta^6}{181,440} \dots \quad (1)$$

The general term of the expansion can be shown to be

$$\frac{(-1)^r (n^{2r}-1) \delta^{2r} B_r}{2r \cdot (2r)!} \dots$$

(see *Hobson's Plane Trigonometry*, p. 343), where  $B_r$  is the  $r$ th of Bernoulli's numbers.

Putting  $(n-1)\delta=6x$ , and  $12n/(n-1)^2=l$ ,

$$t = x - \frac{x^2}{2} + \frac{3}{20} \left( 1 + \frac{l}{6} \right) x^4 - \frac{3}{35} \left( 1 + \frac{l}{3} + \frac{l^2}{48} \right) x^6 + \&c. \quad (2)$$

A first approximation is  $x=t$ , and a second approximation is

$$\begin{aligned} x = \frac{t}{1 - \frac{t}{2}}, \text{ or } \log_{10}(1+i) &= \frac{6(\log_{10} n - \log_{10} a)}{3(n+1) - \frac{n-1}{2} \cdot \left( \frac{\log_{10} n - \log_{10} a}{.4342945} \right)} \\ &\dots \quad (J.I.A., \text{ vol. viii, p. 64}) \end{aligned}$$

By reversion of series—a method re-introduced by De Morgan—we obtain from (2)

$$\frac{(n-1)}{6} \delta = x = t + \frac{t^2}{2} + \frac{t^3}{2} + \frac{(19-l)}{40} t^4 + \frac{(17-3l)}{40} t^5 + \frac{(195-82l+l^2)}{560} t^6 + \dots \quad (3)$$

Hence

$$\log_{10}(1+i) = \frac{\log_{10} n - \log_{10} a}{n+1} \left( 1 + \frac{1}{1-t} - \frac{(1+l)}{20} \frac{t^3}{(1-t)^3} \right) \quad \text{approximately} \quad \dots \quad (a)$$

$$\text{or} \quad = \frac{\log_{10} n - \log_{10} a}{n+1} \left( 1 + \frac{1}{1-t} - \frac{(1+l)}{20} \frac{t^3(1+t)}{(1-t)^2} \right) \quad \text{approximately} \quad \dots \quad (\beta)$$

The true value of  $\log_{10}(1+i)$  lies between the values given by the formulas (a) and (β).

The following table gives a comparison of the results from the above and from De Morgan's formula

$$\log_{10}(1+i) = \frac{\log_{10} n - \log_{10} a}{n+1} \left( 1 + \frac{1}{1-t} - \frac{1}{20} \cdot \frac{t^3(1+t)}{(1-t)^2} \right).$$

<i>n</i>	TRUE RATE OF INTEREST <i>i</i> = .05		
	Rate by De Morgan's Formula	Rate by Formula (α)	Rate by Formula (β)
10	.05000	.05000	.05000
25	.05001	.05000	.05000
50	.05004	.05000	.05001
75	.05014	.04998	.05008
100	.05037	.04990	.05027

NOTE.—The value .05019, for  $n=100$ , found on page 67, *J.I.A.*, vol. viii, is incorrect. The mistake occurs at the end of the calculation in taking out the anti-logarithm of .021344.

(2). The corresponding formulas from the amount of an annuity are

$$\log_{10}(1+i) = \frac{\log_{10} s - \log_{10} n}{n-1} \left( 1 + \frac{1}{1+t} + \frac{1-l}{20} \frac{t^3}{(1+t)^3} \right) \quad \dots \quad (\gamma)$$

$$\text{and } \log_{10}(1+i) = \frac{\log_{10} s - \log_{10} n}{n-1} \left( 1 + \frac{1}{1+t} + \frac{1-l}{20} \frac{t^3(1-t)}{(1+t)^2} \right) \quad \dots \quad (\delta)$$

where  $t = \frac{1}{3} \frac{n+1}{n-1} \log_e \left( \frac{s}{n} \right)$ , and  $l = \frac{12n}{(n+1)^2}$ .



(3). Denoting the value of an annuity for  $n$  years deferred  $m$  years by  $B$ , and putting

$$t = \frac{1}{3} \cdot \frac{n^2 - 1}{(2m + n + 1)^2} \log_e \frac{n}{B}, \quad x = \frac{(n^2 - 1)\delta}{(2m + n + 1)6}$$

we have in a similar way

$$t = x - \frac{x^2}{2} + \frac{3}{20}x^4 \cdot \frac{\left(1 + \frac{1}{n^2}\right)\left(1 + \frac{1}{n} + \frac{2m}{n}\right)^2}{\left(1 - \frac{1}{n^2}\right)^2} - \frac{3}{35} \cdot x^6 \cdot \frac{\left(1 - \frac{1}{n^2}\right)\left(1 + \frac{1}{n} + \frac{2m}{n}\right)^4}{\left(1 - \frac{1}{n^2}\right)^5} \dots$$

If the values of  $m$  and  $n$  are such that this series may be written approximately

$$t = x - \frac{x^2}{2} + \frac{3}{20}x^4 - \frac{3}{35}x^6$$

we shall have

$$\log_{10}(1+i) = \frac{\log_{10}n - \log_{10}B}{2m + n + 1} \left\{ 1 + \frac{1}{1-t} - \frac{1}{20} \frac{t^3(1+t)}{(1-t)^2} \right\}$$

$$\text{or} \quad = \frac{\log_{10}n - \log_{10}B}{2m + n + 1} \left\{ 1 + \frac{1}{1-t} - \frac{1}{20} \frac{t^3}{(1-t)^3} \right\} \text{approximately.}$$

## REVIEW.

### *The British Offices Life Tables, 1893.*

THIS volume contains tables deduced from the graduated experience of Whole-Life Participating Assurances on Male Lives—Aggregate Tables. Briefly, the tables before us will be known as the  $O^M$  and the  $O^{M(5)}$ , and, in many respects, they correspond to the  $H^M$  and the  $H^{M(5)}$  of the 1863 Experience. Mr. Ackland's paper last session has made us acquainted, to some extent, with the principal features of the new tables, but it may be useful, perhaps, to give our readers a short review of the present volume, and to deal with some of the points not specially discussed by Mr. Ackland.

First of all, the graduation of the tables well deserves the most careful study. Mr. G. F. Hardy, we are told in the introduction, has been responsible for this part of the work, and his account of the methods he has employed will be awaited with much interest. So far as can be judged from the volume before us, Mr. Hardy appears to have come to the conclusion that the  $O^{M(5)}$  Table would be the one

most adapted for general use, and accordingly the  $O^{M(5)}$  data were first adjusted. The method of adjustment selected is sure to meet with almost universal approval, though it may surprise those who ranged themselves either on one side or the other in the famous controversy between Mr. Woolhouse and Dr. Sprague some years ago. In considering the question of graduation, one must remember why graduation is necessary at all, and what are the objects at which it aims. Graduation appears to be necessary because in the data collected, however large the experience, there are certain to occur irregularities of considerable magnitude which tend to obscure the true meaning of the data. It is the object of graduation to discover this true meaning, and by the removal of the irregularities to derive from the data a basis for the estimates of the future. For this purpose too rigid an adherence to the actual unadjusted data is unnecessary. The conditions of life assurance are continually changing, and it may very well be that already the "New Experience" is obsolete. It is sufficient, therefore, to try to obtain from the unadjusted data a law following them only approximately, and provided that law lends itself readily to the preparation of tables, &c., to prefer it to one which does not so lend itself, though it may adhere more exactly to the original data. Makeham's first modification of Gompertz's Law of Mortality seems exactly what is required, and it is a matter for congratulation that Mr. Hardy found it possible to adjust the  $O^{M(5)}$  Table by this method. The function chosen for graduation was apparently  $\mu_{x+\frac{1}{2}}$ , rather than  $\mu_x$ . It will be remembered that  $\mu_{x+\frac{1}{2}}$  is very nearly equal to  $\text{colog}_e p_x$ , the difference between the two being approximately  $\frac{1}{12}q_x^3$ , which is very small.\* It is easy, therefore, to derive  $\mu_{x+\frac{1}{2}}$  from the unadjusted data; and using the formula  $\mu_{x+\frac{1}{2}} = A + Bc^{x+\frac{1}{2}}$  it is only necessary to find values of  $A$ ,  $B$ , and  $c$ , which will give a table following the original data with sufficient closeness for our purposes. How these values of  $A$ ,  $B$ , and  $c$ , were found will, no doubt, be explained in the final volume referred to in the introduction to the book under review. The explanation will be read with much interest. For the present we notice that  $\log_{10} c$  is  $\cdot 039$ , differing but little from the value one might have expected.

Turning to the  $O^M$  Table, we find that here a graduation by Makeham's Law has not proved possible. It has been recognized, however, that the connection between the  $O^M$  and the  $O^{M(5)}$  Tables should be observed as far as possible, and the  $O^M$  graduation is therefore based on the  $O^{M(5)}$ . Above age 80, where new entrants cease and selection has no part, the two tables are identical. For the rest of the  $O^M$  Table, the graduation formula is

$$\Delta \text{col}_{10}(p_x)O^M = \Delta \text{col}_{10}(p_x)O^{M(5)} + \phi_x$$

where  $\phi_x = \cdot 0000504e^{-\cdot 0032 \log_e 10(29-x)^2} + \cdot 0000115e^{-\cdot 0060 \log_e 10(66\cdot 5-x)^2}$ ,

that is, the  $O^M$  function is derived from the  $O^{M(5)}$  by superimposing two normal frequency curves. It is difficult to see how the

\* This follows from the approximate relation  $\mu_{x+\frac{1}{2}} = m_x = \frac{q_x}{1 - \frac{1}{2}q_x}$ .

arithmetical coefficients were obtained; possibly the forms of the curves, drawn in the first instance by some graphic method, enabled them to be read off. No doubt, also, the forms of the curves suggested the ages 29 and 66.5. We must wait, however, for Mr. Hardy's explanation of these points, and, for the present, we can only admire the skill he has displayed, and congratulate him on the success of his efforts.

Coming now to the functions tabulated, we are told in the introduction that the initial forms of  $N$  and  $S$  have been adopted throughout, so as to preserve consistency with the Select Tables now being prepared. Another point, not mentioned in the introduction, but consequent on the method of graduation, is that the tabulated values of  $\mu_x$  and  $\bar{A}_x$ , and of  $\bar{a}$  given by the formula  $a + \frac{1}{2} - \frac{1}{12}(\mu + \delta)$ , are as nearly exact as possible. The rates of interest for which all the functions have been tabulated are  $2\frac{1}{2}$ , 3, and  $3\frac{1}{2}$  per-cent. In addition, some of the values are given at 2,  $2\frac{1}{4}$ ,  $2\frac{3}{4}$ , and 4 per-cent. The  $2\frac{3}{4}$  per-cent tables do not include temporary annuities, and this is, perhaps, a little unfortunate, as without these the valuation of endowment assurances is impracticable. A valuation by the  $O^M$  or  $O^{M(5)}$  Table at  $2\frac{3}{4}$  per-cent may very well be the next stage at which the actuary of some office is aiming, and, if for his own purposes he should prepare the necessary temporary annuity-values, it is to be hoped that he may publish them for the benefit of others as well. The omission of these  $2\frac{3}{4}$  per-cent temporary annuity-values suggests that in some points the actual needs of every-day office life have been overlooked, and this is confirmed by an examination of the tables relating to Joint Lives. So far as the  $O^M$  Table is concerned there is nothing to complain of, as we are given the values of annuities on joint lives for all combinations of two ages at  $2\frac{1}{2}$ , 3, and  $3\frac{1}{2}$  per-cent. When we come to the  $O^{M(5)}$  Table, however, we find that advantage has been taken of the Makeham Graduation to calculate the annuities for lives of equal ages only, and to add the necessary tables of Uniform Seniority. This is all very well if an isolated annuity-value is required, but if we are faced with the problem of dealing with several hundred Joint-Life Policies at an annual valuation, the adoption of the  $O^{M(5)}$  Tables is for the present too laborious. It would have been better, we think, to calculate complete tables for two lives, and to rely on the Uniform Seniority Tables only when 3 or 4 lives are involved. It must be admitted, however, that everything has been done to make the use of the Uniform Seniority Tables as easy as possible, and the illustrations and examples on page 252 of the volume make the process perfectly clear.

There is little else in the volume on which we can comment without entering into a discussion of the changes likely to be effected by the New Experience, and this would be beyond the scope of our review. The interesting tables of the ratios of the annuities-due have been before us for some months, and have already been dealt with by Mr. Ackland and others. The question of the reserves, on which this borders, has been fully treated by Mr. King in his paper the other day. The other great question, that of a possible revision of rates of premium, in consequence of the lighter rates of mortality



shown by the new tables as compared with the  $H^M$ , is one to which we cannot here give the attention it deserves. It is to be hoped, however, that no general reduction of rates of premium will be made. It appears at least open to discussion whether the rates of mortality in future will be as light as those indicated by the Experience just completed; and in view of this fact, and of the ever-growing rate of expenditure, it is difficult to see where profits have to come from in future if premiums are reduced.

In concluding, attention should be called to a useful innovation in this volume. The  $O^{M(5)}$  Tables are printed throughout on tinted paper, and this should facilitate reference and add to the usefulness of the book.

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## THE FOURTH INTERNATIONAL CONGRESS OF ACTUARIES, 1903.

THE preliminary Programme of the Fourth International Congress of Actuaries, to be held in New York, 31 August 1903, and following days, has been settled by the Organizing Committee (the Council of the Actuarial Society of America), after consultation with the Permanent Committee in Brussels.

To make the necessary arrangements for the United Kingdom, excluding Scotland, the Council of the Institute of Actuaries has appointed a Committee consisting of the President, the Vice-Presidents, the Honorary Secretaries, the Treasurer, and Messrs. C. D. Higham, H. W. Manly, and E. Woods. All communications should be addressed to the Secretary of the Committee, Mr. Ernest Woods, 28, King Street, Covent Garden, London, W.C., who has been designated Secretary of the Congress for the United Kingdom, excluding Scotland. He will receive subscriptions and answer enquiries.

We print below the Regulations and Programme of the Congress, in the form of an official invitation, issued by the President and Council of the Actuarial Society of America.

At the time of going to press the Committee have received promises of papers from Messrs. J. Burn, G. King, A. G. Mackenzie, W. O. Nash, H. E. Nightingale, W. P. Phelps, F. Schooling, S. G. Warner, A. W. Watson, and T. E. Young.

## THE FOURTH INTERNATIONAL CONGRESS OF ACTUARIES.

NEW YORK,

17 October 1902.

DEAR SIR,—We have the honor to call your attention to the decision taken by the Third International Congress of Actuaries, in its closing Session on 30 June 1900, by which the Fourth Congress is

to be held in New York in September 1903. This decision having been taken on the invitation of the Actuarial Society of America, the Council of the Society has taken upon itself the duties of an Organizing Committee. The names of those composing the Council are subject to change at the meeting of the Society in May 1903. The undersigned are at present authorized by the Organizing Committee to sign this invitation in their name.

You are earnestly requested, if you can possibly make it convenient, to attend the Fourth Congress. The present invitation is sent from New York to the members of the Permanent Committee and to those adherents of the Third Congress who are noted in the list as government delegates, as members or associates of actuarial bodies, as government officials, and as officers of insurance companies. It will be sent also by the Permanent Committee of the International Congresses of Actuaries, and by its designated correspondents, to others who may be eligible for membership, according to the rules heretofore fixed for such Congresses.

The first sitting of the Congress will be held on the morning of Monday, 31 August 1903, and the sittings will be continued during the week. Official and social entertainments are contemplated during the same week, and it is intended to invite the members of the Congress, and the ladies who may accompany them, to an excursion on the Hudson River, to begin on Monday, 7 September, and to reach New York on return early on Wednesday morning, 9 September. The Military Academy at West Point will be an object of interest, and probably Saratoga also, but the chief purpose of the excursion will be to enable the members of the Congress to acquaint themselves with the Insurance Department of the State of New York, at Albany, and with its system for the valuation of reserve liabilities of life companies.

The President of the United States will designate certain high officials as Government delegates to the Congress, and has requested his diplomatic representatives to express to other Governments the pleasure with which he would learn of their having likewise appointed official delegates. It is left to the judgment of the several organized bodies of actuaries in different countries whether to name certain individuals as delegates from those bodies, or to choose that all of their members who may personally be able to be present in New York shall be regarded as their delegates. If no other action be taken, the latter choice will be understood to have been intended. The members of the Actuarial Society of America will not be designated as delegates. All members present, whether officially delegated or not, will, as at previous Congresses, take equal part in the proceedings.

The fee for membership in the Congress, whether attending or not attending, is twenty francs, sixteen marks, or sixteen shillings, or in other currencies the equivalent of twenty francs, to be paid before 1 June 1903, to the correspondent for each country designated by the Permanent Committee. The name of this correspondent for your own country will be transmitted to you somewhat later by the Permanent Committee or by the correspondent himself. Subscriptions are also payable to Mr. A. A. Welch, Treasurer, 49, Pearl Street, Hartford, Conn., who will also have an office in New York at the time of the

Congress. Each member will be entitled to a copy of the Proceedings, which will be published within a few months after the close of the Congress.

Any person interested, not eligible for membership, can become a subscribing Adherent by payment of the same fee, and will be entitled to a copy of the Proceedings.

A further circular will be sent on or about 1 June 1903, to all who have meanwhile become qualified as members by payment of the fee, giving more precise details of the place of meeting and of the programme. Arrangements for members will be made with a first-class hotel in New York, so as to cost each person, for board and lodging, about \$3 per diem.

The programme of the Congress is not fixed, and the subjects hereinafter named will not only be changed in their order, but will also probably be somewhat increased in number, should that course be deemed advisable after consultation of members of the Permanent Committee by its general officers. Apart from such additions, if any, it is desired that arrangements may be made, through the mediation of the Permanent Committee and of its designated correspondents in different countries, by which a Report shall be obtained from each country on each of these four subjects, with special reference to the history of the subject in said country:

(1). On the improvement in longevity during the Nineteenth Century.

(2). On the growth of life insurance, assessment insurance, friendly societies, accident insurance, employers' liability insurance, health insurance, pure endowment business, annuity business, old age pensions, workmen's pensions, and other operations requiring actuarial advice.

(3). On the instruction given in Universities on actuarial subjects.

(4). On the question of health insurance from the practical point of view, including tables (question proposed by Third Congress).

It is also desired that arrangements may be made by the designated correspondents for reports from such countries as may be able to report upon the three following subjects:

(5). War mortality.

(6). Mortality among non-caucasian races.

(7). On existing legislation for the protection of wives and children, as life insurance beneficiaries, against the claims of creditors.

The following are proposed as subjects for memoirs and for discussion:

(8). The probable future course of the interest rate.

(9). The question of life probabilities proposed at the Third Congress, 26 June 1900. This question, too briefly indicated in the preliminary report of the Third Congress, will be stated more fully in the programme after consultation with the General Secretary of the Permanent Committee.

(10). Studies of the New British Experience, in any of its branches.

(11). On different systems for insuring impaired lives.

(12). On the best plan for preparing and publishing Government statistics concerning dangerous occupations and the mortality therein (subject proposed by Third Congress).



(13). On the province of State supervision of life insurance companies, whether

- (1) Confined to securing publicity ; or
- (2) Securing publicity and solvency ; or
- (3) Securing publicity and solvency, and suppressing practices which violate the general principles of equity ; or
- (4) Securing publicity and solvency, and taking part in the determination of methods before their adoption by companies.

(14). On the technical principles which should be observed in State supervision (question proposed by Third Congress).

Memoirs are invited upon other topics of international interest.

There will also be laid before the Congress, a copy being supplied to each member in attendance, the volume to be published by the Actuarial Society of America containing the statistical results of the investigation now in progress concerning the mortality in ninety-eight special classes of insured lives.

Members who are willing to contribute papers, whether they are able to attend the Congress personally or not, are requested to send their names to the General Secretary in Brussels, or to the Secretary of the Organizing Committee in New York, before 1 January 1903. Memoirs not previously arranged for by the designated correspondents should be sent for examination to the General Secretary, or to the Secretary in New York, not later than 1 May 1903, and all reports and papers must reach the Secretary in New York by 1 June 1903, to afford time for printing before the meeting of the Congress. Memoirs will be accepted which are found satisfactory either by the General Secretary or by the Secretary in New York. The General Secretary is M. Amédée Bégault, 48, Rue du Fossé-aux-Loups, Bruxelles, Belgium. It is desired that reports and papers be written in either French, German, or English, or in more than one of these languages, and that each be accompanied by a brief abstract of its purport, written by its author, to be translated by the Organizing Committee into such of these three languages as are not used in the original. Manuscripts are preferred typewritten, if not inconvenient.

Free discussion of all subjects is particularly desired, and in order that undue influence shall not be exerted by reason of any predominance in numbers of the American members present, it will be understood that at this Congress no vote is to be taken concerning any question of opinion.

Renewing our expression of the earnest wish of the Organizing Committee that you accept this invitation to membership in the Fourth Congress, and in particular, that you may find it convenient to attend it personally, we remain,

DEAR SIR,

Yours respectfully,

OSCAR B. IRELAND, *President.*

JOHN TATLOCK, JR., *Secretary.*

32, Nassau Street, New York.

## THE INSTITUTE OF ACTUARIES.

## THE PATRIOTIC FUND.

IN the *Journal* for April 1901 (vol. xxxvi, page 85) and for January last (vol. xxxvi, page 400), abstracts were reproduced of the correspondence between the Royal Commissioners of the Patriotic Fund and the Council of the Institute of Actuaries, as well as of the Report of the Council on the valuation of certain Funds administered by the Royal Commissioners, the valuation having been made as on the 31st December 1900. The Council have made another valuation of the majority of these Funds, and also of the Transvaal War Fund, both as on the 31st December 1901, and the following are the Reports they presented to the Royal Commissioners, and an extract from the letter of acknowledgment from Colonel Young :—

STAPLE INN HALL,  
LONDON.

28th April 1902.

SIR,—In response to the Commissioners' desire for another valuation of certain of the Funds under their administration, we have the honour to submit our Report on those referred to us. The particulars as to the Transvaal War Fund have only lately reached our hands, and we propose to consider that Fund separately in due course. We observe that the Ashantee War Fund, which was among those we valued as on 31st December 1900, is not now included, whereas the Rodriguez Fund and the Royal Naval Relief Fund are brought to our notice for the first time.

Our calculations are based, as before, on the particulars received from you as to the numbers of recipients, their ages and allowances, and the assets applicable to the various Funds.

We have, as we did a year ago and for the same reasons, made no reduction in the liability for the chance of widows re-marrying; and we have used the same Tables of Mortality, viz. :—

For all pensions the Tables deduced from the experience of Government Annuitants (four years after purchase) and published in 1884, distinguishing males from females.

For allowances to children and endowments, Dr. Farr's Healthy English Districts Life Tables, distinguishing males from females.

For allowances to infirm persons, the English Life Table No. 3, distinguishing males and females.

We have again taken 3 per-cent as the rate of interest, for we see no reason for changing the views we previously expressed; and we have provided for future expenses of management as we did before.

We have observed that the dissolution of the Royal Commission of the Patriotic Fund has been under consideration, and, therefore,

think it right to mention that, in default of any instructions, we have made no allowance for a general system of pensions to officials who may in consequence retire.

We annex particular statements as to the various Funds, and append a Summary of Results.\*

VALUATION SUMMARY, 31st December 1901.

Name of Fund	Liabilities	Assets.	Surplus
	£	£	£
PATRIOTIC (Russian War, 1854-6) FUND . . . . .	168,731	174,372	5,641
PATRIOTIC (General) FUND . . . . .	79,411	172,529	93,118
"CAPTAIN" RELIEF FUND . . . . .	17,969	20,115	2,146
"EURYDICE" RELIEF FUND . . . . .	9,495	11,364	1,869
ZULU WAR FUND . . . . .	18,581	21,101	2,520
"ATALANTA" RELIEF FUND . . . . .	6,404	7,700	1,296
SOLDIERS' EFFECTS FUND . . . . .	133,197	162,035	28,838
BALACLAVA (Light Brigade) RELIEF FUND . . . . .	1,729	2,140	411
"VICTORIA" RELIEF FUND . . . . .	47,589	58,276	10,687
RODRIGUEZ FUND . . . . .	11,168	17,021	5,853
ROYAL NAVAL RELIEF FUND . . . . .	5,306	7,813	2,507
	£499,580	£654,466	£154,886

We would again remind the Commissioners that it is practically impossible that Funds so small as are some of those under consideration can be worked out without serious departures from the averages upon which all valuations such as the present are based; and we still think that great advantage would ensue, not only for this reason, but also in facility and economy of administration, if a consolidation of assets were to take place in protection of the varying liabilities under the different Funds.

Should it further be practicable in the case of future charitable subscriptions—seeing that the circumstances of the different beneficiaries are not, probably, very dissimilar—to lay down one general set of rules as to widowhood, re-marriage, orphans' allowances, and such matters (not, of course, including amount of grant), the advantages of such an arrangement would appear to be great.

We are, SIR,

Your obedient Servants,

C. D. HIGHAM, *President.*

ERNEST WOODS } *Honorary*

F. SCHOOLING } *Secretaries.*

*On behalf of the  
Council of the Insti-  
tute of Actuaries.*

To COLONEL J. S. YOUNG,

*Secretary,*

*Royal Commission of the Patriotic Fund.*

\* It has been thought sufficient to give, as before, only the summary of the results.—ED. *J.I.A.*



STAPLE INN HALL,

LONDON.

21st May 1902.

SIR,—It may be for the convenience of the Commissioners if we amplify in a separate communication the closing paragraphs of our Report of the 28th ultimo.

Since it is practically impossible that any annuity or similar fund can be worked out to the end without serious departures from the averages upon which all valuations such as those under consideration are based, we strongly recommend the Commissioners to arrange for the establishment of a single consolidated fund, or account, in respect of each class of benefit for which they are responsible, and the transfer thereto from the various charitable funds, or accounts, of amounts representing the value of such liabilities. In the event of any future increase of weekly payment, such as took place lately in the case of certain beneficiaries, further transfers would be necessary; in fact, on any grant of a pension the proper transfer from one account to the other would be requisite. For the actuarial reasons which concern us we have no hesitation in advising this course; but we think the Commissioners would find that it afforded facility in administration and economy in working, while the consequent diminution of the amounts standing to the credit of the different charitable accounts might tend to remove common misconceptions as to the balances held by the Commissioners.

The Commissioners will doubtless have quinquennial actuarial valuations of their liabilities, which, it may be hoped, will disclose from time to time some small surplus, due to prudent management (especially of the investments) and other causes, and such surplus would be available towards satisfying any special requirements of the beneficent schemes administered by the Commissioners. It is possible also that they may sometimes think it desirable to fully apply the monies subscribed for any particular object, when it is clear that no more claimants can arise to share therein.

To what extent such an organisation might form the nucleus of a national institution for relieving the necessities not only of sailors and soldiers, but also of the sufferers by public calamities, and under what conditions private benefactions might be permitted to supplement more public charities, it is for the Commissioners and others to determine.

We append (p. 449) balance sheet in respect of the various Funds included in the Report to which we have referred, drawn up in the manner suggested; but we would add that, should it become necessary for liabilities incurred in the future to be valued at a different rate of interest, separate accounts in respect thereof would naturally be required.

An incidental advantage of the adoption of a plan such as we have indicated would be the saving of much labour and some expense to future actuaries, but we would repeat our assurance that it has been

PATRIOTIC FUNDS CONSOLIDATED BALANCE SHEET.—31st December 1901.

LIABILITIES.		ASSETS.	
	£		Per- cent
Non-commissioned Officers and Men :—		Consols	23
Annuities (Female)—£37,843 per annum to 2,272 beneficiaries	356,419	New South Wales Inscribed Stock (1918)	93
Annuities (Male)—£234, 4s. 2d. per annum to 16 Balachava survivors	1,674	New Zealand Consolidated Stock (1929)	33
Provision for deferred and contingent allowances	25,700	Do, Inscribed Stock (1910)	100
Infirmary Orphans	2,183	Queensland Inscribed Stock (1916)	105
Children's temporary allowances	36,053	Do, Inscribed Stock (1920)	33
Dependent Relatives	12,948	South Australian Bonds (1899-29)	4
Cambridge Asylum for £85, 12s. 8d. per annum on behalf of a widow	401	Victoria Bonds (1904)	4
Officers :—		West Australian Bonds (1902)	43
Annuities (Female)—£2,339 per annum to 59 beneficiaries	20,197	Canada (Government Bonds (1910)	4
Provision for deferred and contingent allowances	946	Do, Inscribed Stock (1910)	4
Infirmary Orphans	3,122	Cap of Good Hope Bonds (drawings)	4
Children's temporary allowances	448	Ceylon Inscribed Stock (1934)	43
Dependent Relatives	4,764	Great Western Railway Debenture Stock	4
Funeral Allowances	4,718	London, Brighton, and South Coast Railway	4
Sickness Allowances	1,682	Perpetual Debenture Stock	4
Pensions to officials	1,230	North Eastern Railway Debenture Stock	43
Management expenses and commission on Post Office Orders	27,095	East Indian Ry. Co. Annuity, Class C	3
	£499,580	Do, Defd. Anny. Capt., Class D	—
Surplus :—		Great Indian Peninsula Ry. Debenture Stock	4
Patriotic (Russian War, 1854-6) Fund	5,641	Madras Railway Stock	33
Patriotic (General) Fund	93,118	Belfast Corporation Stock (1924)	3
"Captain" Relief Fund	2,146	Leeds Corporation Stock (1927)	33
"Burydice" Relief Fund	1,869	Liverpool Corporation Stock (non-redeemable)	33
Zulu War Fund	2,520	Metropolitan Consolidated Stock	33
"Malanka" Relief Fund	1,296	Portsmouth Corpn. Stock (non-redeemable)	33
Soldiers' Effects Fund	28,838	Wigan Corporation Stock (1921)	3
Balachava (Light Brigade) Relief Fund	411	Manchester Ship Canal 1st Debenture (1914)	4
"Victoria" Relief Fund	10,687	London and India Docks Co. Deb. Stock A	3
Rodriguez Fund	5,853	Do, do, do, B	3
Royal Naval Relief Fund	2,507	Mersey Docks and Harbour Board Bonds	3
		Do, do, do, C	33
		Bristol Waterworks Perpetual Debenture Stock	33
	£654,466		
			£654,466

a great satisfaction to us to have been able to render any service to the Commissioners.

We are, SIR,

Your obedient Servants,

C. D. HIGHAM, *President.*

ERNEST WOODS } *Honorary*

F. SCHOOLING } *Secretaries.*

*On behalf of the  
Council of the Insti-  
tute of Actuaries.*

To COLONEL J. S. YOUNG,

*Secretary,*

*Royal Commission of the Patriotic Fund.*

STAPLE INN HALL,

LONDON.

10th June 1902.

SIR,—Supplementing our previous communication, we now have the honour to submit to the Commissioners our valuation of the Transvaal War Fund, as on the 31st December last, upon the basis of the particulars with which you furnished us of the assets, and of the numbers, ages, and allowances of the beneficiaries.

The allowances are, we are told, in form merely temporary, but our calculations assume the continuance of pension until re-marriage or death, as also that children will still receive grants though their mothers re-marry. Funeral allowances on the death of dependent relatives we have taken on the scale for non-commissioned officers and men.

We have again used the same Tables of Mortality, viz.:—

For all pensions, the tables deduced from the experience of Government Annuitants (four years after purchase) and published in 1884:

For allowances to children and endowments, Dr. Farr's Healthy English Districts Life Tables, distinguishing males from females:

but we think it right to mention that the mortality of the recipients of these pensions is likely to be heavier than that of Government Annuitants, in which case there will be some amount of surplus under this heading.

We have found it necessary to formulate a rate of re-marriage for this Fund, and to this end have analyzed the returns you supplied to us as to the re-marriages which have taken place among the beneficiaries under the Patriotic (Russian War, 1854-6) Fund. Marriage rates, however, vary so greatly from one generation to another in accordance with the social condition and habits of the people, as well as because of regulations in respect of money grant and cessation of pension on re-marriage, that we have not thought it desirable to include all the relief to the Fund which our computations would permit. A recommencement of pension on second widowhood would materially increase the liabilities, but we have made no provision for such a benefit, as we understand that no such allowance is to be granted.



We trust that the Commissioners will permit us to publish, in our *Journal* and elsewhere, any of the results of our investigations which we may think likely to be useful to other enquirers.

We have very carefully considered as to the rate of interest to be employed, for the position of this Fund is somewhat different from that of the others which have been under our notice. It is true that it is one for which further investments would not ordinarily be required, but we observe that a large proportion of the assets is invested in the National War Loan which will be repaid in 1910, and that a considerable investment has been in Local Loans Stock which may be repaid in 1912, at which times the interest to be generally obtained may be less than the rates now current. On reviewing all the circumstances, however, we are of opinion that three per-cent should again be the rate used, especially as the Commissioners' receipts are free from income tax, but we have made certain deductions in valuing the assets so as to provide for the re-investment contingency. Should the difficulty not arise, there will be some profit in this respect.

We would again venture to call attention to the advantages of what are called "wasting" securities for a Fund such as the one under consideration—loans, for instance, repayable in the form of annuities during periods of years, the annuities, Class A, of Indian Railways, and other investments of this character.

We have made what we believe to be a sufficient provision for the expenses of future management.

We append (p. 452) a Balance Sheet of the Fund in question.\*

We are, SIR,

Your obedient Servants,

C. D. HIGHAM,	} <i>On behalf of the</i>
ERNEST WOODS,	
F. SCHOOLING,	

*Council of the Institute of Actuaries.*

To COLONEL J. S. YOUNG,

*Secretary,*

*Royal Commission of the Patriotic Fund.*

In his letter of acknowledgment to Mr. Higham, dated 1 August 1902, Colonel Young said :

I am commanded by His Royal Highness the President to express to you and the Members of the Council of the Institute of Actuaries and the Honorary Secretaries of the Institute the grateful appreciation of the President and the Commissioners for the honorary services given to this Commission, and through this Commission to the public, in making such exhaustive and laborious investigations as those involved in the preparation of the valuations furnished in your reports.

The valuations thus afforded have been of the greatest assistance to the Commissioners, and have been submitted as part of the annual report of the Commissioners to the King, which, in due course, will be published as a Parliamentary paper.

\* See next page.



# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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*On the Comparative Reserves of Life Assurance Companies, according to various Tables of Mortality, at various rates of interest. By GEORGE KING, F.I.A., F.F.A., &c., one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute, 29 December 1902.]

IT is not proposed in the present paper to take up any original investigation, the main object being to bring up to date the papers submitted to the Institute in 1876 and 1877, and which appeared in *J.I.A.*, xix, p. 381, and xx, p. 233. These papers were written not very long after the publication of the Twenty Offices' Experience, with the Monetary Tables based thereon, and to it they were almost entirely devoted. The peculiar rates of mortality prevailing amongst assured lives were investigated, by means of ten mortality tables calculated for central ages at entry, tables which were called by name "Analyzed." That name has since been superseded by the name "Select", and the tables themselves, together with the annuity tables based upon them, have become obsolete, their place having been taken by Dr. Sprague's much more complete, and much better, "Select" Tables. The Analyzed Mortality Tables, however, served their purpose, as the main object in taking the matter up was, to estimate the reserves of life offices when the mortality of lives assured is kept separate according to age at entry.

The second and principal of the two papers above mentioned dealt with a "Model Office", by the valuation of which could be estimated with extreme accuracy the change in the reserve of an actual Company on changing its valuation basis. It will be remembered that there was constructed a table of  $l_x$  for policies,



for each of ten ages at entry, taking as data the experience of the Twenty Companies as regards the three factors, proportionate numbers of entrants at each age, rates of mortality, and rates of discontinuance; and that the Model Office was valued by twenty-six different tables of mortality and rates of interest, a list which included all the principal bases in use in valuations twenty-five years ago. In the paper there were also investigated various formulas, by means of which to make use of the Model Office in connection with Companies not making net premium valuations, and examples showed how these formulas might be employed. The formulas themselves have now been incorporated in the *Institute of Actuaries' Text-Book*, Part ii, Chap. xviii, Arts. 27-32, and some of them will be used again on the present occasion.

The time has now arrived, through the publication of the "British Offices Life Tables, 1893", when the question may with advantage be taken up afresh, but, on account of the very complete way in which that Experience has been prepared, the preliminary enquiries are rendered unnecessary. The Experience has already been published in a Select form, and the mortality according to age at entry has been set forth, so that there is no need to discuss the rates prevailing amongst assured lives. Moreover, special investigations, notably that by Mr. Ackland (*J.I.A.*, xxxvii, p. 113), have given us most copious analyses of the Experience from many points of view; and, therefore, we may legitimately confine ourselves to the bearing of the new tables upon the valuations of Companies, as distinct from their bearing upon individual policy-values. This limitation of the work is a great advantage, because already the mass of arithmetical calculations has involved much greater labour than anyone could imagine who had not attempted a similar task. Here I wish to tender my hearty thanks to Mr. E. C. Thomas for the valuable assistance he has rendered in computing the tables, and in superintending the staff who have done the major portion of the mechanical work. Without his invaluable assistance this paper would never have seen the light.

Although the special object of this paper is to illustrate the effects of the new mortality tables on the valuations of Companies, yet, incidentally, other points arise which it may be interesting to take up first. The old Model Office was, as already remarked, based upon the experience of the Twenty Companies, and it very well might be asked whether we should not do better now by forming a new Model Office, based on the most recently collected data.

That has been done, and in Table 1, the new Model Office, which is styled Model Office No. 2, is given. Model Office No. 2 has been prepared from the O<sup>M</sup> Experience on exactly the same principles, so far as conditions allow, as Model Office No. 1 from the H<sup>M</sup> Experience. In constructing it, use has been made of the records of both the "Old" and the "New" Assurances; for each central age at entry, the experience of that age itself, and of the two ages on each side of it, five ages in all, having been taken into account. The probability of the persistence of a policy was computed for each year from the moment of entering, throughout subsequent life; and for the radix of each of the ten columns there was taken the total number of "New" Assurances entering at the five ages represented by the particular column. It will be noticed that, while, to obtain the probability of persistence of a policy, both groups of data were employed, yet, for the radices, only the New Assurances were taken into account; the reason being that, in the case of the Old Assurances, the policies coming under observation were only those remaining in force in 1863 out of many more which had been issued prior to that year. It therefore did not follow that the proportionate numbers for the various ages at entry brought into account would represent correctly the proportionate numbers of policies actually issued at each age, and therefore, for the radices, it was necessary to include only the New Assurances. The British Offices Experience, however, is of such proportions that even the New Assurances by themselves give nearly four times as many policies as the total number included in the Twenty Offices' Experience.

In taking out the probability of persistence of a policy a difficulty was met with. The O<sup>M</sup> Select Tables were prepared, in so far as withdrawals are concerned, by what is known as the nearest duration method, which means that those policies which were withdrawn in the first half of a policy-year got no credit for the fractional time of their existence during that year, while those policies which were withdrawn in the second half of the year were treated as having existed the whole year. Thus, for instance, taking age at entry 40, the withdrawals against year 5 may be due to policies which were discontinued in the second half of year 4, or in the first half of year 5. It was therefore impossible, from the Select Tables as published in the volume, to apportion accurately the withdrawals according to assurance years, and personally I cannot help thinking it would have been better, even at the cost

of another column, had the facts been given in more detail, so that the withdrawals against any particular year might have been assigned to their correct assurance year. At the end of the volume, however, the necessary information is given for central ages at entry 20, 25, &c., up to 60, for the first ten years of assurance, and this, for that part of the policy histories, exactly met the requirements. There was wanting, however, the same information for central age 65, and that, by the kindness of Mr. Ackland, was supplied. But after ten years, information on this rather important point was entirely absent from the published records, and the tables had to be completed from 10 years onwards by making an adjustment between each two years of duration. This, however, is not of vital importance, and even had there been available the desired analysis of the withdrawals for the whole duration of the policies, the figures of Model Office No. 2 would not have been appreciably affected.

TABLE 1.—*Model Office No. 2.—O<sup>M</sup> Experience. Number of Policies existing at the end of each Year.*

Dura- tion of Policy	20	25	30	35	40	45	50	55	60	65	Dura- tion of Policy
0	35007	98155	110930	88323	62638	40480	24327	12398	5769	1907	0
1	33647	94905	107740	85860	60875	39336	23650	12029	5540	1838	1
2	30209	86361	99742	79765	56794	36779	22194	11222	5180	1700	2
3	28001	80923	94423	75730	54073	35009	21119	10631	4860	1595	3
4	26337	77106	90452	72599	51874	33610	20215	10160	4602	1503	4
5	25127	74145	87273	70116	50097	32409	19423	9754	4404	1386	5
6	24134	71642	84392	67903	48540	31300	18666	9338	4183	1273	6
7	23224	69482	81803	65848	47116	30354	18004	8961	3995	1193	7
8	22511	67442	79526	64072	45749	29372	17382	8609	3760	1094	8
9	21877	65628	77450	62340	44513	28468	16785	8257	3551	1017	9
10	21283	63994	75600	60762	43319	27601	16137	7864	3327	930	10
11	20678	62320	73690	59125	42143	26711	15552	7485	3121	840	11
12	20188	60928	71968	57634	41013	25859	14990	7106	2913	761	12
13	19705	59531	70366	56213	39906	24998	14412	6660	2679	678	13
14	19254	58222	68738	54741	38809	24155	13801	6295	2461	592	14
15	18828	57020	67154	53361	37707	23317	13185	5922	2262	513	15
16	18431	55828	65692	52031	36581	22454	12520	5551	2053	456	16
17	18016	54653	64122	50658	35504	21592	11909	5190	1837	389	17
18	17657	53537	62611	49357	34344	20750	11258	4777	1632	330	18
19	17279	52405	61128	47967	33181	19861	10569	4396	1453	264	19
20	16859	51297	59686	46588	32007	18972	9909	4028	1282	220	20
21	16489	50234	58245	45167	30882	18086	9261	3652	1117	177	21
22	16142	49128	56807	43810	29763	17162	8569	3274	941	144	22
23	15770	48066	55457	42457	28582	16252	7889	2934	799	110	23
24	15450	46986	54033	41183	27483	15348	7242	2578	667	80	24
25	15083	45949	52579	39891	26211	14326	6674	2237	543	66	25
26	14761	44888	51127	38540	24962	13350	6012	1924	437	57	26
27	14481	43861	49622	37205	23666	12385	5409	1629	350	42	27
28	14190	42809	48148	35675	22367	11465	4840	1388	275	35	28
29	13884	41695	46646	34232	21045	10533	4231	1148	210	23	29
30	13545	40683	45119	32693	19764	9562	3705	900	153	13	30



TABLE 1—continued.

*Model Office No. 2—O<sup>M</sup> Experience. Number of Policies existing at the end of each Year.*

Dura- tion of Policy	20	25	30	35	40	45	50	55	60	65	Dura- tion of Policy
31	13219	39641	43596	31203	18477	8620	3171	725	110	6	31
32	12895	38527	42010	29654	17114	7682	2694	550	78	3	32
33	12609	37428	40413	28128	15763	6804	2249	421	51	2	33
34	12287	36304	38836	26525	14498	5984	1850	306	33	2	34
35	11970	35100	37140	24960	13193	5235	1494	232	24	...	35
36	11645	33922	35451	23368	11984	4529	1220	175	13	...	36
37	11298	32700	33776	21716	10724	3836	935	128	8	...	37
38	11004	31410	32027	20024	9561	3217	705	93	7	...	38
39	10688	30014	30168	18370	8383	2657	542	61	4	...	39
40	10359	28677	28371	16737	7256	2109	417	35	3	...	40
41	10051	27379	26491	15193	6229	1712	283	17	...	...	41
42	9691	26078	24608	13581	5309	1298	192	9	...	...	42
43	9256	24655	22749	12067	4432	1014	142	5	...	...	43
44	8768	23279	20843	10574	3649	753	92	5	...	...	44
45	8318	21906	18907	9269	2980	578	61	3	...	...	45
46	7800	20423	17107	8044	2368	427	35	...	...	...	46
47	7281	19108	15328	6701	1886	336	22	...	...	...	47
48	6834	17604	13771	5639	1481	212	8	...	...	...	48
49	6449	16297	12125	4549	1171	142	3	...	...	...	49
50	6059	15052	10599	3755	886	97	2	...	...	...	50
51	5641	13786	8992	3084	644	50	...	...	...	...	51
52	5293	12387	7776	2417	452	40	...	...	...	...	52
53	4936	11096	6471	1908	339	28	...	...	...	...	53
54	4462	9765	5410	1526	224	28	...	...	...	...	54
55	4117	8513	4562	1178	133	12	...	...	...	...	55
56	3785	7430	3719	859	87	...	...	...	...	...	56
57	3511	6306	2823	609	44	...	...	...	...	...	57
58	3031	5378	2115	418	33	...	...	...	...	...	58
59	2609	4468	1622	271	27	...	...	...	...	...	59
60	2415	3490	1152	144	16	...	...	...	...	...	60
61	2128	2756	839	93	11	...	...	...	...	...	61
62	1803	2193	594	50	...	...	...	...	...	...	62
63	1472	1821	400	22	...	...	...	...	...	...	63
64	1253	1376	246	22	...	...	...	...	...	...	64
65	967	1087	141	22	...	...	...	...	...	...	65
66	892	694	106	22	...	...	...	...	...	...	66
67	852	591	63	22	...	...	...	...	...	...	67
68	468	282	63	...	...	...	...	...	...	...	68
69	341	201	21	...	...	...	...	...	...	...	69

In order to see how far Model Office No. 2 differs from Model Office No. 1, there is given in Table 2 a statement showing the distribution of business in the two Model Offices, both as regards policies issued, and as regards policies in force after 50 years.

TABLE 2.—*Showing the Age Distribution of Business in the two Model Offices.*

MODEL OFFICE NO. 1. H <sup>M</sup>			MODEL OFFICE NO. 2. O <sup>M</sup>		
Policies Issued					
Central Age at Entry	Number	Per-cent	Number	Per-cent	Central Age at Entry
20	8882	6·97	35007	7·30	20
25	22631	17·75	98155	20·45	25
30	26821	21·04	110930	23·11	30
35	23462	18·41	88323	18·40	35
40	17610	13·82	62638	13·05	40
45	12044	9·45	40480	8·44	45
50	7945	6·23	24327	5·07	50
55	4473	3·51	12398	2·58	55
60	2516	1·97	5769	1·20	60
65	1087	0·85	1907	0·40	65
Total	127471	100·00	479934	100·00	Total

Policies in Force after 50 Years					
Ages attained	Number	Per-cent	Number	Per-cent	Ages attained
21-25	34035	1·35	143321	1·38	21-25
26-30	118812	4·72	526469	5·07	26-30
31-35	209341	8·32	916471	8·83	31-35
36-40	276054	10·97	1169104	11·26	36-40
41-45	311723	12·39	1293208	12·45	41-45
46-50	320828	12·75	1311917	12·63	46-50
51-55	309368	12·30	1253942	12·08	51-55
56-60	279736	11·12	1132591	10·91	56-60
61-65	239591	9·53	969252	9·34	61-65
66-70	190777	7·59	767432	7·39	66-70
71-75	130698	5·20	521157	5·02	71-75
76-80	66218	2·63	262360	2·53	76-80
81-85	22695	0·90	91808	0·88	81-85
86 & over	5748	0·23	24301	0·23	86 & over
Total	2515624	100·00	10383333	100·00	Total

From the percentage columns in the above Table it will be seen that in Model Office No. 2 the lives entered on the average at a somewhat younger age, and that, in the ultimate state of the Office, the average age of the lives assured is also somewhat less; but the differences are small, and prove that if we still retain Model Office No. 1 for the purpose of estimating the reserves of Companies, we shall not go far astray. That course is rendered desirable by the fact that thereby we can still use Table V of the former paper (*J.I.A.*, xx, pp. 268-272) for those valuation bases which are therein given and not repeated here; whereas, had Model Office No. 2 been now adopted, only the bases now dealt with would have been available.

To measure the extent of the divergence in a valuation between Model Offices Nos. 1 and 2, Table 3 is appended; and in making the comparison the data that would produce the largest possible deviation have been selected. In actual practice it is likely that the deviation would be much smaller. It will be observed that, except perhaps for very young offices, the percentage of deviation is trifling, and throughout this paper Model Office No. 1 has been adhered to. The question is an open one, which Model Office will more nearly represent an actual Company.

TABLE 3—*Illustrating the Deviation that may arise by the use of the Model Office.*

Age of Office	RESERVE OF MODEL OFFICE NO. 2			Deviation in Estimate	
	By OM Table at 2½ % Interest	By OM(5) Table at 4 % Interest		Amount	Per-cent
		By Actual Valuation	Estimated by Model Office No. 1		
5 years	97,816	74,755	75,879	1,124	1·504
10 „	323,831	252,981	256,253	3,272	1·293
15 „	647,828	516,032	521,692	5,660	1·097
20 „	1,039,554	842,400	850,226	7,826	0·929
25 „	1,465,621	1,205,606	1,215,076	9,470	0·785
30 „	1,891,993	1,576,537	1,586,842	10,305	0·654
35 „	2,285,072	1,924,849	1,935,251	10,402	0·540
40 „	2,617,652	2,224,529	2,234,247	9,718	0·437
45 „	2,870,170	2,455,571	2,464,396	8,825	0·359
50 „	3,038,771	2,611,990	2,620,032	8,042	0·308

Tables 4 to 7 illustrate in more detail the qualities of the two Model Offices. Table 4 gives, for each central age at entry, the number of policies in force in Model Office No. 1



after it has been in existence for any period from 5 up to 50 years, and it will be seen later on that this information is required in transforming the valuations of the Office. See, for instance, the examples 2 to 4 below. Table 5 is not of such direct service, but, nevertheless, it may sometimes be found of use. A Company that has been established half a century or more is not necessarily best represented by a Model Office fifty years old. If it has been transacting a progressively increasing new business, it is younger in constitution than its calendar years show, and it will be better represented by a younger Model Office. By seeing from the classification registers the age distribution of the policies on the books, and comparing it with the columns of this Table, an idea may be formed of the age of the Model Office that should be selected to represent it. Tables 6 and 7 give corresponding information regarding Model Office No. 2.

TABLE 4.—Summary of Model Office No. 1.—H<sup>M</sup> Experience.—Existing Policies grouped according to Central Ages at Entry.

Central Age at Entry	AGE OF OFFICE										Central Age at Entry
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years	
20	34035	58215	78086	95204	110295	123747	135937	146964	156689	164584	20
25	94632	168583	232315	288584	338481	382787	420825	452668	478419	497503	25
30	115519	208923	289741	360867	422991	476119	519823	554039	579230	594800	30
35	101800	184780	256489	318501	371730	415932	450394	474940	489323	495736	35
40	76565	138632	191137	235271	271635	300020	320565	332849	338281	339949	40
45	52577	94447	128910	156621	177951	192710	201173	204838	205960	206179	45
50	34361	60968	82223	98523	109982	116642	119619	120441	120587	120589	50
55	19110	33417	44077	51560	55991	58045	58751	58904	58906	58906	55
60	10480	17763	22500	25260	26597	27030	27105	27106	27106	27106	60
65	4495	7389	9056	9873	10204	10271	10272	10272	10272	10272	65
Total	543574	973117	1334534	1640264	1895857	2103303	2264464	2383021	2464773	2515624	Total

TABLE 5.—Summary of Model Office No. 1.—H<sup>M</sup> Experience.—Existing Policies grouped according to Ages Attained.

Ages attained	AGE OF OFFICE										Ages attained
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years	
21-25	34035	34035	34035	34035	34035	34035	34035	34035	34035	34035	21-25
26-30	94632	118812	118812	118812	118812	118812	118812	118812	118812	118812	26-30
31-35	115519	189470	209341	209341	209341	209341	209341	209341	209341	209341	31-35
36-40	101800	195204	258936	276054	276054	276054	276054	276054	276054	276054	36-40
41-45	76565	159545	240363	296632	311723	311723	311723	311723	311723	311723	41-45
46-50	52577	114644	186353	257479	307376	320828	320828	320828	320828	320828	46-50
51-55	34361	76231	128736	190748	252872	297178	309368	309368	309368	309368	51-55
56-60	19110	45717	80180	124314	177543	230671	268709	279736	279736	279736	56-60
61-65	10480	24787	46042	73753	110117	154319	198023	229866	239591	239591	61-65
66-70	4495	11778	22438	38738	60068	88453	122915	157131	182882	190777	66-70
71-75	...	2894	7631	15114	26573	41332	61877	86423	111614	130698	71-75
76-80	...	...	1667	4427	8858	15518	23981	36265	50648	66218	76-80
81-85	...	...	...	817	2154	4208	7185	10850	16282	22695	81-85
86-90	...	...	...	...	331	764	1470	2292	3414	5082	86-90
91 & over	...	...	...	...	...	67	143	297	445	666	91 & over
Total	543574	973117	1334534	1640264	1895857	2103303	2264464	2383021	2464773	2515624	Total

TABLE 6.—Summary of Model Office No. 2.—O<sup>M</sup> Experience.—Existing Policies grouped according to Central Ages at Entry.

Central Age at Entry	AGE OF OFFICE										Central Age at Entry
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years	
20	143321	256350	355003	443245	522179	593040	656020	711014	757098	791521	20
25	413440	751628	1049649	1317369	1557732	1771668	1958668	2115391	2238688	2327172	25
30	479630	878401	1230317	1543556	1820677	2061339	2263334	2423127	2536725	2605655	30
35	384070	704995	986069	1232670	1445178	1623523	1763993	1864208	1924892	1953580	35
40	273713	502950	702558	874175	1017096	1128900	1207945	1255853	1278452	1286244	40
45	177143	324238	449278	552907	634081	691376	725701	742049	747404	748618	45
50	106601	193575	265515	321680	361315	385512	396970	400789	401559	401629	50
55	53796	96825	130293	154235	168910	175899	178133	178625	178664	178664	55
60	24586	43402	56838	65095	69162	70587	70883	70918	70918	70918	60
65	8022	13529	16913	18572	19149	19319	19332	19332	19332	19332	65
Total	2064322	3765893	5242433	6523504	7615479	8521163	9240979	9781306	10153732	10383333	Total

TABLE 7.—Summary of Model Office No. 2.—O<sup>M</sup> Experience.—Existing Policies grouped according to Ages Attained.

Ages attained	AGE OF OFFICE										Ages attained
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years	45 years	50 years	
21-25	143321	143321	143321	143321	143321	143321	143321	143321	143321	143321	21-25
26-30	413440	526469	526469	526469	526469	526469	526469	526469	526469	526469	26-30
31-35	479630	817818	916471	916471	916471	916471	916471	916471	916471	916471	31-35
36-40	384070	782841	1080862	1169104	1169104	1169104	1169104	1169104	1169104	1169104	36-40
41-45	273713	594638	946554	1214274	1293208	1293208	1293208	1293208	1293208	1293208	41-45
46-50	177143	406380	687454	1000693	1241056	1311917	1311917	1311917	1311917	1311917	46-50
51-55	106601	253696	453304	699905	977026	1190962	1253942	1253942	1253942	1253942	51-55
56-60	53796	140770	265810	437427	649935	890597	1077597	1132591	1132591	1132591	56-60
61-65	24586	67615	139555	243184	386105	564450	766445	923168	969252	969252	61-65
66-70	8022	26838	60306	116471	197645	309449	449919	609712	733009	767432	66-70
71-75	...	5507	18943	42885	82520	139815	218860	319075	432673	521157	71-75
76-80	...	...	3384	11641	26316	50513	84838	132746	193430	262360	76-80
81-85	...	...	...	1659	5726	12715	24173	40521	63120	91808	81-85
86-90	...	...	...	...	577	2002	4236	8055	13410	21202	86-90
91 & over	...	...	...	...	...	170	479	1006	1815	3099	91 & over
Total	2064322	3765893	5242433	6523504	7615479	8521163	9240979	9781306	10153732	10383333	Total



The new British Offices Experience enables us for the first time to deduce the rates of discontinuance for each of the first ten years of the duration of ordinary whole-life participating policies, and these rates are given in Table 8. They may be compared with those computed from the Twenty Offices Experience, and given in Table G of my former paper (*J.I.A.*, xix, p. 392). It would appear as if the rate of discontinuance had diminished, but it must be remembered that the H<sup>M</sup> Experience included policies of all kinds, many of which would necessarily be of shorter duration than whole-life policies, and, moreover, the H<sup>M</sup> Experience did not run accurately by assurance years. The comparison, therefore, is scarcely relevant.

TABLE 8.—*Rates of Discontinuance per-cent according to the O<sup>M</sup> Experience.*

Year	CENTRAL AGE AT ENTRY										Year
	20	25	30	35	40	45	50	55	60	65	
1	3·64	3·05	2·55	2·42	2·40	2·25	2·06	1·98	2·31	1·89	1
2	9·77	8·59	7·02	6·56	6·09	5·70	5·03	5·17	4·35	4·37	2
3	6·77	5·81	4·79	4·44	4·03	3·88	3·59	3·46	3·79	3·17	3
4	5·35	4·21	3·63	3·40	3·19	2·92	2·77	2·64	2·44	1·89	4
5	3·94	3·27	2·87	2·66	2·55	2·44	2·31	1·95	1·41	2·18	5
6	3·34	2·78	2·52	2·34	2·11	2·09	1·85	1·77	1·52	1·93	6
7	3·00	2·30	2·23	2·08	1·87	1·70	1·48	1·41	0·85	0·90	7
8	2·36	2·17	1·98	1·72	1·66	1·49	1·45	1·11	1·01	0·80	8
9	2·18	1·91	1·72	1·67	1·41	1·33	1·33	1·02	0·83	0·87	9
10	1·85	1·68	1·49	1·52	1·31	1·21	1·09	1·11	0·88	0·93	10

As throwing further light on the question of the persistence of life policies, there are given, in Table 9, the Curtate Expectations of Existence of policies according to the O<sup>M</sup> Experience, and these are compared with the same function derived from the H<sup>M</sup> Experience, and with the Expectations of Life of persons, also from the O<sup>M</sup> and the H<sup>M</sup> Select Experience. Policies remain, on the average, in force for much longer periods than many people suppose.

TABLE 9.—*A Comparison of the Curtate Expectations of Life of Persons, ( $e_x$ ), and the Curtate Expectations of Existence of Policies, ( $e'_x$ ), at the Date of Assuring.*

Central Age at Entry	H <sup>M</sup> SELECT		O <sup>M</sup> SELECT		Central Age at Entry
	Persons $e_x$	Policies $e'_x$	Persons $e_x$	Policies $e'_x$	
20	40.109	19.850	42.80	24.038	20
25	37.238	22.830	39.09	24.663	25
30	33.929	22.540	35.11	23.914	30
35	30.415	21.230	31.25	22.262	35
40	26.980	19.323	27.74	20.565	40
45	22.952	17.119	24.04	18.497	45
50	20.302	15.178	20.44	16.509	50
55	16.935	13.169	17.61	14.411	55
60	14.225	10.773	14.30	12.293	60
65	12.093	9.450	12.07	10.137	65

The main object of this paper is to display the Comparative Reserves of Life Assurance Companies by the old and the new Experience at various rates of interest, and that object is attained in Tables 10 to 19.

Table 12 is a continuation of Table V of my former paper (*J.I.A.* xx, pp. 268–272), and may be read along with it. At the present day the only two sets of mortality tables in ordinary use by British Companies for their valuations are the Institute of Actuaries' Twenty Offices Experience and the new British Offices Experience, and the great question with the Companies during the next few years will be, what results will be produced in passing in their valuations from the former to the latter? Therefore, to avoid inconvenient reference to an old volume, there have been reproduced in the present Table 12 the valuations by the H<sup>M</sup> Table, and by the Combined H<sup>M</sup> and H<sup>M(5)</sup> Tables, but it has not been thought necessary to repeat the figures for the other bases of valuation given in the former Table V. When the former paper was written, no Company thought of valuing at a lower rate of interest than 3 per-cent, but now numerous Offices adopt 2½ per-cent or 2¾ per-cent, and the practice may still further extend. Therefore these rates are now included. Formerly there were given three sets of Comparative Reserves, taking as standards the Combined H<sup>M</sup> and H<sup>M(5)</sup> valuations at 3 per-cent, 3½ per-cent, and 4 per-cent, respectively. Now the only standard is the O<sup>M</sup> Table at 3 per-cent, as probably in the near future

that will be the valuation with which others will be compared. If, however, any other standard be required in a particular case, it can easily be derived, simply by dividing the actual reserve of the Model Office on the one basis by the actual reserve on the other. The only other change in the Table that need be mentioned is, that the comparative reserves are now given per 10,000, instead of per 1,000 as formerly, the object being to secure greater accuracy in using the Table.

Tables 13 to 18 are merely extensions of Table 12, and have been added in order to assist in transforming the valuations of the Model Office. They show for each central age at entry what Table 12 shows for the Model Office as a whole. Such tables were omitted from the former paper, with the consequence that in certain cases formulas of approximation had to be resorted to. It is hoped that the few extra pages of tabular matter will not be considered to be superfluous.

Tables 10 and 11 relate to Reversionary Bonuses, and their construction and use will be explained presently.

In my former paper there were demonstrated certain very powerful formulas, useful in transforming the valuations of the Model Office, and these, as above mentioned, have since been incorporated in the *Text-Book*, Part II, chap. xviii, Arts. 26 to 32. It is therefore unnecessary to repeat the proofs. For convenience, however, in what follows, it will be useful simply to state the principal formulas. If, in a valuation by any mortality table, a valuation premium other than the net premium be brought into account, let  $P_x$  be the usual net premium, and  $V_x$  the net policy-value, and let  $P'_x$  be the altered valuation premium and  $V'_x$  the corresponding altered policy-value. Also let  $(P_x - P'_x) = \phi_x$ , and  $\phi_x(1 + a_x) = \psi_x$ . Then

$$V'_x = V_x(1 - \psi_x) + \psi_x.$$

Also, if there be  $r_x$  policies, all effected at age  $x$ , but of various durations, then, if  $\Sigma V_x$  be the sum of their net-premium values, and  $\Sigma V'_x$  the sum of their altered values corresponding to the altered premium,  $P'_x$ , then

$$\Sigma V'_x = \Sigma V_x \times (1 - \psi_x) + r_x \psi_x *$$

The use of Tables 12 to 18 is more or less obvious, but a few examples may be given to illustrate it.

\* The sign of  $\psi_x$  here is opposite to that given in the demonstration in the *Text-Book*, because, for present purposes, that is a more convenient arrangement.



*Example 1.*—An old-established Company values by the  $H^M$  Table at 3 per-cent interest, and its reserve is £2,385,272. What will be the cost of passing to the  $O^M$  Table, also at 3 per-cent?

To answer the question, we have only to divide the reserve of the actual Company by the Comparative Reserve, 9,906, given in Table 12 in the column for 50 years against the line  $H^M$  3 per-cent. The result is £2,407,908, and the increase in the estimate of liability is £22,636.

For many practical purposes this very simple method of using the Table will prove sufficient, but in other cases it will be found advisable to apply certain corrections, and these are discussed later on.

*Example 2.*—An Office has valued by the  $O^{M(5)}$  Table alone, at 3 per-cent interest. By how much does its reserve fall short of that by the  $O^M$  and  $O^{M(5)}$  Tables, combined throughout, also at 3 per-cent interest?

This question may be answered briefly as in Example 1, but it will be instructive to work it out in full by the second formula above given, applying the formula to each central age at entry separately. In the formula we must take  $P_x$  and  $a_x$  by the  $O^{M(5)}$  Table, and  $P'_x$  by the  $O^M$ . In the following scheme the calculations are given in detail. The numbers in the column  $\Sigma V_x$  are taken from Table 16, and those in the column  $r_x$  from Table 4.

Central Age at Entry $x$	$P_x$ $OM^{(5)}$	$P'_x$ $OM$	$P_x - P'_x$ $= \phi_x$	$1 + a_x$ $OM^{(5)}$	$\phi_x(1 + a_x)$ $= \psi_x$
20	·01410	·01306	·00104	23·132	·024057
25	·01603	·01524	·00079	22·145	·017495
30	·01846	·01790	·00056	21·013	·011767
35	·02156	·02116	·00040	19·728	·007891
40	·02555	·02524	·00031	18·291	·005670
45	·03072	·03046	·00026	16·709	·004344
50	·03573	·03730	·00023	15·002	·003450
55	·04661	·04641	·00020	13·204	·002641
60	·05887	·05872	·00015	11·364	·001705
65	·07567	·07557	·00010	9·542	·000954

Central Age at Entry $x$	$\Sigma V_x$ = Reserve $OM^{(5)}$	$r_x$ = Number of Policies	$\Sigma V_x(1 - \psi_x)$	$r_x \psi_x$	$\Sigma V'_x$ = Reserve $OM$ & $OM^{(5)}$
20	36658	164584	35776	3959	39735
25	124593	497503	122413	8704	131117
30	159804	594800	157924	6999	164923
35	139503	495736	138402	3912	142314
40	97866	339949	97311	1928	99239
45	58670	206179	58415	896	59311
50	34470	120589	34351	416	34767
55	16676	58906	16632	156	16788
60	7272	27106	7260	46	7306
65	2710	10272	2707	3	2710
				Total	698210

The reserve of the Model Office comes out at 698,210 by the Combined Tables, against 678,222 by the  $OM^{(5)}$  Table alone, the increase being 19,988, and the increase in the case of the actual Company will be in the same proportion.

This example has been given to show how certain of the valuations in Table 12 have been arrived at. It was not necessary, for the Combined  $OM$  and  $OM^{(5)}$  Tables, to make detailed valuations of the Model Office. There is a small difference of 4 between the  $OM$  and  $OM^{(5)}$  reserve just found, and that appearing in Table 12, because in preparing the Table the formula was applied to each quinquennium of the Model Office separately, and the sum of the ten reserves so found was taken for that of an Office fifty years old, whereas now we have found the reserve for the whole of the fifty years by one process.

*Example 3.*—A Company values by the  $H^M$   $2\frac{1}{2}$  per-cent Table, but brings into account only the  $H^M$   $3\frac{1}{2}$  per-cent net

premiums. Its estimated liability is £1,572,018. How much is this in excess of the  $H^M 2\frac{1}{2}$  per-cent net premium reserve? In answering this question, our first step must be to find the reserve of the Model Office under the conditions of the real Company, and to do this we must, in the formula, take  $P_x$  and  $a_x$  at  $H^M 2\frac{1}{2}$  per-cent, and  $P'_x$  at  $H^M 3\frac{1}{2}$  per-cent. The following scheme gives the calculations:

Central Age at Entry $x$	$P_x$ $H^M 2\frac{1}{2} \%$	$P'_x$ $H^M 3\frac{1}{2} \%$	$P_x - P'_x$ $= \phi_x$	$1 + a_x$ $H^M 2\frac{1}{2} \%$	$\phi_x(1 + a_x)$ $= \psi_x$
20	·01538	·01330	·00208	25·145	·052302
25	·01742	·01521	·00221	23·916	·052854
30	·02003	·01769	·00234	22·515	·052685
35	·02322	·02076	·00246	21·006	·051675
40	·02723	·02465	·00258	19·371	·049977
45	·03253	·02985	·00268	17·570	·047088
50	·03943	·03667	·00276	15·669	·043246
55	·04879	·04588	·00282	13·683	·038586
60	·06134	·05848	·00286	11·665	·033362
65	·07853	·07564	·00289	9·716	·028079
Central Age at Entry $x$	$\Sigma V_x$ = Reserve $H^M 2\frac{1}{2} \%$	$r_x$ = Number of Policies	$\Sigma V_x(1 - \psi_x)$	$r_x \psi_x$	$\Sigma V'_x$ = Reserve, new conditions
20	39409	164584	37384	8608	45956
25	132706	497503	125692	26295	151987
30	168246	594800	159382	31337	190719
35	146063	495736	138515	25617	164132
40	102297	339949	97185	16990	114175
45	61162	206179	58282	9709	67991
50	35891	120589	34329	5215	39554
55	17330	58906	16661	2273	18934
60	7533	27106	7282	904	8186
65	2807	10272	2728	288	3016
	742,444			Total	804650

The reserve of the Model Office under the conditions of the real Company is thus 804,650, or 91,206 above the  $H^M 2\frac{1}{2}$  per-cent reserve. The reserve of the real Company is therefore £178,187 above the  $H^M 2\frac{1}{2}$  per-cent reserve.

*Example 4.*—A Company values by the  $H^M 3$  per-cent Table, and the liability under its non-participating business is £344,188. What are the values of the sums assured, and of the net premiums, separately?



To answer the question, we must obtain the information in respect of the Model Office, with liabilities of 682,535, and the calculation is easy. In calculating the value of the sums assured,  $P'_x$  in the formula vanishes, and  $\phi_x$  becomes  $P_x$ , and  $\psi_x$  becomes  $A_x$  by the  $H^M$  Table. The following are the detailed figures :

Central Age at Entry $x$	$\Sigma V_x$ = Reserve $H^M$ 3 %	$r_x$ = Number of Policies	$\psi_x$ = $A_x$	$\Sigma V_x(1 - A_x)$	$r_x \times A_x$	$\Sigma V'_x$ = Reserve for Sums Assured
20	37014	164584	32886	24842	54125	78967
25	125663	497503	35812	80661	178166	258827
30	160356	594800	39221	97463	233287	330750
35	139977	495736	42950	79857	212919	292776
40	98535	339949	47060	52164	159989	212144
45	59130	206179	51669	28578	106531	135109
50	34846	120589	56613	15119	68269	83388
55	16893	58906	61863	6442	36441	42883
60	7367	27106	67274	2411	18235	20646
65	2754	10272	72569	755	7454	8209
Total						1463699

It thus appears that the value of the sums assured in the Model Office is 1,463,699, and, as the net liability is 682,535, it follows that the value of the future premiums is 781,164. The reserve of the actual Company is £344,188, and by simple proportion, the value of the sums assured is £738,112, and of the future net premiums £393,924. The corresponding figures given in the Board of Trade Returns are £734,277 and £390,089 respectively, so that there is a deviation of only £3,835 in the estimate by the Model Office, or 0.522 per-cent. This question puts the Model Office to the most severe test possible, and, unless the actual Company had transacted a nearly uniform business for many years, the calculations would have broken down. A fairly accurate result could, however, be obtained in the majority of cases by examining into the history as regards new business of the actual Company, and selecting accordingly the age of the Model Office to be operated on, or by applying Table 19.

Table 19 shows the liability of the Model Office in each quinquennium of insurance, and has been introduced to provide a correction, if desired, for irregular flow of new business. The Model Office represents a Company which transacts exactly the

same amount of new business every year, and if an actual Company has experienced wide variations in that respect, the estimates by the Model Office will be thrown out. There are, however, so many other factors involved, that irregularity in the flow of new business disturbs the calculations to a surprisingly small extent, and as good results as those provided by Table 19 may usually be obtained from Table 12 alone by selecting a Model Office of suitable age. In *J.I.A.*, xx, p. 254, an example of the use of Table 19 was given, there called Table W, and we need not repeat one here.

When we are dealing with only one Mortality Table, but at different rates of interest—for instance, if we wish to pass from a valuation by the  $O^M$  Table at  $3\frac{1}{2}$  per-cent to one by the same Table at 3 per-cent—it is not of much consequence in what proportion the new entrants come in, because, no matter what may be the age at entry, policy-values are affected in the same direction, and to nearly the same extent. Also, with certain Mortality Tables, the same conditions hold, and we may pass from one Mortality Table to another without much risk of error. But under other circumstances this is not so. For instance, the  $H^M$  and the  $O^M$  Tables differ materially, and, analyzing the reserves of an Office of fifty years' standing, we find as follows :

Ages of Entrants	RESERVE		Ratios
	$H^M$ 3 %	$O^M$ 3 %	
20 to 40	561545	568702	·9874
45 to 65	120990	120297	1·0058
Total	682535	688999	·9906

It will be observed that, while the  $O^M$  Table gives increased reserves for policies at the younger ages at entry, it gives diminished reserves for those at the older ages, and the influence is reversed. In discussing these Tables, therefore, it is necessary to bear in mind the average age at entry. With the Model Office that age is 35·3 years, but if a Company for any reason secures its policyholders at an average age materially higher, the Model Office would exaggerate the difference between the two Tables, and it may even so misrepresent the real difference as to exhibit it in the wrong direction. By the help of Tables 13 to 18,

however, a correction may be applied to overcome this difficulty. Taking out from these Tables the reserve for each age at entry separately, the different figures may be multiplied by suitable factors so as to give increased weight to any particular age at entry, and a new Model Office may thus be very easily formed, which will represent with greater accuracy the actual Company in question. Thus, if a Company secures its policyholders at a high average age, the reserves of the Model Office against the higher ages at entry may be increased, and, adding together the figures thus corrected for all ages at entry, we shall have the total reserve of a Model Office with a higher average at entry. Each case would have to be dealt with on its merits; but, with care, it would not be difficult so to plan that the Model Office should represent each individual case.

The Model Office is the type of a Company which either declares no bonuses, or pays away all its surplus in cash. If, therefore, heavy reversionary bonuses attach to the policies of a Company, care must be exercised in making estimates by means of the Model Office. In reply to question 2 of the sixth schedule of the Act of 1870, Companies must furnish a return, arranged according to age, of the sums assured for the whole of life, and must give the reversionary bonuses separately. It is, therefore, very easy to make a valuation of the bonuses by each of the two bases which it is intended to compare, and these valuations need be only approximate, for instance, being made by grouping the bonuses into quinquennial age periods. The Model Office may then be applied to the sums assured alone, the difference in reserves in respect of these being ascertained, and the actual difference in reserves in respect of bonuses being added thereto. If, however, it is merely a question of passing from one rate of interest to another by the same mortality table, there is no great need to trouble about the refinement, but the Model Office may be applied to the whole liabilities of the Company, including bonuses. For instance, the reserve of the Model Office at  $H^M 2\frac{1}{2}$  per-cent is greater than at  $H^M 4$  per-cent by 14·1 per-cent for the sums assured, and by 18·3 per-cent for the bonuses, and the corresponding ratios for the  $O^M$  Table are 14·1 per-cent and 18·6 per-cent respectively. Moreover, the reserve for bonuses in a Company is generally only a fraction of the total liability, seldom more than a sixth. The differences in these percentages may therefore be neglected, especially when it is remembered that in practice we shall have to deal probably with a change of



not more than .5 per-cent in the rate of interest, and not with 1.5 per-cent as above.

The case is, however, different when we are concerned with two Mortality Tables of such opposite characteristics as the  $H^M$  and the  $O^M$ , because, while the latter gives the larger reserve for the sums assured, it gives the smaller reserve for the bonuses. By far the best course in such circumstances will be to deal with the bonuses apart, as above described, even at the cost of a little trouble, but, as that may not always be convenient, I have given some thought to the construction of a Model Office for bonuses, and the results are embodied in Tables 10 and 11. These almost speak for themselves. Five representative Companies were selected, and the bonuses taken out in quinary age groups, as shown in the second column of Table 10. The bonuses amounted to 13.911 per-cent of the corresponding sums assured. Then, to make the information available for use with the Model Office, the amount of bonuses in each age period was reduced in the ratio of 2,515,624 to 6,962,220. Therefore, in the third column of Table 10, we have the bonus in each age period, which gives a total bonus equal to the total sums assured in the Model Office of fifty years' standing, and from this we may pass to any percentage of bonus we may require.

The bonuses in column 3 of Table 10 were then valued on the various bases displayed in Table 12, and the results entered in Table 11. This is the Table to be used in practice.

A single example will be sufficient to illustrate the application of this Table.

*Example 5.*—A Company, with bonuses amounting to 8.12 per-cent of its sums assured, values by the  $H^M$  Table at 3 per-cent interest, and brings out a liability of £3,150,000. What will be its liability by the  $O^M$  Table at 3 per-cent?

Looking first at the Model Office, we have

	H <sup>M</sup> 3 PER-CENT		O <sup>M</sup> 3 PER-CENT	
	Sums Assured	Bonuses	Sums Assured	Bonuses
	682,535	1,703,183	688,999	1,692,107
8.12 per-cent of Bonuses	138,298	...	137,399	...
Total Reserve	<u>820,833</u>	...	<u>826,398</u>	...

In passing to the  $O^M$  Table the Model Office must increase its reserves by 5,565, or .678 per-cent, and the actual Company must increase its reserves by £21,357 to £3,171,357.

Permanent reductions of premium are not generally of such amount as to be of importance from this point of view, but if in any case they are of any great magnitude, they must, like reversionary bonuses, be allowed for. When a change in the valuation rate of interest is alone in question, reductions of premium, even of large amount, would scarcely affect the estimates, but, in passing from the  $H^M$  to the  $O^M$  Table, it would be better to give them attention, because the  $O^M$  Table assigns to them a considerably larger value. Unfortunately, the Act of 1870 does not ask for any separate statement of bonus reductions of premium, and therefore the Board of Trade Returns seldom yield information on the point.

In applying the Model Office, all the minor classes of business of a Company may be included, and the conclusions will not be thereby vitiated; but there is one class, Endowment Assurances, no longer minor, which must be considered independently. I had hoped to take this matter up on the present occasion, but the pressure of work which the paper has already involved has prevented me, and this branch of the subject must be reserved for a future communication. It may, however, be well to say now that, for the present, a good practical rule for using the Model Office will be to apply it in two sections, first to the general business of the Company, and next to the Endowment Assurances; and then to take only half the difference in the Endowment Assurance section which the operation indicates. This method will give results sufficiently accurate for usual purposes. Changes in valuation bases affect Endowment Assurances much less than ordinary whole-life policies—in fact, to but little more than half the extent. If we take from Mr. Carment's book, *Values of Endowment Assurance Policies*, the sum of the policy-values for ages at entry 20, 30, and 40, and maturing at 50 and 65, we shall find that at 3 per-cent the sum is 6521·328, and at 4 per-cent 6158·044. The latter must therefore be increased by 5·90 per-cent to equal the former. But the Model Office reserves by  $H^M$  4 per-cent for whole-life policies without bonus must be increased by 9·15 per-cent to equal the  $H^M$  3 per-cent reserve, or nearly double the difference involved in the Endowment Assurances.

It would have been interesting to obtain valuations of the Model Office by the  $O^M$  Select Tables, but the volume of monetary values is not yet available. I hope, later on, to be able to supply the, at present, unavoidable omission.

TABLE 10.—REVERSIONARY BONUSES.

*The actual amount of Reversionary Bonuses, grouped according to Ages attained, existing at the Dates of the last Valuations of five representative Companies, and the proportionate Amounts, assuming the total Bonuses to be equal to the total Sums Assured in the Model Office of 50 Years' standing.*

Ages attained	Actual Amount of Bonuses	Proportionate Amount of Bonuses	Ages attained
18-22	745	269	18-22
23-27	5360	1937	23-27
28-32	29529	10670	28-32
33-37	115776	41833	33-37
38-42	295321	106707	38-42
43-47	602256	217610	43-47
48-52	812828	293695	48-52
53-57	996548	360078	53-57
58-62	998508	360786	58-62
63-67	986736	356532	63-67
68-72	873361	315567	68-72
73-77	630994	227994	73-77
78-82	390303	141026	78-82
83-87	180036	65051	83-87
88-92	38731	13994	88-92
93 & over	5188	1875	93 & over
Total	6962220	2515624	Total



TABLE 11.—*Showing the Reserves of the Model Office for Reversionary Bonuses, by various Mortality Tables, at various Rates of Interest, the total amount of Reversionary Bonuses being assumed to be equal to the total amount of the Sums Assured.*

Mortality Table and Rate of Interest	Reserve for Reversionary Bonuses	Mortality Table and Rate of Interest	Reserve for Reversionary Bonuses
H <sup>M</sup> 2½ per-cent	1805429	O <sup>M</sup> 2½ per-cent	1795376
„ 2¾ „	1753037	„ 2¾ „	1742436
„ 3 „	1703183	„ 3 „	1692107
„ 3½ „	1610476	„ 3½ „	1598562
„ 4 „	1526168	„ 4 „	1513610
H <sup>M(5)</sup> 2½ per-cent	1812254	O <sup>M(5)</sup> 2½ per-cent	1797381
„ 2¾ „	1760238	„ 2¾ „	1744556
„ 3 „	1710720	„ 3 „	1694327
„ 3½ „	1618584	„ 3½ „	1600947
„ 4 „	1534735	„ 4 „	1516118

N.B.—In case of valuations by the Combined H<sup>M</sup> and H<sup>M(5)</sup> Tables, or the Combined O<sup>M</sup> and O<sup>M(5)</sup> Tables, the H<sup>M(5)</sup> or the O<sup>M(5)</sup> Table, respectively, would be used alone to value the Bonuses.

TABLE 12.—Showing the Reserves of the Model Office by various Mortality Tables, at various Rates of Interest, and a Comparison of each with the Reserve by the O<sup>M</sup> Table at 3 per-cent Interest.

AGE OF OFFICE =	Tables and Rate of Interest	5 YEARS		10 YEARS		15 YEARS		20 YEARS		25 YEARS	
		Actual Reserve	Com- parative Reserve O <sup>M</sup> 3% =10,000	Actual Reserve	Com- parative Reserve O <sup>M</sup> 3% =10,000	Actual Reserve	Com- parative Reserve O <sup>M</sup> 3% =10,000	Actual Reserve	Com- parative Reserve O <sup>M</sup> 3% =10,000	Actual Reserve	Com- parative Reserve O <sup>M</sup> 3% =10,000
H <sup>M</sup>	. . . . .	25,978	10,425	83,702	10,427	164,056	10,423	258,765	10,416	359,725	10,406
	. . . . .	25,051	10,053	80,907	10,078	158,933	10,098	251,197	10,111	349,860	10,121
	. . . . .	24,166	9,698	78,223	9,744	153,988	9,783	243,870	9,816	340,282	9,813
	. . . . .	22,503	9,031	73,158	9,113	144,630	9,189	229,960	9,256	322,035	9,316
	. . . . .	20,968	8,415	68,483	8,531	135,947	8,637	216,983	8,734	304,936	8,821
Combined H <sup>M</sup> and H <sup>M(5)</sup> (H <sup>M</sup> alone 1st 5 years)	. . . . .	25,978	10,425	88,574	11,033	172,223	10,942	269,196	10,836	371,671	10,751
	. . . . .	25,051	10,053	85,701	10,676	166,984	10,609	261,493	10,526	361,663	10,462
	. . . . .	24,166	9,698	82,951	10,333	161,949	10,289	254,062	10,226	351,989	10,182
	. . . . .	22,503	9,031	77,752	9,685	152,388	9,682	239,925	9,657	333,507	9,647
	. . . . .	20,968	8,415	72,933	9,085	143,492	9,117	226,728	9,126	316,185	9,146
O <sup>M</sup>	. . . . .	26,754	10,737	85,810	10,689	167,545	10,645	263,419	10,603	365,246	10,566
	. . . . .	25,811	10,358	82,974	10,336	162,358	10,315	255,778	10,296	355,289	10,278
	. . . . .	24,918	10,000	80,278	10,000	157,397	10,000	248,435	10,000	345,693	10,000
	. . . . .	23,211	9,315	75,121	9,358	147,902	9,397	234,344	9,433	327,247	9,466
	. . . . .	21,661	8,693	70,388	8,768	139,124	8,839	221,250	8,906	310,006	8,968
O <sup>M(5)</sup>	. . . . .	25,777	10,345	83,138	10,356	163,072	10,361	257,293	10,357	357,703	10,347
	. . . . .	24,831	9,965	80,320	10,005	157,905	10,032	249,669	10,050	347,764	10,060
	. . . . .	23,954	9,613	77,648	9,672	152,977	9,719	242,350	9,755	338,188	9,783
	. . . . .	22,279	8,941	72,573	9,010	143,611	9,124	228,425	9,195	319,924	9,255
	. . . . .	20,754	8,329	67,903	8,458	134,923	8,572	215,444	8,672	302,808	8,759
Combined O <sup>M</sup> and O <sup>M(5)</sup> throughout	. . . . .	31,229	12,533	92,541	11,528	175,543	11,153	272,154	10,955	374,395	10,830
	. . . . .	30,195	12,118	89,582	11,159	170,204	10,814	264,341	10,640	364,260	10,537
	. . . . .	29,248	11,738	86,801	10,813	165,147	10,492	256,886	10,340	354,545	10,256
	. . . . .	27,427	11,007	81,495	10,152	155,501	9,880	242,656	9,767	335,970	9,719
	. . . . .	25,755	10,336	76,590	9,541	146,523	9,309	229,353	9,232	318,518	9,214
Combined O <sup>M</sup> and O <sup>M(5)</sup> (O <sup>M</sup> alone 1st 5 years)	. . . . .	26,754	10,737	88,066	10,970	171,068	10,869	267,679	10,775	369,920	10,701
	. . . . .	25,811	10,358	85,198	10,613	165,820	10,535	259,957	10,464	359,876	10,410
	. . . . .	24,918	10,000	82,471	10,273	160,817	10,217	252,556	10,166	350,215	10,131
	. . . . .	23,211	9,315	77,279	9,626	151,285	9,612	238,440	9,598	331,754	9,597
	. . . . .	21,661	8,693	72,496	9,031	142,429	9,049	225,259	9,067	314,424	9,095

AGE OF OFFICE = Tables and Rate of Interest	30 YEARS		35 YEARS		40 YEARS		45 YEARS		50 YEARS	
	Actual Reserve	Com- parative Reserve On 3% =10,000	Actual Reserve	Com- parative Reserve On 3% =10,000	Actual Reserve	Com- parative Reserve On 3% =10,000	Actual Reserve	Com- parative Reserve On 3% =10,000	Actual Reserve	Com- parative Reserve On 3% =10,000
$H^M$ . . . . .	458,364 446,526 435,007 412,995 392,288	$2\frac{1}{2}\%$ $2\frac{3}{4}\%$ 3% $3\frac{1}{2}\%$ 4%	547,144 533,748 520,708 495,722 472,134	10,383 10,129 9,881 9,407 8,959	620,553 606,057 591,921 564,778 539,097	10,372 10,130 9,893 9,440 9,010	676,133 660,917 646,061 617,508 590,447	10,362 10,129 9,901 9,464 9,049	713,444 697,820 682,535 653,185 625,308	10,355 10,128 9,906 9,480 9,076
Combined $H^M$ and $H^{M(5)}$ } ( $H^M$ alone 1st 5 years) }	471,256 459,271 447,659 425,421 404,499	$2\frac{1}{2}\%$ $2\frac{3}{4}\%$ 3% $3\frac{1}{2}\%$ 4%	560,580 547,047 533,920 508,711 484,919	10,638 10,381 10,132 9,654 9,202	634,290 619,652 605,433 578,090 552,202	10,601 10,357 10,119 9,662 9,229	690,023 674,664 659,743 630,986 603,721	10,575 10,340 10,111 9,670 9,252	727,409 711,641 696,313 666,746 638,680	10,557 10,329 10,106 9,677 9,270
$O^M$ . . . . .	464,449 452,504 440,973 418,739 397,876	$2\frac{1}{2}\%$ $2\frac{3}{4}\%$ 3% $3\frac{1}{2}\%$ 4%	553,525 540,033 526,969 501,743 477,989	10,504 10,248 10,000 9,521 9,071	627,074 612,472 598,306 570,918 545,055	10,481 10,237 10,000 9,542 9,110	682,703 667,377 652,496 623,681 596,425	10,463 10,228 10,000 9,558 9,141	720,035 704,289 688,999 659,362 631,297	10,450 10,222 10,000 9,570 9,163
$O^{M(5)}$ . . . . .	455,770 443,836 432,315 410,271 389,540	$2\frac{1}{2}\%$ $2\frac{3}{4}\%$ 3% $3\frac{1}{2}\%$ 4%	543,995 530,495 517,442 492,403 468,786	10,323 10,067 9,819 9,344 8,896	616,937 602,318 588,160 560,954 535,227	10,311 10,067 9,830 9,376 8,946	672,175 656,826 641,947 613,305 586,185	10,302 10,066 9,838 9,399 8,983	709,279 693,515 678,222 648,750 620,815	10,294 10,066 9,844 9,416 9,010
Combined $O^M$ and $O^{M(5)}$ throughout	473,822 461,693 450,034 427,682 406,612	$2\frac{1}{2}\%$ $2\frac{3}{4}\%$ 3% $3\frac{1}{2}\%$ 4%	563,008 549,317 536,131 510,791 486,840	10,684 10,424 10,174 9,693 9,238	636,595 621,791 607,504 580,005 553,949	10,610 10,393 10,154 9,694 9,259	692,239 676,710 661,704 632,777 605,336	10,609 10,371 10,141 9,698 9,277	729,572 713,631 698,214 668,462 640,210	10,589 10,358 10,134 9,702 9,292
Combined $O^M$ and $O^{M(5)}$ } ( $O^M$ alone 1st 5 years) }	469,347 457,309 445,704 423,466 402,518	$2\frac{1}{2}\%$ $2\frac{3}{4}\%$ 3% $3\frac{1}{2}\%$ 4%	558,533 544,933 531,801 506,575 482,746	10,599 10,341 10,092 9,613 9,161	632,120 617,407 603,174 575,789 549,855	10,565 10,319 10,081 9,624 9,190	687,764 672,326 657,374 628,561 601,242	10,541 10,304 10,075 9,633 9,214	725,097 709,247 693,884 664,246 636,116	10,524 10,294 10,071 9,641 9,232



Central Age at Entry	AGE OF OFFICE				
	5 years	10 years	15 years	20 years	25 years
	H <sup>M</sup> Mortality Table—Interest $2\frac{1}{2}$ per-cent				
20	916	2872	5641	9109	13202
25	3143	10172	20470	33630	49206
30	4414	14611	29658	48805	70900
35	4545	15120	30655	49840	71398
40	4066	13387	26563	42216	58833
45	3274	10545	20482	31597	42439
50	2498	7860	14842	22156	28587
55	1624	4917	8843	12568	15246
60	1006	2884	4829	6309	7161
65	492	1334	2073	2535	2753
Total	25978	83702	164056	258765	359725
	H <sup>M</sup> Mortality Table—Interest $2\frac{3}{4}$ per-cent				
20	865	2719	5357	8677	12616
25	2990	9697	19571	32244	47318
30	4217	14000	28503	47046	68539
35	4365	14568	29627	48307	69383
40	3929	12973	25811	41123	57440
45	3181	10272	19996	30913	41599
50	2439	7691	14553	21763	28123
55	1591	4828	8701	12384	15038
60	989	2842	4764	6231	7078
65	485	1317	2050	2509	2726
Total	25051	80907	158933	251197	349860
	H <sup>M</sup> Mortality Table—Interest 3 per-cent				
20	816	2574	5089	8267	12058
25	2845	9250	18712	30917	45502
30	4028	13414	27393	45353	66259
35	4194	14037	28630	46810	67417
40	3800	12574	25083	40061	56082
45	3089	10005	19522	30245	40774
50	2380	7524	14266	21372	27657
55	1558	4740	8559	12203	14834
60	977	2803	4705	6158	6999
65	479	1302	2029	2484	2700
Total	24166	78223	153988	243870	340282
	H <sup>M</sup> Mortality Table—Interest $3\frac{1}{2}$ per-cent				
20	726	2308	4591	7503	11010
25	2576	8413	17111	28432	42090
30	3682	12327	25316	42159	61942
35	3877	13045	26758	43990	63677
40	3549	11814	23688	38023	53466
45	2916	9488	18606	28951	39176
50	2269	7199	13711	20617	26763
55	1499	4573	8285	11846	14430
60	943	2720	4579	6007	6835
65	466	1271	1985	2432	2646
Total	22503	73158	144630	229960	322035
	H <sup>M</sup> Mortality Table—Interest 4 per-cent				
20	647	2071	4145	6814	10058
25	2335	7660	15659	26162	38947
30	3366	11331	23405	39201	57920
35	3583	12126	25015	41347	60162
40	3319	11107	22383	36099	50986
45	2756	9008	17746	27729	37660
50	2159	6890	13178	19887	25894
55	1439	4410	8018	11501	14038
60	911	2638	4455	5857	6674
65	453	1242	1943	2386	2597
Total	20968	68483	135947	216983	304936

AGE OF OFFICE					Central Age at Entry
30 years	35 years	40 years	45 years	50 years	
H <sup>M</sup> Mortality Table—Interest 2½ per-cent					
17854	23015	28565	34224	39409	20
66640	84799	102642	119099	132706	25
94534	117817	138911	156421	168246	30
93464	113755	130271	140945	146063	35
74562	87824	96672	100915	102297	40
51422	57278	60060	60970	61162	45
32946	35117	35764	35889	35891	50
16665	17200	17328	17330	17330	55
7471	7532	7533	7533	7533	60
2806	2807	2807	2807	2807	65
458364	547144	620553	676133	713444	Total
H <sup>M</sup> Mortality Table—Interest 2¾ per-cent					
17114	22128	27545	33092	38194	20
64269	82001	99496	115693	129131	25
91632	114473	135241	152539	164251	30
91041	111024	127352	137926	143008	35
72937	86041	94808	99021	100397	40
50479	56278	59039	59943	60134	45
32444	34599	35243	35368	35370	50
16446	16978	17106	17108	17108	55
7385	7446	7447	7447	7447	60
2779	2780	2780	2780	2780	65
446526	533748	606057	660917	697820	Total
H <sup>M</sup> Mortality Table—Interest 3 per-cent					
16405	21273	26559	31995	37014	20
61979	79296	96453	112393	125663	25
88819	111224	131671	148757	160356	30
88673	108355	124480	134951	139977	35
71349	84301	92986	97168	98535	40
49551	55295	58039	58939	59130	45
31940	34081	34721	34844	34846	50
16233	16763	16891	16893	16893	55
7305	7366	7367	7367	7367	60
2753	2754	2754	2754	2754	65
435007	520708	591921	646061	682535	Total
H <sup>M</sup> Mortality Table—Interest 3½ per-cent					
15072	19668	24702	29921	34778	20
57656	74164	90653	106088	119026	25
83473	105029	124840	141506	152880	30
84143	103227	118965	129240	134208	35
68283	80931	89454	93575	94926	40
47752	53392	56093	56982	57170	45
30970	33079	33712	33835	33836	50
15810	16334	16460	16462	16462	55
7139	7200	7201	7201	7201	60
2697	2698	2698	2698	2698	65
412995	495722	564778	617508	653185	Total
H <sup>M</sup> Mortality Table—Interest 4 per-cent					
13854	18186	22977	27987	32667	20
53657	69390	85235	100180	112789	25
78466	99196	118391	134645	145798	30
79870	98374	113729	123807	128701	35
65364	77715	86079	90139	91474	40
46036	51573	54235	55114	55302	45
30021	32100	32725	32847	32849	50
15397	15915	16040	16042	16042	55
6974	7035	7036	7036	7036	60
2649	2650	2650	2650	2650	65
92288	472134	539097	590447	625308	Total

Central Age at Entry	AGE OF OFFICE				
	5 years	10 years	15 years	20 years	25 years
	Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest $2\frac{1}{2}$ per-cent				
20	916	3326	6363	10018	14249
25	3143	11224	22257	35946	51939
30	4414	15759	31610	51390	73911
35	4545	16008	32225	51867	73703
40	4066	14030	27627	43529	60298
45	3274	10917	21067	32310	43214
50	2498	8046	15138	22504	28964
55	1624	5003	8971	12716	15403
60	1006	2916	4876	6363	7218
65	492	1345	2089	2553	2772
Total	25978	88574	172223	269196	371671
	Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest $2\frac{3}{4}$ per-cent				
20	865	3167	6068	9574	13647
25	2990	10731	21329	34526	50017
30	4217	15128	30426	49600	71519
35	4365	15441	31175	50305	71658
40	3929	13608	26864	42422	58890
45	3181	10639	20576	31621	42370
50	2439	7874	14845	22107	28494
55	1591	4911	8824	12528	15190
60	989	2873	4811	6284	7134
65	485	1329	2066	2526	2744
Total	25051	85701	166984	261493	361663
	Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest 3 per-cent				
20	816	3015	5788	9149	13073
25	2845	10263	20443	33160	48161
30	4028	14523	29292	47881	69215
35	4194	14902	30169	48798	69688
40	3800	13204	26127	41354	57528
45	3089	10367	20095	30946	41537
50	2380	7705	14556	21716	28030
55	1558	4823	8681	12343	14982
60	977	2834	4751	6212	7055
65	479	1315	2047	2503	2720
Total	24166	82951	161949	254062	351989
	Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest $3\frac{1}{2}$ per-cent				
20	726	2733	5267	8358	11997
25	2576	9393	18787	30616	44684
30	3682	13402	27159	44618	64827
35	3877	13880	28255	45936	65903
40	3549	12428	24712	39290	54887
45	2916	9845	19173	29645	39931
50	2269	7381	14002	20961	27135
55	1499	4656	8409	11991	14585
60	943	2751	4623	6057	6889
65	466	1283	2001	2453	2669
Total	22503	77752	152388	239925	333507
	Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest 4 per-cent				
20	647	2483	4802	7645	11017
25	2335	8605	17280	28279	41476
30	3366	12371	25194	41603	60742
35	3583	12933	26474	43252	62355
40	3319	11704	23382	37343	52386
45	2756	9356	18298	28406	38399
50	2159	7066	13462	20224	26258
55	1439	4494	8140	11659	14205
60	911	2668	4501	5913	6731
65	453	1253	1959	2404	2616
Total	20968	72933	143492	226728	316185



AGE OF OFFICE					Central Age at Entry
30 years	35 years	40 years	45 years	50 years	
Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest $2\frac{1}{2}$ per-cent					
19008	24243	29841	35531	40732	20
69660	87992	105942	122453	136090	25
97803	121244	142413	159965	171809	30
95941	116314	132877	143569	148693	35
76103	89406	98271	102519	103905	40
52229	58094	60880	61793	61985	45
33333	35508	36158	36283	36285	50
16824	17362	17490	17492	17492	55
7530	7591	7592	7592	7592	60
2825	2826	2826	2826	2826	65
471256	560580	634290	690023	727409	Total
Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest $2\frac{3}{4}$ per-cent					
18252	23340	28804	34382	39499	20
67253	85161	102765	119016	132484	25
94868	117866	138710	156050	167782	30
93485	113552	129918	140510	145598	35
74463	87609	96394	100611	101989	40
51282	57095	59861	60768	60960	45
32825	34984	35631	35756	35758	50
16602	17137	17265	17267	17267	55
7444	7505	7506	7506	7506	60
2797	2798	2798	2798	2798	65
459271	547047	619652	674664	711641	Total
Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest 3 per-cent					
17527	22474	27809	33280	38317	20
64924	82414	99679	115677	128978	25
92033	114600	135122	152253	163873	30
91114	110881	127051	137543	142592	35
72871	85862	94565	98754	100124	40
50346	56105	58850	59752	59943	45
32323	34467	35111	35236	35238	50
16384	16917	17045	17047	17047	55
7364	7426	7427	7427	7427	60
2773	2774	2774	2774	2774	65
447659	533920	605433	659743	696313	Total
Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest $3\frac{1}{2}$ per-cent					
16167	20837	25920	31171	36046	20
60541	77229	93835	109328	122299	25
86616	108331	123224	144937	156331	30
86546	105716	121500	131795	136769	35
69778	82468	91011	95138	96492	40
48539	54194	56900	57792	57982	45
31351	33466	34101	34224	34226	50
15968	16493	16621	16623	16623	55
7193	7254	7255	7255	7255	60
2722	2723	2723	2723	2723	65
425421	508711	578090	630986	666746	Total
Combined HM and HM <sup>(5)</sup> Mortality Tables—Interest 4 per-cent					
14921	19330	24171	29216	33930	20
56471	72381	88342	103345	115990	25
81551	102451	121727	138029	149203	30
82240	100830	116234	126332	131231	35
66841	79235	87616	91681	93019	40
46808	52360	55027	55909	56096	45
30399	32482	33109	33231	33233	50
15567	16087	16212	16214	16214	55
7033	7094	7095	7095	7095	60
2668	2669	2669	2669	2669	65
404499	484919	552202	603721	638680	Total

Central Age at Entry	AGE OF OFFICE				
	5 years	10 years	15 years	20 years	25 years
O <sup>M</sup> Mortality Table—Interest $2\frac{1}{2}$ per-cent					
20	1031	3171	6151	9831	14118
25	3337	10777	21585	35242	51226
30	4672	15343	30903	50471	72904
35	4722	15581	31350	50733	72393
40	4114	13486	26719	42370	58952
45	3291	10591	20509	31583	42376
50	2497	7828	14762	22026	28381
55	1605	4867	8760	12429	15085
60	1001	2864	4773	6239	7094
65	484	1302	2033	2495	2717
Total	26754	85810	167545	263419	365246
O <sup>M</sup> Mortality Table—Interest $2\frac{3}{4}$ per-cent					
20	977	3012	5857	9388	13520
25	3183	10298	20673	33845	49323
30	4469	14720	29731	48692	70522
35	4541	15028	30321	49195	70371
40	3979	13072	25964	41270	57548
45	3194	10308	20012	30887	41523
50	2436	7653	14465	21624	27904
55	1573	4779	8619	12246	14878
60	983	2819	4706	6159	7007
65	476	1285	2010	2472	2693
Total	25811	82974	162358	255778	355289
O <sup>M</sup> Mortality Table—Interest 3 per-cent					
20	926	2862	5581	8969	12952
25	3033	9837	19801	32499	47488
30	4282	14136	28622	46997	68244
35	4371	14499	29324	47704	68405
40	3845	12668	25230	40202	56184
45	3103	10041	19541	30221	40699
50	2379	7489	14182	21238	27446
55	1542	4695	8480	12068	14676
60	966	2779	4646	6087	6929
65	471	1272	1990	2450	2670
Total	24918	80278	157397	248435	345693
O <sup>M</sup> Mortality Table—Interest $3\frac{1}{2}$ per-cent					
20	828	2581	5061	8175	11870
25	2753	8975	18158	29961	44014
30	3924	13022	26508	43759	63876
35	4046	13490	27433	44859	64646
40	3596	11909	23836	38160	53563
45	2930	9526	18623	28924	39099
50	2264	7163	13623	20479	26543
55	1479	4522	8200	11703	14263
60	934	2695	4518	5931	6762
65	457	1238	1942	2393	2611
Total	23211	75121	147902	234344	327247
O <sup>M</sup> Mortality Table—Interest 4 per-cent					
20	746	2333	4599	7467	10897
25	2505	8202	16673	27647	40826
30	3601	12009	24567	40770	59818
35	3748	12560	25676	42200	61109
40	3364	11197	22522	36231	51074
45	2770	9044	17760	27696	37573
50	2159	6859	13096	19754	25677
55	1421	4361	7935	11358	13872
60	903	2615	4396	5783	6602
65	444	1208	1900	2344	2558
Total	21661	70388	139124	221250	310006

AGE OF OFFICE					Central Age at Entry
30 years	35 years	40 years	45 years	50 years	
OM Mortality Table—Interest $2\frac{1}{2}$ per-cent					
18928	24217	29860	35581	40802	20
68991	87365	105334	121853	135445	25
96745	120133	141267	158726	170537	30
94490	114761	131192	141836	146962	35
74630	87790	96601	100846	102238	40
51277	57099	59882	60798	60986	45
32709	34881	35534	35657	35659	50
16506	17046	17170	17172	17172	55
7407	7466	7467	7467	7467	60
2766	2767	2767	2767	2767	65
464449	553525	627074	682703	720035	Total
OM Mortality Table—Interest $2\frac{3}{4}$ per-cent					
18174	23316	28826	34434	39571	20
66603	84552	102175	118435	131852	25
93818	116765	137572	154816	166512	30
92059	112029	128262	138804	143893	35
72992	85995	94723	98938	100321	40
50318	56084	58848	59760	59948	45
32193	34349	34998	35120	35122	50
16287	16824	16948	16950	16950	55
7318	7376	7377	7377	7377	60
2742	2743	2743	2743	2743	65
452504	540033	612472	667377	704289	Total
OM Mortality Table—Interest 3 per-cent					
17461	22460	27839	33337	38391	20
64295	81824	99105	115111	128363	25
91012	113520	134003	151033	162614	30
89689	109356	125390	135831	140883	35
71397	84243	92889	97075	98451	40
49390	55103	57847	58752	58938	45
31696	33837	34482	34604	34606	50
16075	16609	16733	16735	16735	55
7239	7297	7298	7298	7298	60
2719	2720	2720	2720	2720	65
440973	526969	598306	652496	688999	Total
OM Mortality Table—Interest $3\frac{1}{2}$ per-cent					
16088	20810	25935	31215	36106	20
59909	76627	93240	108736	121650	25
85608	107262	127110	143715	155069	30
85138	104207	119849	130090	135065	35
68324	80862	89342	93463	94823	40
47586	53193	55897	56794	56979	45
30715	32824	33462	33583	33585	50
15643	16171	16295	16297	16297	55
7069	7127	7128	7128	7128	60
2659	2660	2660	2660	2660	65
418739	501743	570918	623681	659362	Total
OM Mortality Table—Interest 4 per-cent					
14846	19306	24190	29260	33993	20
55858	71801	87767	102771	115354	25
80566	101395	120622	136808	147934	30
80842	99326	114582	124622	129521	35
65392	77629	85946	90007	91352	40
45858	51358	54021	54907	55091	45
29772	31850	32481	32602	32604	50
15232	15756	15877	15879	15879	55
6904	6961	6962	6962	6962	60
2606	2607	2607	2607	2607	65
397876	477989	545055	596425	631297	Total



Central Age at Entry	AGE OF OFFICE				
	5 years	10 years	15 years	20 years	25 years
OM <sup>(5)</sup> Mortality Table—Interest $2\frac{1}{2}$ per-cent					
20	900	2803	5518	8946	13014
25	3023	9899	20095	33190	48687
30	4396	14607	29692	48831	70899
35	4582	15210	30744	49914	71381
40	4057	13329	26456	42002	58493
45	3263	10514	20376	31394	42146
50	2483	7784	14685	21923	28262
55	1596	4842	8721	12381	15033
60	995	2851	4755	6220	7074
65	482	1299	2030	2492	2714
Total	25777	83138	163072	257293	357703
OM <sup>(5)</sup> Mortality Table—Interest $2\frac{3}{4}$ per-cent					
20	846	2647	5229	8508	12419
25	2865	9420	19186	31791	46783
30	4196	13988	28524	47057	68524
35	4399	14658	29715	48376	69362
40	3920	12917	25703	40909	57097
45	3168	10236	19885	30706	41300
50	2422	7612	14392	21524	27787
55	1562	4751	8574	12191	14817
60	977	2808	4690	6141	6988
65	476	1283	2007	2466	2687
Total	24831	80320	157905	249669	347764
OM <sup>(5)</sup> Mortality Table—Interest 3 per-cent					
20	797	2504	4961	8096	11861
25	2719	8970	18324	30458	44959
30	4013	13411	27427	45373	66254
35	4232	14132	28727	46889	67396
40	3790	12518	24975	39846	55738
45	3078	9969	19410	30036	40474
50	2363	7444	14105	21132	27323
55	1532	4668	8439	12016	14620
60	961	2765	4625	6063	6903
65	469	1267	1984	2441	2660
Total	23954	77648	152977	242350	338188
OM <sup>(5)</sup> Mortality Table—Interest $3\frac{1}{2}$ per-cent					
20	708	2238	4464	7334	10812
25	2452	8135	16725	27974	41545
30	3659	12312	25334	42161	61915
35	3912	13134	26850	44063	63660
40	3540	11760	23583	37808	53122
45	2906	9457	18501	28748	38882
50	2249	7120	13550	20377	26424
55	1468	4496	8160	11653	14209
60	929	2682	4501	5913	6743
65	456	1239	1943	2394	2612
Total	22279	72573	143611	228425	319924
OM <sup>(5)</sup> Mortality Table—Interest 4 per-cent					
20	630	2003	4019	6647	9862
25	2210	7379	15268	25695	38397
30	3344	11317	23423	39206	57897
35	3617	12214	25105	41421	60147
40	3310	11052	22277	35885	50639
45	2743	8972	17634	27518	37348
50	2144	6817	13023	19653	25558
55	1413	4338	7897	11311	13819
60	899	2604	4379	5766	6585
65	444	1207	1898	2342	2556
Total	20754	67903	134923	215444	302808

AGE OF OFFICE					Central Age at Entry
30 years	35 years	40 years	45 years	50 years	
OM <sup>(5)</sup> Mortality Table—Interest $2\frac{1}{2}$ per-cent					
17637	22772	28289	33912	39065	20
66049	84106	101839	118194	131685	25
94445	117610	138590	155956	167720	30
93327	113496	129865	140479	145596	35
74105	87225	96014	100254	101643	40
51022	56833	59611	60527	60715	45
32584	34753	35404	35526	35528	50
16451	16990	17114	17116	17116	55
7387	7446	7447	7447	7447	60
2763	2764	2764	2764	2764	65
455770	543995	616937	672175	709279	Total
OM <sup>(5)</sup> Mortality Table—Interest $2\frac{3}{4}$ per-cent					
16888	21876	27260	32771	37842	20
63657	81284	98667	114763	128083	25
91523	114243	134895	152042	163692	30
90895	110756	126922	137434	142512	35
72475	85435	94141	98349	99729	40
50071	55826	58584	59494	59682	45
32068	34220	34869	34991	34993	50
16224	16761	16885	16887	16887	55
7299	7357	7358	7358	7358	60
2736	2737	2737	2737	2737	65
443836	530495	602318	656826	693515	Total
OM <sup>(5)</sup> Mortality Table—Interest 3 per-cent					
16178	21024	26275	31672	36658	20
61360	78567	95605	111446	124593	25
88723	111006	131333	148265	159804	30
88525	108082	124052	134462	139503	35
70885	83689	92313	96491	97866	40
49140	54841	57580	58485	58670	45
31565	33702	34346	34468	34470	50
16017	16550	16674	16676	16676	55
7213	7271	7272	7272	7272	60
2709	2710	2710	2710	2710	65
432315	517442	588160	641947	678222	Total
OM <sup>(5)</sup> Mortality Table—Interest $3\frac{1}{2}$ per-cent					
14844	19410	24407	29587	34409	20
57033	73428	89797	105122	117932	25
83350	104779	124469	140975	152279	30
84000	102959	118535	128743	133708	35
67815	80311	88770	92885	94242	40
47344	52938	55636	56530	56715	45
30587	32693	33330	33451	33453	50
15588	16116	16240	16242	16242	55
7050	7108	7109	7109	7109	60
2660	2661	2661	2661	2661	65
410271	492403	560954	613305	648750	Total
OM <sup>(5)</sup> Mortality Table—Interest 4 per-cent					
13627	17932	22686	27656	32317	20
53026	68646	84367	99196	111672	25
78348	98951	118018	134105	145182	30
79727	98106	113297	123306	128194	35
64891	77084	85380	89434	90776	40
45608	51097	53755	54641	54825	45
29645	31721	32352	32473	32475	50
15177	15700	15822	15824	15824	55
6887	6944	6945	6945	6945	60
2604	2605	2605	2605	2605	65
389540	468786	535227	586185	620815	Total

Central Age at Entry	AGE OF OFFICE				
	5 years	10 years	15 years	20 years	25 years
	Combined OM and OM <sup>(5)</sup> (throughout) Mortality Tables—Interest $2\frac{1}{2}$ per-cent				
20	1728	4188	7332	11102	15446
25	4675	12761	23922	37796	53913
30	5757	16987	32877	52653	75212
35	5364	16574	32560	52075	73797
40	4481	14062	27419	43132	59739
45	3490	10900	20875	31970	42771
50	2594	7969	14920	22189	28546
55	1642	4917	8814	12484	15141
60	1012	2878	4787	6254	7109
65	486	1305	2037	2499	2721
Total	31229	92541	175543	272154	374395
	Combined OM and OM <sup>(5)</sup> (throughout) Mortality Tables—Interest $2\frac{3}{4}$ per-cent				
20	1664	4016	7024	10644	14830
25	4495	12247	22971	36351	51963
30	5532	16327	31658	50822	72777
35	5175	16013	31521	50527	71770
40	4331	13629	26640	42010	58312
45	3380	10598	20354	31248	41888
50	2535	7801	14632	21796	28077
55	1609	4828	8669	12296	14927
60	994	2834	4721	6174	7022
65	480	1289	2014	2473	2694
Total	30195	89582	170204	264341	364260
	Combined OM and OM <sup>(5)</sup> (throughout) Mortality Tables—Interest 3 per-cent				
20	1596	3844	6720	10193	14230
25	4327	11762	22067	34973	50093
30	5325	15711	30513	49085	70451
35	5002	15479	30525	49033	69798
40	4203	13233	25917	40954	56962
45	3294	10337	19887	30587	41072
50	2473	7628	14339	21398	27607
55	1578	4744	8533	12120	14729
60	977	2790	4655	6095	6936
65	473	1273	1991	2448	2667
Total	29248	86801	165147	256886	354545
	Combined OM and OM <sup>(5)</sup> (throughout) Mortality Tables—Interest $3\frac{1}{2}$ per-cent				
20	1489	3550	6189	9393	13143
25	4020	10864	20392	32407	46596
30	4927	14541	28331	45773	66007
35	4670	14463	28628	46188	66046
40	3929	12436	24476	38861	54287
45	3118	9820	18973	29295	39477
50	2354	7296	13775	20633	26698
55	1515	4573	8256	11760	14321
60	945	2707	4531	5945	6776
65	460	1245	1950	2401	2619
Total	27427	81495	155501	242656	335970
	Combined OM and OM <sup>(5)</sup> (throughout) Mortality Tables—Interest 4 per-cent				
20	1387	3277	5698	8654	12138
25	3723	10018	18821	29999	43310
30	4592	13516	26387	42786	61960
35	4348	13499	26828	43484	62467
40	3690	11714	23153	36919	51785
45	2945	9318	18084	28040	37917
50	2249	6994	13249	19911	25834
55	1458	4412	7989	11413	13926
60	915	2629	4409	5798	6618
65	448	1213	1905	2349	2563
Total	25755	76590	146523	229353	318518



AGE OF OFFICE					Central Age at Entry
30 years	35 years	40 years	45 years	50 years	
Combined O <sup>M</sup> and O <sup>M(5)</sup> (throughout) Mortality Tables—Interest 2½ per-cent					
20290	25601	31256	36982	42204	20
71761	90178	108165	124689	138281	25
99120	122537	143679	161141	172952	30
95922	116206	132641	143285	148412	35
75426	88589	97398	101645	103036	40
51674	57497	60279	61196	61384	45
32876	35048	35700	35822	35824	50
16561	17100	17224	17226	17226	55
7422	7481	7482	7482	7482	60
2770	2771	2771	2771	2771	65
473822	563008	636595	692239	729572	Total
Combined O <sup>M</sup> and O <sup>M(5)</sup> (throughout) Mortality Tables—Interest 2¾ per-cent					
19520	24685	30208	35823	40964	20
69324	87313	104953	121220	134642	25
96138	119110	139925	157169	168866	30
93484	113461	129694	140237	145326	35
73764	86767	95493	99708	101090	40
50685	56452	59214	60125	60313	45
32366	34521	35171	35293	35295	50
16336	16873	16997	16999	16999	55
7333	7391	7392	7392	7392	60
2743	2744	2744	2744	2744	65
461693	549317	621791	676710	713631	Total
Combined O <sup>M</sup> and O <sup>M(5)</sup> (throughout) Mortality Tables—Interest 3 per-cent					
18767	23790	29180	34681	39737	20
66982	84553	101850	117864	131115	25
93281	115816	136306	153335	164922	30
91109	110784	126822	137263	142315	35
72184	85032	93677	97862	99239	40
49764	55477	58220	59126	59311	45
31857	33997	34642	34764	34766	50
16128	16661	16785	16787	16787	55
7246	7304	7305	7305	7305	60
2716	2717	2717	2717	2717	65
450034	536131	607504	661704	698214	Total
Combined O <sup>M</sup> and O <sup>M(5)</sup> (throughout) Mortality Tables—Interest 3½ per-cent					
17396	22141	27279	32565	37459	20
62574	79337	95969	111471	124388	25
87801	109482	129337	145941	157293	30
86571	105650	121295	131535	136511	35
69053	81592	90071	94193	95552	40
47966	53572	56274	57169	57354	45
30869	32978	33616	33737	33739	50
15702	16230	16351	16356	16356	55
7083	7141	7142	7142	7142	60
2667	2668	2668	2668	2668	65
427682	510791	580005	632777	668462	Total
Combined O <sup>M</sup> and O <sup>M(5)</sup> (throughout) Mortality Tables—Interest 4 per-cent					
16123	20607	25503	30581	35315	20
58425	74412	90397	105405	117989	25
82775	103635	122868	139056	150183	30
82230	100729	115990	126032	130931	35
66110	78346	86663	90724	92068	40
46203	51704	54366	55253	55437	45
29929	32008	32640	32761	32763	50
15286	15810	15932	15934	15934	55
6920	6977	6978	6978	6978	60

Central Age at Entry	AGE OF OFFICE				
	5 years	10 years	15 years	20 years	25 years
	Combined $OM$ and $OM^{(5)}$ ( $OM$ 1st 5 years) Mortality Tables—Interest $2\frac{1}{2}$ per-cent				
20	1031	3491	6635	10405	14749
25	3337	11423	22584	36458	52575
30	4672	15902	31792	51568	74127
35	4722	15932	31918	51433	73155
40	4114	13695	27052	42765	59372
45	3291	10701	20676	31771	42572
50	2497	7872	14823	22092	28449
55	1605	4880	8777	12447	15104
60	1001	2867	4776	6243	7098
65	484	1303	2035	2497	2719
Total	26754	88066	171068	267679	369920
	Combined $OM$ and $OM^{(5)}$ ( $OM$ 1st 5 years) Mortality Tables—Interest $2\frac{3}{4}$ per-cent				
20	977	3329	6337	9957	14143
25	3183	10935	21659	35039	50651
30	4469	15264	30595	49759	71714
35	4541	15379	30887	49893	71136
40	3979	13277	26288	41658	57960
45	3194	10412	20168	31062	41702
50	2436	7702	14533	21697	27978
55	1573	4792	8633	12260	14891
60	983	2823	4710	6163	7011
65	476	1285	2010	2469	2690
Total	25811	85198	165820	259957	359876
	Combined $OM$ and $OM^{(5)}$ ( $OM$ 1st 5 years) Mortality Tables—Interest 3 per-cent				
20	926	3174	6050	9523	13560
25	3033	10468	20773	33679	48799
30	4282	14668	29470	48042	69408
35	4371	14848	29894	48402	69167
40	3845	12875	25559	40596	56604
45	3103	10146	19696	30396	40881
50	2379	7534	14245	21304	27513
55	1542	4708	8497	12084	14693
60	966	2779	4644	6084	6925
65	471	1271	1989	2446	2665
Total	24918	82471	160817	252556	350215
	Combined $OM$ and $OM^{(5)}$ ( $OM$ 1st 5 years) Mortality Tables—Interest $3\frac{1}{2}$ per-cent				
20	828	2889	5528	8732	12482
25	2753	9597	19125	31140	45329
30	3924	13538	27328	44770	65004
35	4046	13839	28004	45564	65422
40	3596	12103	24143	38528	53954
45	2930	9632	18785	29107	39289
50	2264	7206	13685	20543	26608
55	1479	4537	8220	11724	14285
60	934	2696	4520	5934	6765
65	457	1242	1947	2398	2616
Total	23211	77279	151285	238440	331754
	Combined $OM$ and $OM^{(5)}$ ( $OM$ 1st 5 years) Mortality Tables—Interest 4 per-cent				
20	746	2636	5057	8013	11497
25	2505	8800	17603	28781	42092
30	3601	12525	25396	41795	60969
35	3748	12899	26228	42884	61867
40	3364	11388	22827	36593	51459
45	2770	9143	17909	27865	37742
50	2159	6904	13159	19821	25744
55	1421	4375	7952	11376	13889
60	903	2617	4397	5786	6606
65	444	1209	1901	2345	2559
Total	21661	72496	142429	225259	314424

AGE OF OFFICE					Central Age at Entry
30 years	35 years	40 years	45 years	50 years	
Combined OM and OM <sup>(5)</sup> (OM 1st 5 years) Mortality Tables—Interest 2½ per-cent					
19593	24904	30559	36285	41507	20
70423	88840	106827	123351	136943	25
98035	121452	142594	160056	171867	30
95280	115564	131999	142643	147770	35
75059	88222	97031	101278	102669	40
51475	57298	60080	60997	61185	45
32779	34951	35603	35725	35727	50
16524	17063	17187	17189	17189	55
7411	7470	7471	7471	7471	60
2768	2769	2769	2769	2769	65
469347	558533	632120	687764	725097	Total
Combined OM and OM <sup>(5)</sup> (OM 1st 5 years) Mortality Tables—Interest 2¾ per-cent					
18833	23998	29521	35136	40277	20
68012	86001	103641	119908	133330	25
95075	118047	138862	156106	167803	30
92850	112827	129060	139603	144692	35
73412	86415	95141	99356	100738	40
50499	56266	59028	59939	60127	45
32267	34422	35072	35194	35196	50
16300	16837	16961	16963	16963	55
7322	7380	7381	7381	7381	60
2739	2740	2740	2740	2740	65
457309	544933	617407	672326	709247	Total
Combined OM and OM <sup>(5)</sup> (OM 1st 5 years) Mortality Tables—Interest 3 per-cent					
18097	23120	28510	34011	39067	20
65688	83259	100556	116570	129821	25
92238	114773	135263	152292	163879	30
90478	110153	126191	136632	141684	35
71826	84674	93319	97504	98881	40
49573	55286	58029	58935	59120	45
31763	33903	34548	34670	34672	50
16092	16625	16749	16751	16751	55
7235	7293	7294	7294	7294	60
2714	2715	2715	2715	2715	65
445704	531801	603174	657374	693884	Total
Combined OM and OM <sup>(5)</sup> (OM 1st 5 years) Mortality Tables—Interest 3½ per-cent					
16735	21480	26618	31904	36798	20
61307	78070	94702	110204	123121	25
86798	108479	128334	144938	156290	30
85947	105026	120671	130911	135887	35
68720	81259	89738	93860	95219	40
47778	53384	56086	56981	57166	45
30779	32888	33526	33647	33649	50
15666	16194	16318	16320	16320	55
7072	7130	7131	7131	7131	60
2664	2665	2665	2665	2665	65
423466	506575	575789	628561	664246	Total
Combined OM and OM <sup>(5)</sup> (OM 1st 5 years) Mortality Tables—Interest 4 per-cent					
15482	19966	24862	29940	34674	20
57207	73194	89179	104187	116771	25
81784	102644	121877	138065	149192	30
81630	100129	115390	125432	130331	35
65784	78020	86337	90398	91742	40
46028	51529	54191	55078	55262	45
29839	31918	32550	32671	32673	50
15249	15773	15895	15897	15897	55
6908	6965	6966	6966	6966	60
2607	2608	2608	2608	2608	65
402518	482746	549855	601242	636116	Total



TABLE 19.—*Showing the Liability under Policies existing*

Quin- quennium	2½ per-cent	2¾ per-cent	3 per-cent	3½ per-cent	4 per-cent
	H <sup>M</sup>				
1	25978	25051	24166	22503	20968
2	57724	55856	54057	50655	47515
3	80354	78026	75765	71472	67464
4	94709	92264	89882	85330	81036
5	100960	98663	96412	92075	87953
6	98639	96666	94725	90960	87352
7	88780	87222	85701	82727	79846
8	73409	72309	71213	69056	66963
9	55580	54860	54140	52730	51350
10	37311	36903	36474	35677	34861
Total	713444	697820	682535	653185	625308
	O <sup>M</sup>				
1	26754	25811	24918	23211	21661
2	59056	57163	55360	51910	48727
3	81735	79384	77119	72781	68736
4	95874	93420	91038	86442	82126
5	101827	99511	97258	92903	88756
6	99203	97215	95280	91492	87870
7	89076	87529	85996	83004	80113
8	73549	72439	71337	69175	67066
9	55629	54905	54190	52763	51370
10	37332	36912	36503	35681	34872
Total	720035	704289	688999	659362	631297
	Combined O <sup>M</sup> and O <sup>M(5)</sup> throughout				
1	31229	30195	29248	27427	25755
2	61312	59387	57553	54068	50835
3	83002	80622	78346	74006	69933
4	96611	94137	91739	87155	82830
5	102241	99919	97659	93314	89165
6	99427	97433	95489	91712	88094
7	89186	87624	86097	83109	80228
8	73587	72474	71373	69214	67109
9	55644	54919	54200	52772	51387
10	37333	36921	36510	35685	34874
Total	729572	713631	698214	668462	640210

*e Model Office No. 1, in each Quinquennium of Insurance.*

2½ per-cent	2¾ per-cent	3 per-cent	3½ per-cent	4 per-cent	Quinquennium
Combined H <sup>M</sup> and H <sup>M(5)</sup>					
25978	25051	24166	22503	20968	1
62596	60650	58785	55249	51965	2
83649	81283	78998	74636	70559	3
96973	94509	92113	87537	83236	4
102475	100170	97927	93582	89457	5
99585	97608	95670	91914	88314	6
89324	87776	86261	83290	80420	7
73710	72605	71513	69379	67283	8
55733	55012	54310	52896	51519	9
37386	36977	36570	35760	34959	10
727409	711641	696313	666746	638680	Total
O <sup>M(5)</sup>					
25777	24831	23954	22279	20754	1
57361	55489	53694	50294	47149	2
79934	77585	75329	71038	67020	3
94221	91764	89373	84814	80521	4
100410	98095	95838	91499	87364	5
98067	96072	94127	90347	86732	6
88225	86659	85127	82132	79246	7
72942	71823	70718	68551	66441	8
55238	54508	53787	52351	50958	9
37104	36689	36275	35445	34630	10
709279	693515	678222	648750	620815	Total
Combined O <sup>M</sup> and O <sup>M(5)</sup> —O <sup>M</sup> alone first 5 years					
26754	25811	24918	23211	21661	1
61312	59387	57553	54068	50835	2
83002	80622	78346	74006	69933	3
96611	94137	91739	87155	82830	4
102241	99919	97659	93314	89165	5
99427	97433	95489	91712	88094	6
89186	87624	86097	83109	80228	7
73587	72474	71373	69214	67109	8
55644	54919	54200	52772	51387	9
37333	36921	36510	35685	34874	10
725097	709247	693884	664246	636116	Total

## DISCUSSION.

Mr. G. GREEN, in opening the discussion, said it was 25 years since Mr. King wrote his first papers on the model office. At the close of the discussion Mr. King referred to the kindness with which the Institute encouraged its young members, and he thought he (Mr. Green) was in somewhat of a similar position that evening. In considering the paper they naturally compared it with Mr. King's former papers, and were struck with the similarity between the two model offices, a point which was a striking testimony to both of them. It seemed to him there was one point of difference in the construction of the second model office, namely, that the author had been able to use select functions throughout the tables; whereas, from the scarcity of the data in the former case he was obliged to do a certain amount of grouping at the higher ages. Further, a point of difficulty was raised with regard to the method in which the withdrawals were tabulated in the new experience. Turning to the valuations of the model office, which were really the crux of the whole paper, it would be found that the results were very similar to what they had been led to expect from Mr. Ackland's paper. To pass from an  $H^M$  to an  $O^M$  valuation, meant, in practically every instance, a certain amount of increase in the reserves; to change from  $H^M$  and  $H^{M(5)}$  to  $O^M$  and  $O^{M(5)}$  would mean less increase, or possibly a slight decrease. One point had struck him in looking through the valuation of the model office, namely, the extraordinarily heavy values which would be given to new business by employing the table known as the  $O^M$  and  $O^{M(5)}$  throughout. It was obvious that the reason was that the net premium under the  $O^M$  was less than under the  $O^{M(5)}$ ; and it would seem a very heavy reserve indeed to make if the company was transacting a large amount of new business. It seemed to him that practically the data for a model office for endowment assurances were not yet available, because in the British Offices Experience no information was given about the age at which the policies matured, which was one of the most essential points in the valuation of endowment assurances. Although endowment assurances seemed to be extremely numerous, they were not so yet in the total; in fact he believed they only amounted to something between 20 and 25 per-cent of the whole, both in respect of the sums assured and of the reserves held against them. Inasmuch as many questions of changes of basis would be gone into within a few years, it was not likely that the question of endowment assurances would be of such great importance as one might at first suppose. There was a further use of the tables which he had not noticed as being pointed out, although doubtless it had occurred to many, namely, that when at a valuation it was decided to lower the rate of interest, it was well known that the whole of the strain would not come on the first valuation, and that at the second and possibly third valuations, if the company was doing a large amount of new business, still further amounts would have to be put aside, and he thought it could be said that the tables gave one the means of examining into the question. For instance, if one valued by the  $O^M$  table it would be seen that the difference between the 3 per-cent and the  $2\frac{1}{2}$  per-cent reserves was about 1,800 at the end of five years,



and 5,500 after 10 years. The only extra source to meet the latter was the extra reserve of 1,800 together with the interest on it. It was quite obvious that at the end of the 10 years there would be something like another 3,000 to put away to meet the extra reserves. At the end of 15 years it had gone up from 5,500 to no less than 10,000. Of course, that was an extreme case of a company which had only commenced business about 5, 10 or 15 years ago, but if the amount of new business was very large it was quite plain that there would be a considerable strain on the resources of the company at the second and other valuations.

Mr. R. P. HARDY said it would be scarcely doing justice to the paper, or to its author, if the members were to confine themselves to congratulating the Institute that the opening night of the session was signalized by the first great practical application of the results of the recent mortality investigation jointly conducted by the Institute and Faculty of Actuaries. While they could well do that, still more was due from them. They ought to recognize with every feeling of gratitude that the Institute numbered in its ranks such self-devoted students, and that in such processes as those illustrated in the paper the choicest products of the actuarially-trained mind were exhibited. If one asked what was the great art which they all would fain possess and to which they all would willingly yield their homage, it might be replied that, in the main, it was partly the original gift of an inborn capacity for seizing the salient points and for generalization upon them. That was not given to all of them; but, for the encouragement of others, it might be said that it was the result of unsparing study of particulars, and of the constant exercise of the salutary habit of diligently comparing results and of endeavouring to account for them. The reward, though it might linger, came in the shape of that enlarged view which marked the ripened mind, that used formulas as mere humble instruments and not as masters, and finally rose to an instantaneous and instinctive selection of the dominating essentials, relegating the mere adjuncts to the actuarial scrap-heap. A quarter of a century ago, Mr. King followed up Mr. Manly's fertile idea of a model office, improving upon the methods employed, by substituting an actual experience of entrants and lapses drawn from the then recent  $H^M$  investigation, in lieu of the fair assumption that Mr. Manly, in the absence of materials, was forced to employ. It was instructive to read what the author said upon that occasion. He said, "It is necessary to have a hypothetical office as closely as possible resembling an average real company, in the proportionate number of policies of every duration on lives of every age:" "My model office is built of blocks cut from the 'Twenty Offices' Experience." Speaking of the use of such a standard, he said, "It supplies, it is true, an empirical method of valuation and not a scientific one, but I think, notwithstanding, we may congratulate ourselves that by so easy a process we can arrive so nearly at perfection." Then, speaking in the spirit of prophecy, Mr. King said, "It is my hope that it will prove an implement by the judicious and cautious use of which may be solved with sufficient accuracy some of the most important problems that come before an actuary of a life office." Time had not put that paper, nor its

methods, out of date. The members now had replaced in their hands the powerful instrument of 1877, already well tested in past years, but now re-verified by data which all present would accept. By its help, they could approach with greater confidence those questions as to the cost of a change of basis which were now causing such searchings of hearts. But, like all highly-perfected machinery, it required, as the author pointed out in 1877, a "judicious and cautious use." There was one table incidentally given, namely, No. 9, the Mean Policy life-time, the importance of which should not be overlooked, especially in the United States, Canada, and Australia, where they were cursed with an aggressive assessmentism in its most pretentious and specious shape. The fact that there was a centre of gravity to be found in the groupings of assurance companies and of like averaging associations, corresponding to the outside social aggregations, had been scientifically employed by Mr. King. But the interested persons that ran these assessment concerns had continued wilfully to misuse the principle, to the great damage of the uninstructed portions of society. Such a table as that given by the author, confirming the experience shown in 1877, should assist those who were fighting the battle of right, against those subtle powers which relied upon ignorance, and hasten the inevitable end. Only a little patience need be exercised, for as sure as Fate, with its silent but ever-advancing step, that contemptible travesty of the great scheme of life assurance was doomed to perish under the light afforded by such tables as those given by Mr. King.

Mr. T. G. ACKLAND said that he felt there was much ground covered by the paper which could not be adequately discussed until it had been considered more fully and carefully than he had personally had the opportunity of doing. At the same time it was evident that the results given would be of extreme value to all who had need to compare the policy valuations of an office in any way akin to the model office. It was satisfactory to find that the "Model Office No. 2" was so closely allied, in its nature and characteristics, to the former model office, that the author practically set it aside, and proceeded upon the basis of using the old model office, so that the paper, as he understood it, was limited to a development of the old model office by valuations upon the basis of the new tables. It was singular to remark, as Mr. King stated, that after the lapse of a period of 25 years (which was not such a very long period), so marked a change had taken place in the valuation bases of the offices. If reference were made to Mr. King's paper in the 19th volume of the *Journal*, such antiquated tables as Davies' Equitable, and the Carlisle and Northampton Tables, were then included, as forming bases for the general valuation of an office; and it was surprising to find that they had now altogether disappeared from the returns of the offices. He thought that the Life Assurance Companies Act of 1870, and the periodical publication of the valuation returns, must have been a factor largely contributing to that end. His main object in rising was to refer to the question of withdrawals mentioned by the author, and to which Mr. Green in his opening speech had also alluded. The author stated that the particulars as to the withdrawals, classified according to separate policy-years,



were only published for the first 10 years of insurance. But in a ponderous volume at the Institute, the whole of the details were supplied for at least the first 30 years of assurance, and he thought also for longer durations. It was satisfactory, however, to find the author thought that, on the whole, the available material was sufficient to enable him to work out an accurate result, although he (Mr. Ackland) would have been most pleased to obtain for the author any further material which he desired in that connection. The author thought it was rather unfortunate that, in the tabular results in the published volumes of the unadjusted data, the withdrawals had been mixed in adjacent policy-years, so that those arising, for instance, in the latter half of the policy-year 4 had been grouped with those arising in the earlier half of policy-year 5, and the combined total treated as passing out of observation at the point of duration 5. He ventured to think that, on consideration, it would be found that no material difficulty was produced by this grouping of the data. The method which had been followed was not precisely the nearest duration method. It was a modification of the nearest duration method, and did not quite have the effect which the author stated in the paper. For instance, if one took the volume of unadjusted data for whole-life assurances, males, in the participating group, and dealt with the cases entering at the grouped ages 18 to 22, with central age at entry 20, it would be found that the withdrawals, tabulated at the end of the volume, passing out of observation during the first year of assurance, were, in the first six months of the year, 318, and in the second six months, 955 in number; but those numbers did not by any means represent the numbers that were carried respectively to the beginning and end of the year for tabulation purposes, these latter numbers being 721 and 552 respectively. In fact, a totally different method was followed, which was, however, based upon the principles of the nearest duration method; and the modification of the method was introduced because the Committee determined that the days of grace should be included in the experience; and it was evident that, when the days of grace were brought into consideration, the whole basis of the nearest duration method had to be shifted or modified; as indicated in a note appended to the grouped data at the end of the volume. Those remarks had some bearing upon the comparison referred to by Mr. King, between the rate of discontinuance as deduced from the new experience, and the  $H^M$  rate, as published in his earlier paper. He thought that one had not only to consider the facts which the author had set out as demanding consideration in connection with this comparison, but also the fact that a totally different method was followed in tabulating the withdrawals. In comparing the rates of discontinuance given in Table S of the present paper with the rates of discontinuance given in the former paper, it must also be borne in mind that the lapses in the first year in the  $H^M$  experience would, generally speaking, have to be compared with the lapses in the second year in the new experience, because the lapses fell at the end of the first 12 months in the  $H^M$  experience, whilst in the new experience they would fall after 13 months, allowing for the days of grace, *i.e.*, within the second year of experience; and that point was, he thought, material to the



comparison. The results of the method actually followed in the new experience were so closely accurate, as compared with the true exposures of the cases, that he thought it would be found that in the form in which they were tabulated, they gave all needed information as to the incidence of the withdrawals.

Mr. E. C. THOMAS said he was glad to take the opportunity of expressing his appreciation of the further obligation under which the author had laid the Institute by the timely production of his interesting paper. Like the previous speakers, he was quite unable on the present occasion to omit all reference to the former paper which the author published twenty-five years ago. The author had referred to a difficulty which was met with in constructing the new model office in reference to the withdrawals, a point that had been already very fully dealt with by Mr. Ackland. He ventured to think that Mr. Ackland had slightly misunderstood Mr. King's reference to the matter. As he understood it, it was not the method of obtaining the exposed to risk with which Mr. King had any quarrel, but the absence of the detailed particulars of the distribution of the withdrawals in their respective assurance years. He thought that Mr. King had clearly demonstrated the great usefulness of the small table, showing such distribution in grouped ages at the end of the volume. It was rather surprising at the first blush to notice how widely the figures given in the final table differed from what would be obtained by a simple interpolation between the figures at the beginning and the end of each assurance year, as tabulated in detail. That, as Mr. Ackland had pointed out, was due largely to the fact that the days of grace were included in the term of exposure. The very large number of policies which lapsed on the first anniversary of the policy were thus drafted into the second year of assurance, and therefore any interpolation between the figures at the beginning and end of the first year would considerably overstate the withdrawals in the first year. There was an incidental advantage in that method of treating the days of grace, in that it removed all ambiguity as to whether a policy lapsed in the first or second year of assurance. After ten years recourse was had to an interpolation between the figures at the beginning of the volume, and there they were met with the difficulty of a small carry-over from the previous ten years to be accounted for later on. That was only a small matter, which was brought into account in the eleventh year. Reference had been made in the paper and by Mr. Ackland to the rate of discontinuance. The author remarked that a comparison between the rates now shown and those given in his former paper based on the  $H^M$  was irrelevant, and that point was rather emphasized by Mr. Ackland. It appeared to him, however, that the differences between the methods of obtaining the various years of exposure in the two cases to a certain extent counter-balanced, and that the comparison was more permissible than it might seem to be. From the fact that the days of grace were included in the new experience they were left with only those policies carrying fractional premiums which could possibly lapse in the first year of assurance, and it might very well be that those policies corresponded fairly well to the numbers included in the fractional part of a calendar year represented by the year 0 in the

previous experience. That view was confirmed by the remarkable similarity between the two sets of figures. Both were characterized by a fairly small initial rate, with a violent upward jump on the second figure, from that point decreasing with a fairly regular progression. At some ages the figures were almost identical. For instance, at age 25 the rates per-cent now given for assurance years 1 and 2 were respectively 3.05 and 8.32 compared with 3.07 and 8.59 for the old years 0 and 1 respectively; for age 30 they were 2.55 and 7.02 against 2.66 and 6.47, the agreement being maintained at ages 35, 40 and 45. Those five groups of entry ages accounted for the bulk of the whole business, and he thought that agreement considerably supported the contention he had made. The author drew attention to a formula of his own devising for passing from a pure premium valuation to a modified premium valuation. If the result of the valuation alone were required without any details, the work was very simple, merely the few multiplications shown by the author in the example. If, however, for any reason the figures were required in detailed or grouped durations, the work could be simplified very much by throwing the expression into the form

$$\Sigma V'_x = \Sigma V_x + \psi_x (r_x - \Sigma V_x).$$

All they had to do then was to tabulate the difference between the total number of policies and reserves for each duration and set up one figure on the machine, making one multiplication for each duration. It occurred to him that it might be useful to show the results of a valuation on the  $O^{M(5)}$  Table with  $H^M$  net premiums. The ordinary  $H^M$  and  $H^{M(5)}$  combination might be defended on the ground that while keeping the premium margin intact it was desirable to value the single premiums and annuities by the best exponent of the mortality likely to be experienced in the future, and the same argument would apply to substituting  $O^{M(5)}$  for  $H^{M(5)}$  in the formula. He had made a valuation of the model office at 3 per-cent on that basis. The first surprise he had was to notice that the  $O^{M(5)}$  net premiums were actually less than the  $H^M$  net premiums. That fact was shown in another way by the annuities tabulated in Mr. Ackland's paper, but he had not seen the fact commented on before. The result was that the reserve by the combined tables was actually less than by  $O^{M(5)}$ . By using the above-mentioned basis throughout and taking  $O^M$  3 per-cent as the standard at 10,000, the reserve was 9,675 for an office fifty years old. Substituting  $H^M$  alone for the first five years the figure became 9,724, which was approximately equal to  $H^M$  and  $H^{M(5)}$  at  $3\frac{1}{2}$  per-cent. They were thus led to the conclusion that a valuation on the combined  $H^M$  and  $H^{M(5)}$  3 per-cent basis contained a concealed surplus from mortality (always assuming that the  $O^{M(5)}$  Table would represent the mortality in the future) nearly equal to the total effect of a variation of  $\frac{1}{2}$  per-cent in the valuation rate of interest.

Mr. H. P. CALDERON said that the question which he had hoped to go into when he heard the title of the paper was as to whether the best basis for a model office mortality table would not be found to be that one in the published volumes known as the "old" assurances, which represented the true after-history of the past thirty years of a certain number of policies in force upon their anniversary in one given



calendar year. The same idea could also be worked out by applying the rates of mortality, &c., to the cases "existing" in 1893 in the combined "old" and "new" table. There was another point to which he thought it also right to draw attention. The Mortality Committee had considered a suggestion for taking out details of the number of entrants in different calendar years, but, unfortunately perhaps, this was not done on account of the time which would be required, but in the course of sorting the cards had been arranged in calendar years of entry, and the gradual rise and, later, a sudden drop in the size of the different heaps in the whole-life class on account of the newer fashion of endowment assurances was a very noticeable feature.

Mr. H. W. MANLY thought that when most of the members had read the paper they must have put it down with a great sigh of relief when they saw that the difference between an  $H^M$  and  $H^{M(5)}$  and  $O^M$  and  $O^{M(5)}$  valuation at the same rate of interest was very small indeed, so small that one could almost feel satisfied that the basis could be changed to the new tables without any preliminary investigation. The mode of analysis, by means of a model office, was the most powerful that could be used, because it represented the conditions in an average stationary office, and it was possible, by some simple adjustment, to determine, with sufficient accuracy, what the valuation record of a particular office would be by any of the bases used in Mr. King's papers. Reference had been made to the Table 9, showing the curtate expectations of existence of policies, and he had something to say about it himself. When he was once under cross-examination in an important case, which had reference to an assessment company, he was asked whether he was not aware that the life of a policy had been as accurately determined as the life of the policyholder, and he said "No." He was then asked if he knew Mr. Teece's work, and replied that he was not aware of his work on that subject. There was then produced to him one of Mr. Teece's valuation reports, in which he had set out the average number of years during which the policies had been under observation in the various mortality tables which had been constructed. He (Mr. Manly) laughed, and said that that was not the average life of a policy; it was the average number of years the policies had been under observation, most of which were still in existence. He commented upon that fact, and suggested that he did not think the learned counsel would consider that his life had ceased if he (Mr. Manly) took an observation of it at that moment. That settled the matter, but he determined that, in case he was asked such a question again, he would be in a position to give an answer, and with that object he ascertained from the  $O^M$  unadjusted data the average duration of a policy taken out at the age of 30. His figure differed slightly from Mr. King's, being 24.09 against the author's 23.914 at the age of entrance. That was an insignificant difference, probably arising from the different way in which the withdrawals were treated, but it was quite near enough to say that the average life of a policy taken out at age 30 was 24 years. There was one point about this average duration which had not been mentioned, namely, that the expectation was very much like the expectation of infants in an infantile mortality table, namely, that the expectation increased as the policy got older, the same as the



expectation of a life at 3 was greater than the expectation of a life at 0, and so on. The expectation at entrance in his table was 24·09, after a year it was 24·96, after two years 25·43, and after three years 25·45; then it gradually decreased. This was caused by the heavy lapsing among policies in the early years, and it was not until after the third premium had been paid that a policy attained its maximum expectation. No stronger condemnation of the basis upon which assessment companies had worked could be found. They had deliberately stated that the average life of a policy did not exceed 10 years, and, as all their statements and figures had been based on the assumption that the policies would work off in ten years, the figures now produced most effectually prove the fallacy upon which their calculations were based. A good deal of discussion had taken place on the treatment of withdrawals, but he thought, from his own observations with regard to the construction of a model office, that the difference caused by different modes of treatment must be exceedingly small, and altogether insignificant in practical results. Neither Mr. King nor anyone else who had had to use such tables would expect to produce a valuation by his method which would agree with an individual valuation to the 6th, 7th, or 8th figure. They could very well deal in thousands when using these tables, and the question whether the withdrawals were all in the one year, or half in the one year and half in the other, had very little effect indeed.

The PRESIDENT (Mr. W. Hughes), said he thought it was one of the penalties of such services as the author had been wont to render to the Institute, that the writers were always expected to render yet further services of the same kind. Personally he should look forward with considerable interest to the promised extension of the paper to the subject of endowment assurance, because he thought the remark made by Mr. Green that the prevalence of endowment assurance and its influence was much less than was generally supposed was somewhat premature. Mr. Green had stated that endowment assurances now amounted to about 25 per-cent of the total insurances in force. It was not such a very long time ago that the figure was 5 per-cent, and if it had extended from 5 to 25 it might easily extend from 25 to something a great deal larger. And he thought it would, because as he had remarked on a recent occasion, the idea of life insurance as an investment was taking firm hold of the public mind, and, in addition to the normal increase in that class of business, he thought it would be found that such insurances would extend, not merely to those payable at 50 or 60 years of age, which had been customary hitherto, but to those payable at much earlier ages, and in a good many cases at later ages also, so as at least to avoid having to pay premiums, under a whole-life policy, up to the very end of a long life.

Mr. KING, in reply, said he thought the proceedings of the evening illustrated the fact that history repeated itself, in that he was asked to repeat the thanks he gave twenty-five years ago for the very cordial way in which the Institute recognized any loyal efforts to do work for it. It had been remarked that he had more or less committed himself to papers to be given later on, and the President, especially, had spoken of the interest which would be felt in dealing

with endowment assurances. That led him to ask the members to help him. It had been truly stated by one of the speakers that the published data as regards endowment assurance did not enable one to prepare a model office for them. But there was one way in which, to a certain extent, that could be done, namely, if those gentlemen who had made valuations of a batch of endowment assurances on more than one basis would be good enough to send him a summary of the results, that would help him exceedingly in such a useful and interesting enquiry. He had had one or two such summaries sent to him, but would like a great many more, which would not, of course, be used so as to disclose identity. Valuations on one basis only would not, so far as he could see at present, be of any service, but where the valuation had been made on more than one basis it would be very useful indeed that he should have the particulars of the two sets of valuations so as to be able to compare them. It was only by processes of that kind that they would be able to measure with any accuracy the effect of a change of valuation basis in dealing with endowment assurance, and the more of such examples that were available the more accurate would the deductions from them be. The discussion had been of an extremely flattering character, and practically there had been no criticisms. He was very much obliged for the suggestion that he might have had at his disposal all the data with regard to withdrawals that were available at the Institute, but although he was quite aware that from the schedules in the large volume to which Mr. Ackland referred they might have been obtained, he really had not the time to do it, and he had not the assurance to suggest to the staff that were working that they should take such a very large amount of trouble over the matter. After all, as Mr. Manly had said, it was not of great consequence, especially as they were able to get the withdrawals accurately assigned to the assurance years for the first ten years. It was very interesting to hear from Mr. Manly about the increasing expectation of existence of policies after the first few years of duration. He might point out that the Model Office No. 2 as given in the paper would enable all questions of that kind to be answered. It was a kind of D and N column, and it was possible to at once get at any point the expectation of further existence of a policy. In speaking about the nearest duration method having been employed, he need hardly assure Mr. Ackland that there was no idea of in any way questioning the accuracy of the methods followed in tabulating the results, beyond the fact that personally he liked to get as much information as he could; and it would have been interesting to him and useful in the enquiry, if they had obtained accurately the withdrawals belonging to each assurance year.

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Temporary Assurances. By WILLIAM PALIN ELDERTON, F.I.A.,  
of the Guardian Assurance Company.

[Read before the Institute, 26 January 1903.]

THE subject of Temporary Assurances has been suggested as one that would be of interest to Members of the Institute of Actuaries, and the present paper is an attempt to examine the mortality among lives who have been assured by this class of policy. All the particulars that have been used are given in the "Combined Experience of Assured Lives—Endowment Assurances and Minor Classes of Assurance" (C. & E. Layton, 1900), but for convenience of reference Table I has been extracted and gives the experience in a slightly altered form for the durations 0-4 years, and Table II shows the groups into which the data were thrown so as to render them less uneven. The reason for omitting the experience beyond duration "4" was that it is generally assumed for practical purposes that selection has worn off by that time.

TABLE I.

Years elapsed since date of Assurance	GROUPED AGES AT DATE OF ASSURANCE																						Years elapsed since date of Assurance
	0-17		18-22		23-27		28-32		33-37		38-42		43-47		48-52		53-57		58-62		63-89		
	θ	E	θ	E	θ	E	θ	E	θ	E	θ	E	θ	E	θ	E	θ	E	θ	E			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
0	1	267	10	1,178	3	1,083	4	1,159	8	1,341	5	1,417	11	1,275	11	1,060	10	720	8	420	12	367	0
1	1	255	4	770	5	726	4	821	5	970	4	1,066	12	946	14	799	8	514	3	269	8	195	1
2	..	243	5	494	5	519	2	671	8	766	5	874	9	766	14	598	7	415	4	197	6	128	2
3	4	232	3	259	7	356	2	512	5	608	7	656	7	592	7	449	2	298	5	141	4	76	3
4	..	183	4	159	2	292	6	422	6	494	5	564	1	511	12	384	10	254	3	109	3	52	4
Total	6	1,180	26	2,860	22	2,976	18	3,585	32	4,179	26	4,577	40	4,090	58	3,290	37	2,201	23	1,136	33	818	Total

θ and E are written for the Headings instead of "No. of Deaths" and "No. Exposed to Risk" in order to save space.

TABLE II.

Years elapsed since date of Assurance	GROUPED AGES AT DATE OF ASSURANCE										Years elapsed since date of Assurance
	0-17		18-32		33-47		48-62		63-89		
	θ	E	θ	E	θ	E	θ	E	θ	E	
0	1	267	17	3,420	24	4,033	29	2,200	12	367	0
1	1	255	13	2,317	21	2,982	25	1,582	8	195	1
2	...	243	12	1,684	22	2,406	25	1,210	6	128	2
3	4	232	12	1,127	19	1,856	14	888	4	76	3
4	...	183	12	873	12	1,569	25	747	3	52	4
Total	6	1,180	66	9,421	98	12,846	118	6,627	33	818	Total



The irregularity of the data, even in the 15-year groups, is very noticeable, and when the work was begun it was feared that it would prove an insuperable difficulty. Existing methods of gradation were considered but discarded. Higham's Method (including the particular case known as Woolhouse's Method) is quite unsuitable for the graduation of short series, and it seemed out of the question to form a Table on Makeham's Hypothesis with Mr. G. F. Hardy's Modification for selection in such a case as the one under consideration. The only other well-known method (Dr. Sprague's Graphic)\* leaves too much to the taste or fancy of the calculator, especially when it is used for a small uneven experience. It was necessary to graduate somewhat extensively, but it was quite as important not to destroy the character of the Table, and in the hope of dealing with the matter in a fairly satisfactory way the following method was devised: The total exposed in the groups under observation were graduated by means of a frequency curve; then the exposed in each group were graduated by a cubic parabola and the figures altered in proportion to make the total equal to the figure given by the frequency curve. A similar graduation was made for the deaths, and the values of  $q_{[x]+n}$  were then obtained by division. The description of the method is given in general terms because there seems no *à priori* reason why it could not be used for fuller Tables than those to which it is now applied if curves can be found to fit the experience satisfactorily; in fact, since the paper was originally drafted, further work on the subject has led me to think that the method might be re-stated in a slightly altered form, giving a graduation suitable to almost any kind of table.

The chief assumption involved in the method is that the total numbers of the exposed (or deaths) in the consecutive groups represent a chance distribution, that is to say, a distribution "in accordance with law, and one the nature of which, as a whole, can for all practical purposes be closely predicted." The binomial and the normal curve of error are doubtless the best known examples of chance distributions, but they are practically useless in a large number of cases. By considering

\* Mr. Lidstone's suggested method for graduating a small Table was considered, but it seems to force the small Table into the form of the larger one from which the ratios are obtained. It was also difficult to find a suitable Table on which to work. I am doubtful if the method is sound unless the small Table is of such a description that it might be considered as a random sample from the larger Table.

cases arising from throwing dice or tossing coins, for example, we see how the binomial form can arise, and by recording the number of black balls which result from drawing  $r$  balls from a bag containing  $b$  black and  $a$  white balls (where  $r > b$ ) we can obtain a hypergeometrical series. The normal curve can be derived from the symmetrical binomial, and Professor Karl Pearson† has shown that, by using the hypergeometrical, we can obtain new types of frequency curves. Analytically we may consider that the slopes of the types of curves correlated to the hypergeometrical series are given by a relation of the form  $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{-x}{c_1 + c_2x + c_3x^2}$ . The integrals of this differential equation give the frequency curves required. The following Table will show clearly the types of frequency curves—it is based on a Table given by Mr. G. F. Hardy,|| in his review of Professor Pearson's "Chances of Death", but is rather fuller. The origin is at the mode, or position of the maximum ordinate in the fitted curve, in all types except IV, V, and VI.

TABLE III.—Frequency Curves.

No. of Type	Description of Curve	Examples	CRITERION		
			$\kappa_2$ *	$\beta_1$	$\beta_2$
I	Limited range in both directions (skew)	$y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2\dagger}$	$< 0$	...	...
II	Do. do. do. (symmetrical)	$y = y_0 \left(1 - \frac{x^2}{a^2}\right)^m$	$= 0$	$= 0$	not = 3
III	Limited range in one direction (skew)	$y = y_0 \left(1 + \frac{x}{a}\right)^{\gamma a} e^{-\gamma x}$	$\infty$	...	...
IV	Unlimited range . . (skew)	$y = y_0 \left(1 + \frac{x^2}{a^2}\right)^{-m} e^{-\nu \tan^{-1} \frac{x}{a}}$	$> 0$ and $< 1$	...	...
V	Limited range in one direction (skew)	$y = y_0 x^{-p} e^{-\gamma x}$	$= 1$	...	...
VI	Do. do. do. (skew)	$y = y_0 (x - a)^{m_1} x^{-m_2}$	$> 1$ and $< \infty$	...	...
VII	Unlimited range symmetrical (normal curve)	$y = y_0 e^{-cx}$	$= 0$	$= 0$	$= 3$

\*  $\kappa_2 = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$ ,  $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$  and  $\beta_2 = \frac{\mu_4}{\mu_2^2}$ .

†  $\frac{a_1}{m_1} = \frac{a_2}{m_2}$

† "Skew Variation in Homogeneous Material" (*Phil. Trans.*, 1895, Vol. 186A, pp. 343, &c).

|| *J.I.A.*, xxxiii, p. 531.

Types V and VI in this Table were not given in Professor Pearson's 1895 paper, but have been published in *Phil. Trans.* 1901, Vol. 197A, pp. 443, &c.; they will not be found in Mr. Hardy's Table, which was given before the later paper was written. The types in question fill in two gaps which arise owing to certain functions in the calculation of Types I and IV becoming imaginary under some circumstances.

It will be noticed that if we put  $c_2=c_3=0$  in the differential equation given above we obtain the normal curve (Type VII), and if  $c_3=0$ , Type III (skew binomial) is the result of the solution.\*

In order to calculate a curve to fit actual statistics the best method is to find the mean and then multiply each ordinate by the powers of the distance of the ordinate from the mean. Thus, if  $X_1, X_2$ , &c., are the frequencies that actually occur, and  $x_1, x_2$ , &c., their distances from the mean, we calculate  $X_1x_1^n, X_2x_2^n$ , &c., and the sum of these quantities divided by the total frequencies ( $X_1 + X_2 + \dots$ ) is the  $n$ th moment ( $\mu_n$ ) of the whole distribution about the mean.† The curve which would have the same moments as those of the actual statistics is the one to be used. If the moments were calculated by taking the ordinates multiplied by the various powers of the distances we neglect the fact that the curve is really continuous, and Professor Pearson suggested that we should assume the curve to be made up of a series of trapezia. Since this suggestion was made Mr. W. F. Sheppard has shown‡ how the moments may be modified on the assumption that the ordinates represent areas, but his corrections are only accurate when there is high contact at each end of the curve. Having calculated the moments the type to be used is found by examining the column headed "Criterion," in Table III.

Having dealt with the theory on which it rests, we can now turn to the application of the method of graduation to the "Temporary Assurances" experience. As will be remembered, we must first fit frequency curves to the totals of the Deaths and Exposed in the 15-year groups shown in Table II. The constants for the curves are given in Table IV, and the graduated and ungraduated figures are put side by side in Table V.

\* It is also worth noting that if  $c_2=c_1=0, y=y_0x^{-\frac{1}{c_3}}$ , if  $c_3=c_1=0, y=e^{-\frac{x}{c_2}}$ , i.e.,  $y=y_0a^x$ ; if  $c_1=0, y=y_0\left(1+\frac{x}{a_1}\right)^{m_1}$  and if  $c_2=0, y=y_0\left(1+\frac{x^2}{a^2}\right)^m$ .

† In practice we calculate the moments about some central point so as to avoid fractions and then make adjustments to get the moments about the mean.

‡ *Proc. London Math. Soc.*, Vol. XXIX, p. 368. Prof. Pearson has since given a general solution in *Biometrika*, Part III, Vol. I.



TABLE IV.

Constant	Exposed	Deaths
$\mu_2$	·68189	·8670
$\mu_3$	·09021	—·1431
$\mu_4$	1·18225	1·629
$m_1$	2·95097	1·1627
$m_2$	4·22629	·7088
$\alpha_1$	2·18748	2·5713
$\alpha_2$	3·13283	1·5675
$\eta_0$	13859	116·64
Mean-Age	38·2918	44·953
Mode-Age	36·7467	48·837

TABLE V.

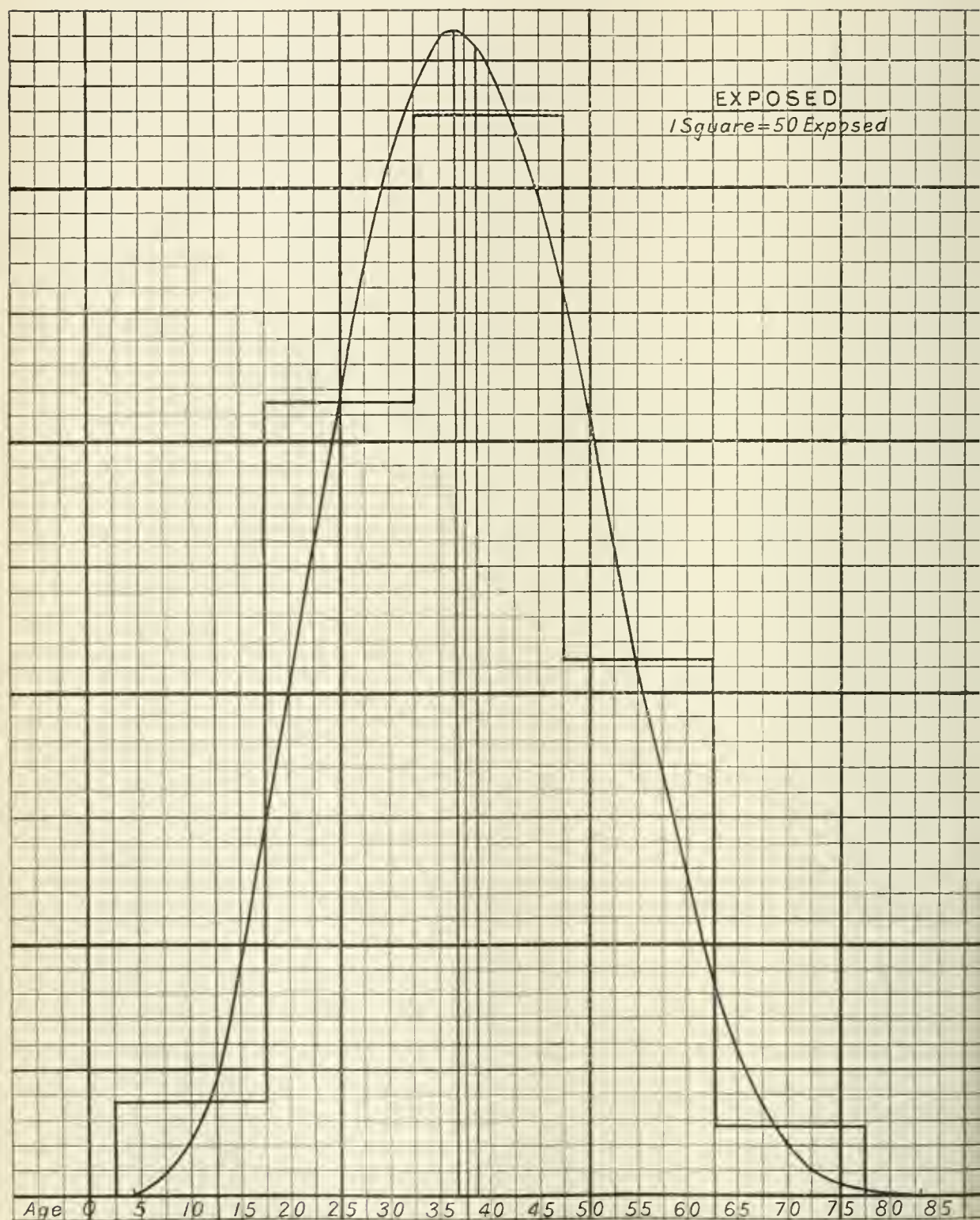
Grouped Ages at Entry	EXPOSED		DEATHS	
	Ungraduated	Graduated*	Ungraduated	Graduated*
0-17	1,180	1,148	6	6·2
18-32	9,421	9,373	66	61·7
33-47	12,846	13,056	98	106·5
48-62	6,627	6,484	118	109·1
63-49	818	831	33	37·5
Totals ...	30,892	30,892	321	321·0

\* In reading the values, areas, not ordinates, should be used.

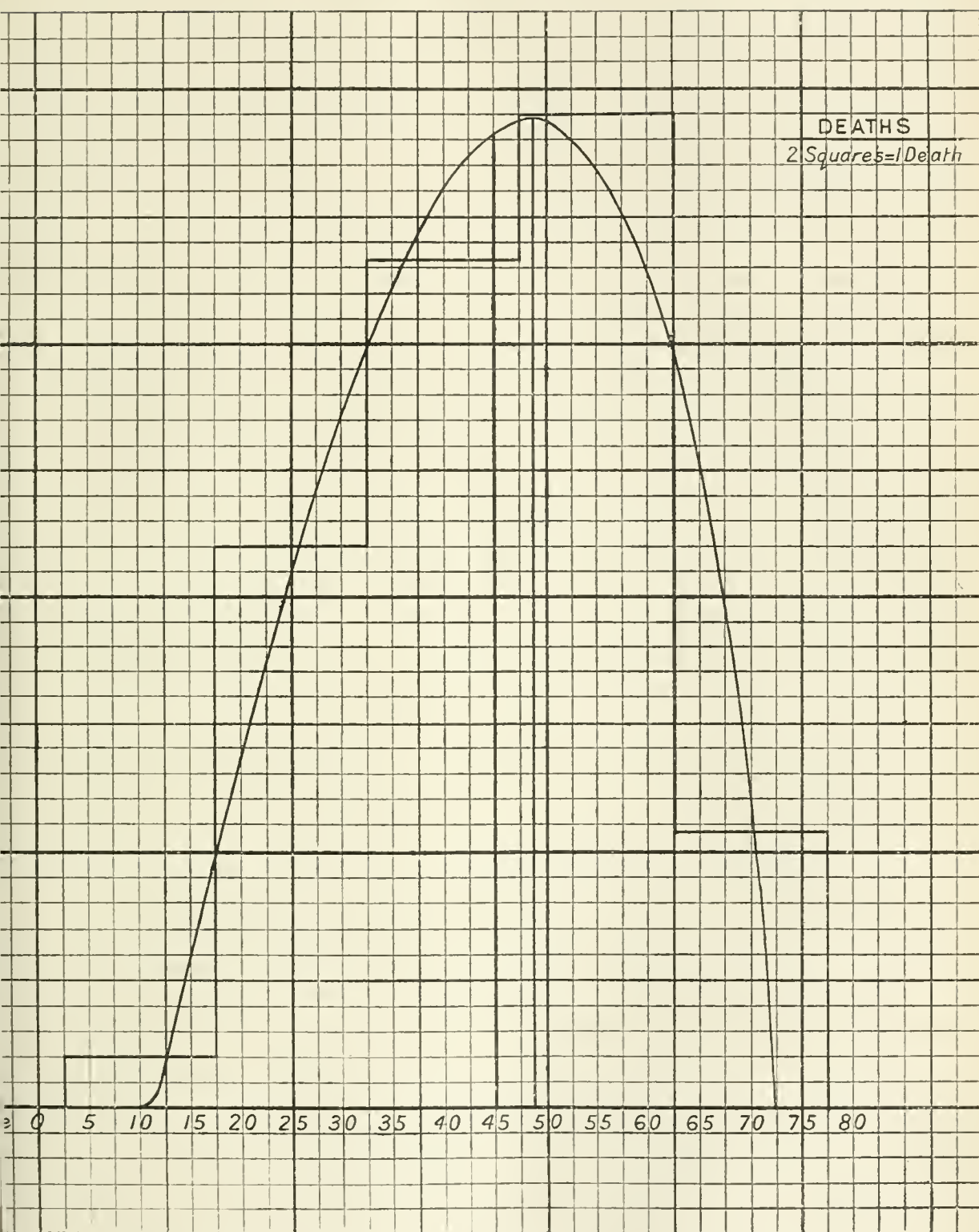
When calculating the moments from the data (Table II) it was assumed that the central ages of the groups were 10, 25, 40, 55, and 70. It will be noticed that in both cases curves of Type I were used, and it may seem strange that the “death” curve only starts at about age 10 and ends at 73, while the “exposed” curve runs from age 4 to age 84, but it should be remembered that the numbers exposed are very small at the ends of the curve and we should therefore expect still smaller, and ultimately insignificant, values for the deaths at the corresponding ages. It is also worth remark that though a curve of Type I must give an absolutely limited range, the use of such a curve does not necessarily mean that we assert that the actual statistics cannot continue beyond the range.

The moments were modified by using Sheppard’s corrections, and a better result was obtained than that which was brought out by the “ordinates” or “trapezia” methods, even though there is not *high* contact at either end of the range. The other methods lengthened the range but made the 0-17 and 63-89 groups too large. The assumption that 10 and 70 were

the central ages may have affected the matter. The following are the curves actually used : \*



\* For a full description of the method and of the equations by which the constants are obtained from the moments, reference must be made to Professor Pearson's papers quoted above. The following may also be referred to for examples, &c.: Pearson & Filon, "Probable Errors of Frequency Constants", *Phil. Trans.*, 1897; Pearson & Lee, "Distribution of Frequency of the Barometric Height, &c.", *Phil. Trans.*, 1898; Yule, "History of Pauperism", *Journal Statistical Society*, Vol. LIX; and Pearson, "Chances of Death." Since this paper was written Professor Pearson has dealt with Systematic Curve Fitting, see *Biometrika*, Vol. I, Part III, and Vol. II, Part I.



Having fitted a curve to any experience, we must decide if it really does fit the statistics satisfactorily. Professor Pearson has shown (*Phil. Mag.*, July 1900) that if  $m_r$  and  $m_s$  be the observed and theoretical frequencies, and  $(m_r - m_s)^2/m_s$  be calculated for each entry, the sum of these values ( $\chi^2$ ) is a measure of the



“goodness of fit”, and if the number of errors  $n'$  be odd, the improbability of the system observed is given by

$$P = e^{-\frac{1}{2}\chi^2} \left( 1 + \frac{\chi^2}{2} + \frac{\chi^4}{2.4} + \dots + \frac{\chi^{n'-3}}{2.4.6 \dots (n'-3)} \right)$$

and if  $n'$  be even, by

$$P = \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} d\chi + \sqrt{\frac{2}{\pi}} e^{-\frac{1}{2}\chi^2} \left( \frac{\chi}{1} + \frac{\chi^3}{1.3} + \dots + \frac{\chi^{n'-3}}{1.3.5 \dots (n'-3)} \right)$$

In the paper in the *Phil. Mag.* referred to, one or two examples are given of the application of the method, and a Table of  $P$  for various values of  $\chi^2$  and  $n'$  is given in *Biometrika*,\* vol. i, part 2, p. 155.

All the curves used in the present paper have been tested by these formulæ, and the Table in *Biometrika* has been used.

The values of  $\chi^2$ † were 7.85 and 2.25 for the exposed and death curves respectively, and, using first differences, the corresponding values of  $P$  are found to be .098225 and .691275. This means that the odds are 10 to 1 against a system of deviations as improbable as or more improbable than the one given by the exposed graduation. In the case of the deaths the odds are 14 to 10 against the system. Considering the data to which the curves have been fitted, I think they may be considered fairly satisfactory.

Turning to the short description of the method given above, we see that it is now necessary to graduate the exposed in each group by the curve  $y = a + bx + cx^2 + dx^3$ . In each case duration “2” was taken as the origin, and the curves were then obtained by finding the sums of the products of the frequencies into the powers of their distances from “2” and equating. Professor Pearson points out to me that I should probably have obtained a slightly better result by using areas for each of the five years, and then by equating to the actual figures, finding  $a$ ,  $b$ ,  $c$ , and  $d$  by the method of least squares. It should be borne in mind when Tables VI and VII, which give the results of the work, are examined, that, while they do not necessarily give the best fitting

\* This periodical, the first number of which appeared in October 1901, is published four times a year by the Cambridge University Press. It contains much that should be of great interest to actuaries, and it is hoped that the Institute may see its way to add it to the library.

† Two examples of the method with all the working will be found in Tables XI and XII.

parabolas, the values of P shown in Table VII point to the fact that the curves give a good graduation of the original data.

Table VIII and the first line of Table VII give the graduation of the total deaths in Table II summed across. It is obvious that no good result could be obtained by using each series of figures in Table II, and the assumption made seems reasonable.

Table IX was formed by proportion so as to make the totals of the exposed and deaths agree with the figures given in Table IV. The "deaths" group 0-17 was put in the form given so as to make the table of probabilities (Table X) run evenly, and, considering the original data, this course does not appear to be open to criticism. The deaths were altered in the decimal place in two cases to improve the final table.

The frequency curves and parabolas will enable us to form a complete table of probabilities for all ages at entry from, say, 20 to 65, and all durations from 0 to 4. Earlier than age 20 and later than 65 it would be unsafe to rely on the deaths curve except, of course, for the central ages 10 and 70. As I only wish to show the general trend of the tables an enlargement of Table X is unnecessary.

TABLE VI.

Years elapsed since Assurance	0-17		18-32		33-47		48-62		63-89	
	Exposed	Graduated	Exposed	Graduated	Exposed	Graduated	Exposed	Graduated	Exposed	Graduated
0	267	266	3,420	3,380	4,033	4,057	2,200	2,192	367	362
1	255	252	2,317	2,348	2,982	3,025	1,582	1,604	195	200
2	243	246	1,684	1,631	2,406	2,347	1,210	1,183	128	119
3	232	230	1,127	1,166	1,856	1,891	888	904	76	83
4	183	186	873	896	1,569	1,526	747	744	52	54
Total	1,180	1,180	9,421	9,421	12,846	12,846	6,627	6,627	818	818

TABLE VII.

Constants (to nearest Integer) used in above Graduation.  
( $\chi^2$  and P are also given.)

Table	a	b	c	d	$\chi^2$	P
Deaths . . .	62	- 7	1	0	·50	·9549
Exposed 0-17	246	- 8	- 5	- 3	·15	·9865
„ 18-32	1,631	- 581	127	- 10	4·50	·3467
„ 33-47	2,347	- 545	111	- 22	4·09	·3953
„ 48-62	1,182	- 346	71	- 4	1·24	·8680
„ 63-89	119	- 53	22	- 6	1·57	·8106

TABLE VIII.

Duration	0	1	2	3	4	Total
Deaths . . .	83	68	65	53	52	321
Graduated .	80	70	62	56	53	321

TABLE IX.—*Graduated "Exposed" and "Deaths."*

Years elapsed since Assurance	0-17		18-32		33-47		48-62		63-89	
	Deaths	Exposed	Deaths	Exposed	Deaths	Exposed	Deaths	Exposed	Deaths	Exposed
0	1.19	259	15.5	3,362	26.4	4,124	27.0	2,144	9.4	368
1	1.22	245	13.4	2,336	23.2	3,074	23.7	1,569	8.2	203
2	1.24	239	11.9	1,623	20.6	2,386	21.1	1,158	7.3	121
3	1.26	224	10.8	1,160	18.6	1,922	19.2	885	6.5	84
4	1.29	181	10.0	892	17.8	1,550	18.1	728	6.1	55
Total	6.2	1,148	61.6	9,373	106.6	13,056	109.1	6,484	37.5	831

TABLE X.—*Table of  $q_{[x]+n}$  formed from the above.*

Years elapsed since Assurance	0-17	18-32	33-47	48-62	63-89
0	.00459	.00461	.00640	.01259	.02554
1	.00498	.00574	.00755	.01510	.04039
2	.00519	.00733	.00863	.01822	.06033
3	.00562	.00931	.00968	.02169	.07738
4	.00713	.01121	.01148	.02486	.11091

Before finally testing the goodness of fit of the graduated to the ungraduated Table, it will be well to refer to a criticism that may be urged—that the method seems complicated and costs more trouble than the experience to which it has been applied is worth. The best reply to such a criticism is to refer to the volumes of the *Journal* giving the first descriptions of other methods, and to remind the critic that a new method, almost always, seems more difficult in print than it is in application. The most pressing question in graduation to-day, seems to be "How are we to graduate a Select Mortality Table"? Not "How are we to graduate an aggregate Table"? and there does



not appear to be any answer which is entirely satisfactory. From the point of view of graduation, a Select Table is a large series of Tables requiring consistent treatment, and the difficulty of managing it is increased because it depends on two forces—mortality and selection—of both of which we know very little.

We now come to the last part of the work required by the graduation—to see whether the whole graduated Table fits the original on which it is based. The method is exactly like that already used for the smaller series, but to make it clearer the work is given in full. It would probably be sufficient to test the complete Tables only, instead of also testing every curve as has been done in the present graduation.

TABLE XI.—*Deaths.*

Group	Duration	Observed Frequency $m_r$	Theoretical Frequency $m_s$	$m_r - m_s$		$(m_r - m_s)^2$	$\frac{(m_r - m_s)^2}{m_s}$
				+	—		
0-17	0	1	1.19	...	.19	.04	.03
	1	1	1.22	...	.22	.04	.03
	2	...	1.24	...	1.24	1.54	1.24
	3	4	1.26	2.74	...	7.51	5.96
	4	...	1.29	...	1.29	1.66	1.29
18-32	0	17	15.5	1.5	...	2.25	.15
	1	13	13.4	...	.4	.16	.01
	2	12	11.9	.1	...	.01	.00
	3	12	10.8	1.2	...	1.44	.13
	4	12	10.0	2.0	...	4.00	.40
33-47	0	24	26.4	...	2.4	5.76	.22
	1	21	23.2	...	2.2	4.84	.21
	2	22	20.6	1.4	...	1.96	.10
	3	19	18.6	.4	...	.16	.01
	4	12	17.8	...	5.8	33.64	1.89
48-62	0	29	27.0	2.0	...	4.00	.15
	1	25	23.7	1.3	...	1.69	.07
	2	25	21.1	3.9	...	15.21	.72
	3	14	19.2	...	5.2	27.04	1.41
	4	25	18.1	6.9	...	47.61	2.63
63-89	0	12	9.4	2.6	...	6.76	.72
	1	8	8.2	...	.2	.04	.00
	2	6	7.3	...	1.3	1.69	.23
	3	4	6.5	...	2.5	6.25	.96
	4	3	6.1	...	3.1	9.61	1.58
$n' = 25$		321	321.00	26.04	26.04	...	$\chi^2 = 20.14$

TABLE XII.—Exposed to Risk.

Group	Duration	Observed Frequency $m_r$	Theoretical Frequency $m_s$	$m_r - m_s$		$(m_r - m_s)^2$	$\frac{(m_r - m_s)^2}{m_s}$
				+	—		
0-17	0	267	259	8	...	·64	·25
	1	255	245	10	...	1·00	·41
	2	243	239	4	...	·16	·07
	3	232	224	8	...	·64	·29
	4	183	181	2	...	·04	·02
18-32	0	3,420	3,362	58	...	33·64	1·00
	1	2,317	2,336	...	19	3·61	·15
	2	1,684	1,623	61	...	37·21	2·30
	3	1,127	1,160	...	33	10·89	·94
	4	873	892	...	19	3·61	·41
33-47	0	4,033	4,124	...	91	82·81	2·01
	1	2,982	3,074	...	92	84·64	2·76
	2	2,406	2,386	20	...	4·00	·17
	3	1,856	1,922	...	66	43·56	2·27
	4	1,569	1,550	19	...	3·61	·23
48-62	0	2,200	2,144	56	...	31·36	1·47
	1	1,582	1,569	13	...	1·69	·11
	2	1,210	1,158	52	...	27·04	2·33
	3	888	885	3	...	·09	·01
	4	747	728	19	...	3·61	·50
63-89	0	367	368	...	1	·01	·00
	1	195	203	...	8	·64	·32
	2	128	121	7	...	·49	·41
	3	76	84	...	8	·64	·76
	4	52	55	...	3	·09	·02
	$n' = 25$	30,892	30,892	340	340	...	$\chi^2 = 19·21$

Turning to the Tables in *Biometrika*, to which reference has been made above, we find that, when  $n'$  is 25, the values of P corresponding to  $\chi^2=19$ ,  $\chi^2=20$ , and  $\chi^2=21$  are ·751990, ·696776, and ·638725 respectively, and we reach the satisfactory conclusion that the whole graduated Table (No. IX) fits the original data, as given in Table II, very closely.

Whatever may be the disadvantages of the graduation adopted, it is fairly clear that, for the particular case, the result does not differ unreasonably from the rough statistics, and the method has the advantage of being comparatively simple in application.

Turning from the graduation to the Table itself, it will be interesting to compare the values of  $q$  with those given by the Whole-Life Tables. The rates of mortality seem distinctly higher than those given on page 478 of the New Whole-Life Experience, where a “Select” Table for with-profit policies is given, but the

Table on the following page (without-profit policies) seems to show a mortality closer to and sometimes heavier than that shown by the Temporary Assurances. It is, however, very difficult to compare the rates until the without-profit Table has been graduated, but it hardly seems, from a first examination, that the Temporary Experience can be correctly considered as nothing more or less than a “random sampling” from either of the two larger Tables. The rates given in Table IX for the mortality in the fifth year seem distinctly higher than those given in the aggregate Tables excluding the first five years of assurance (*see pp.* 488, 489, 504, 505).

TABLE XIII.  $A^1_{[x]:\overline{n}|}$  *Three per-cent.*

$\overline{n}$	$x$ 0-17	$x$ 18-32	$x$ 33-47	$x$ 48-62	$x$ 63-89	$n$
1	·00446	·00448	·00621	·01221	·02481	1
2	·00914	·00986	·01328	·02628	·06190	2
3	·01384	·01650	·02106	·04250	·11352	3
4	·01876	·02462	·02947	·06089	·17393	4
5	·02479	·03403	·03904	·08091	·25150	5

TABLE XIV.  $a_{[x]:\overline{n}|}$  *Three per-cent.*

$n$	$x$ 0-17	$x$ 18-32	$x$ 33-47	$x$ 48-62	$x$ 63-89	$n$
1	0·966	0·966	0·965	0·959	0·946	1
2	1·900	1·900	1·894	1·875	1·827	2
3	2·802	2·799	2·789	2·749	2·631	3
4	3·672	3·664	3·649	3·579	3·352	4

TABLE XV.  $\pi^1_{[x]:\overline{n}|}$  *Three per-cent.*

$n$	$x$ 0-17	$x$ 18-32	$x$ 33-47	$x$ 48-62	$x$ 63-89	$n$
1	·00446	·00449	·00621	·01221	·02481	1
2	·00464	·00501	·00676	·01341	·03181	2
3	·00477	·00570	·00728	·01478	·04015	3
4	·00494	·00648	·00778	·01624	·04790	4
5	·00531	·00730	·00839	·01767	·05779	5



TABLE XVI. *Sprague's Select Tables.*  $\pi_{[x]:\overline{n}}^1$  *Three per-cent.*

$n$	$x=15$	$x=25$	$x=40$	$x=55$	$x=70$	$n$
1	·00291	·00437	·00534	·01010	·02845	1
2	·00328	·00574	·00673	·01299	·03949	2
3	·00360	·00671	·00772	·01547	·04994	3
4	·00400	·00733	·00848	·01756	·05668	4
5	·00447	·00767	·00911	·01932	·06200	5

In order to form a rough idea of the premiums, Tables XIII, XIV, and XV (giving term premiums, and annuities), have been calculated from Table X. The values of  $\pi$  given by Dr. Sprague are shown in Table XVI. It is very interesting to note that the new premiums for 1-year assurances are higher than Dr. Sprague's (except for 63-89 group), and in three cases the excess is more than 20 per-cent.

The question as to whether the office premiums for term assurances are sufficient may now be discussed. Dr. Sprague gave (*J.I.A.*, xxii, p. 439) a list of premiums for this class of policy, but I do not see a remark as to the method of loading adopted in calculating them. We may, however, be able to form an opinion of a reasonable loading by considering the methods of calculating whole-life without-profit premiums. The best known loadings are those given by Dr. Sprague in the paper referred to, and by Mr. Rothery in his paper on bonuses (*J.I.A.*, xxx., p. 135), but before they can be used for the present purpose, they require modifications to allow for a reduction in the amount of commission paid, and after allowing for this, Dr. Sprague's loading becomes (1) in respect of the first premium, £1 per-cent on the sum assured, (2) in respect of all premiums, including the first, 2s. 6d. per-cent and 2 per-cent on the gross premium, while Mr. Rothery's loading reduces to (1) in respect of the first premium, £1 per-cent on the sum assured and 5 per-cent on the premium, and (2) in respect of all premiums, including the first, 8 per-cent on the gross premium. These loadings will, at most ages, bring out a lower premium for a 5-years' term policy than for a 1-year term policy, and they will therefore be considered unsatisfactory, though they serve to remind us that if a loading of £1 per-cent on the sum assured is not excessive for initial expenses, then the premiums charged for term policies are too low. But the most convincing evidence that the term premiums need re-consideration, is that in small cases there is not sufficient

premium to provide for the current risk and the actual expenses, including share of general expenses of the office (*e.g.*, 25 next birthday for 1 year for a £500 case and for a 5 years' term for a smaller amount). It is also worth remark that if each class of policy is supposed to meet its own claims, more allowance for deviation from the expected number of claims must be made in a small class than would be required in a large one (*see* p. 516).

Of course it is quite possible to increase the rates, but an increase sufficient to deal satisfactorily with a small case becomes a burden in a large one, for initial expenses are at a rate per policy rather than per sum assured, and the increased term premium would become very close to the whole-life without-profit rate.

The following suggestions are offered in case they may help to afford a solution to the problem before us:—

- (1) Vary the rate according to the amount of the sum assured. The rates might be quoted for a £1,000 sum assured, and a note added that assurances may be effected for smaller sums at an increased rate.
- (2) Make the assured pay the medical fee in small cases (say, under £600). This has already been carried out in practice to a certain extent, but there seem some objections to it, for chance deviation in large cases is more serious than in small cases, and a small case can afford to contribute to the fee from the premium to some extent.
- (3) Refuse to take small cases altogether. A whole-life policy could be effected, and the cost, if the assurance were required for three or more years, would be practically the same as by Nos. 1 and 2, because of the surrender-value of the policy.
- (4) Adopt a modification of the assessment system, *i.e.*, a sum must be paid down for expenses when the policy is effected, and a very low premium charged for actual assurance.

In dealing with the practical aspect of the question, I have so far confined my attention to office premiums, but there are also matters of interest in connection with the net premiums on which the office rates are based. Dr. Sprague suggested that in calculating the net premium for a term longer than one year it would

be well to assume that all healthy lives withdraw at the end of the first year. Whether this is strictly correct would seem to depend on the much larger question as to whether the lives who withdraw are subject to a higher or lower rate of mortality than those who continue; if the former be the actual state of affairs, it would be unfair to adopt the suggestion referred to.

There is another question which I wish to raise, and it is of more practical import than it appears to be at first sight. The mean number of claims among  $n$  policies is  $nq$ , but there may be a proportionately large deviation from this mean, so that we might write for the mean number  $nq \pm nr$ , or, dividing by the number of policies,  $q \pm r$ , where  $nr$  is the probable error. Now, would it not be possible to discover a reasonable loading to cover adverse fluctuations in mortality in small classes by using  $q + r$  for  $q$  when calculating office premiums? As an example, let us consider an office which has 200 term assurances on its books, the average age at entry having been 35 and the average term 3 years, then, as the probable error for a binomial distribution is  $\cdot6745 \sqrt{pqn}$ , we have  $\cdot6745 \sqrt{\cdot025 \times \cdot975 \div 200} = \cdot0074$  as our loading for a 3-year case, or, say, 5s. per-cent per annum. Of course, this example only roughly represents the theory referred to in the question stated above.

The questions as to whether alterations in the treatment of temporary assurances are desirable, and, if so, whether any of the above methods are feasible, can only be answered by those who have to deal with such matters in office practice; and I would close my paper by making the apology, which I feel is necessary, for expressing an opinion on the subject.

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#### DISCUSSION.

Mr. G. E. MAY said that the paper had afforded him great interest and instruction. The author first referred to various well-known methods for the purpose of graduating the limited and very irregular experience at his disposal, and said that both the late Mr. J. A. Higham's method, and Mr. G. F. Hardy's modification of Makeham's hypothesis, were unsuitable to the graduation of short series, and had therefore been discarded. He (Mr. May) thought that this statement would meet with general assent. The objection raised to the graphic method of Dr. Sprague, that it left too much to the taste or fancy of the calculator, he could not altogether agree with, as he was of opinion that in order to obtain a smooth curve, and at the same time retain any marked feature of a limited experience, it was necessary to leave a great deal to the skill and



knowledge of the operator. The method finally decided upon was by means of frequency curves. Professor Karl Pearson had shown that the normal error curve, which was based on correct *à priori* reasoning, had been found in many cases to produce results very wide of the mark. He had, therefore, from an empirical formula, evolved a varied system of curves; and, by a suitable selection, it had been found that the original observations were very closely followed. Any judgment of the method must therefore be based almost entirely upon the results brought out. The chief assumption made by Mr. Elderton in his application of the method was that there was a law connecting the numbers entering at the various ages; and, similarly, a relationship between the deaths, and they had been treated separately. He (Mr. May) had not at first thought that this law would be very strongly marked in the case under consideration. However, an examination of the original data showed a very great similarity in the curves representing the exposed to risk in years of assurance, for each of the age groups taken. The deaths, however, were very irregular, but this, no doubt, was due to the very limited number of cases. The method from its nature reproduced the original number of observations, that was to say, any error was not eradicated, but was distributed over the curve. That being the case, should there be a comparatively large error in the deaths in any group, it would be necessary to make some adjustment at the outset, or the rates of mortality in the adjoining groups might be seriously affected. That was particularly the case owing to the inclusion of the first and last groups, on which absolutely no reliance could be placed. First graduating the exposed to risk in years of assurance by means of a parabolic curve, and then increasing the numbers so as to agree with the aggregate brought out for each group by the first graduation, was very ingenious; but to treat the deaths differently from the exposed to risk was very much open to question. The method adopted of graduating the deaths, in one group, for years of assurance, and then assuming that the same distribution prevailed in the different age groups, thereby involved the further assumption that there was a marked similarity in the curves representing the select rates of mortality in each age group. There was, however, no very marked similarity in the author's graduated figures. A rough examination of the original data showed a high rate of mortality at age 20, decreasing to about age 30, and then increasing in almost all years of assurance. That, he thought, was not due to the paucity of the facts, but rather to a marked feature of the experience. This peculiarity was not shown in the graduated table. If his conclusion was correct, and it was desired to retain any such marked feature, it would be necessary to use more than one frequency curve. The general effect of the graduation had been to take the deaths from groups 18 to 32, and 48 and 62, and to throw them into the central group, 33 to 47, thereby reducing the rates of mortality in the first-mentioned groups. That was more particularly marked in group 48 to 62, the rates of mortality in four of the insurance years having been unduly decreased. At the beginning of his paper the author stated that his reason for neglecting the experience beyond duration 4 was that it was generally assumed for practical purposes that selection had worn off

by that time. Referring to his rates of mortality on page 510 it would be seen that in group 33 to 47 the rate of mortality increased from year of assurance 0 to 1 by 18 per-cent; 1 to 2, 14 per-cent; 2 to 3, 12 per-cent; and 3 to 4, 18 per-cent. Surely, in the face of those figures, it was impossible to assume, even for practical purposes, in the case of temporary assurances, that the effect of selection had worn off after five years. The reason that he (Mr. May) had referred to group 33 to 47 was that it contained the main body of the facts. The same variations were quite as marked, if not more so, in the other groups. The question of adverse selection must also be considered. He believed it was usual to find, in term assurances of from five to ten years, that, in a considerable number of cases, the last premium was not paid, which fact must have a considerable effect on the rate of mortality. In connection with that point, it was interesting to note that no inconsiderable portion of the initial expenses included in the loading of the last premium was lost to the office. He feared that many of the objections he had raised to the method were not so much objections to the method itself as to the particular experience to which it had been applied, and he would look forward with very great interest to seeing the results brought out by its application to a much fuller body of facts.

Mr. GEORGE KING said the paper divided itself naturally into two parts: (1) the mathematical methods which had been followed in preparing the tables, and (2) the practical results. As to the first, he could not follow step by step all that had been written in the paper, for the simple reason that he did not feel equal to the task. That led him to put in a plea for students. Frequency curves were now always cropping up, yet very few of the Fellows understood what they were. Possibly when Mr. Bowley's lectures were published some light might be thrown on the subject. At present it was necessary to study extremely difficult papers in order to get an idea of the subject. He would therefore ask Mr. Elderton whether he could not prepare for the *Journal* an explanatory paper dealing with frequency curves, and giving illustrations of their use drawn from actuarial sources; and whether that paper could not be so written that those with only a working knowledge of mathematics could understand it. Coming to the practical application, he agreed with the remarks which had been made by Mr. May regarding limiting the tables to four years. He felt sure that, especially in term assurances, the force of selection was much stronger, and lasted longer, and that it would be very desirable to carry out the investigation further. He did not know how far the data at the disposal of the Institute permitted it to be done, but if it were possible it would be very useful. His own experience of term assurances had been very unsatisfactory; and he was surprised at the comparatively low premiums which the new combined experience brought out. It was found that for the very young ages, *i.e.*, under 17, Dr. Sprague's tables gave lower premiums than those by the British Offices; but he imagined that in both the experiences the statistics at those very young ages were altogether deficient, and there was no ground to go upon. For terms of one year, also, Sprague's tables gave lower premiums almost throughout



than the new tables submitted by Mr. Elderton, but for longer terms the condition was reversed. He was surprised at that result. His own personal view, *à priori*, would have been that the rates of the combined experience would come out greater. To return to the younger ages, he had always considered it desirable that for term assurances, even for a long term—say, up to ten or fifteen years—the same rates should be charged for all ages below thirty, because, after all, the difference in the mortality was not very great, and at young ages term assurances were not satisfactory, and the rates ran irregularly. It was well known that at the ages from about 25 to 30 there was a dip down in the mortality curve to which effect could scarcely be given in the premiums. He therefore thought that up to age 30, unless the term was a long one, it was desirable to charge a uniform rate.

Mr. R. TODHUNTER said they must all feel indebted to Mr. Elderton for his interesting illustration of modern statistical methods. At the same time it might be felt by many that his application of those methods to the particular problem under consideration was not altogether satisfactory. The method used, like the simpler methods with which they were familiar, had its limitations; it was not infallible; and it exposed them to risks not less, perhaps, than those they incurred when they trusted to the "taste or fancy" of the graphical graduator. He (Mr. Todhunter) felt there was a danger of falling too much under the sway of these modern statistical methods. The authority with which they were introduced was so eminent, the nomenclature used in their development was so impressive, and the mathematical analysis upon which they depended was so imposing, that one was rather prone to surrender one's private judgment, and to yield an assent not entirely based on conviction. Here, however, they had a case in regard to which they might feel a reasonable doubt whether the results were, in fact, right, *i.e.*, right in such a sense as was of practical importance to actuaries. He would ask them to look a little closely at the actual process followed in the paper. To begin with, the total deaths and the exposures for the five years for the respective grouped ages at entry had been subjected to a process of graduation, with the result that the heavy mortality at the younger ages at entry—which, if the experience had any features at all, was surely one of its most pronounced features—had been entirely smoothed away. That appeared to be due mainly to the grouping of the facts for ages 18 to 32, and to their subsequent treatment as a single group without any reference to the contributions of the smaller age-groups to the total, and partly also, he thought, to the treatment of the deaths and exposures as entirely independent chance distributions. But, whatever the cause, they saw that, notwithstanding the fact that the heavy mortality in group 18 to 32 was due almost entirely to the early ages at entry, 4·3 deaths had been shifted from that group—right across the light-mortality group 28–32—into the central group, 33–47. He was aware that Milne's graphic method dealt with the deaths and exposures independently in a somewhat similar way; but he ventured to think that in the application of that method to the data under consideration, what Mr. Elderton



called the "taste or fancy of the calculator"—but what he (the speaker) would prefer to call his judgment and his comprehension of the data with which he was dealing—would have intervened to preserve the true character of the experience. The exception which he took to the method of the paper was, that it dealt with the figures too mechanically, not as deaths and exposures, but as irregular series of numbers to be fitted with the latest statistical uniform. Still, the graduation of the totals was a minor matter. By far the most important step in the process, if he understood it correctly, was that briefly disposed of by the author in the following words:—"It is obvious that no good result could be obtained by using each series of figures in Table II, and the assumption made seems reasonable." It was not very clearly stated in the paper what that assumption was, but he understood it to be that the total deaths in the period of five years of each age group, might be distributed in identical proportions over the successive years of assurance. In accordance with that assumption—the total deaths observed being 321, and the adjusted totals for the successive years of assurance being 80, 70, 62, 56 and 53, the total deaths in each group—for example, the 61·7 for ages 18 to 32—had been distributed over the successive years of assurance in the proportion  $\frac{80}{321}$  and so on. It seemed to him that by the side of that boldly empirical hypothesis, the whole apparatus of frequency curves and cubical parabolas shrank into insignificance and relative unimportance. Leaving out of account the exposures (which were large enough to take care of themselves), Table IX depended ultimately on *ten* numbers, namely, the sums of the deaths shown in Table II, and the cross totals shown in Table VIII, without reference to the composition of those numbers. To draw any conclusions of practical value from a table constructed in that way seemed to him to be almost useless. He agreed with Mr. Elderton in his statement that no good result could be obtained by using the separate series of figures in Table II. But ought not that fact to have suggested to him that it would be a useless expenditure of labour to attempt to graduate the experience entirely on its own basis, without reference to any other knowledge one might have of the progression of mortality in successive years of assurance? His own feeling was that the proper method of dealing with an experience of that sort was that referred to by the writer in a footnote on page 502, namely, Mr. Lidstone's method of graduation by reference to a larger experience. The data under consideration, as they stood, told them little or nothing about the progression of the rate of mortality in the successive years of assurance. The most they did was to indicate the general nature of the mortality under that particular class of assurances. That being so, the obvious course was to go to some other source for information in regard to the progression of the rate of mortality, to find some suitable standard by which to test the mortality under that class of assurances. He thought they had, in the New Experience, a standard suitable for that purpose. He had calculated the expected deaths under the temporary assurances, in successive years of assurance, by the whole-life non-profit ungraduated experience. Omitting the initial group, he found that the actual deaths were 282, expected

deaths 303 nearly. Looking at the successive years of assurance, in the first year the expected deaths by the non-profit experience were 74 and the actual deaths 71. In the second year the expected were 71 and the actual 59; in the third year the expected and actual deaths both came out at 59; in the fourth year they were 52 and 45 respectively; in the fifth 53 and 45, showing a fair agreement except in the second year of assurance. This seemed to him to indicate that the first step in dealing with the temporary assurance experience should be to graduate the non-profit experience.

Mr. H. P. CALDERON said there were one or two points that had not yet been alluded to about which he would like to hear some clearer statement than the paper afforded. In the first place, in trying to make a series of what the author called a chance distribution fit a series already showing some degree of regularity, it was perfectly possible, given a certain number of constants in the formula for such distribution, to put the exposures and deaths into two separate series (as the author had done) of chance distributions, and then, according to the number of unknown terms in the expression for the frequency curve, there were so many points of agreement which could be obtained. In looking at the series employed, it was not quite clear to him, seeing that Mr. Elderton used a type of curve with four unknowns, what four separate points of agreement the author had employed in finding his respective constants. He would be very glad if some further information on that point could be given. In the next place, Mr. Elderton said, on page 502 and on page 508, that he had grouped all the years of assurance in his first age-groups, and had then graduated the exposed to risk in the several years by a cubic parabola. There were five years of assurance, whilst the cubic parabola had only four unknowns; with only four unknowns it was naturally impossible to make such cubic parabola exactly fit the five several years, and they were not told with sufficient fulness what equations had been used in finding the constants on page 509. It was also not quite clear from the paper, and perhaps Mr. Elderton would make it so in his reply, as to whether  $a, b, c, d$ , were found separately for each of the five groups of fifteen years of age, or whether one uniform series was obtained for the whole. That seemed to him to be a most important point, because if the exposures to risk in successive years were treated by the same cubic parabola in successive age-groups, the constants being the same, a uniform rate of lapse at each age would be the resulting assumption, which he thought would hardly be justified by an analysis of the experience. It was, of course, possible to obtain a uniform series to represent the exposures, and an independent uniform series to represent the deaths, and the rates of mortality resulting would likewise become a uniform series; but it appeared to him better to adopt a formula for the deaths whereby the rate of mortality would be incorporated into the expression for the death curve, and connecting it with the exposed curve, so that the two were graduated together rather than separately. He thought Tables XI and XII would be better if set out as a table of two dimensions, so that one could compare the results at successive ages for each of the different years. It was not easy to see how these deviations really ran in the successive years as the table was at present set out.



Mr. G. J. LIDSTONE said he thought that some of the previous speakers had done somewhat less than justice to Mr. Elderton in regard to the processes employed by him. Anyone who was in the least familiar with Professor Pearson's papers would know that the points of agreement were that the successive "moments," as he termed them, were identical in the graduated curve which he produced, and in the original data from which he started. He (Mr. Lidstone) did not wish to express an opinion as to how far the methods were applicable to assurance work. He quite agreed with Mr. Todhunter that, if one were not careful, these waves of frequency curves would carry one into very deep water indeed; but he thought it was desirable for someone to intervene in the discussion to point out that there was a real scientific basis in the way in which they were fitted, whether they were the best curves for the purpose or not. He gathered, with reference to Mr. Elderton's cubic parabolas, that a separate one had been fitted for each different group of entry ages, though Mr. Calderon rather spoke as though the same form of parabola applied to each age at entry. Mr. May had remarked, in reference to Table X, that Mr. Elderton's assumption that selection was exhausted after five years was conclusively disproved by Table X. He (Mr. Lidstone) thought Mr. Elderton had made a mistake in his assumption, for, if one thing was clearer than another in the new experience, it was that the selection was not exhausted after five years, and not fully even after ten years. But he did not think this was proved by Table X, as Mr. May suggested. The ages at the head of the columns in that table were *ages at entry*; and, therefore, taking successive years of assurance, there was not only a wearing-out of selection, but also an increased *age* year by year; and that must inevitably result in a steadily increasing rate of mortality. Some of those misunderstandings lent additional weight to what Mr. King had said as to the desirability of Mr. Elderton favouring the Institute with a further paper dealing with the elementary theory of Karl Pearson's frequency curves. He would not, as Mr. King had done, ask Mr. Elderton to put the matter into a simple mathematical form, as he believed that to be impossible. But Mr. Elderton might very well omit some of the more difficult demonstrations, giving the principal results likely to be of service to members of the Institute. The very wealth of physical illustration in Professor Pearson's papers was rather a disadvantage to the actuarial student.

Mr. T. G. ACKLAND wished to deal for a moment with the problem which Mr. Elderton had attempted. It seemed to him, as to other speakers, that there was special difficulty in dealing with the data for temporary assurances. The published tables setting forth the aggregate experience of the sixty contributing offices in this class were rather disappointing, and the more one probed it, the less one was satisfied with the results brought out by that experience. The extraordinary wave of variation in the mortality, as indicated at different grouped ages at entry, was somewhat difficult to deal with and to account for; and it seemed to him that, in attempting to graduate that particular experience, Mr. Elderton had bravely tackled a very difficult question; and one could not be altogether surprised if his strictly scientific methods did not always produce what would,



perhaps, be considered the best results. As to the methods which Mr. Elderton adopted for dealing with the particular problem, he had been very much interested, like Mr. King, in that matter. When one remembered that the  $O^M$  Table, which would doubtless become in due course a standard table for valuations, was graduated by modifying the Makeham curve of the  $O^{M(5)}$  Table, by the addition of a double frequency curve (he thought of type No. VII in Mr. Elderton's Table III), it was very important that actuaries should have some measure of intelligent acquaintance with the matter. He dissented from Mr. Elderton, and also from Mr. May, in thinking that Makeham's formula was "out of the question" in such a case as that under consideration; nor did he agree with the remark, made later in the paper, that the graduation of select tables was a question which actuaries were at present unable to deal with. He would be most interested to see the application of some such method as that devised by Mr. G. F. Hardy, who had indicated a method of applying Makeham's formula in its first development, to the construction of select tables. In a few days the members would have before them an example of the graduation of an annuity experience in the form of a select curve extending over the first five years, and then merging into an ultimate curve, with progressive variations in the values of the constants  $A$  and  $B$ , the value of the constant  $c$  being uniform throughout. It would be of great interest to see that method applied to other classes of assurance, and perhaps to some where the material was more tractable than in the case of temporary assurances. One would then see exactly how those constants varied at different durations of the assurances, and there might be a good deal of teaching in the results so brought out. He hoped that some of the younger members would work out the matter on these lines for the benefit of the Institute generally. Before the Second Congress in Brussels, Dr. Karup said that, in his opinion, the day had entirely gone past in which Makeham's graduation would be practically applied in the graduation of tables; and it was a singular commentary upon this statement that the annuity tables just about to be produced were graduated by Makeham's curve, and that the select assurance tables which were preparing for publication were also graduated by Mr. G. F. Hardy by a most beautiful example of the application of the same formula, extending over ten years, and then merging into an ultimate table, the principle of uniform seniority being preserved throughout. He would not make much comment upon the application of Mr. Elderton's methods, since other speakers had dealt pretty fully with the subject. But he entirely agreed with the remarks which had been made as to the method of dealing with the deaths. He feared that was the one point in which the author laid himself most open to criticism. Perhaps it was difficult to deal with the matter in any other way, but the grouping of the deaths, the objections to which had been so clearly indicated by Mr. Todhunter, seemed to him to have forced the numbers representing the deaths into a curve which had very little regard for the original distribution of the facts. He sympathized with Mr. Elderton in the difficulty he had experienced in the matter; and with such paucity of data it was perhaps impossible for him to have done otherwise. It would,

however, have been interesting to see the resulting series of graduated deaths both at ages at entry and durations of assurance. Amongst the many things he did not understand as to the formulas and methods employed in the paper, was one which he would very much like Mr. Elderton to throw some light upon in his reply, namely, as to the precise significance of  $P$ , which purported to test the accuracy of the graduated curve. It appeared to him that throughout the paper the value of  $P$ , as deduced, varied from  $\cdot 09$  as a minimum up to  $\cdot 98$  as a maximum. That was supposed to represent the "improbability" of something or other, and Mr. Elderton expressed himself as being entirely satisfied with the result, whatever the value of  $P$  might happen to be. Mr. Elderton referred in his paper to the table given in the volume for ungraduated data for whole-life assurances, setting forth the select rates of mortality, and attempted some comparisons of that table with the rate shown in the class of term assurances. In a paper which he (Mr. Ackland) had the honour to submit to the Institute in March last, there was included in the appendix Table 39, which set out the graduated rate of mortality, as deduced by Mr. G. F. Hardy for the Select Assurance Tables in the first ten years. That would furnish the author of the paper, in a better form, with graduated results perhaps more suitable as a basis for comparison. With regard to the resulting term premiums themselves, he had been interested in comparing his own results, in the paper to which he had referred, with Mr. Chatham's, as submitted in March last to the Faculty of Actuaries, and also with those got out by Dr. Sprague from the  $H^M$  Table. It was very satisfactory that, on the whole, there was very little difference between those results. It was sometimes a little difficult to compare them, because the ages at entry were grouped in different ways, and Mr. Chatham adopted a rate of interest as the basis for his premiums different from that adopted by Mr. Elderton and himself. Where, however, one could compare the results there was a very close similarity; and whatever criticisms might properly be offered as to Mr. Elderton's methods, it appeared to him at any rate his results were not very far out. Mr. Chatham seemed somewhat to exaggerate the premiums at the early ages, especially for short terms. He also seemed to exaggerate the premiums for older ages, especially where the terms were long; the two ends of his curve perhaps departing a little from the data in this respect. Mr. Elderton's results, however, agreed very satisfactorily with the others, and with the rates deduced from the ungraduated data. He (Mr. Ackland) had arrived at the conclusion that the best way, if one wished to get out the net annual premiums for these term assurances, was to take the ungraduated data for each separate group of entry ages, to deduce the net premiums from those data by the ordinary formula, and then to group the results for ages at entry in fifteen-year groups, giving to each result a weight corresponding to the number of entrants or exposures included in the particular groups. He had tested that method, and found it to give results which on the whole closely approximated to those arrived at by Mr. Elderton, and also, he thought, to those published in his own paper to which he had referred.



The PRESIDENT said the meeting could congratulate Mr. Elderton upon having provoked a very interesting discussion. He did not himself intend to say anything on the merits of the paper, but it was very consolatory to him to find that some of the speakers, and those not the least learned, had confessed to a certain amount of ignorance of the methods dealt with by Mr. Elderton, and he feared that if he spoke on those methods he might display a greater ignorance. He hoped the author would not fail to act upon the suggestion which had been made as to giving the Institute some more light on such subjects as frequency curves and curves of error. As far as he had been able to follow the discussion, it seemed that a feature of the paper which had been subjected to criticism was that it was an attempt to apply an elaborate method to a very small numerical basis, and it would perhaps be well if Mr. Elderton would try his method on something larger, such as the as yet unadjusted non-profit tables of the new mortality experience.

Mr. ELDERTON, in reply, feared he had made one or two unfortunate remarks in his paper. One of those was that in applying the Graphic method of graduation, too much was left to the taste or fancy of the calculator. He regretted he could not withdraw the statement; he believed it to be true. When he plotted out the values of  $q_x$  he felt it would be quite wrong to rely on drawing a simple curve through them without any algebraic formula at all. He thought many of the criticisms arose because he had not sufficiently explained the curve fitting. It was, of course, quite true, as Mr. Lidstone had said, that he had used the method of moments; but it would be necessary to go through a rather lengthy explanation to show exactly how everything was done in connection with the method. With regard to using one cubic parabola for the deaths, he adopted that course only as an approximation to the method used for the exposures. He did not think it was right to use only one curve, but looking at the series of deaths he did not see that he could have done otherwise. It was only meant as an approximation to the larger method. Mr. Todhunter had said it was of no use relying on the results, or had used words to that effect, while, on the other hand, Mr. Ackland had said he found the results agreed with his own. With regard to the values of  $P$ , he must refer Mr. Ackland to Professor Pearson's paper. But if any of the members had been listening to Mr. Bowley's lectures he thought some of their difficulties as to the divergent values of  $P$  would have disappeared; in fact, if he had done, as Mr. Bowley had, in testing the fit of many of his curves, namely, reduced the numbers to something like one thousand and then applied tests as to goodness of fit, it would have been found that his probabilities came out at something like .99. Mr. Calderon made a mistake when he said there were five constants in the Type I curves used in the paper, for there was a note to Table III which took away one of them; and the  $y$ 's could practically be discarded. There were  $a_1, a_2$ , and then a common factor  $n$ , where  $a_{1,m1} = a_{2,m2} = n$ . Mr. Ackland had referred to his paper. Perhaps he (Mr. Elderton) ought to have put a footnote to his paper saying that it was written about fifteen months ago, before Mr. Ackland's paper was published, but he had not thought it necessary to re-write his paper, which he would have had to do in order to refer to subsequent work.



*On the Graduation of the British Offices Annuity Experience, (1863-1893), by the Graphic Method, by JAMES CHATHAM, F.I.A., F.F.A.\**

THE Joint Committee on Mortality Investigation asked Dr. Sprague some time ago to graduate the Annuity Experience by the Graphic Method with the view of determining the period during which selection lasts ; but as he was too fully occupied to allow of his personally undertaking the proposed graduation, the work was undertaken by the writer on principles arranged in consultation with Dr. Sprague.

As the investigation was to a great extent experimental, the female annuity experience, in which class the observations are most numerous, was dealt with first. The numbers entering at each age are, comparatively speaking, small, and the numbers at risk and the deaths, for five ages at entry, were therefore added together for each of the 20 years following the date of purchase ; the groups so dealt with being those for ages at entry 50-54, 55-59 . . . 80-84, and the results have been treated as if they were for ages at entry 52, 57 . . . 82. The numbers at risk and the deaths thus obtained were then grouped in the manner described in Dr. Sprague's Paper on "The Graphic Method of Adjusting Mortality Tables" (*J.I.A.*, xxvi, 77), so as to bring out the law which they follow. The results showed that, although selection endures at some ages at entry for a longer period than 10 years, yet for all practical purposes it may be taken as ceasing after that period. As a rule, the period diminishes as the age at entry becomes greater. The graduation was therefore proceeded with on the assumption that the period during which selection endures was 10 years, and the ultimate curve formed.

What may be called the initial curve, showing the rate of mortality in the year immediately following the date of purchase, was so regular that very little adjustment of it was necessary. The partial curves showing how the adjusted mortality proceeds during the 2nd, 3rd . . . 10th years after date of purchase were then formed. The high death-rate in the second year after entry is a feature of the observations, as it occurs in most of the partial curves. A reference to the observations showed that in four of

\* This memorandum was prepared at the request of the Joint Committee of the Institute and the Faculty, and space is gladly found for it in the *Journal of the Institute of Actuaries*.—ED. *J.I.A.*

the seven groups of ages at entry, the death-rate for the second year after entry is actually higher than the rate for the third year, while in a fifth it is only a little less. The rates of mortality during each year following purchase were thus obtained for quinquennial ages at entry.

As the assumed age at entry was not the true average age, before interpolating for the intermediate ages, it was necessary first of all to correct the rates so as to obtain the rates during each year for exact quinquennial ages at entry. This was done by first differences. Dr. Sprague's formula of interpolation (*J.I.A.*, xxii, 270) was then applied to the corrected rates so found, and the rates at intermediate ages obtained. When the data were too scanty to admit of its application, the interpolation proceeded on the assumption of constant fourth or third differences, as there explained. It was found necessary, however, to slightly adjust some of the figures thus obtained, in order to have a smoother progression in the rates of mortality.

The male annuity experience was dealt with in a similar manner. The regularity in the rates observed among the female annuitants was not experienced in this instance, as the numbers at risk and the deaths are much smaller, being only about one-third; but the same features were found to be common to both, namely, a rapid increase in the rate of mortality immediately after entry, and a slackening of it afterwards, although they were not so pronounced as in the case of the females.

Tables of  $l_{[x-t]+t}$ ,  $d_{[x-t]+t}$ ,  $p_{[x-t]+t}$ ,  $q_{[x-t]+t}$ ,  $e_{[x-t]+t}$ , for both males and females are appended.

The rates of Mortality for males and females may now be compared, and this is done in the following Table for quinquennial ages :

*New Annuity Experience—Males and Females.*

Age <i>x</i>	FIRST YEAR $q[x]$			ULTIMATE $q_x$			DIFFERENCE BETWEEN FIRST YEAR & ULTIMATE $\times 10,000$	
	Males	Females	Difference $\times 10,000$	Males	Females	Difference $\times 10,000$	Males	Females
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
52	·0100	·0085	15	·0224	·0152	72	124	67
57	·0146	·0098	48	·0314	·0172	142	168	74
62	·0228	·0139	89	·0417	·0236	181	189	97
67	·0336	·0214	122	·0524	·0371	153	188	157
72	·0479	·0341	138	·0719	·0561	158	240	220
77	·0743	·0529	214	·1112	·0861	251	369	332
82	·1055	·0761	294	·1653	·1355	298	598	594

This is a severe test of the method of graduation adopted, but the differences show that the rates proceed with great regularity. The superiority of female annuitant life to male is well-known, and is clearly shown in the above Table ; but it is not so generally known that, as shown by the last two columns, the difference between the first year's rate and the ultimate is greater in the case of males than females. If instead of the difference between these two rates the ratio is compared, the above feature still holds good at the lower ages, but not at the higher. The difference between the two sexes for the first year, given in the fourth column, steadily increases throughout, and is in every case less than the difference between the ultimate rates in the seventh column.

It will be interesting to see how the Government Annuity Rates compare with those of the new experience, and the same particulars relating to them are given in the following Table :

*Government Experience (1883)—Males and Females.*

Age $x$	FIRST YEAR $q[x]$			ULTIMATE $q_x$			DIFFERENCE BETWEEN FIRST YEAR AND ULTIMATE $\times 10,000$	
	Males	Females	Difference $\times 10,000$	Males	Females	Difference $\times 10,000$	Males	Females
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
52	·0152	·0082	70	·0229	·0151	78	77	69
57	·0196	·0093	103	·0263	·0194	69	67	101
62	·0252	·0125	127	·0301	·0248	53	49	123
67	·0324	·0220	104	·0490	·0375	115	166	155
72	·0422	·0359	63	·0734	·0565	169	312	206
77	·0562	·0523	39	·1085	·0921	164	523	398

The rates for Government males in the first year are very much higher at the younger ages, and considerably less at the older ages, than in the new experience ; but this is probably due to the method of adjustment adopted. It is hardly likely that the rate for males at age 52 at entry is 85 per-cent greater than that for females, while at age 77 it is only  $7\frac{1}{2}$  per-cent greater. The ultimate rate for males is, as was to be expected, almost invariably less than that of the new experience, because it applies to four years and upwards from purchase ; but at age 62 it is very considerably less, and this is due to a sudden break in the curve of mortality at that age, caused probably by the joining of two different methods of graduation. The rates for females, on



the other hand, agree very closely with those of the new experience, notwithstanding the greater period during which selection has been given effect to in it. The result is, however, that the differences between the two sexes do not run regularly—for instance, those in the fourth column attain a maximum and then diminish; but on the whole the results of the new experience are confirmed.

It will also be interesting to see how the rates of Mortality among assured lives compare with those of annuitants. The graduated rates for the former are not published yet; but it will be sufficient to use in the meantime rates obtained from the Unadjusted Whole-Life Assurance Experience recently published, relating to males and females who effected policies with participation in profits. The method by which they were obtained is similar to that employed in the New Annuity Experience, and they are given in the following Table along with those for annuitants. The ultimate rate for assured lives has been taken from the “Aggregate Tables”, excluding the first ten years of assurance.

*New Experience—Annuitants and Assured Lives.*

Age $x$	FIRST YEAR $q[x]$			ULTIMATE $q_x$			DIFFERENCE BETWEEN FIRST YEAR AND ULTIMATE	
	Annui- tants	Assured Lives	Difference $\times 10,000$	Annui- tants	Assured Lives	Difference $\times 10,000$	Annui- tants	Assured Lives
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
MALES								
52	·0100	·0091	9	·0224	·0171	53	124	80
57	·0146	·0123	23	·0314	·0241	73	168	118
62	·0228	·0172	56	·0417	·0332	85	189	160
67	·0336	·0232	104	·0524	·0487	37	188	255
72	·0479	·0366	113	·0719	·0745	-26	240	379
77	·0743	·0266	477	·1112	·1091	21	369	825
82	...	...	...	·1653	·1629	24	...	...
FEMALES								
52	·0085	·0089	- 4	·0152	·0190	- 38	67	101
57	·0098	·0125	-27	·0172	·0228	- 56	74	103
62	·0139	·0186	-47	·0236	·0280	- 44	97	94
67	·0214	·0225	-11	·0371	·0441	- 70	157	216
72	·0341	·0312	29	·0561	·0647	- 86	220	335
77	...	...	...	·0861	·0952	- 91	...	...
82	...	...	...	·1355	·1564	-209	...	...

Dealing first with males, it will be seen that both the first year's and the ultimate rate among assured lives, is, with one exception, less than that for annuitants, showing that the medical examination in the case of assured lives is much more efficacious than the self-selection in the case of annuitants. With females it is exactly the opposite. In their case the rate, whether first year's or ultimate, is, with one exception, greater among assured lives than annuitants. These results may be somewhat modified when the adjusted rates are obtained, but it is probable that the rates for males and females will be similarly affected.





BRITISH OFFICES ANNUITY EXPERIENCE, 1863-1893.

*Graduated by the Graphic Method.*—FEMALES.

Age attained	YEARS ELAPSED SINCE PURCHASE										
	0	1	2	3	4	5	6	7	8	9	10 and upwards
$x$	$l_x$	$l_{[x-1]+1}$	$l_{[x-2]+2}$	$l_{[x-3]+3}$	$l_{[x-4]+4}$	$l_{[x-5]+5}$	$l_{[x-6]+6}$	$l_{[x-7]+7}$	$l_{[x-8]+8}$	$l_{[x-9]+9}$	$l_x$
50	100000	...	...	...	...	...	...	...	...	...	102750
51	98401	99159	...	...	...	...	...	...	...	...	101210
52	96857	97571	98078	...	...	...	...	...	...	...	99683
53	95389	96037	96505	96863	...	...	...	...	...	...	98168
54	93990	94580	94986	95308	95550	...	...	...	...	...	96656
55	92640	93177	93545	93806	94013	94202	...	...	...	...	95139
56	91142	91816	92157	92383	92529	92679	92847	...	...	...	93607
57	89495	90296	90778	91012	91126	91201	91327	91485	...	...	92053
58	87712	88622	89220	89607	89741	89782	89842	89954	90103	...	90471
59	85773	86802	87490	87995	88288	88349	88383	88438	88532	88671	88842
60	83680	84824	85609	86201	86612	86824	86880	86908	86954	87036	87155
61	81424	82686	83562	84246	84742	85067	85218	85265	85283	85316	85385
62	79037	80380	81352	82120	82705	83109	83357	83462	83491	83503	83524
63	76528	77940	78975	79833	80495	80980	81294	81476	81542	81549	81553
64	73923	75375	76463	77378	78125	78683	79066	79296	79418	79446	79449
65	71234	72713	73828	74793	75594	76229	76678	76959	77112	77177	77193
66	68487	69963	71095	72088	72936	73622	74140	74475	74662	74752	74784
67	65692	67152	68276	69288	70160	70891	71458	71852	72076	72190	72234
68	62913	64288	65394	66403	67293	68047	68656	69096	69363	69501	69554
69	60152	61437	62458	63459	64343	65113	65749	66225	66527	66688	66751
70	57413	58602	59533	60459	61337	62098	62752	63254	63583	63756	63828
71	54733	55788	56621	57470	58279	59028	59675	60194	60541	60717	60790
72	52026	53031	53727	54493	55227	55907	56537	57051	57409	57582	57647
73	49247	50249	50879	51523	52178	52787	53343	53838	54193	54361	54414
74	46454	47396	47999	48578	49126	49660	50140	50563	50900	51062	51100
75	43636	44540	45046	45580	46085	46518	46918	47264	47538	47694	47717
76	40787	41674	42092	42497	42976	43373	43671	43934	44144	44265	44276
77	37739	38794	39135	39409	39775	40149	40412	40568	40711	40787	40792
78	34423	35741	36175	36320	36555	36829	37072	37181	37238	37279	37279
79	31025	32454	33070	33240	33328	33484	33645	33716	33747	33761	33761
80	27650	29112	29776	30050	30104	30132	30213	30250	30266	30266	30266
81	24342	25817	26467	26726	26791	26793	26817	26827	26834	26834	26834
82	21142	22610	23241	23437	23502	23504	23504	23504	23504		

## BRITISH OFFICES ANNUITY EXPERIENCE, 1863-1893.

*Graduated by the Graphic Method.—MALES.*

Age attained	YEARS ELAPSED SINCE PURCHASE										
	0	1	2	3	4	5	6	7	8	9	10 and upwards
$x$	$d_x$	$d_{[x-1]+1}$	$d_{[x-2]+2}$	$d_{[x-3]+3}$	$d_{[x-4]+4}$	$d_{[x-5]+5}$	$d_{[x-6]+6}$	$d_{[x-7]+7}$	$d_{[x-8]+8}$	$d_{[x-9]+9}$	$d_x$
50	883	...	...	...	...	...	...	...	...	...	2040
51	910	1256	...	...	...	...	...	...	...	...	2160
52	941	1280	1585	...	...	...	...	...	...	...	2256
53	978	1307	1613	1881	...	...	...	...	...	...	2366
54	1018	1336	1636	1917	2068	...	...	...	...	...	2474
55	1063	1375	1664	1941	2113	2170	...	...	...	...	2578
56	1117	1419	1691	1961	2147	2224	2322	...	...	...	2681
57	1176	1470	1725	1978	2170	2268	2368	2466	...	...	2779
58	1241	1531	1764	2000	2191	2305	2402	2509	2634	...	2863
59	1313	1596	1811	2025	2210	2335	2427	2536	2672	2814	2932
60	1387	1671	1866	2058	2230	2363	2448	2557	2695	2852	2997
61	1459	1746	1930	2098	2252	2388	2468	2574	2710	2872	3047
62	1527	1818	1994	2148	2280	2411	2488	2590	2720	2881	3081
63	1591	1880	2057	2202	2314	2435	2509	2607	2729	2882	3100
64	1649	1931	2113	2219	2351	2461	2536	2624	2738	2883	3108
65	1699	1975	2160	2289	2385	2488	2567	2649	2752	2884	3100
66	1735	2008	2203	2321	2412	2513	2599	2679	2769	2889	3080
67	1759	2032	2236	2345	2434	2535	2630	2715	2793	2900	3060
68	1776	2046	2257	2363	2452	2554	2656	2745	2818	2915	3049
69	1778	2057	2266	2373	2466	2570	2676	2768	2846	2935	3050
70	1777	2061	2271	2375	2473	2583	2692	2786	2874	2959	3058
71	1779	2061	2269	2381	2472	2590	2702	2801	2899	2982	3076
72	1787	2056	2260	2382	2474	2591	2705	2808	2915	3002	3100
73	1806	2051	2251	2380	2476	2596	2701	2808	2920	3017	3126
74	1823	2057	2240	2374	2475	2595	2698	2799	2910	3018	3142
75	1828	2057	2241	2362	2468	2586	2689	2790	2881	3005	3142
76	1813	2044	2237	2361	2455	2564	2669	2774	2898	2973	3116
77	1776	2011	2223	2355	2451	2527	2635	2746	2906	2957	3057
78	1711	1957	2187	2337	2439	2543	2583	2700	2881	2908	2961
79	1623	1877	2130	2300	2413	2551	2581	2633	2801	2810	2828
80	1513	1778	2016	2243	2363	2518	2531	2559	2658	2658	2658
81	1382	1662	1943	2155	2287	2412	2416	2424	2459	2459	2459
82	1232	1529	1822	2045	2142	2236	2236	2236	2236	2236	2236
83	...	1380	1682	1904	1951	1998	1998	1998	1998	1998	1998
84	...	...	1524	1725	1740	1756	1756	1756	1756	1756	1756
85	...	...	...	1515	1515	1515	1515	1515	1515	1515	1515
86	...	...	...	...	1282	1282	1282	1282	1282	1282	1282
87	...	...	...	...	...	1067	1067	1067	1067	1067	1067
88	...	...	...	...	...	...	870	870	870	870	870
89	...	...	...	...	...	...	...	698	698	698	698
90	...	...	...	...	...	...	...	...	551	551	551
91	...	...	...	...	...	...	...	...	...	426	426
92	...	...	...	...	...	...	...	...	...	...	323
93	...	...	...	...	...	...	...	...	...	...	241
94	...	...	...	...	...	...	...	...	...	...	177
95	...	...	...	...	...	...	...	...	...	...	126
96	...	...	...	...	...	...	...	...	...	...	89
97	...	...	...	...	...	...	...	...	...	...	62
98	...	...	...	...	...	...	...	...	...	...	41
99	...	...	...	...	...	...	...	...	...	...	27
100	...	...	...	...	...	...	...	...	...	...	19
101	...	...	...	...	...	...	...	...	...	...	14
102	...	...	...	...	...	...	...	...	...	...	8

BRITISH OFFICES ANNUITY EXPERIENCE, 1863-1893.

*Graduated by the Graphic Method.*—FEMALES.

[illegible]





BRITISH OFFICES ANNUITY EXPERIENCE, 1863-1893.

*Graduated by the Graphic Method.*—FEMALES.

Age attained	YEARS ELAPSED SINCE PURCHASE										
	0	1	2	3	4	5	6	7	8	9	10 and upwards
$x$	$P[x]$	$P[x-1]+1$	$P[x-2]+2$	$P[x-3]+3$	$P[x-4]+4$	$P[x-5]+5$	$P[x-6]+6$	$P[x-7]+7$	$P[x-8]+8$	$P[x-9]+9$	$P_x$
50	99158	...	...	...	...	...	...	...	...	...	98500
51	99156	98909	...	...	...	...	...	...	...	...	98490
52	99154	98907	98761	...	...	...	...	...	...	...	98480
53	99151	98905	98759	98644	...	...	...	...	...	...	98460
54	99136	98905	98757	98642	98590	...	...	...	...	...	98430
55	99110	98905	98757	98640	98580	98561	...	...	...	...	98390
56	99072	98870	98757	98640	98565	98541	98532	...	...	...	98340
57	99023	98807	98709	98603	98525	98511	98497	98490	...	...	98280
58	98963	98724	98628	98529	98448	98443	98437	98420	98410	...	98200
59	98893	98625	98526	98429	98343	98337	98330	98322	98310	98290	98100
60	98812	98513	98407	98308	98216	98150	98140	98129	98117	98103	97970
61	98718	98388	98274	98171	98072	97989	97939	97919	97912	97900	97820
62	98613	98252	98131	98022	97916	97817	97743	97700	97675	97666	97640
63	98495	98105	97978	97862	97749	97637	97541	97475	97430	97425	97420
64	98363	97946	97816	97694	97574	97451	97335	97246	97179	97165	97160
65	98216	97775	97643	97516	97391	97259	97127	97015	96939	96899	96880
66	98050	97588	97458	97326	97197	97059	96914	96779	96690	96631	96590
67	97863	97383	97257	97120	96989	96848	96693	96537	96427	96348	96290
68	97654	97154	97039	96897	96762	96622	96459	96283	96143	96043	95970
69	97423	96901	96799	96656	96512	96374	96206	96010	95833	95711	95620
70	97169	96621	96533	96393	96237	96096	95923	95711	95492	95347	95240
71	96890	96306	96240	96096	95932	95780	95602	95375	95114	94944	94830
72	96586	95943	95898	95753	95583	95413	95228	94990	94692	94497	94390
73	96240	95522	95478	95348	95175	94984	94788	94543	94223	94000	93910
74	95881	95043	94959	94867	94692	94478	94265	94018	93699	93448	93380
75	95506	94505	94341	94289	94115	93880	93640	93398	93116	92834	92790
76	95115	93907	93625	93595	93423	93172	92894	92663	92395	92153	92130
77	94706	93248	92806	92760	92593	92335	92004	91793	91572	91400	91390
78	94279	92528	91887	91761	91598	91355	90947	90765	90660	90560	90560
79	93834	91749	90868	90568	90411	90229	89911	89768	89685	89650	89650
80	93370	90913	89755	89153	89001	88997	88796	88708	88660	88660	88660
81	92889	90023	88554	87939	87730	87724	87645	87611	87590	87590	87590
82	92392	89085	87275	86689	86457	86450	86450	86450	86450	86450	86450
83	...	88102	85929	85396	85256	85250	85250	85250	85250	85250	85250
84	...	...	84530	84049	83983	83980	83980	83980	83980	83980	83980
85	...	...	...	82640	82640	82640	82640	82640	82640	82640	82640
86	...	...	...	...	81220	81220	81220	81220	81220	81220	81220
87	...	...	...	...	...	79720	79720	79720	79720	79720	79720
88	...	...	...	...	...	...	78140	78140	78140	78140	78140
89	...	...	...	...	...	...	...	76480	76480	76480	76480
90	...	...	...	...	...	...	...	...	74750	74750	74750
91	...	...	...	...	...	...	...	...	...	72960	72960
92	...	...	...	...	...	...	...	...	...	...	71120
93	...	...	...	...	...	...	...	...	...	...	69240
94	...	...	...	...	...	...	...	...	...	...	67330
95	...	...	...	...	...	...	...	...	...	...	65330
96	...	...	...	...	...	...	...	...	...	...	63420
97	...	...	...	...	...	...	...	...	...	...	61430
98	...	...	...	...	...	...	...	...	...	...	59430
99	...	...	...	...	...	...	...	...	...	...	57330
100	...	...	...	...	...	...	...	...	...	...	55000
101	...	...	...	...	...	...	...	...	...	...	52000
102	...	...	...	...	...	...	...	...	...	...	48000
103	...	...	...	...	...	...	...	...	...	...	30000
104	...	...	...	...	...	...	...	...	...	...	...







BRITISH OFFICES ANNUITY EXPERIENCE, 1863-1893.

*Graduated by the Graphic Method.*—FEMALES.

Age attained	YEARS ELAPSED SINCE PURCHASE										
	0	1	2	3	4	5	6	7	8	9	10 and upwards
$x$	$q$	$q[x-1]+1$	$q[x-2]+2$	$q[x-3]+3$	$q[x-4]+4$	$q[x-5]+5$	$q[x-6]+6$	$q[x-7]+7$	$q[x-8]+8$	$q[x-9]+9$	$q_x$
50	00842	...	...	...	...	...	...	...	...	...	01500
51	00844	01091	...	...	...	...	...	...	...	...	01510
52	00846	01093	01239	...	...	...	...	...	...	...	01520
53	00849	01095	01241	01356	...	...	...	...	...	...	01540
54	00864	01095	01243	01358	01410	...	...	...	...	...	01570
55	00890	01095	01243	01360	01420	01439	...	...	...	...	01610
56	00928	01130	01243	01360	01435	01459	01468	...	...	...	01660
57	00977	01193	01291	01397	01475	01489	01503	01510	...	...	01720
58	01037	01276	01372	01471	01552	01557	01563	01580	01590	...	01800
59	01107	01375	01474	01571	01657	01663	01670	01678	01690	01710	01900
60	01188	01487	01593	01692	01784	01850	01860	01871	01883	01897	02030
61	01282	01612	01726	01829	01928	02011	02061	02081	02088	02100	02180
62	01387	01748	01869	01978	02084	02183	02257	02300	02325	02334	02360
63	01505	01895	02022	02138	02251	02363	02459	02525	02570	02575	02580
64	01637	02054	02184	02306	02426	02549	02665	02754	02821	02835	02840
65	01784	02225	02357	02484	02609	02741	02873	02985	03061	03101	03120
66	01950	02412	02542	02674	02803	02941	03086	03221	03310	03369	03410
67	02137	02617	02743	02880	03011	03152	03307	03463	03573	03652	03710
68	02346	02846	02961	03103	03238	03378	03541	03717	03857	03957	04030
69	02577	03099	03201	03344	03488	03626	03794	03990	04167	04289	04380
70	02831	03379	03467	03607	03763	03904	04077	04289	04508	04653	04760
71	03110	03694	03760	03904	04068	04220	04398	04625	04886	05056	05170
72	03414	04057	04102	04247	04417	04587	04772	05010	05308	05503	05610
73	03760	04478	04522	04652	04825	05016	05212	05457	05777	06000	06090
74	04119	04957	05041	05133	05308	05522	05735	05982	06301	06552	06620
75	04494	05495	05659	05711	05885	06120	06360	06602	06884	07166	07210
76	04885	06093	06375	06405	06577	06828	07106	07337	07605	07847	07870
77	05294	06752	07194	07240	07407	07665	07996	08207	08428	08600	08610
78	05721	07472	08113	08239	08402	08645	09053	09235	09340	09440	09440
79	06166	08251	09132	09432	09589	09771	10089	10232	10315	10350	10350
80	06630	09087	10245	10847	10999	11003	11204	11292	11340	11340	11340
81	07111	09977	11446	12061	12270	12276	12355	12389	12410	12410	12410
82	07608	10915	12725	13311	13543	13550	13550	13550	13550	13550	13550
83	...	11898	14071	14604	14744	14750	14750	14750	14750	14750	14750
84	...	...	15470	15951	16017	16020	16020	16020	16020	16020	16020
85	...	...	...	17360	17360	17360	17360	17360	17360	17360	17360
86	...	...	...	...	18780	18780	18780	18780	18780	18780	18780
87	...	...	...	...	...	20280	20280	20280	20280	20280	20280
88	...	...	...	...	...	...	21860	21860	21860	21860	21860
89	...	...	...	...	...	...	...	23520	23520	23520	23520
90	...	...	...	...	...	...	...	...	25250	25250	25250
91	...	...	...	...	...	...	...	...	...	27040	27040
92	...	...	...	...	...	...	...	...	...	...	28880
93	...	...	...	...	...	...	...	...	...	...	30760
94	...	...	...	...	...	...	...	...	...	...	32670
95	...	...	...	...	...	...	...	...	...	...	34670
96	...	...	...	...	...	...	...	...	...	...	36580
97	...	...	...	...	...	...	...	...	...	...	38570
98	...	...	...	...	...	...	...	...	...	...	40570
99	...	...	...	...	...	...	...	...	...	...	42670
100	...	...	...	...	...	...	...	...	...	...	45000
101	...	...	...	...	...	...	...	...	...	...	48000
102	...	...	...	...	...	...	...	...	...	...	52000
103	...	...	...	...	...	...	...	...	...	...	70000
104	...	...	...	...	...	...	...	...	...	...	...





BRITISH OFFICES ANNUITY EXPERIENCE, 1863-1893.

*Graduated by the Graphic Method.*—FEMALES.

Age attained	YEARS ELAPSED SINCE PURCHASE										
	0	1	2	3	4	5	6	7	8	9	10 and upwards
$x$	$e[x]$	$e[x-1]+1$	$e[x-2]+2$	$e[x-3]+3$	$e[x-4]+4$	$e[x-5]+5$	$e[x-6]+6$	$e[x-7]+7$	$e[x-8]+8$	$e[x-9]+9$	$e_x$
50	23·005	...	...	...	...	...	...	...	...	...	22·475
51	22·352	22·200	...	...	...	...	...	...	...	...	21·817
52	21·684	21·542	21·445	...	...	...	...	...	...	...	21·152
53	21·000	20·869	20·780	20·714	...	...	...	...	...	...	20·479
54	20·296	20·179	20·100	20·041	19·999	...	...	...	...	...	19·799
55	19·576	19·473	19·402	19·353	19·317	19·285	...	...	...	...	19·115
56	18·869	18·752	18·689	18·646	18·620	18·595	18·567	...	...	...	18·428
57	18·183	18·046	17·966	17·924	17·903	17·891	17·870	17·844	...	...	17·739
58	17·505	17·362	17·264	17·201	17·178	17·171	17·162	17·143	17·117	...	17·049
59	16·848	16·689	16·586	16·504	16·458	16·448	16·443	16·434	16·418	16·394	16·362
60	16·209	16·037	15·922	15·834	15·768	15·735	15·726	15·722	15·714	15·700	15·679
61	15·585	15·404	15·279	15·180	15·106	15·055	15·032	15·024	15·022	15·016	15·004
62	14·975	14·787	14·656	14·547	14·463	14·403	14·364	14·348	14·343	14·342	14·338
63	14·379	14·186	14·050	13·935	13·841	13·771	13·724	13·696	13·686	13·685	13·685
64	13·791	13·599	13·460	13·340	13·240	13·160	13·105	13·070	13·051	13·047	13·047
65	13·213	13·021	12·884	12·760	12·655	12·569	12·504	12·464	12·440	12·430	12·428
66	12·643	12·453	12·317	12·195	12·085	11·994	11·923	11·874	11·847	11·833	11·828
67	12·076	11·894	11·761	11·638	11·530	11·434	11·357	11·302	11·269	11·253	11·246
68	11·511	11·340	11·214	11·093	10·983	10·888	10·806	10·745	10·708	10·687	10·679
69	10·948	10·788	10·672	10·556	10·448	10·351	10·268	10·203	10·160	10·137	10·127
70	10·387	10·238	10·133	10·025	9·921	9·826	9·741	9·673	9·627	9·602	9·591
71	9·824	9·689	9·596	9·497	9·400	9·309	9·225	9·155	9·106	9·081	9·070
72	9·267	9·139	9·061	8·971	8·883	8·799	8·719	8·650	8·599	8·574	8·565
73	8·723	8·595	8·526	8·448	8·368	8·293	8·222	8·156	8·106	8·081	8·074
74	8·189	8·063	7·998	7·929	7·861	7·793	7·731	7·674	7·626	7·603	7·597
75	7·670	7·541	7·484	7·423	7·358	7·301	7·248	7·201	7·162	7·139	7·136
76	7·169	7·031	6·980	6·933	6·872	6·818	6·777	6·741	6·710	6·692	6·690
77	6·703	6·537	6·488	6·455	6·407	6·356	6·318	6·296	6·274	6·263	6·262
78	6·282	6·077	6·011	5·990	5·959	5·920	5·883	5·867	5·858	5·852	5·852
79	5·892	5·663	5·568	5·541	5·528	5·505	5·480	5·469	5·464	5·462	5·462
80	5·527	5·279	5·173	5·128	5·118	5·115	5·101	5·095	5·092	5·092	5·092
81	5·185	4·919	4·807	4·763	4·751	4·751	4·747	4·745	4·744	4·744	4·744
82	4·867	4·582	4·464	4·428	4·416	4·416	4·416	4·416	4·416	4·416	4·416
83	...	4·267	4·143	4·115	4·108	4·108	4·108	4·108	4·108	4·108	4·108
84	...	...	3·844	3·822	3·819	3·819	3·819	3·819	3·819	3·819	3·819
85	...	...	...	3·547	3·547	3·547	3·547	3·547	3·547	3·547	3·547
86	...	...	...	...	3·292	3·292	3·292	3·292	3·292	3·292	3·292
87	...	...	...	...	...	3·053	3·053	3·053	3·053	3·053	3·053
88	...	...	...	...	...	...	2·830	2·830	2·830	2·830	2·830
89	...	...	...	...	...	...	...	2·622	2·622	2·622	2·622
90	...	...	...	...	...	...	...	...	2·428	2·428	2·428
91	...	...	...	...	...	...	...	...	...	2·248	2·248
92	...	...	...	...	...	...	...	...	...	...	2·082
93	...	...	...	...	...	...	...	...	...	...	1·927
94	...	...	...	...	...	...	...	...	...	...	1·783
95	...	...	...	...	...	...	...	...	...	...	1·648
96	...	...	...	...	...	...	...	...	...	...	1·523
97	...	...	...	...	...	...	...	...	...	...	1·401
98	...	...	...	...	...	...	...	...	...	...	1·281
99	...	...	...	...	...	...	...	...	...	...	1·155
100	...	...	...	...	...	...	...	...	...	...	1·015
101	...	...	...	...	...	...	...	...	...	...	·844
102	...	...	...	...	...	...	...	...	...	...	·624
103	...	...	...	...	...	...	...	...	...	...	·300
104	...	...	...	...	...	...	...	...	...	...	·000



## REVIEW.

*The British Offices Life Annuity Tables, 1893.*

THE Tables now published will form for many years to come the basis of the bulk of the annuity transactions of this country, and they are, therefore, of the utmost importance. They are derived from the unadjusted data already noticed in the *Journal* (*J.I.A.*, xxxv, p. 147), and they comprise the graduated mortality tables and the values of annuities on single lives, and on two joint lives, at various rates of interest. The synopsis at the beginning of the volume shows that the Tables are as complete as one can reasonably expect. The annuity-values on single lives are given at rates of interest ranging from  $2\frac{1}{4}$  per-cent to 5 per-cent. At all these rates, both "select" and "ultimate" values (*i.e.*, after five years from purchase) are published; and, in addition, at  $2\frac{1}{2}$ , 3, and  $3\frac{1}{2}$  per-cent, we have the values one year, two, three, and four years after purchase. Male and Female Lives are of course treated separately throughout. The Two-Life Tables are necessarily less complete. The discrimination of sex renders it impracticable nowadays to prepare tables so convenient as those to which we have been accustomed in Jones's well-known volumes. The result is that one has to interpolate for most things that one requires in the way of joint-life values, and to console oneself with the recognition of the fact that this is inevitable. On one point, however, some may possibly regret the views of the Joint Committee responsible for the present volume. It appears to have been felt that the most important rates of interest are  $2\frac{1}{2}$ , 3, and  $3\frac{1}{2}$  per-cent, and the suggestion is made that annuity-values at 4 and 5 per-cent "are not likely to be very frequently required." No doubt for valuation purposes the  $2\frac{1}{2}$  and 3 per-cent values are essential, and the 3 per-cent probably also for the calculation of rates. It does not appear to us, however, that the  $3\frac{1}{2}$  per-cent values are of the same importance. Under modern conditions  $3\frac{1}{2}$  per-cent is too high a rate both for the calculation of Office Tables and for valuations. On the other hand, 4 and  $4\frac{1}{2}$  per-cent values are of greater importance in every-day office life than would appear from the suggestion quoted above. In the valuation of reversions it has hitherto been customary to adopt either the Carlisle Table or the Government Annuity Experience, 1883. The paper read by Mr. Neil Campbell before the Faculty of Actuaries in Edinburgh shows that both these Tables under-estimate the vitality of life-tenants, and it appears reasonable that in future the severest standard possible, which is afforded by the new Tables now published, should be adopted. For this a more complete set of Tables at 4 and  $4\frac{1}{2}$  per-cent would be necessary.

The graduation has been carried out by Mr. G. F. Hardy and applied to the Select Tables. The Tables for Male Lives, we are told, have been graduated by the application of Makeham's Law. For Female Lives similar methods were adopted, but it was found necessary to introduce a supplementary series. It follows that the law of uniform seniority applies under the Male Table, but not under the Female Table. The problem of the graduation of Select Tables is one of extreme difficulty, and it is another triumph for Mr. Hardy

that he has been successful in applying Makeham's formula to so important a set of Tables. It may safely be said that that "later volume" so often mentioned, which is to give us the detailed account of the methods followed in the graduation, will not be the least interesting of the series. For the present we must defer any comment on this subject, apart from noticing the point that  $\log_{10} c$  is  $\cdot 038$  both for the Male and Female Tables as compared with  $\cdot 039$  in the  $O^{M(5)}$  Table.

An examination of the graduated results proves how dangerous it is to draw conclusions from the unadjusted data. We must confess that it has been a surprise to us to find how widely different are some of the annuity-values now published from the tentative values which appeared on pp. 214-5 of the volume of unadjusted data.

The following table may be interesting:—

*3 per-cent Annuity-Values as at Date of Purchase.*

Age	MALES—B.O.L.A.		Government Experience 1883	FEMALES—B.O.L.A.		Government Experience 1883	Age
	Unadjusted Data	Graduated Result		Unadjusted Data	Graduated Result		
40	16·360	17·604	16·376	18·184	18·257	18·180	40
45	14·600	16·061	15·152	16·993	16·931	16·820	45
50	13·867	14·403	13·813	15·585	15·514	15·271	50
55	12·667	12·660	12·310	13·971	13·965	13·607	55
60	10·862	10·882	10·601	12·158	12·231	11·791	60
65	9·029	9·121	8·902	10·281	10·333	9·909	65
70	7·313	7·441	7·299	8·471	8·406	8·000	70
75	6·095	5·898	5·809	6·644	6·614	6·367	75
80	4·530	4·537	4·553	4·941	5·054	4·937	80

The differences in the Female Table are, perhaps, not greater than might have been expected, but in the Male Table, at ages 40, 45, and 50, the differences can only be described as remarkable. No doubt they are due to the paucity of the data.

Comparing the graduated values with the Government Experience of 1883, we see that the Male Table shows the greater differences below age 55, and the Female Table above that age. Very roughly it may be said that the new Female Table gives select annuity-values equal to those of the 1883 experience at ages one year younger. In the Male Table the difference, at the higher ages, averages about half a year. The effect on reserves in passing from a valuation by the Government Experience to one by the new Tables will depend very much on the relative age distribution of the annuity business.

A fairly average annuity business, comprising 130 annuities for £6,126. 13s. 11d. per annum on Female Lives, and 49 annuities for £3,949. 12s. 2d. per annum on Male Lives, has been valued on both bases with the following result:

		Govt. Experience, 1883	B.O.L.A.
3 per-cent reserve for	Female Annuities	50,965	53,088
3       "              "	Male Annuities	26,942	27,643
Total		77,907	80,731

The additional reserve required by the New Tables is in this case more than  $3\frac{1}{2}$  per-cent.

In the valuation, account was taken, of course, of the date of purchase, and in this connection it may be noted that, in the Government Experience, the "ultimate" values commence with a period of four years only after purchase, as compared with five years in the new Tables.

Turning to the joint-life functions, it will be remembered that, for the ordinary annuity business of an office, these are required principally on account of the *last survivor* annuities. We, therefore, give a small table of last survivor annuities on two lives of equal age, which will give some idea of the difference between the Government Experience of 1883 and the new Tables in this respect.

*3 per-cent. Annuity-Values at Date of Purchase.*

$$a_{[x][x]}$$

Ages	TWO MALES		TWO FEMALES		MALE AND FEMALE	
	Government Experience 1883	B.O.L.A.	Government Experience 1883	B.O.L.A.	Government Experience 1883	B.O.L.A.
40·40	20·204	20·980	21·613	21·764	20·987	21·393
50·50	17·284	17·696	18·530	18·830	17·962	18·302
60·60	13·598	13·886	14·742	15·203	14·213	14·600
70·70	9·703	9·900	10·474	10·935	10·106	10·450
80·80	6·268	6·295	6·724	6·920	6·504	6·620

Probably the average age of the annuitants in an ordinary office who have effected last survivor annuities is well over sixty, and a glance at the above figures would appear to indicate that the addition to the reserve involved in passing to the new Tables will be at least 3 per-cent. A small group of 21 annuities of this kind for £2,029. 17s. 6d. per annum has been valued by both Tables at 3 per-cent, with due allowance for selection, and the comparative results are as follows :—

3 per-cent reserve by Government 1883 Tables	21,764
3 " " B.O.L.A. Tables	22,560

The difference, it will be noticed, is 3·7 per-cent, but this figure will vary with the nature of the business of this class, which any office may have on its books.

In addition to the Two-Life Tables analogous to those based on the Government Experience published by the Institute in 1895, the present volume contains a Table of Uniform Seniority, applicable at all ages to the calculation of joint annuities on two male lives, and, to facilitate the calculation, special tables of select and ultimate annuity-values on two males of equal age are appended, at  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$ , 4, and 5 per-cent, the age being expressed to two decimal places. Two further sets of tables are given, showing, at 4 and 5 per-cent the select and ultimate annuities on two females of equal age, and on a male and a female of equal age, and the volume concludes with a memorandum by Mr. Lidstone, explaining all the Two-Life Tables.



This memorandum is invaluable. It makes as clear as possible the various processes of interpolation necessary for the calculation of such joint-life annuity-values as are not tabulated. Mr. Lidstone's examples are well chosen, and his characteristic thoroughness proved again, if that were necessary, by his Table of the values of 1,000 ( $w_2 - w_1$ ) where at 3 per-cent  $a_{xy} = a_{w_1:w_1}$  and at  $3\frac{1}{2}$  per-cent  $a_{xy} = a_{w_2:w_2}$ . One point only is not touched upon; how should one calculate  $a_{[x]+2:[y]+2}$ , say? What kind of interpolation between the select and ultimate values should one attempt if, in any case, great accuracy were required? The point is not, perhaps, of much practical importance, though it arises when one attempts to value last survivor annuities by analyzed tables, and we should have been glad if it had been possible to make it clear. Of course, it is not within the scope of Mr. Lidstone's memorandum.

An interesting innovation in the present volume calls for mention. The analyzed values  $a_{[x]}$ ,  $a_{[x]+1}$ ,  $a_{[x]+2}$ , and so on, are to be read *horizontally* instead of diagonally down as in previous volumes of select tables. The change appears a distinct improvement, as it greatly facilitates the reading off of successive values.

It need scarcely be added that, as in the  $O^M$  and  $O^{M(5)}$  Tables, the Joint Committee have done everything they could in the way of using different kinds of type, &c., to make the present volume as convenient as possible.

## THE INSTITUTE OF ACTUARIES.

### *Indexes of the JOURNAL.*

IN the preparation of the Card Index of the *Journal* the following errata and misprints were discovered in the two Indexes, for Vols. I to XX, and XXI to XXX, respectively:

#### LIST OF ERRATA.

##### VOLS. I-XX.

1. Page 23, line 15. For xiii-225, read xiii-325.
2. „ 24 „ 15. For ii-118, read ii-115.
3. „ 65 „ 27. For “above,” read “below.”
4. „ 79 „ 12. For viii-245, read viii-241.

##### VOLS. XXI-XXX.

1. Page 57, line 36. For “Re-marriage,” read “marriage.”

#### LIST OF MISPRINTS.

##### VOLS. I-XX.

1. Page 4, line 51. For “H. W. Manby,” read “H. W. Manly.”
2. „ 22 „ 21. For “W. H. Porter,” read “H. W. Porter.”
3. „ 63 „ 39. For “Norwich Union Fire,” read “Union Fire.”
4. „ 63 „ 59. For “United Mntual,” read “United Mutual.”

##### VOLS. XXI-XXX.

1. Page 40, line 14. For “quinquenniad,” read “quinquennial.”

# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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*The Mortality Experience of the Imperial Forces during the War in South Africa, 11 October 1899 to 31 May 1902.*  
 By FREDERICK SCHOOLING, F.I.A., one of the Honorary Secretaries of the Institute of Actuaries and Actuary of the Prudential Assurance Company, and EDWARD A. RUSHER, F.I.A., Assistant-Actuary, Prudential Assurance Company.

[Read before the Institute, 30 March 1903.]

THE subject of War Mortality has not often been before the actuarial profession; there is a reference to the subject in Vol. iv of the *Journal*, and in Vol. vii there appear Mr. W. B. Hodge's papers on "The Mortality arising from Military Occupations", which give statistics ending with the Crimean War. The next reference to the subject is in Vol. xxxiv of the *Journal*, where Messrs. Smee and Ackland's valuable paper is given; this was first published in pamphlet form some years before its appearance in the *Journal*. In the same volume Mr. McLaughlin contributed a paper on general naval and military statistics, dealing principally with the ten years 1886-95, and an abstract of the tables is given in the *Transactions of the Second International Actuarial Congress*. There is also the well-known paper by Mr. A. G. Mackenzie published in the *Transactions of the Actuarial Society of Edinburgh* in 1881; and, finally, last year Mr. A. T. Anderson read a paper before the Actuarial Society of New South Wales which dealt with the mortality during the war now under consideration.

For the purpose of enabling general comparisons to be readily made, we append the following two tables, the first being extracted from Herr Klang's *Kriegs-Versicherungs-Vorlagen*, and the other from Mr. Mackenzie's paper. It should be borne in mind that the rates are for the whole duration of the campaigns unless otherwise specified, and not per annum or for any limited period. It may also be pointed out that Herr Klang's rate of mortality would probably be better defined as loss-rate, as in Mr. Mackenzie's paper, for the Text-Book definition of a rate of mortality certainly does not fit Herr Klang's facts.

*Rates of War Mortality in Different European Campaigns, 1854-1878 (HERR KLANG).*

Campaign and Date	Rate of Mortality
France in the Crimea, 1854-55 . . . . .	·103
France in Campaign of 1859 . . . . .	·056
Italy       "       "       . . . . .	·055
Austria     "       "       . . . . .	·047
North American Army, 1861-65 . . . . .	·054
Prussia in Campaign of 1864 . . . . .	·033
Austria     "       "       . . . . .	·040
Italy       "       1866 . . . . .	·040
Prussia     "       "       . . . . .	·034
Bavaria    "       "       . . . . .	·051
Anstria     "       "       . . . . .	·056
Germany    "       1870-71 . . . . .	·042
Austria     "       1878 (occupation of Bosnia-Herzegovina)	·015
General Average . . . . .	·048

*Comparative Losses from Battle and Disease (MACKENZIE).*

	Losses per 100 from Battle	Proportion of Total Losses from Battle	Losses per 100 from Disease	Proportion of Total Losses from Disease	Total Losses per 100
Crimean War—					
English per annum . . . . .	3·3	·262	9·3	·738	12·6
French       "       . . . . .	3·4	·219	12·1	·781	15·5
American War—					
North, 1st year . . . . .	1·7	·254	5·0	·746	6·7
"     4 years . . . . .	3·9	·345	7·4	·645	11·2
Austro-Prussian War, 1866	1·4	·438	1·8	·562	3·2
Prussian Losses, 7 weeks }					

In this paper no attempt has been made to calculate the probability of war occurring, neither has attention been given to



the interesting question of the proportion of deaths to casualties. It was decided to confine its scope to the mortality experience of all classes of the Imperial Forces, distinguishing deaths by accident or disease from deaths by wounds, the latter class being understood to include all deaths on the battle-field itself.

For this purpose the Imperial Forces were divided into three classes, the combined experience being also given ; in each class the experience of the officers was distinguished from that of the N.C.O's and men.

- (a) Regulars, Volunteers, and Militia.
- (b) Imperial Yeomanry.
- (c) Colonials, including the Forces raised in South Africa.

It was originally intended to give British Volunteers separately, but this was found to be impossible, as, with the exception of the C.I.V.'s, the Volunteers were attached to particular regiments. This fact prevented us obtaining with any degree of accuracy the number of Volunteers exposed to risk, so that the idea of giving a separate return for members of British Volunteer corps who went to South Africa had to be abandoned. The proportion of Volunteers to Regulars was a very small one, and consequently the effect of their inclusion amongst the Regulars would not be great, nor is it probable that their separate experience would yield important results.

Before closing these introductory remarks, we wish to place upon record our cordial thanks to the Secretary of State for War, and to the War Office authorities generally, for their kindness in giving us all the information in their power, and also for letting us have access to their returns and books. Without this help the paper would have lost much of its value, for, although we had records of the deaths, the published official returns did not give the numbers engaged at sufficiently frequent intervals to enable us to obtain with any approach to accuracy the "Exposed to risk."

Previous investigators have been much embarrassed by the large numbers of men returned as missing,—a difficulty which was happily absent on this occasion. In the war under consideration, the final total under the heading of missing and prisoners for the whole period of the war was only 105. Although many more were reported missing at various periods, courts-martial were held and the missing were traced in nearly every case. This may be looked upon as evidence that the sick and wounded were not lost sight of by those whose duty it was to tend them.

In addition to the experience deduced from published returns or particulars supplied to us by the War Office, we were fortunately in a position to trace accurately the experience of nearly 20,000 Imperial Yeomanry and Volunteers who were assured lives, exposed to over 250,000 months of risk of war, having an average duration of 13·3 months. As the ages were recorded in all these cases, the monetary results have been tabulated for seven groups of ages.

#### GENERAL MORTALITY OF IMPERIAL FORCES.

The first requisite was to obtain, if possible, the number engaged for each month of the war; for this purpose the following schedules were kindly filled up by the War Office.

#### SCHEDULE A.

#### *Strength of Forces in South Africa.*

	REGULARS AND VOLUNTEERS		MILITIA		IMPERIAL YEOMANRY		C. I. VOLUNTEERS	
	Officers	N.C.O's and Men	Officers	N.C.O's and Men	Officers	N.C.O's and Men	Officers	N.C.O's and Men
11 October 1899	...	...	...	...	...	...	...	...
1 November	1,869	53,234	...	...	...	...	...	...
1 December	2,764	81,252	...	...	...	...	...	...
1 January 1900	3,218	100,111	...	...	...	...	...	...
1 February	3,714	117,597	181	4,900	84	1,426	56	1,519
1 March	3,971	134,585	603	14,480	284	5,006	56	1,519
1 April	4,387	152,148	753	18,209	417	7,786	56	1,517
1 May	4,478	156,763	745	18,778	521	10,115	56	1,512
1 June	4,636	159,429	765	19,145	522	10,122	56	1,513
1 July	4,586	163,835	770	19,266	523	10,175	56	1,500
1 August	4,619	158,241	760	19,790	523	10,125	55	1,436
1 September	4,642	155,422	751	19,477	526	10,096	55	1,421
1 October	4,732	155,685	745	20,079	526	9,470	55	1,464
1 November	4,715	155,472	725	19,708	516	9,027	...	...
1 December	4,574	153,692	730	19,312	530	8,891	...	...
1 January 1901	4,551	149,801	725	19,081	530	8,024	...	...
1 February	4,550	149,618	715	18,988	530	7,882	...	...
1 March	4,579	150,047	700	18,402	571	10,565	...	...
1 April	4,659	154,824	732	19,367	845	21,982	...	...
1 May	4,746	148,440	779	19,726	960	23,376	...	...
1 June	4,793	145,597	740	17,900	876	20,349	...	...
1 July	4,795	143,422	837	20,734	813	18,603	...	...
1 August	4,801	140,711	727	17,300	793	17,955	...	...
1 September	4,788	141,681	711	17,003	783	17,330	...	...
1 October	4,829	140,492	693	16,452	773	16,962	...	...
1 November	4,750	138,426	665	15,932	773	16,125	...	...
1 December	4,757	139,761	658	15,728	773	15,758	...	...
1 January 1902	4,876	144,581	764	19,550	780	15,149	...	...
1 February	4,869	144,020	814	22,010	630	14,170	...	...
1 March	4,870	146,475	790	21,437	640	14,372	...	...
1 April	4,845	144,640	966	23,840	640	13,720	...	...
1 May	4,916	146,643	886	22,276	700	14,733	...	...
1 June	4,867	146,046	771	20,156	1,000	19,117	..	...

SCHEDULE B.

*Approximate Statement showing Strength of Colonial Forces  
all kinds in South Africa on the date stated.*

Date		Colonials	South African Constabulary
1 December	1899	15,278	...
1 January	1900	18,762	...
1 February		22,796	...
1 March		26,069	...
1 April		26,944	...
1 May		31,037	...
1 June		33,512	...
1 July		35,747	...
1 August		43,150	...
1 September		42,775	...
1 October		42,097	...
1 November		42,097	...
1 December		39,302	...
1 January	1901	30,306	...
1 February		29,450	...
1 March		52,995*	1,467
1 April		54,852	5,634
1 May		56,086	5,234
1 June		54,909	7,401
1 July		54,159	7,430
1 August		53,625	7,461
1 September		52,912	7,516
1 October		51,952	8,743
1 November		51,527	9,117
1 December		51,448	8,883
1 January	1902	49,398	9,116
1 February		46,560	8,935
1 March		47,113	9,081
1 April		48,803	9,316
1 May		48,386	9,364
1 June		45,646	9,372

In cases where the figures on 1st of the month are not available, those for the nearest approximate date are given.

We find, upon enquiry at the War Office, that the numbers given in Schedule A represent, as the heading indicates, the strength of the forces in South Africa, and do not include drafts on their way out or returning. It will be observed that in Schedule B, referring to Colonials, commissioned officers have not been distinguished from N.C.O's and men. An investigation of returns made to Parliament gives the following approximate numbers of officers and men respectively.

\* Includes 10,000 men raised in February locally by General Brabant, and additional oversea Colonials, *i.e.*, Canada, New Zealand, &c.



*Approximate Strength of Forces (Colonials).*

	Officers	Men	Total
1 February 1901 . . .	1,339	27,000	28,339
1 May 1901 . . . . .	2,478	56,343	58,821
1 January 1902 . . . .	2,300	55,000	57,300
	6,117	138,343	144,460

This shows a ratio of one officer to 22·6 men, the percentage of officers in this section to the total Colonial forces being 4·2344. By the use of this ratio, the figures given in Schedule B were divided into officers and men respectively, the resulting numbers being shown in the second column of Table II.

It may be observed here that amongst the Regulars and Volunteers the ratio was one officer to 30·6 men, and amongst the Yeomanry one officer to 20·6 men. Thus it will be seen that the proportion of officers amongst the Colonials approximated closely to that amongst the Yeomanry.

The South African Constabulary were included in Schedule A amongst the "Regulars and Volunteers"; consequently we have excluded them from the Colonials.

The deaths were ascertained from three independent sources:—  
 (a) a register wherein all deaths recorded in official lists supplied to the *Times* newspaper were tabulated in regiments daily from the commencement of the war, a folio being opened for each regiment or distinct corps, the cause of death being stated in each case, and we would here express our thanks to Mr. J. Blennerhassett of the Prudential for planning and keeping this register; (b) a printed list of casualties during the war, supplied by the South African authorities to the War Office; (c) the manuscript returns of casualties comprised in ten thick volumes belonging to the War Office. The card system was adopted as far as the deaths were concerned, a card being written for each of the 21,945 deaths observed. The facts recorded upon the cards were the regimental number, name, regiment, whether commissioned officer or not, date of death and cause of death. The deaths from wounds or on the battle-field were throughout distinguished from those from other causes, the word "Wounds" being employed to denote those from wounds or on the battle-field, and "Illness" those from other causes.

The comparison of these three records proved to be extremely laborious. It will be readily understood by those who have had any experience in compiling statistics that the comparison of the same details recorded by three altogether independent methods must necessarily produce many points for investigation. For example, it was found that the same death was occasionally reported twice over, the reason of this being that it was reported by different officers. Although the labour was great, it was not labour lost, for the results may now be looked upon with perfect confidence, a degree of accuracy having by means of these comparisons been attained which would be altogether impossible if figures were taken from official returns without analysis.

The following Table shows the distribution of these deaths for each calendar month of the war.

Tables II and III show the exposed to risk, the deaths, and the resulting monthly death-rates for each month of the war for each separate class; the exposed to risk being taken as the arithmetic mean of the figures supplied for each month in the first Schedule.

TABLE I.—Deaths from all causes during the War.

Month <i>n</i>	OFFICERS						NON-COMMS. AND MEN					
	Regulars, &c.			Yeomanry			Regulars, &c.			Yeomanry		
	Wounds	Illness	Total	Wounds	Illness	Total	Wounds	Illness	Total	Wounds	Illness	Total
October 11 to 31 1899	21	1	22	...	...	...	146	15	161	...	...	...
November . . .	17	...	17	...	...	...	215	40	255	...	...	...
December . . .	37	...	37	...	...	...	426	86	512	...	...	...
January . . . 1900	35	6	41	...	...	...	417	280	697	...	...	...
February . . .	58	15	73	...	...	...	688	375	1,063	...	...	...
March . . .	15	17	32	...	...	...	238	623	861	1	8	9
April . . .	14	12	26	...	...	...	105	823	928	2	17	19
May . . .	11	38	49	...	...	...	201	1,201	1,402	52	35	87
June . . .	23	16	39	...	...	...	145	937	1,082	25	43	68
July . . .	15	11	26	...	...	...	151	514	665	17	36	53
August . . .	7	2	9	...	...	...	110	236	346	11	18	29
September . . .	3	1	4	...	...	...	53	173	226	10	18	28
October . . .	15	4	19	...	...	...	126	195	321	14	22	36
November . . .	8	6	14	...	...	...	99	287	386	12	25	37
December . . .	14	12	26	...	...	...	193	439	632	20	54	74
January . . . 1901	8	14	22	...	...	...	114	530	644	18	47	65
February . . .	6	13	19	...	...	...	103	508	611	15	30	45
March . . .	4	10	14	...	...	...	74	359	433	12	29	41
April . . .	4	10	14	...	...	...	67	321	388	9	41	50
May . . .	4	6	10	...	...	...	106	426	532	43	65	108
June . . .	8	7	15	...	...	...	86	261	347	27	75	102
July . . .	6	1	7	...	...	...	82	150	232	22	30	52
August . . .	6	4	10	...	...	...	72	115	187	27	20	47
September . . .	16	4	20	...	...	...	175	105	280	41	18	59
October . . .	22	3	25	...	...	...	118	135	253	38	22	60
November . . .	8	3	11	...	...	...	64	232	296	14	28	42
December . . .	16	8	24	...	...	...	96	350	446	82	60	142
January . . . 1902	10	8	18	...	...	...	64	409	473	22	89	111
February . . .	18	13	31	...	...	...	120	377	497	52	106	158
March . . .	12	8	20	...	...	...	68	313	381	27	62	89
April . . .	12	15	27	...	...	...	63	257	320	17	23	40
May . . .	1	9	10	...	...	...	26	245	271	7	16	23



Month n (1)	Strength of Force on 1st of Month (2)	Sum of Column (2) for months n and n+1 (3)	E (4)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
				Wounds (5)	Illness (6)	Total (7)	Wounds (8)	Illness (9)	Total (10)
October 11 to 31 1899	857	2,726	909	21	1	22	23,102	1,100	24,202
November . . .	1,869	4,633	2,317	17	...	17	7,337	...	7,337
December . . .	2,764	5,982	2,991	37	...	37	12,370	...	12,370
January . . 1900	3,218	7,169	3,584	35	6	41	9,763	1,674	11,437
February . . .	3,951	8,581	4,291	58	15	73	13,517	3,496	17,013
March . . .	4,630	9,826	4,913	15	17	32	3,053	3,460	6,513
April . . .	5,196	10,475	5,237	14	12	26	2,673	2,291	4,964
May . . .	5,279	10,736	5,368	11	38	49	2,049	7,079	9,128
June . . .	5,457	10,869	5,435	23	16	39	4,232	2,944	7,176
July . . .	5,412	10,846	5,423	15	11	26	2,766	2,028	4,794
August . . .	5,434	10,882	5,441	7	2	9	1,287	368	1,655
September . . .	5,448	10,980	5,490	3	1	4	546	182	728
October . . .	5,532	10,972	5,486	15	4	19	2,734	729	3,463
November . . .	5,440	10,744	5,372	8	6	14	1,489	1,117	2,606
December . . .	5,304	10,580	5,290	14	12	26	2,617	2,268	4,915
January . . 1901	5,276	10,541	5,270	8	14	22	1,518	2,656	4,174
February . . .	5,265	10,544	5,272	6	13	19	1,138	2,466	3,604
March . . .	5,279	10,670	5,335	4	10	14	750	1,874	2,624
April . . .	5,391	10,916	5,458	4	10	14	733	1,832	2,565
May . . .	5,525	11,058	5,529	4	6	10	723	1,085	1,808
June . . .	5,533	11,165	5,583	8	7	15	1,433	1,254	2,687
July . . .	5,632	11,160	5,580	6	1	7	1,075	179	1,254
August . . .	5,528	11,027	5,513	6	4	10	1,088	725	1,813
September . . .	5,499	11,021	5,511	16	4	20	2,903	726	3,629
October . . .	5,522	10,937	5,468	22	3	25	4,023	549	4,572
November . . .	5,415	10,830	5,415	8	3	11	1,477	554	2,031
December . . .	5,415	11,055	5,528	16	8	24	2,894	1,447	4,341
January . . 1902	5,640	11,323	5,661	10	8	18	1,766	1,413	3,179
February . . .	5,683	11,343	5,672	18	13	31	3,173	2,292	5,465
March . . .	5,660	11,471	5,735	12	8	20	2,092	1,395	3,487
April . . .	5,811	11,613	5,807	12	15	27	2,066	2,583	4,649
May . . .	5,802	11,440	5,720	1	9	10	175	1,573	1,748
June . . .	5,638	...	...	...	...	...	...	...	...
	...	...	161,604	454	277	731	2,809	1,714	4,523



TABLE II (continued).—Officers—Colonials.

Month <i>n</i> (1)	Strength of Force on 1st of Month (2)	Sum of Column (2) for months <i>n</i> and <i>n</i> +1 (3)	E (4)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
				Wounds (5)	Illness (6)	Total (7)	Wounds (8)	Illness (9)	Total (10)
October 11 to 31 1899	...	...	...	...	...	...	...	...	...
November . . .	647	1,441	720	...	...	...	...	...	...
December . . .	794	1,759	880	2	1	3	2,778	1,389	4,167
January 1900	965	2,069	1,034	...	...	...	...	...	...
February . . .	1,104	2,245	1,123	13	...	13	12,573	...	12,573
March . . .	1,141	2,455	1,227	5	3	8	4,452	2,671	7,123
April . . .	1,314	2,733	1,367	5	2	7	4,075	1,630	5,705
May . . .	1,419	2,933	1,466	5	4	9	3,658	2,926	6,584
June . . .	1,514	3,341	1,671	6	7	13	4,093	4,775	8,868
July . . .	1,827	3,638	1,819	7	...	7	4,189	...	4,189
August . . .	1,811	3,594	1,797	7	...	7	3,848	...	3,848
September . . .	1,783	3,566	1,783	2	...	2	1,113	...	1,113
October . . .	1,783	3,447	1,723	1	1	2	561	561	1,122
November . . .	1,664	2,947	1,474	8	1	9	4,643	580	5,223
December . . .	1,283	2,530	1,265	2	3	5	1,357	2,035	3,392
January 1901	1,247	3,491	1,745	9	3	12	7,115	2,372	9,487
February . . .	2,244	4,567	2,284	1	4	5	573	2,292	2,865
March . . .	2,323	4,698	2,349	10	2	12	4,378	876	5,254
April . . .	2,375	4,700	2,350	2	3	5	851	1,277	2,128
May . . .	2,325	4,618	2,309	7	3	10	2,979	1,277	4,256
June . . .	2,293	4,564	2,282	5	1	6	2,165	433	2,598
July . . .	2,271	4,512	2,256	4	1	5	1,753	438	2,191
August . . .	2,241	4,441	2,220	5	4	9	2,216	1,773	3,989
September . . .	2,200	4,382	2,191	6	...	6	2,703	...	2,703
October . . .	2,182	4,361	2,181	7	1	8	3,195	456	3,651
November . . .	2,179	4,271	2,135	4	4	8	1,834	1,834	3,668
December . . .	2,092	4,064	2,032	2	1	3	937	468	1,405
January 1902	1,972	3,967	1,984	...	4	4	...	1,969	1,969
February . . .	1,995	4,062	2,031	4	4	8	2,016	2,016	4,032
March . . .	2,067	4,116	2,058	4	2	6	1,969	985	2,954
April . . .	2,049	3,982	1,991	...	3	3	...	1,458	1,458
May . . .	1,933	...	...	2	2	4	1,005	1,005	2,010
June . . .	...	...	53,747	...	...	...	...	...	...
	...	...	53,747	135	64	199	2,512	1,191	3,703



TABLE II (continued).—Officers—All Classes Combined.

Month " (1)	Strength of Force on 1st of Month (2)	E (3)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
			Wounds (4)	Illness (5)	Total (6)	Wounds (7)	Illness (8)	Total (9)
October 11 to 31 1899	857	909	21	1	22	23·102	1·100	24·202
November . . .	1,869	2,317	17	...	17	7·337	...	7·337
December . . .	3,411	3,711	39	1	40	10·509	·269	10·778
January 1900	4,012	4,464	35	6	41	7·841	1·344	9·185
February . . .	5,000	5,509	71	15	86	12·888	2·723	15·611
March . . .	6,018	6,387	20	20	40	3·132	3·132	6·264
April . . .	6,754	6,933	21	14	35	3·029	2·019	5·048
May . . .	7,114	7,257	20	43	63	2·756	5·926	8·682
June . . .	7,398	7,424	29	25	54	3·906	3·367	7·273
July . . .	7,449	7,617	24	13	37	3·151	1·707	4·858
August . . .	7,784	7,785	17	2	19	2·184	·257	2·441
September . . .	7,785	7,813	6	1	7	·768	·128	·896
October . . .	7,841	7,790	18	5	23	2·311	·642	2·953
November . . .	7,739	7,618	17	8	25	2·232	1·050	3·282
December . . .	7,498	7,294	18	15	33	2·468	2·056	4·524
January 1901	7,089	7,065	17	18	35	2·406	2·548	4·954
February . . .	7,042	7,568	8	19	27	1·057	2·511	3·568
March . . .	8,094	8,327	15	14	29	1·801	1·681	3·482
April . . .	8,559	8,710	6	14	20	·689	1·608	2·297
May . . .	8,860	8,797	19	11	30	2·160	1·250	3·410
June . . .	8,734	8,737	18	11	29	2·060	1·259	3·319
July . . .	8,738	8,665	16	2	18	1·847	·231	2·078
August . . .	8,592	8,557	12	8	20	1·402	·935	2·337
September . . .	8,523	8,509	25	5	30	2·938	·588	3·526
October . . .	8,495	8,432	35	4	39	4·151	·474	4·625
November . . .	8,370	8,419	16	8	24	1·900	·950	2·850
December . . .	8,367	8,490	27	10	37	3·180	1·178	4·358
January 1902	8,512	8,398	13	13	26	1·548	1·548	3·096
February . . .	8,285	8,291	29	23	52	3·498	2·774	6·272
March . . .	8,295	8,406	16	12	28	1·903	1·428	3·331
April . . .	8,518	8,535	14	18	32	1·640	2·109	3·749
May . . .	8,551	8,561	3	12	15	·350	1·402	1·752
June . . .	8,571	...	...	...	...	...	...	...
		933 105	662	271	1 022	9·920	1·501	4·420

Month <i>n</i> (1)	Strength of force on 1st of Month (2)	Sum of Column (2) for months <i>n</i> and <i>n</i> +1 (3)	E (4)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
				Wounds (5)	Illness (6)	Total (7)	Wounds (8)	Illness (9)	Total (10)
October 11 to 31 1899	21,629	74,863	24,954	146	15	161	5,851	601	6,452
November . . .	53,234	134,486	67,243	215	40	255	3,197	595	3,792
December . . .	81,252	181,363	90,681	426	86	512	4,698	948	5,646
January 1900	100,111	224,127	112,064	417	280	697	3,721	2,439	6,220
February . . .	124,016	274,600	137,300	688	375	1,063	5,011	2,731	7,742
March . . .	150,584	322,458	161,229	238	623	861	1,476	3,864	5,340
April . . .	171,874	348,927	174,463	105	823	928	602	4,717	5,319
May . . .	177,053	357,140	178,570	201	1,201	1,402	1,126	6,726	7,852
June . . .	180,087	364,688	182,344	145	937	1,082	795	5,139	5,934
July . . .	184,601	364,068	182,034	151	514	665	830	2,824	3,654
August . . .	179,467	355,757	177,878	110	236	346	618	1,327	1,945
September . . .	176,290	353,518	176,759	53	173	226	300	979	1,279
October . . .	177,228	352,408	176,204	126	195	321	715	1,107	1,822
November . . .	175,180	348,184	174,092	99	287	386	569	1,649	2,218
December . . .	173,004	341,886	170,943	193	439	632	1,129	2,568	3,697
January 1901	168,882	337,488	168,744	114	530	644	676	3,141	3,817
February . . .	168,606	337,055	168,527	103	508	611	611	3,014	3,625
March . . .	168,449	342,640	171,320	74	359	433	432	2,095	2,527
April . . .	174,191	342,357	171,179	67	321	388	391	1,875	2,266
May . . .	168,166	331,663	165,832	106	426	532	639	2,569	3,208
June . . .	163,497	327,653	163,827	86	261	347	525	1,593	2,118
July . . .	164,156	322,167	161,083	82	150	232	509	931	1,440
August . . .	158,011	316,695	158,348	72	115	187	455	726	1,181
September . . .	158,684	315,628	157,814	175	105	280	1,109	665	1,774
October . . .	156,944	311,302	155,651	118	135	253	758	867	1,625
November . . .	154,358	309,847	154,924	64	232	296	413	1,498	1,911
December . . .	155,489	319,620	159,810	96	350	446	601	2,190	2,791
January 1902	164,131	330,161	165,080	64	409	473	388	2,478	2,866
February . . .	166,030	333,942	166,971	120	377	497	719	2,258	2,977
March . . .	167,912	336,392	168,196	68	313	381	404	1,861	2,265
April . . .	168,480	337,399	168,700	63	257	320	373	1,523	1,896
May . . .	168,919	335,121	167,561	26	245	271	155	1,462	1,617
June . . .	166,202	...	...	...	...	...	...	...	...
...	...	...	4,980,325	4,811	11,317	16,128	966	2,272	3,238

TABLE II (continued).—N.C.O's and Men—Imperial Yeomanry.

Month <i>n</i> (1)	Strength of Force on 1st of Month (2)	Sum of Column (2) for months <i>n</i> and <i>n</i> +1 (3)	E (4)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
				Wounds (5)	Illness (6)	Total (7)	Wounds (8)	Illness (9)	Total (10)
October 11 to 31 1899									
November . . .	...	...	...	...	...	...	...	...	...
December . . .	...	...	...	...	...	...	...	...	...
January 1900	...	...	...	...	...	...	...	...	...
February . . .	1,426	6,432	3,216	...	5	5	...	1,555	1,555
March . . .	5,006	12,792	6,396	1	8	9	156	1,251	1,407
April . . .	7,786	17,901	8,951	2	17	19	223	1,899	2,122
May . . .	10,115	20,237	10,119	52	35	87	5,139	3,459	8,598
June . . .	10,122	20,297	10,149	25	43	68	2,463	4,237	6,700
July . . .	10,175	20,300	10,150	17	36	53	1,675	3,547	5,222
August . . .	10,125	20,221	10,111	11	18	29	1,088	1,780	2,868
September . . .	10,096	19,566	9,783	10	18	28	1,022	1,840	2,862
October . . .	9,470	18,497	9,249	14	22	36	1,514	2,379	3,893
November . . .	9,027	17,918	8,959	12	25	37	1,339	2,790	4,129
December . . .	8,891	16,915	8,458	20	54	74	2,365	6,384	8,749
January 1901	8,024	15,906	7,953	18	47	65	2,263	5,910	8,173
February . . .	7,882	18,447	9,224	15	30	45	1,626	3,252	4,878
March . . .	10,565	32,547	16,274	12	29	41	737	1,782	2,519
April . . .	21,982	45,358	22,679	9	41	50	397	1,808	2,205
May . . .	23,376	43,725	21,863	43	65	108	1,967	2,973	4,940
June . . .	20,349	38,952	19,476	27	75	102	1,386	3,851	5,237
July . . .	18,603	36,558	18,279	22	30	52	1,204	1,641	2,845
August . . .	17,955	35,285	17,613	27	20	47	1,530	1,134	2,664
September . . .	17,330	34,292	17,146	41	18	59	2,391	1,050	3,441
October . . .	16,962	33,087	16,544	38	22	60	2,297	1,330	3,627
November . . .	16,125	31,883	15,942	14	28	42	878	1,756	2,634
December . . .	15,758	30,907	15,454	82	60	142	5,306	3,882	9,188
January 1902	15,149	29,319	14,660	22	89	111	1,501	6,071	7,572
February . . .	14,170	28,542	14,271	52	106	158	3,644	7,428	11,072
March . . .	14,372	28,092	14,046	27	62	89	1,922	4,414	6,336
April . . .	13,720	28,493	14,247	17	23	40	1,193	1,614	2,807
May . . .	14,773	33,890	16,945	7	16	23	413	944	1,357
June . . .	19,117	...	...	...	...	...	...	...	...
			308,187	637	1,042	1,679	1,730	2,830	4,560



TABLE II (continued).—N.C.O.s and Men—Colonials.

Month n (1)	Strength of force on 1st of Month (2)	Sum of Column (2) for months n and n+1 (3)	E (4)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
				Wounds (5)	Illness (6)	Total (7)	Wounds (8)	Illness (9)	Total (10)
December 1899	14,631	32,599	16,299	58	13	71	3,559	798	4,357
January 1900	17,968	39,799	19,900	17	17	34	854	854	1,708
February . . .	21,831	46,796	23,398	77	22	99	3,291	910	4,231
March . . .	24,965	50,768	25,384	40	45	85	1,576	1,773	3,349
April . . .	25,803	55,526	27,763	53	77	130	1,909	2,773	4,682
May . . .	29,723	61,816	30,908	42	84	126	1,359	2,718	4,077
June . . .	32,093	66,326	33,163	29	101	130	874	3,046	3,920
July . . .	34,233	75,556	37,778	49	53	102	1,297	1,403	2,700
August . . .	41,323	82,287	41,143	51	25	76	1,240	608	1,848
September . . .	40,964	81,278	40,639	39	26	65	960	640	1,600
October . . .	40,314	80,628	40,314	54	31	85	1,339	769	2,108
November . . .	40,314	77,952	38,976	45	44	89	1,155	1,129	2,284
December . . .	37,638	66,661	33,331	43	44	87	1,290	1,320	2,610
January 1901	29,023	57,226	28,613	62	87	149	2,167	3,041	5,208
February . . .	28,203	78,954	39,477	40	53	93	1,013	1,343	2,356
March . . .	50,751	103,280	51,640	47	68	115	910	1,317	2,227
April . . .	52,529	106,240	53,120	43	55	98	809	1,035	1,844
May . . .	53,711	106,295	53,147	55	74	129	1,035	1,392	2,427
June . . .	52,584	104,450	52,225	82	70	152	1,570	1,340	2,910
July . . .	51,866	103,220	51,610	53	39	92	1,027	756	1,783
August . . .	51,354	102,025	51,013	31	48	79	608	941	1,549
September . . .	50,671	100,423	50,211	59	37	96	1,175	737	1,912
October . . .	49,752	99,097	49,549	80	42	122	1,615	848	2,463
November . . .	49,345	98,614	49,307	34	59	93	690	1,197	1,887
December . . .	49,269	96,575	48,287	27	86	113	559	1,781	2,340
January 1902	47,306	91,894	45,947	52	83	135	1,132	1,806	2,938
February . . .	44,588	89,706	44,853	58	103	161	1,293	2,296	3,589
March . . .	45,118	91,854	45,927	73	66	139	1,589	1,437	3,026
April . . .	46,736	93,073	46,537	22	52	74	473	1,117	1,590
May . . .	46,337	90,050	45,025	9	48	57	200	1,066	1,266
June . . .	43,713	...	...	...	...	...	...	...	...
		...	1,215,484	1,424	1,652	3,076	1,172	1,359	2,531

TABLE II (continued).—*N.C.O's and Men—All Classes Combined.*

Month " (1)	Strength of Force on 1st of Month (2)	K (3)	DEATHS DURING MONTH			MONTHLY DEATH-RATE PER 1,000		
			Wounds (4)	Illness (5)	Total (6)	Wounds (7)	Illness (8)	Total (9)
October 11 to 31 1899	21,629	24,954	146	15	161	5851	601	6452
November . . . . .	53,234	67,243	215	40	255	3197	595	3792
December . . . . .	95,883	106,980	484	99	583	4524	925	5449
January 1900	118,079	131,964	434	297	731	3289	2250	5539
February . . . . .	147,273	163,914	765	402	1,167	4667	2453	7120
March . . . . .	180,555	193,009	279	676	955	1446	3502	4948
April . . . . .	205,463	211,177	160	917	1,077	758	4342	5100
May . . . . .	216,891	219,597	295	1,320	1,615	1343	6011	7354
June . . . . .	222,302	225,656	199	1,081	1,280	882	4790	5672
July . . . . .	229,009	229,962	217	603	820	944	2622	3566
August . . . . .	230,915	229,132	172	279	451	750	1218	1968
September . . . . .	227,350	227,181	102	217	319	449	955	1404
October . . . . .	227,012	225,767	194	248	442	859	1098	1957
November . . . . .	224,521	222,027	156	356	512	702	1603	2305
December . . . . .	219,533	212,732	256	537	793	1203	2524	3727
January 1901	205,929	205,310	194	664	858	945	3234	4179
February . . . . .	204,691	217,228	158	591	749	727	2721	3448
March . . . . .	229,765	239,234	133	456	589	556	1906	2462
April . . . . .	248,702	246,978	119	417	536	482	1688	2170
May . . . . .	245,253	240,842	204	565	769	847	2345	3192
June . . . . .	236,430	235,528	195	406	601	828	1724	2552
July . . . . .	234,625	230,972	157	219	376	680	948	1628
August . . . . .	227,320	227,004	130	183	313	573	806	1379
September . . . . .	226,685	225,171	275	160	435	1221	711	1932
October . . . . .	223,658	221,744	236	199	435	1064	897	1961
November . . . . .	219,828	220,173	112	319	431	508	1449	1957
December . . . . .	220,516	223,551	205	496	701	917	2219	3136
January 1902	226,586	225,687	138	581	719	611	2574	3185
February . . . . .	224,788	226,095	230	586	816	1017	2592	3609
March . . . . .	227,402	228,169	168	441	609	736	1933	2669
April . . . . .	228,936	229,484	102	332	434	444	1447	1891
May . . . . .	230,029	229,531	42	309	351	183	1346	1529
June . . . . .	229,032	...	...	...	...	...	...	...





Scale 3 spaces to each unit of 1,000 q.

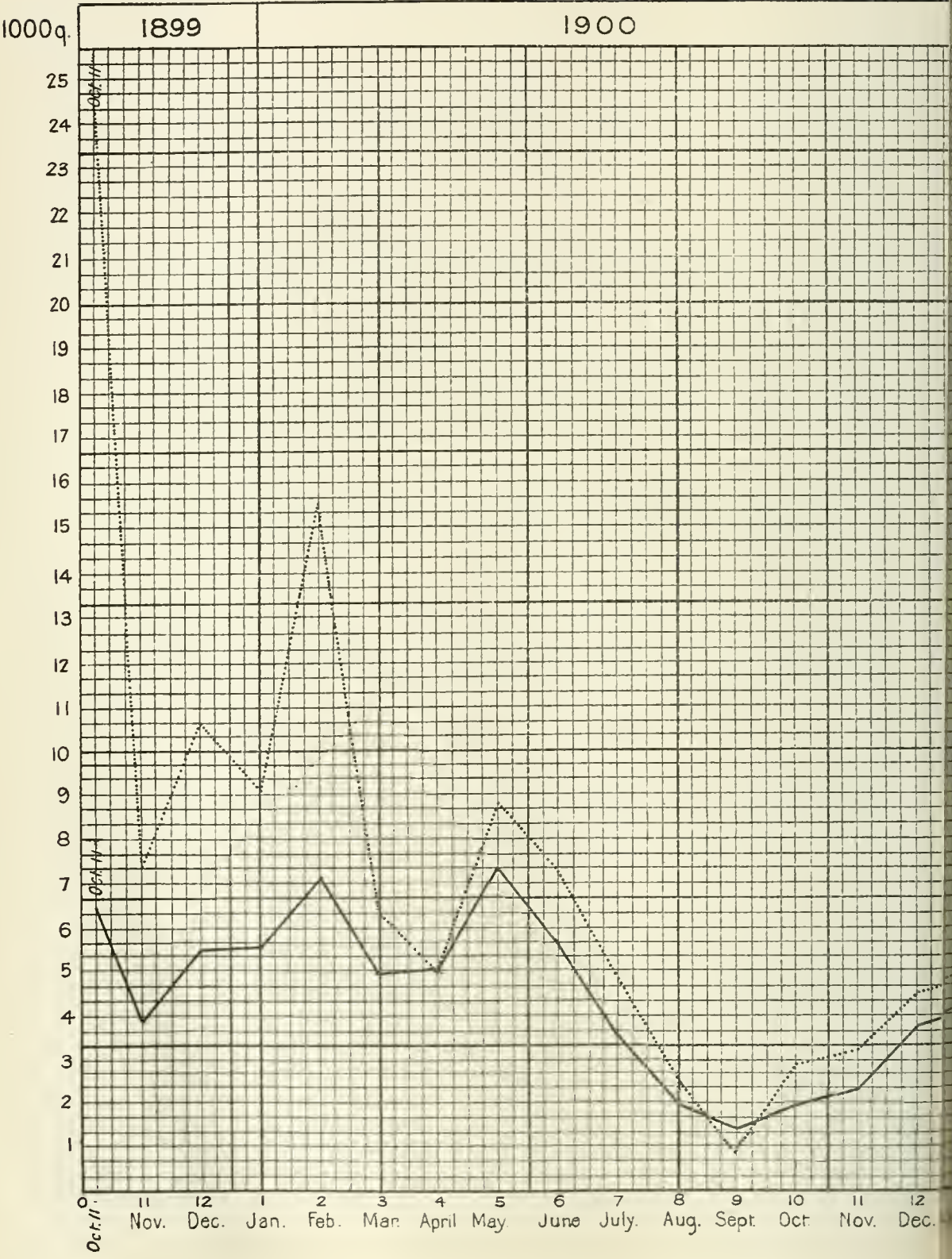
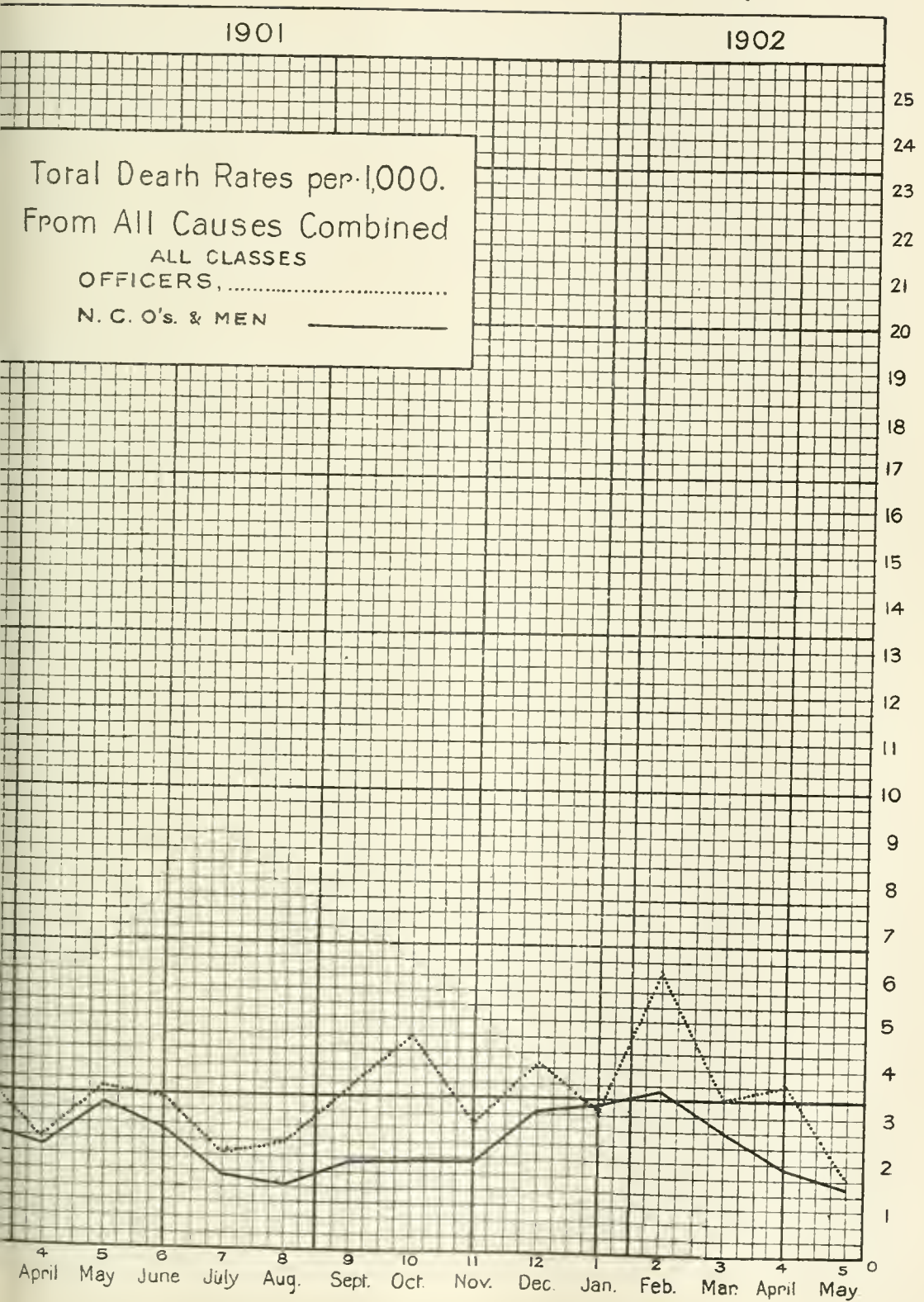


Diagram A.









Scale - 3 Spaces to each unit of 1000 g

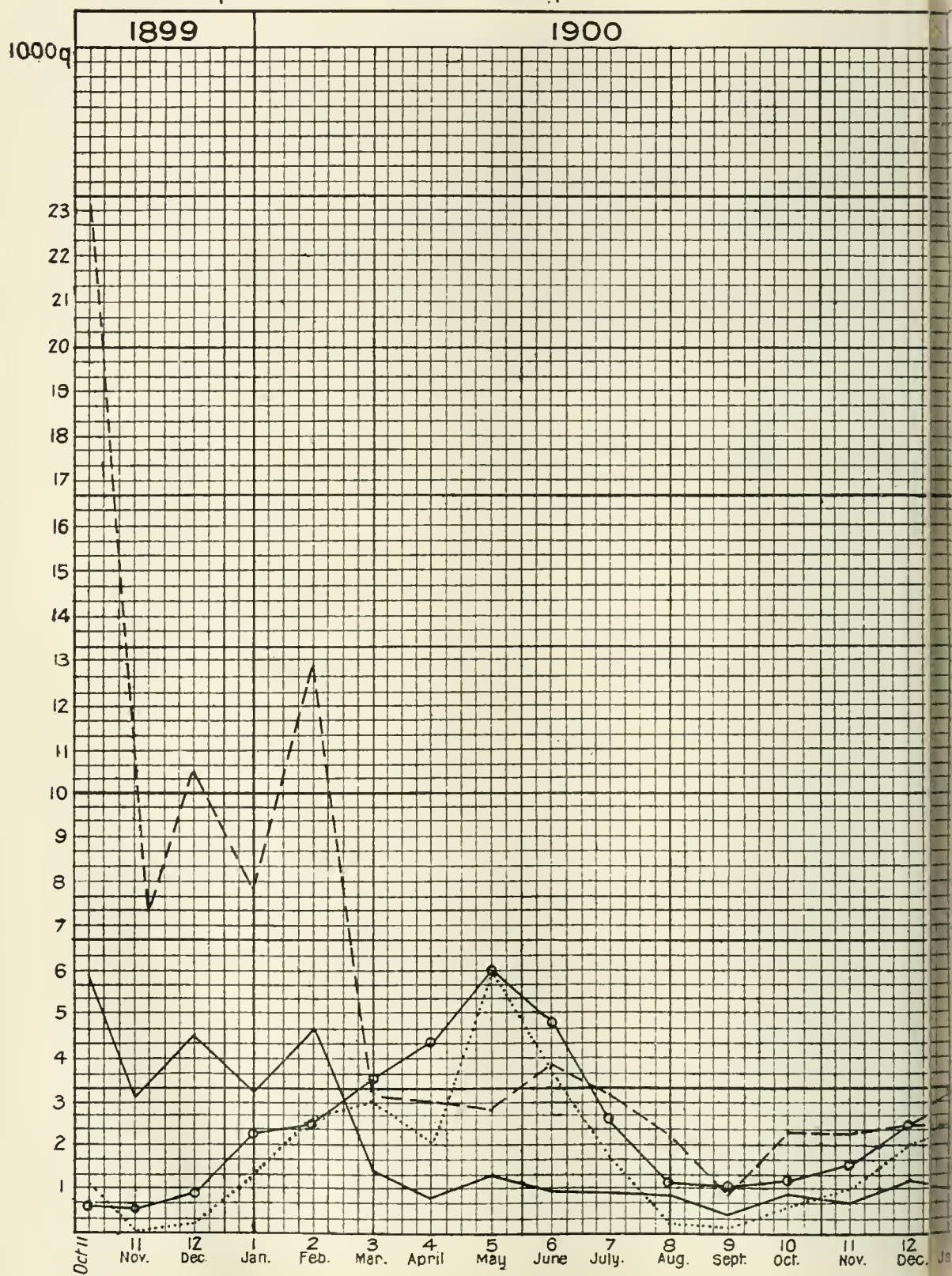


Diagram B.

1901

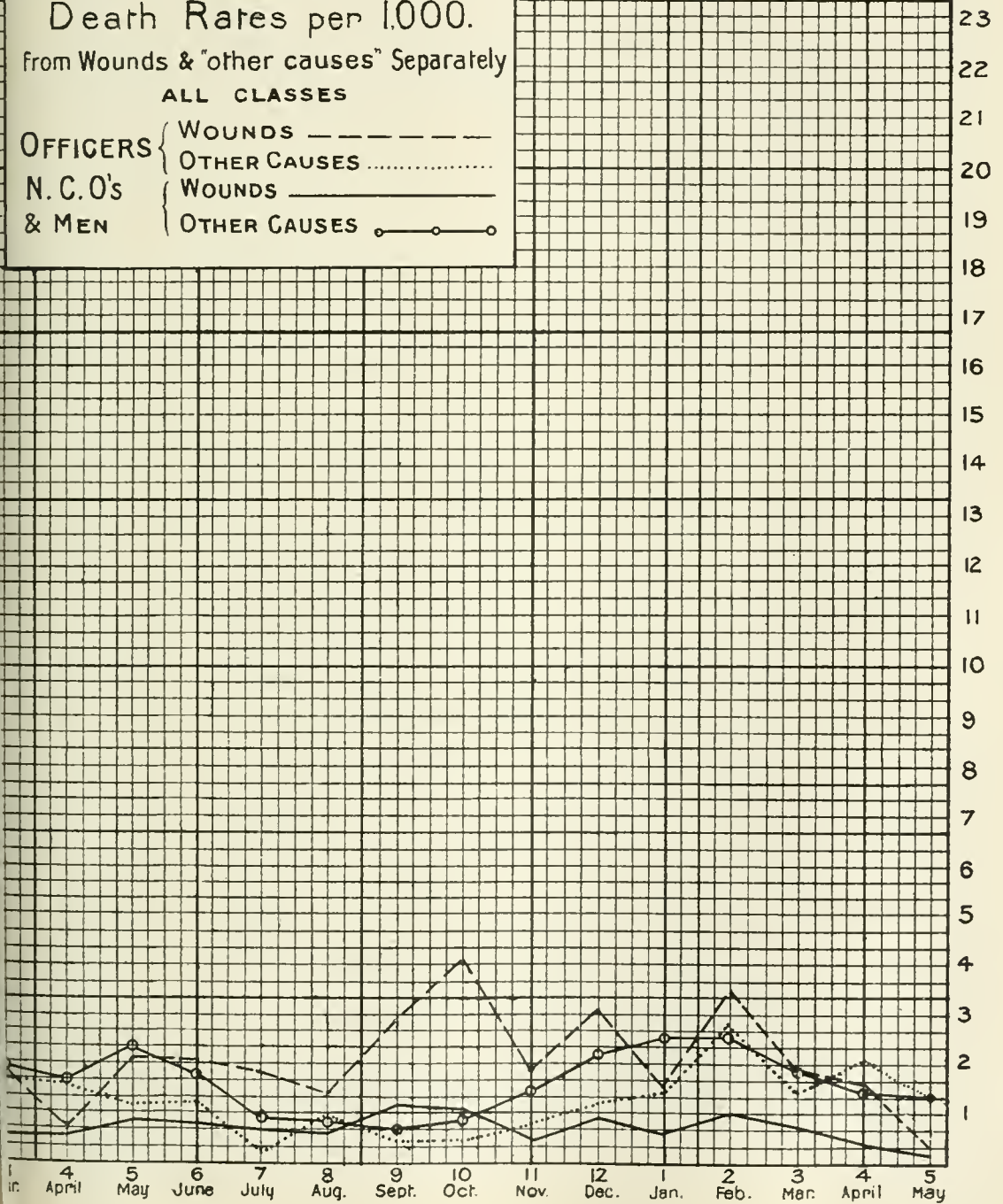
1902

# Death Rates per 1,000.

from Wounds & "other causes" Separately

ALL CLASSES

OFFICERS { WOUNDS -----  
 OTHER CAUSES .....  
 N. C. O's { WOUNDS -----  
 & MEN { OTHER CAUSES o—o—o







In Table II the second column shows the "Strength of Forces", taken from Schedules A and B, for officers and men respectively, further sub-dividing each of these into (1) Regulars, &c.; (2) Imperial Yeomanry; and (3) Colonials.

Column 3 gives the sum of every two consecutive values of column 2, combining months  $n$  and  $n+1$ . Column 4 is one-half of column 3 (except for October 1899), being the arithmetic mean between the numbers exposed on the first of any one month, and on the first of the succeeding month. This gives E, the number of months' exposure during the period. The strength of forces on 11 October 1899 was obtained from the Parliamentary returns, but no reliable information could be obtained as to the strength of the Colonial forces engaged prior to December 1899, so that no rates of mortality could be given for them for the months of October and November, and the corresponding deaths are accordingly omitted. The deaths are reproduced from Table I in columns 5, 6 and 7; and in columns 8, 9 and 10, are shown the resulting death-rates per month per 1,000 exposed to risk, divided into rates from wounds, rates from other causes, and total rates. They are obtained by dividing columns 5, 6 and 7, by the corresponding figures in column 4, and multiplying the result by 1,000. It has been thought advisable, throughout the paper, to follow the practice of the Registrar-General in his weekly returns by giving the death-rates per 1,000, as thereby we are better able to appreciate the significance of the figures, which would be almost entirely in decimals were a percentage method adopted.

For the purpose of seeing how the various phases of the war affected the mortality, we append a concise epitome of the chief events—events likely to increase the death-rate from disease being distinguished from those likely to increase the death-rate from wounds. As an example of the use that may be made of this epitome, we might point out that deaths from disease were most prevalent when the army rested after Paardeberg, owing, as is stated in the Report of the Hospitals Commission, to the bad water drunk by the troops, and also to the scarcity of food and medical stores occasioned by the difficulties of transport in keeping up with the previous rapid movements of the forces at this period of the war; a point referred to more in detail further on.

*South African War 1899-1902—Epitome of Chief Events.*

Month	Events affecting rate of Mortality from Wounds	Events bearing upon rate of Mortality from Disease
1899		
Oct. .	Invasion of Natal. Retreat of British to Ladysmith. Battles of Dundee, Elandslaagte, and Nicholson's Nek. Investment of Kimberley and Mafeking	
Nov. .	British advance on Kimberley. Battles of Belmont, Enslin, and Modder River	Nov. 1 — Ladysmith invested.
Dec. .	Further advance to relieve Kimberley. Battle of Magersfontein. Attempt to relieve Ladysmith. Battle of Colenso. Battle of Stormberg . . . . .	Ladysmith invested.
1900		
Jan. .	Boer assault on Ladysmith. Battle of Spion Kop . . . . .	Ladysmith invested.
Feb. .	Battle of Vaal Krantz. Heavy fighting at Pieter's Hill and relief of Ladysmith. Relief of Kimberley. Battle of Paardeberg . . . . .	Feb. 28 — Ladysmith relieved. Halt at Paardeberg 16-27 Feb. Forced march to Bloemfontein.
March	Bloemfontein occupied. Battle of Sanna's Post . . . . .	Mar. 13—Army halts at Bloemfontein. Insufficient food during part of the time.
April .	Battle of Reddersberg . . . . .	Waterworks cut off at Bloemfontein.
May .	Advance on Pretoria resumed. Lines of communication attacked. Fighting at Lindley . . . . .	12-22 May, halt at Kroonstad.
June .	Pretoria occupied. Continuous guerilla fighting, and Battle of Roodeval . . .	
July .	Further guerilla fighting. Operations along Delagoa Bay Railway. Surrender of General Prinsloo . . . . .	
Aug. .	Battle of Bergendal, and final break-up of the organized Boer Army. . . . .	
	From September 1900 to May 1902 there was continuous skirmishing with the guerilla forces of the enemy, the following being the more important engagements :	
Oct. .	Frederickstad . . . . .	
Nov. .	Bothaville. Dewetsdorp . . . . .	
Dec. .	Nooitgedacht. Helvetia . . . . .	
1901		
Jan. .	Determined attacks on British posts, Modderfontein . . . . .	
March	Lichtenburg (Yeomanry engaged) . . .	
May .	Vlakfontein . . . . .	
June .	Wilmanrust (Victorians engaged). Reitz	
Sept. .	De Jager's Drift. Mland's River Poort. Fort Italia. Moedwill. . . . .	
Oct. .	Zeerust (Yeomanry engaged). Brakenlaagte	
Nov. .	Villiersdorp . . . . .	
Dec. .	Tafelkop. Tweefontein . . . . .	
1902		
Jan. .	Lichtenburg . . . . .	
Feb. .	Calvinia. Klerksdorp . . . . .	
March	Tweebosch . . . . .	



The rates vary so considerably that their general significance is lost in the tabulated form of Table II, and to overcome this difficulty they have been put into geometric form in diagrams A and B.

In these diagrams the base line represents the various calendar months of the war, commencing with 11 October 1899. The ordinates marked 1,000*g* represent the monthly death-rates obtained in Table II. In order to show the effect more clearly, the scale for the N.C.O's and men has been taken as four times that of the commissioned officers. It was not possible to adopt a larger scale for the latter, owing to the heavy death-rate from wounds experienced by them as compared with the N.C.O's and men, particularly in the early stages of the war. In the first diagram also, it was found necessary to adopt the same scale as that used for the commissioned officers. As the scale is inserted on each diagram, it is hoped that no difficulty may be experienced from the use of this dual basis.

No attempt whatever has been made, either in these diagrams or in any other part of the paper, to smooth the angularities by any method of graduation. The death-rates we are dealing with depended upon the ever-varying incidents of the campaign, and a single day's battle involving heavy loss, or the exposure to extreme hardships and want of food for a few days, immediately increased them out of all proportion to surrounding rates, which may possibly refer to a time of comparative peace and plenty. The graduation of such results would assume some form of a law of progression, would tone down the effects of the events of the war, and would thus minimize their significance.

The first diagram (A) contrasts the total mortality from all causes between commissioned officers and N.C.O's and men, in each case all branches of the Service being combined. It will be seen that the diagram shows very clearly that the officers experienced a considerably higher death-rate than the N.C.O's and men.

Diagram B carries the comparison a step further, by subdividing the causes of death into those from wounds, and those from other causes, and enables us better to follow the rates with respect to the various events of the war. Here we notice at once that the death-rates amongst the two classes under consideration vary in opposite directions. Taking the deaths from wounds, we observe that the commissioned officers invariably experienced a higher death-rate throughout the whole period of

the war, in most cases considerably higher, than the other classes of combatants. If, however, we take the deaths from other causes, we observe that almost invariably the rate for the N.C.O's and men is the higher. This fact, so clearly demonstrated by these diagrams, is a marked feature, not only of the recent war, but of war statistics generally, and has usually been attributed to the care taken by officers to avoid those prevalent evils of campaign life which breed disease and death.

These results fit in with general expectations, it being well known that officers were marked men on the battle field, while it is probable that as a body they were men of better stamina than the rank and file, and perhaps were the better cared for when ill.

We can read also, in this illustration, the general rough outline of the history of the war as affected by battle or by disease. It will be observed that, as a rule, the curves for officers and men respectively proceed in the same general direction. The ordinates, for instance, of the death-rates from wounds rise generally to a maximum or fall to a minimum in the same calendar month, and the same may be observed of the two curves relating to the death-rates from other causes. This tends to show that the same causes were operating upon both officers and men, and we may accordingly safely say that the predisposing causes must be looked for, as might naturally be expected, in the varying incidents of the campaign.

Referring to the "Epitome of Events" given on page 562, we are able very easily to follow in the diagram many of the chief events in their effect upon the death-rate. Thus from the beginning of January 1900 a steady rise is observable in the death-rate from wounds, reaching its maximum at the end of the month, and falling rapidly in February. It will at once be seen that the heavy fighting around Ladysmith and connected with its relief caused the rise referred to. The rate practically ceases to fall in March and April, and rises only slowly in May. A reference to the Epitome shows us that this was the period of the long halt at Bloemfontein, when there was comparatively little fighting. We are, however, unpleasantly reminded, upon looking at the curves for "Other Causes", that this period of comparative inactivity was not a period of immunity from death, for a sharp turn upwards occurs in March, reaching a maximum in May, and then slowly descending as the army marched forward

to Pretoria. The reason for this heavy rate has already been touched upon in referring to the fact that the rapid movement to Paardeberg made it impossible for transport to keep pace with the demands made upon it. It may be observed, moreover, that the disease contracted at that place only developed to its fullest extent during the halt at Bloemfontein, three weeks being generally considered by the medical profession as the period of incubation of enteric fever. To this may be added the effects of the engagement at Sanna's Post, resulting in the cutting off for a time of the water supply to Bloemfontein, which must have had a considerable effect upon the deaths from disease.

In reference to the subject of disease generally, we are informed by the Army Medical Department that "Enteric was rife among the Boers around Kimberley before our army reached its neighbourhood, and that Harrismith has always been regarded as an enteric centre. Paardeberg is regarded as the source of the Bloemfontein trouble—decaying animal and vegetable matter in the stream there—and that we brought it from Paardeberg to Bloemfontein. Wherever there has been lack of water for sanitary purposes there has been trouble. Excess of meat is also deemed a factor."

Another interesting point may be mentioned here. It will be seen that the death-rates from other causes uniformly tended to rise during December and January. This is the height of the South African summer, a season which is generally considered by the medical profession as productive of excessive mortality from enteric fever. It may also be noticed that there is a rise in the death-rates about the month of June both for 1900 and for 1901. Dr. J. W. Washbourn, in his presidential address to the British Medical Association, stated that "a chill, which in England would probably give rise to an attack of nasal catarrh, in South Africa would cause an intestinal catarrh"; and this may possibly be the explanation, as June is the period of mid-winter in South Africa.

These points, together with many others equally interesting, are brought out in the succeeding diagrams, in which the three branches of the forces are subdivided and compared. Curves giving the comparisons for officers were prepared, but as they followed very generally the same form as the rates for N.C.O's and men, they have not been introduced, but Diagrams C, D, and E, referring to the latter, are given here that we



may obtain a clearer idea of the facts brought out. In Diagram C, we have a general comparison of the death-rates from all causes for each of the three groups of the Imperial forces. The fourth curve, dealing with assured lives, will be dealt with later on.

The first point that strikes the eye as we look at the curves is that their general conformation is similar, the maxima and minima occurring always about the same period of time. This shows, perhaps, more clearly than anything else that the rates were affected entirely by the incidents of the war, and that we must look for their interpretation only in the light of those incidents.

The next noticeable point is that, almost from the moment of their arrival in South Africa, the Imperial Yeomanry rapidly took the first place in the casualty lists and, practically till the end of the war, held that place as an easy first. The reason for this is probably almost entirely due to their being mounted men; the war, from the time that they went out (February 1900), being largely of such a nature as to keep this arm of the Service continually employed in the most arduous duties, and calling them constantly to positions of danger in almost every action of importance.

It would have been interesting, had time allowed, and had statistics been available, to compare the death-rate of cavalry, as distinguished from the other branches of the Regulars, with the Yeomanry.

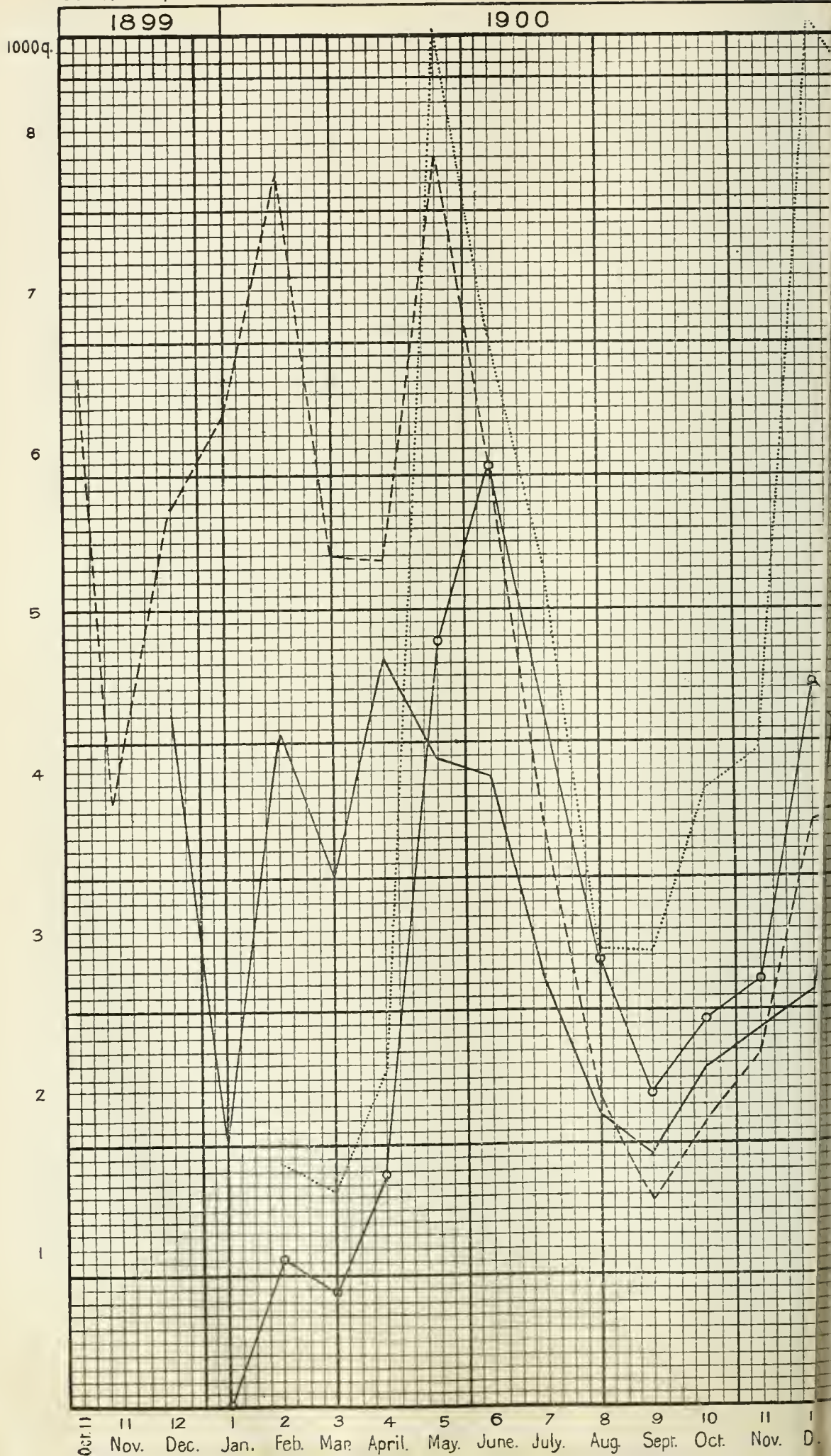
The Regulars, as was to be expected, suffered very severely in the early stages of the war. But as they learnt the lessons taught by the nature of the warfare they had to face, the rate seems very rapidly to have diminished, and thereafter to have fluctuated within much narrower limits, the minima being reached in September 1900 and August 1901. Generally speaking, these rates fall between those for the Imperial Yeomanry on the one side and the Colonials on the other.

In the latter part of the war, *i.e.*, from the middle of May 1901, it will be observed that the rates for the Regulars were the lowest. This may probably be accounted for by their having been employed to hold the lines of communication and to garrison the block-houses, while the Yeomanry and Colonials, being mounted men, were engaged more actively in the field.

Diagrams D and E enable us to analyze the mortality with more regard to its causes, the first diagram giving the death-rates from wounds only, and the second those from other causes.



Scale. 12 Spaces to each unit of 1000 q.





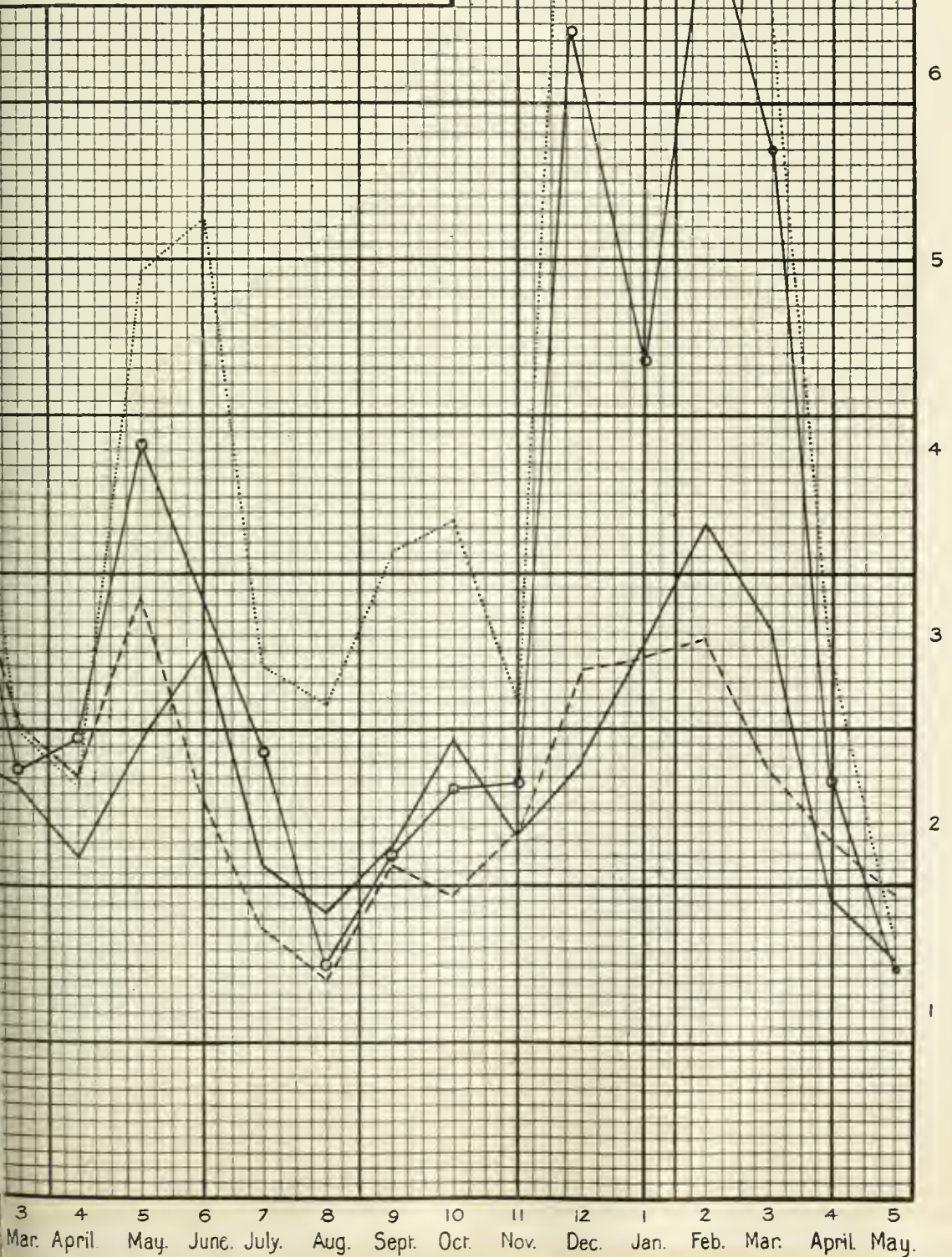
1901

1902

## N. C. O's &amp; MEN.

Death Rates per 1,000.  
From all Causes Combined.

REGULARS &c. -----  
IMPERIAL YEOMANRY, .....  
COLONIALS \_\_\_\_\_  
ASSURED YEOMANRY } ○ ○ ○ ○  
AND VOLUNTEERS. }

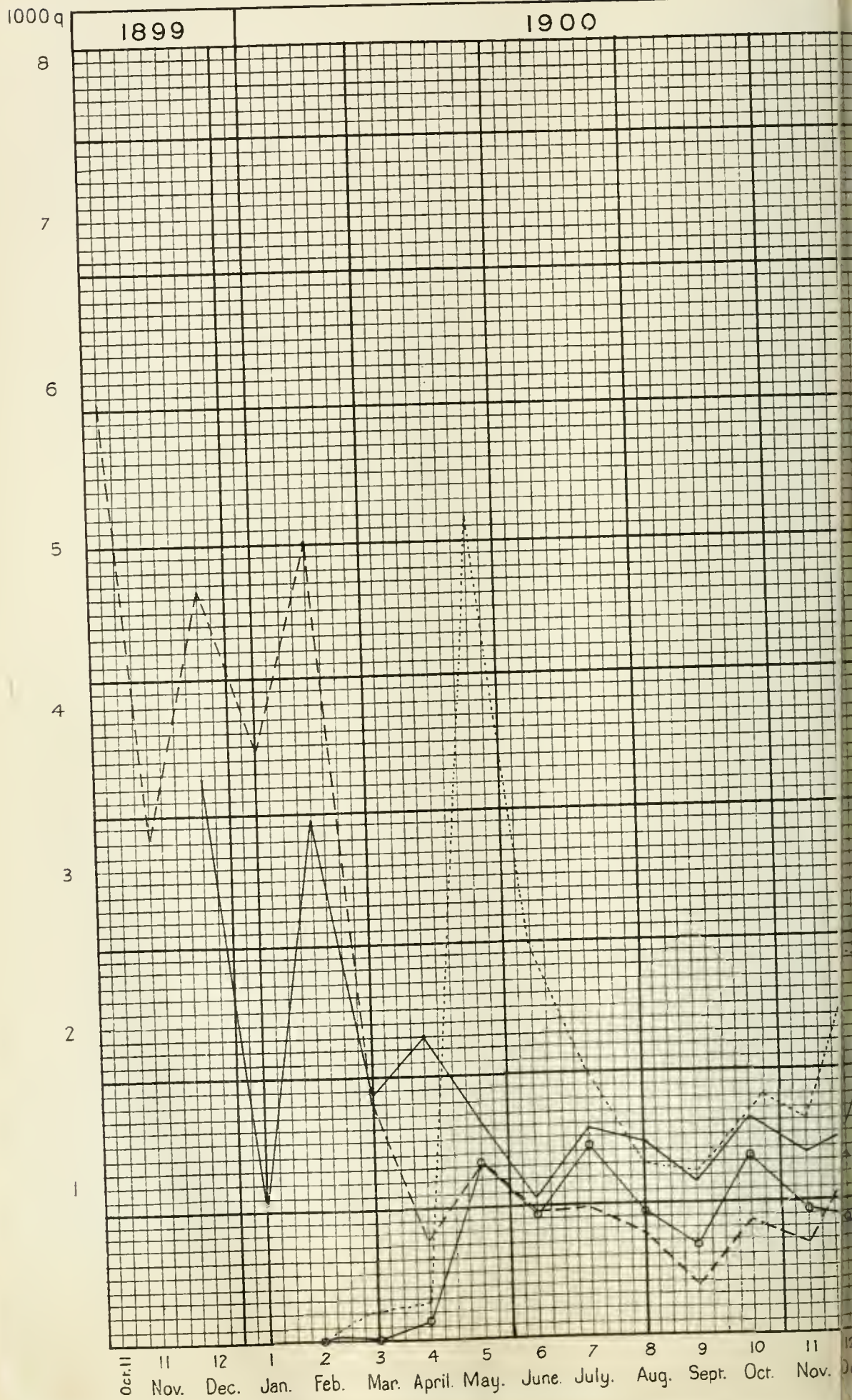








Scale 12 spaces to each unit of 1000 q.





1901

1902

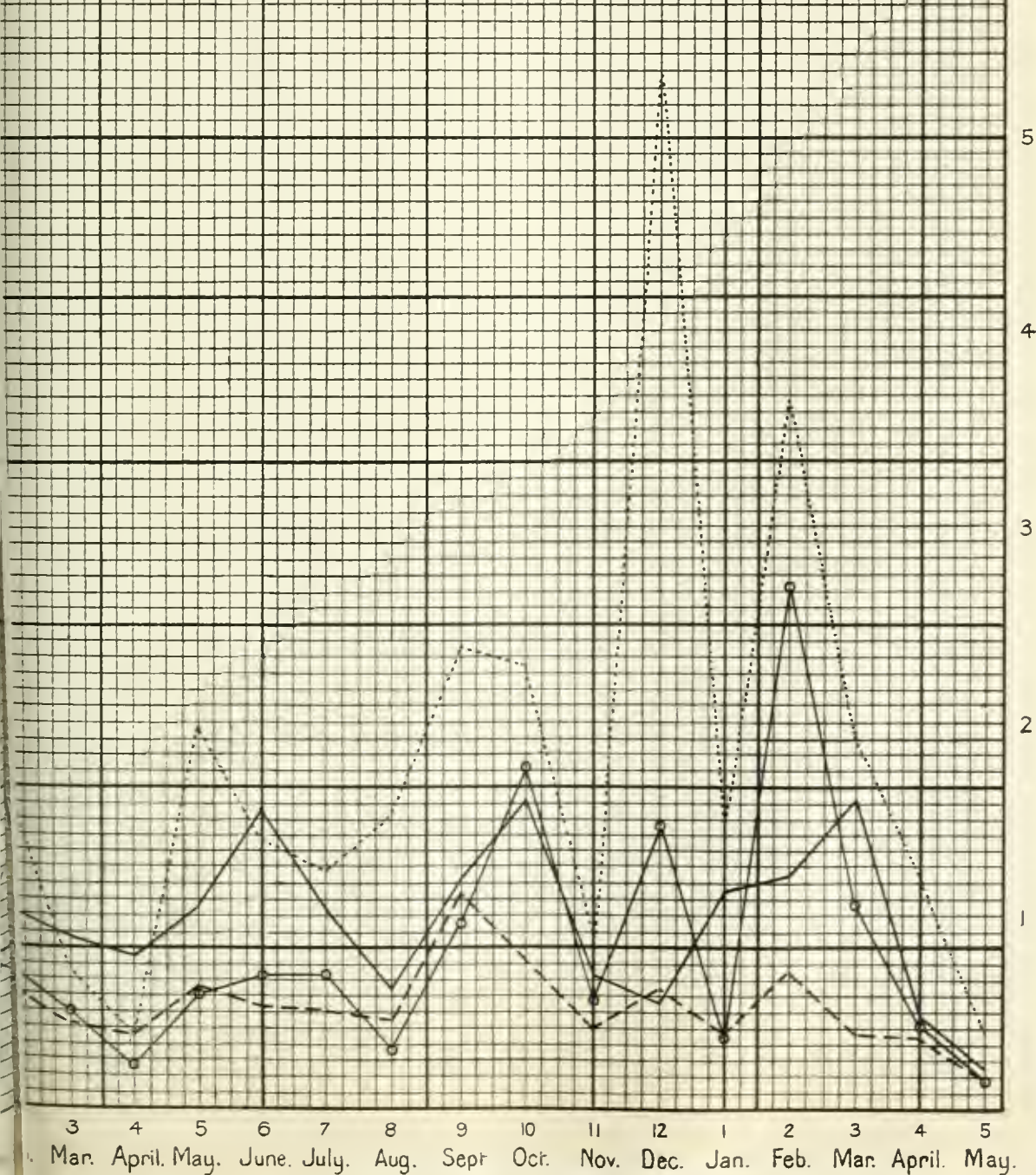
## N.C.O's &amp; MEN.

Death Rates per 1000 from Wounds,

REGULARS &amp;c.-----

IMPERIAL YEOMANRY.....

COLONIALS -----

ASSURED YEOMANRY }  
AND VOLUNTEERS }







Scale 12 spaces to each unit of 1000 q.

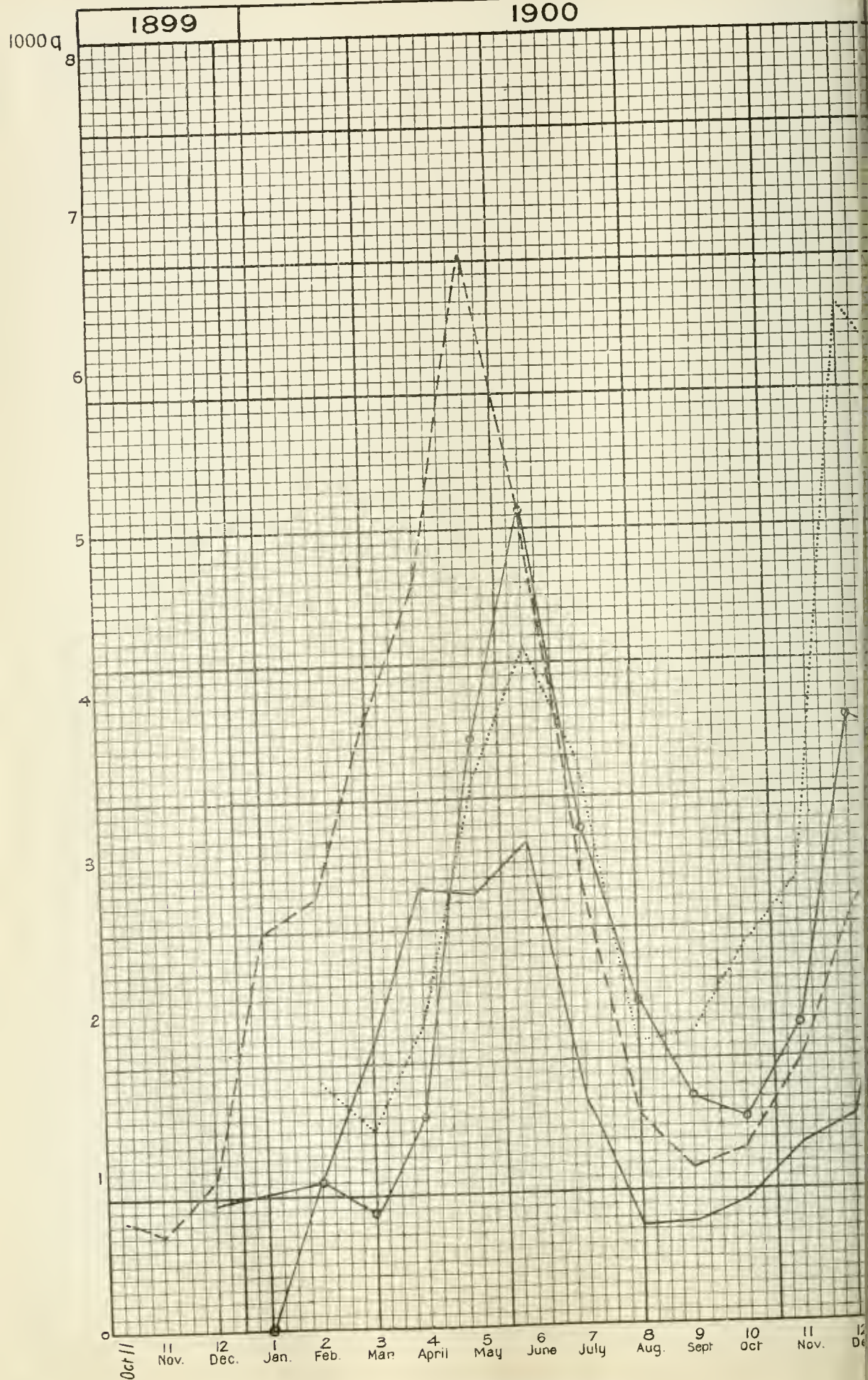
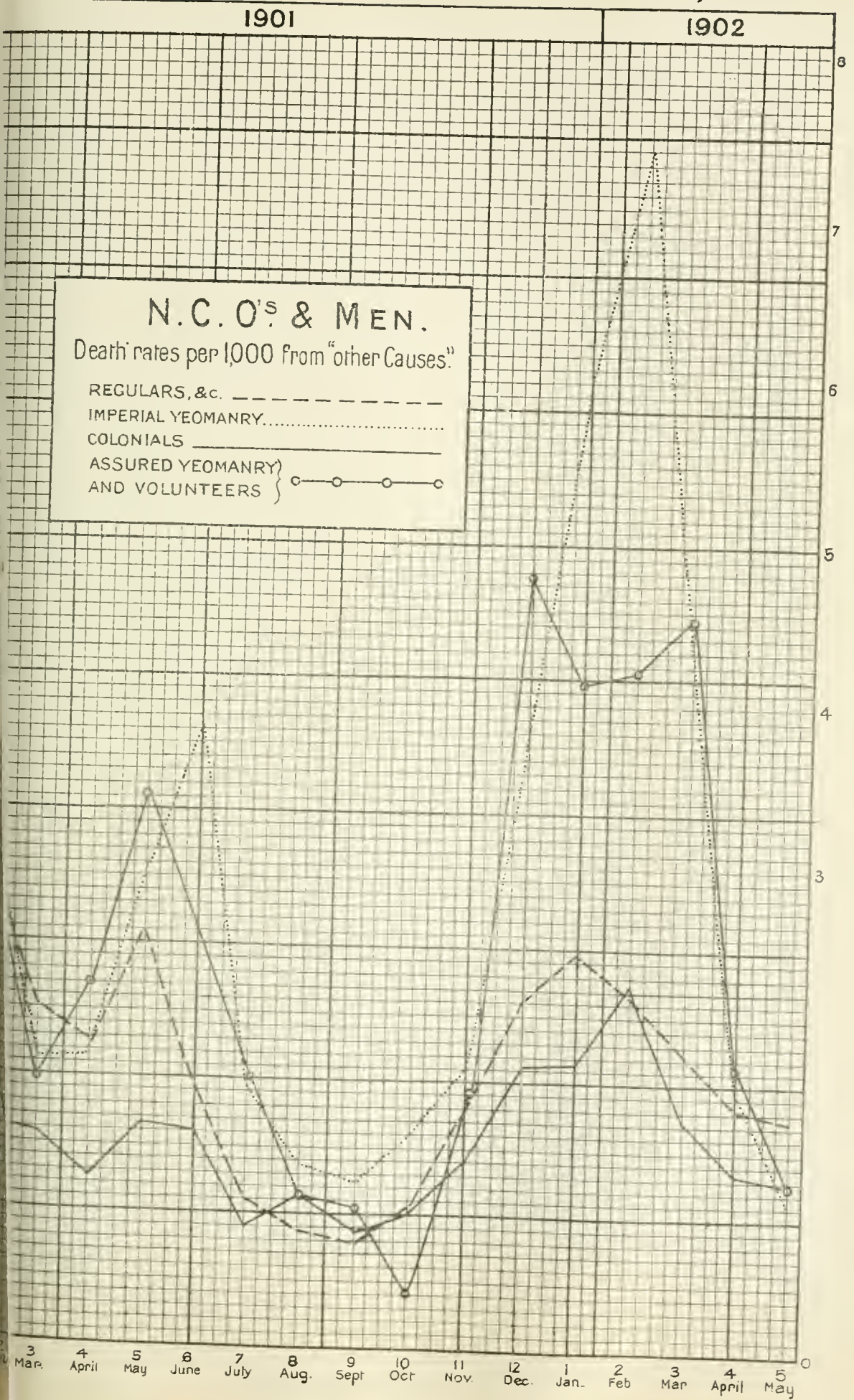




Diagram E.









Scale 12 spaces to each unit of 1000 q.

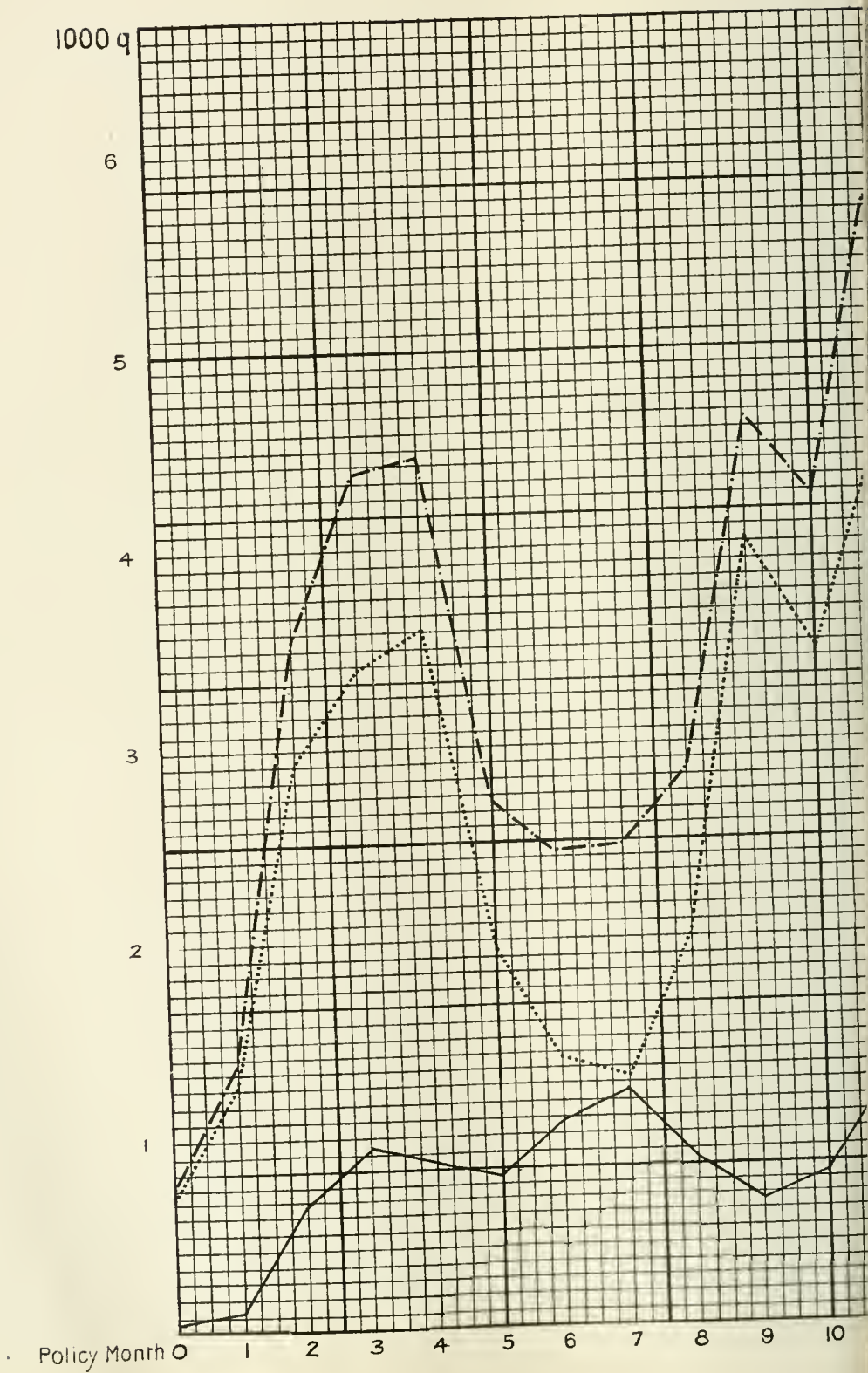
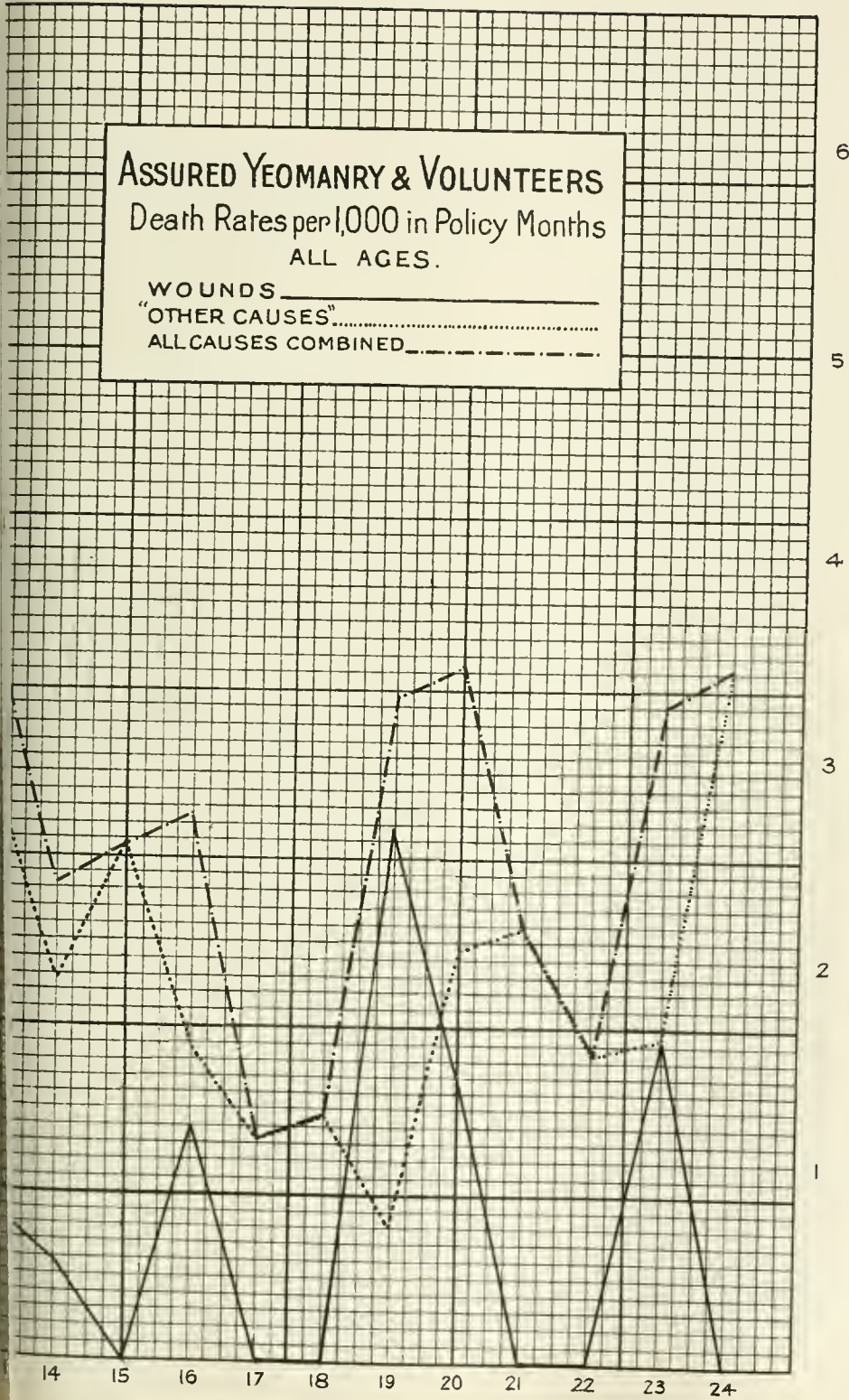




Diagram F.

**ASSURED YEOMANRY & VOLUNTEERS**  
Death Rates per 1,000 in Policy Months  
ALL AGES.

WOUNDS \_\_\_\_\_  
"OTHER CAUSES" .....  
ALL CAUSES COMBINED - - - - -





Looking at Diagram D, we observe that after the first stage of the war, for part of which time there were very few Colonials and no Yeomanry engaged, the Regulars actually had a lower death-rate from wounds than either of the two former bodies, and that this low rate continued throughout the remaining period of the war. This would seem to confirm the suggestion already made, that at this time a large proportion were engaged on service which brought them as a whole less into contact with the enemy, or in which they fought under the advantageous cover of block-houses. The Yeomanry and Colonials follow the same general lines, but the former are again first in the magnitude of the rate. It will be observed that the death-rate of the Yeomanry rises sharply to a high point in May 1900, and again from October 1901 to January 1902. The former was the period of the advance from Bloemfontein to Pretoria, when mounted troops were constantly engaged in skirmishes and actions. The latter period covers the engagements at Zeerust and Tweefontein, in which, it will be remembered, the companies of Yeomanry taking part suffered very severely.

When we turn, however, to the death-rates from "Other Causes" the comparison between the various curves is not quite so simple. The heavy rates during the South African summer are here again very prominent, all the branches of the Service contributing to the rise. The high rate amongst the Regulars up to April 1900 was, of course, occasioned by the hardships of the siege of Ladysmith, the rate amongst all arms immediately after this period being caused (as already referred to) by the rapid advance to Paardeberg and Bloemfontein. The most prominent feature of this diagram is the low death-rate amongst the Colonials. This is in accordance with the common observation that troops engaged in operations in their own country suffer less from disease than when engaged abroad. As the Australian, Canadian and other Colonial troops are included in addition to those raised locally in South Africa, we may take it roughly that the conditions of life in these Colonies approximate so closely to those in South Africa that the Colonials of all kinds were better prepared to fight the causes that produce disease than were the troops from the Mother Country. The rise in the death-rate in the winter (May and June) is also brought out more clearly in the diagram. The extremely heavy death-rate amongst the Yeomanry from November 1900 to March 1901, April 1901 to July 1901, and October 1901 to April 1902, may possibly be explained by the



hardships of the campaigning at these periods, when the mounted troops were often far away from their base, existing on half or quarter rations.

If we exclude the first period of the war, ending after the relief of Ladysmith, it will be observed that the rates amongst the Regulars have a general and steady tendency to decrease. This no doubt may be ascribed to the efficiency with which the medical department of the army did its work.

Table III gives summaries for each subdivision for the whole duration of the war. It consists simply of the totals found in the last lines of Table II. It brings out the same points regarding the death-rates as have been commented upon in reviewing the diagrams, and therefore these need not be further dwelt upon.

Messrs. Smeed and Ackland arrive at the conclusion, based on campaigns extending over nearly one hundred years, that the average death-rate of the army in the field is about five per-cent per annum; they also specially note that the death-rate of the American Civil War was seven per-cent. The death-rate per annum in the recent war for all the forces engaged, according to Table III, including officers and men, was 3·87 per-cent. Taking officers alone, it was 5·3 per-cent, and taking non-commissioned officers and men, 3·81 per-cent.

These annual death-rates practically give the required annual premium per £100 assured according to the actual war experience, including the risk of death from normal mortality, and make allowance for the return of a portion of the premium for any unexpired fraction of a year. They fall considerably below the rates deduced from the mortality of the German Army during the Franco-German War, and compare even more favourably with the death-rates experienced by the Northern Army during the American Civil War.

It should be noted, for it is most important, that the rates are for the period of the war only, and cannot be said to represent the correct extra premium for existing policies at the commencement of the war, or for a new policy to be continued after the expiration of the war. They are, in fact, the term premiums for a temporary assurance. On the one hand, it is true the rates cover the risk of death from normal mortality, but, on the other hand, they do not take into account the probability of deterioration through wounds or disease contracted during the campaign. What the value of this liability to deterioration is, it is impossible to say, but no doubt it is very considerable.

TABLE III.  
*Comparison of the Totals for all Classes for the whole duration of the War.*

	OFFICERS						N.C.O's AND MEN					
	Months of Exposure to Risk	Years of Exposure to Risk	Deaths			Death-Rate per 1,000 per annum	Months of Exposure to Risk	Years of Exposure to Risk	Deaths			Death-Rate per 1,000 per annum
			Wounds	Illness	Total				Wounds	Illness	Total	
Regulars and Volunteers	161,604	13,467	454	277	731	33·712	4,980,325	415,027	4,811	11,317	16,128	11·592
Yeomanry . . .	17,844	1,487	73	30	103	49·092	368,187	30,682	637	1,042	1,679	20·761
Colonials . . .	53,747	4,479	135	64	199	30·141	1,215,484	101,290	1,424	1,652	3,076	14·059
Total (all classes) .	233,195	19,433	662	371	1,033	34·066	6,563,996	546,999	6,872	14,011	20,883	12·563
Total Officers & Men } (all classes) }	6,797,191	566,432	7,534	14,382	21,916	13·301						
						25·391						
						38·692						

We find that the death-rates varied considerably at different periods of the war, and Tables IV and V have been inserted for the purpose of illustrating this point. For example, in the later stages of the war the mortality among the officers was very much less than in the earlier, owing in some degree no doubt to their ceasing to wear their customary distinctive uniform and badges.

The importance of this feature of the war from an assurance point of view is at once apparent. Any premium calculated on the average death-rate for the whole war might be misleading, for a policyholder has the option of discontinuing his assurance at the end of the first year, and a glance at Table IV will show that the average yearly premium for officers of 5·3 per-cent would have been altogether too small for the first year's risk taken by itself. In this connection, we may note that the death-rate for the first year of the war obtained from the experience of the officers of the combined forces was 7·8 per-cent, falling in the second year to 4·13, remaining almost at the same rate for the third year, namely, 4·24.

Mr. Anderson states in his paper "that an extra premium of "5 per-cent per annum for officers and  $2\frac{1}{2}$  per-cent for N.C.O's "and men would have met the case, and that generally an extra "rate of £3. 10s. 0d. per-cent would have been sufficient", and later on he remarks that an office "that charged a premium of "5 per-cent per annum throughout the war would undoubtedly "come out on the right side, and could, in fact, have charged a "much smaller war rate." These quotations are given because we cannot help thinking that they are, to some extent, misleading. From an assurance point of view, officers and men must nearly always be kept apart, and to give premiums founded upon their combined experience is likely to lead officers to imagine that a war risk can be covered by such premiums, and to be disappointed when they find out their mistake. The risk of death to officers and men differs considerably, and it would only be possible, from the point of view of an Assurance Company, to charge the same extra premium for both, if the numbers of officers and men effecting assurances were in the same proportion as exists between officers and men in the whole army, and if the average sum assured were the same for officers as for men. It is difficult to imagine such a state of affairs existing in England, although it is, perhaps, possible in a country where conscription is the rule.



TABLE IV.—Comparison of Death-Rates for various periods of the War.

Period	REGULARS, &c.										
	OFFICERS.					N.C.O's AND MEN					
	Months of Exposure	Years of Exposure	Deaths			Yearly Death-Rate per 1,000	Months of Exposure	Years of Exposure	Deaths		
			Wounds	Illness	Total				Wounds	Illness	Total
11 Oct. 1899 to 30 Sept. 1900	51,399	4,283	256	119	375	59-771	27-784	87-555	2,895	5,303	8,198
1 Oct. 1900 to 30 Sept. 1901	65,199	5,433	99	91	190	18-222	16-749	34-971	1,297	3,696	4,993
1 Oct. 1901 to 31 May 1902	45,006	3,751	99	67	166	26-393	17-862	44-255	619	2,318	2,937
	161,604	13,467	454	277	731	33-712	20-569	54-281	4,811	11,317	16,128
YEOMANRY											
1 Feb. 1900 to 31 Jan. 1901	5,727	477	17	7	24	35-639	14-675	50-314	182	328	510
1 Feb. 1901 to 31 Jan. 1902	9,322	777	47	14	61	60-489	18-018	78-507	352	507	859
1 Feb. 1902 to 31 May 1902	2,795	233	9	9	18	38-627	38-627	77-254	103	207	310
	17,844	1,487	73	30	103	49-092	20-175	69-267	637	1,042	1,679
									637	1,042	1,679
									20,761	33,961	54,722
COLONIALS											
1 Dec. 1899 to 30 Nov. 1900	16,610	1,384	61	19	80	44-075	13-728	57-803	554	538	1,092
1 Dec. 1900 to 30 Nov. 1901	24,906	2,076	62	29	91	29-865	13-969	43-834	629	676	1,305
1 Dec. 1901 to 31 May 1902	12,231	1,019	12	16	28	11-776	15-702	27-478	241	438	679
	53,747	4,479	135	64	199	30-141	14-289	44-430	1,424	1,652	3,076
									1,424	1,652	3,076
									16,310	19,004	35,314
									16,310	19,004	35,314
									30,369	34,883	65,252



IMPERIAL YEOMANRY AND VOLUNTEERS—ASSURED LIVES.

Here a card was written for each of the 19,269 lives under observation, showing the policy number, the name, age at entry, date of entry, date of exit, and duration. The summary of the facts dealt with was as follows :—

Number of Policies	...	...	...	19,269
Of which there—				
Left Active Service, or otherwise passed from observation	...	...	...	12,515
Died from Wounds	...	...	193	
Died from other causes	...	...	638	
			—	831
Existing in South Africa on 31 May 1902, the end of period of observation	...	...	...	5,923
Total months of exposure to risk	...	...	...	256,450
Average duration	...	...	...	13·31 months.

The cards were treated in two distinct ways. They were first sorted into calendar months of the war, so as to compare with the figures given in the general mortality of the Imperial Forces, and then sorted into months of assurance for the purpose of obtaining monetary results, and of tracing the effects of continued exposure to the risks of war, in order to test whether the lives became hardened against disease as time passed. Table VI gives the preliminary facts in calendar months of the war, and is here appended, as it gives information that could be used by anyone desiring to further investigate the subject.



CALENDAR MONTHS.

AGES 23-27.

TABLE VI.—Assured Yeomanry and Volunteers.

AGES 17-22.

Month (1)	No. of Policies Issued (2)	Left Active Service or otherwise withdrew from observa- tion (3)	Died		Sums of Columns (3), (4), (5) (6)	(2)-(6) for preceding Month (7)	Months of Exposure to Risk (8)	No. of Policies Issued (2)	Left Active Service or otherwise withdrew from observa- tion (3)	Died		Sums of Columns (3), (4), (5) (6)	(2)-(6) for preceding Month (7)	Months of Exposure to Risk (8)
			Wounds (4)	Illness (5)						Wounds (4)	Illness (5)			
January 1900 .	32	...	...	...	...	32	32	41	...	...	...	...	41	41
February .	1,090	...	...	...	...	1,090	1,122	1,228	2	...	...	...	1,228	1,269
March .	1,274	1	...	...	2	1,274	2,396	1,339	3	...	3	6	1,336	2,605
April .	367	2	...	1	5	365	2,761	466	1	1	3	5	460	3,065
May .	296	1	...	3	10	291	3,052	344	6	5	13	24	339	3,404
June .	50	12	2	8	30	40	3,092	78	21	2	22	45	54	3,458
July .	36	20	4	8	32	6	3,098	52	27	2	14	43	7	3,465
August .	31	23	3	6	32	1	3,097	33	29	2	9	40	10	3,455
September .	3	17	3	7	27	29	3,068	3	14	1	3	18	37	3,418
October .	8	38	2	7	47	19	3,049	7	35	4	1	40	11	3,407
November .	...	110	1	7	118	47	3,002	1	117	...	6	123	39	3,368
December .	3	58	2	8	68	115	2,887	5	56	2	16	74	118	3,250
January 1901 .	12	44	2	14	60	56	2,831	8	58	2	13	73	66	3,184
February .	554	230	2	8	240	494	3,325	489	265	...	16	281	416	3,600
March .	2,316	341	1	14	356	2,076	5,401	1,742	337	3	4	344	1,461	5,061
April .	619	230	3	7	240	263	5,664	392	292	...	17	309	48	5,109
May .	141	786	4	20	810	99	5,565	85	835	2	21	858	224	4,885
June .	21	468	3	11	482	789	4,776	17	502	3	10	515	841	4,044
July .	12	180	3	12	195	470	4,306	14	224	4	3	231	501	3,543
August .	4	94	...	2	96	191	4,115	1	109	2	6	117	230	3,313
September .	3	39	3	3	45	93	4,022	...	64	5	4	73	117	3,196
October .	3	71	5	...	76	42	3,980	2	62	6	...	68	71	3,125
November .	1	79	4	11	94	75	3,905	...	66	...	...	66	68	3,057
December .	57	83	4	21	108	37	3,868	26	83	5	10	98	40	3,017
January 1902 .	6	54	1	16	71	102	3,766	7	54	1	14	69	91	2,926
February .	115	272	10	17	299	44	3,810	81	269	9	11	289	12	2,938
March .	474	756	3	12	771	175	3,985	229	615	5	22	642	60	2,878
April .	227	385	3	2	390	544	3,441	99	261	...	7	268	543	2,335
May .	71	234	...	2	236	319	3,122	65	205	1	4	210	203	2,132
							100,538	6,854	4,612	67	253	4,932	2,132	92,548

Month (1)	No. of Policies Issued (2)	Left Active Service or otherwise withdrew from observation (3)	Died		Sums of Columns (3), (4), (5) (6)	(2)-(6) for preceding Month (7)	Months of Exposure to Risk (8)	No. of Policies Issued (2)	Left Active Service or otherwise withdrew from observation (3)	Died		Sums of Columns (3), (4), (5) (6)	(2)-(6) for preceding Month (7)	Months of Exposure to Risk (8)
			Wounds (4)	Illness (5)						Wounds (4)	Illness (5)			
January 1900	20	...	...	...	...	20	7	20	...	...	...	...	20	7
February	479	...	...	1	1	479	260	499	...	...	...	...	499	267
March	568	...	...	1	1	567	300	1,066	...	...	...	...	1,066	567
April	230	...	...	3	3	229	144	1,295	...	...	...	...	1,295	710
May	143	...	...	6	9	140	76	1,435	...	...	...	...	1,435	786
June	64	3	3	7	13	55	63	1,490	...	...	...	...	1,490	842
July	40	12	1	7	20	27	27	1,517	...	...	...	...	1,517	866
August	20	14	1	3	18	...	9	1,517	...	...	...	...	1,517	868
September	...	10	...	2	12	18	...	1,499	...	...	...	...	1,499	859
October	...	19	...	3	22	10	...	1,489	...	...	...	...	1,489	851
November	1	38	4	3	45	21	...	1,468	...	...	...	...	1,468	838
December	...	24	1	4	29	45	...	1,423	...	...	...	...	1,423	809
January 1901	4	25	...	3	28	25	2	1,398	...	...	...	...	1,398	790
February	154	102	4	3	109	126	85	1,524	...	...	...	...	1,524	854
March	616	156	1	4	161	507	310	2,031	...	...	...	...	2,031	1,134
April	113	146	...	4	150	48	71	1,983	...	...	...	...	1,983	1,139
May	48	305	2	3	310	102	45	1,881	...	...	...	...	1,881	1,112
June	2	242	...	6	248	308	1	1,573	...	...	...	...	1,573	903
July	4	115	...	2	117	244	2	1,329	...	...	...	...	1,329	748
August	2	45	1	...	46	115	1	1,214	...	...	...	...	1,214	694
September	...	29	...	1	30	46	...	1,168	...	...	...	...	1,168	679
October	3	21	...	1	24	27	...	1,141	...	...	...	...	1,141	665
November	1	29	1	3	33	23	...	1,118	...	...	...	...	1,118	641
December	10	28	2	4	34	23	2	1,095	...	...	...	...	1,095	622
January 1902	4	24	1	3	28	30	...	1,065	...	...	...	...	1,065	602
February	20	97	3	5	105	8	8	1,057	...	...	...	...	1,057	591
March	73	241	...	4	245	32	38	1,025	...	...	...	...	1,025	562
April	29	98	...	2	100	216	28	809	...	...	...	...	809	450
May	24	80	...	1	81	76	19	733	...	...	...	...	733	420
	2,674	1,906	27	89	2,022	733	1,530	36,862	1,085	20	39	1,144	420	20,876

TABLE VI (continued).—Assured Yeomanry and Volunteers.

CALENDAR MONTHS.

AGES 43-47.

AGES 38-42.

Month (1)	No. of Policies Issued (2)	Left Active Service or otherwise withdrew from observa- tion (3)	DIED		Sums of Columns (3), (4), (5) (6)	(2)-(6) for preceding Month (7)	Months of Exposure to Risk (8)	Left Active Service or otherwise withdrew from observa- tion (3)	DIED		Sums of Columns (3), (4), (5) (6)	(2)-(6) for preceding Month (7)	Months of Exposure to Risk (8)
			Wounds (4)	Illness (5)					Wounds (4)	Illness (5)			
January 1900 .	53	1	...	1	2	53	53	...	...	...	...	13	...
February .	59	...	...	...	...	57	110	...	...	...	...	24	37
March .	51	...	...	1	1	51	161	1	...	1	2	15	52
April .	10	...	...	1	1	9	170	...	2	...	2	3	55
May .	25	...	...	...	...	23	193	...	...	...	...	7	62
June .	3	...	...	...	...	3	196	...	1	...	1	1	63
July .	6	1	...	1	2	6	202	...	...	...	...	...	63
August .	...	1	...	...	1	2	200	1	...	...	1	...	63
September .	...	4	...	...	4	...	199	...	...	...	...	1	62
October .	...	3	...	1	4	4	195	...	...	...	...	...	62
November .	...	3	...	1	4	4	191	2	...	...	2	...	62
December .	...	3	...	1	1	1	187	2	...	...	2	...	60
January 1901 .	12	1	...	...	14	11	198	5	...	...	5	1	61
February .	35	13	...	1	18	21	219	5	...	...	5	3	64
March .	6	17	...	1	22	12	207	8	...	...	8	3	61
April .	7	21	...	1	20	15	192	1	...	...	1	4	57
May .	1	19	...	1	36	19	173	10	2	...	12	1	56
June .	...	36	...	...	23	36	137	6	...	...	6	12	44
July .	...	23	...	1	8	22	115	2	...	...	2	5	39
August .	1	7	...	...	3	8	107	1	...	...	1	2	37
September .	...	2	1	...	6	3	104	3	1	...	4	1	36
October .	...	6	...	...	4	6	98	...	...	...	...	4	32
November .	...	4	...	...	6	3	95	1	1	...	2	...	32
December .	1	5	1	...	6	6	89	...	...	...	...	2	30
January 1902 .	...	1	...	...	1	2	91	...	...	...	...	...	30
February .	3	14	...	...	14	10	81	1	...	...	7	...	23
March .	4	13	...	...	13	10	71	7	...	...	4	7	19
April .	3	16	...	...	16	15	56	4	...	...	1	4	...
May .	1	4	...	...	4	15	...	1	...	...	...	...	...
	291	215	3	11	229	56	4,090	61	7	1	69	19	1,305



TABLE VI (continued).

*Assured Yeomanry and Volunteers.*

CALENDAR MONTHS.

AGES 48 AND UPWARDS.

Month	No. of Policies Issued.	Left Active Service or otherwise withdrew from observation	DIED		Sums of Cols. (3), (4), (5)	(2) - (6) for pre- ceding month	Exposed to Risk
			Wounds	Illness			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
January 1900	...	...	...	...	...	...	...
February . .	2	...	...	...	...	2	2
March . . .	2	...	...	...	...	2	4
April . . .	5	...	...	...	...	5	9
May . . . .	...	...	...	1	1	...	9
June . . . .	1	...	...	...	...	...	9
July . . . .	...	...	...	...	...	...	9
August . . .	...	1	...	...	1	...	9
September .	1	...	...	...	...	...	9
October . . .	...	1	...	...	1	...	9
November . .	...	...	...	...	...	-1	8
December . .	...	...	...	...	...	...	8
January 1901	...	...	...	...	...	...	8
February . .	...	...	...	1	1	...	8
March . . . .	5	2	...	...	2	4	12
April . . . .	...	...	...	...	...	-2	10
May . . . . .	...	...	...	...	...	...	10
June . . . . .	...	...	...	...	...	...	10
July . . . . .	...	1	...	...	1	...	10
August . . . .	...	...	...	...	...	-1	9
September . .	...	...	...	...	...	...	9
October . . . .	...	...	...	...	...	...	9
November . . .	...	2	...	...	2	...	9
December . . .	...	...	...	...	...	-2	7
January 1902	...	...	...	...	...	...	7
February . . .	...	...	...	...	...	...	7
March . . . . .	...	1	...	...	1	...	7
April . . . . .	1	...	...	...	...	...	7
May . . . . .	...	...	...	...	...	...	7
	17	8	...	2	10	7	231

The figures obtained by this Table are described in the paper as those referring to the "Calendar Month" method, the term "Select" being used to denote those obtained in the form of "Policy Months." The age was taken as the age next birthday at entry. The date of entry was taken as the first day of the month in which the policy was issued. It was assumed that the date upon which active service commenced was the same as the date of entry. Thus the results are of double value, enabling us to obtain monetary values, and to trace the effect upon the death-rate of duration of service.

The period at which the lives passed from observation was affected in two ways. In some few cases, the policy was lapsed whilst the assured was still on active service. In these the date of lapse was taken as the last day of the month of renewal, that is, the period at which the days of grace expired. The other way in which lives passed from observation was by the life assured leaving active service whilst the policy still remained in force. In this class we experienced some difficulty in determining the exact date upon which the risk ceased. The date obtained was in some instances that upon which the life assured arrived in England, and in others that upon which he left South Africa. As, however, the forces were not actually disbanded for some little time after, it has been assumed that they remained exposed till the end of the month, an assumption which considerably simplified the work of obtaining the "Exposed to Risk."

The deaths were treated as being exposed for the whole of the month in which they occurred.

These assumptions enabled the "Exposed to Risk" to be obtained by a very simple process, the resulting formula being—

Exposed to risk for calendar month  $n$ .

Equals Exposed to risk for month  $n-1$ .

Minus discontinuances and deaths for month  $n-1$ .

Plus policies issued during month  $n$ .

Table VII summarizes these particulars for all months for each of the various groups of ages.

TABLE VII.

*Assured Yeomanry and Volunteers.*

CALENDAR MONTHS.

SUMMARY.

ALL MONTHS.

(1)	Number of Policies Issued (2)	Left Active Service or otherwise withdrew from observa- tion (3)	DIED		Sum of Columns (3), (4), (5) (6)	(2)—(6) for pre- ceding month (7)	Exposed to Risk (8)
			Wounds (4)	Illness (5)			
Ages 17-22 . .	7,826	4,628	69	243	4,940	3,122	100,538
Ages 23-27 . .	6,854	4,612	67	253	4,932	2,132	92,548
Ages 28-32 . .	2,674	1,906	27	89	2,022	733	36,862
Ages 33-37 . .	1,530	1,085	20	39	1,144	420	20,876
Ages 38-42 . .	281	215	3	11	229	56	4,090
Ages 43-47 . .	87	61	7	1	69	19	1,305
Ages 48 & upwards	17	8	...	2	10	7	231
Total . .	19,269	12,515	193	638	13,346	6,489	256,450

$$\frac{256,450}{19,269} = 13.31 = \text{Average duration in months.}$$

Table VIII gives the monthly death-rates for the seven groups of ages for each calendar month of the war.



TABLE VIII.—*Assured Yeomanry and Volunteers.*

CALENDAR MONTHS.

JANUARY 1900										FEBRUARY 1900				
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000			Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 .	32	...	...	...	...	...	...	1,122	...	...	...	...	...	...
23-27 .	41	...	...	...	...	...	...	1,269	...	1	...	...	...	...
28-32 .	20	...	...	...	...	...	...	499	...	1	...	...	...	...
33-37 .	7	...	...	...	...	...	...	267	...	...	...	...	...	...
38-42 .	...	...	...	...	...	...	...	53	...	1	...	...	...	...
43-47 .	...	...	...	...	...	...	...	13	...	...	...	...	...	...
48 upwards	...	...	...	...	...	...	...	2	...	...	...	...	...	...
All ages .	100	...	...	...	...	...	...	3,225	...	3	...	...	...	...

MARCH 1900										APRIL 1900				
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000			Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 .	2,396	...	1	1	...	.417	.417	2,761	...	3	3	...	1,087	
23-27 .	2,605	...	3	3	...	1.152	1.152	3,065	1	3	4	.326	.979	
28-32 .	1,066	...	1	1	...	.938	.938	1,295	...	3	3	...	2,317	
33-37 .	567	...	...	...	...	...	...	710	...	...	...	...	...	
38-42 .	110	...	...	...	...	...	...	161	...	1	1	...	6,211	
43-47 .	37	...	...	...	...	...	...	52	...	1	1	...	19,231	
48 upwards	4	...	...	...	...	...	...	9	...	...	...	...	...	
All ages .	6,785	...	5	5	...	.737	.737	8,053	1	11	12	.124	1,366	
													1,490	

TABLE VIII (continued).—Assured Yeomanry and Volunteers.

CALENDAR MONTHS.

MAY 1900										JUNE 1900									
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000			Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000							
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (6)	Illness (7)	Total (8)								
17-22 .	3,052	1	8	9	328	2,621	2,949	3,092	2	16	18	647	5,175	5,822					
23-27 .	3,404	5	13	18	1,469	3,819	5,288	3,458	2	22	24	578	6,362	6,940					
28-32 .	1,435	...	6	6	...	4,181	4,181	1,490	3	7	10	2,013	4,698	6,711					
33-37 .	786	1	4	5	1,272	5,089	6,361	842	...	2	2	...	2,375	2,375					
38-42 .	170	1	1	2	5,882	5,882	11,764	193	...	...	...	...	...	...					
43-47 .	55	2	...	2	36,364	...	36,364	62	...	...	...	...	...	...					
48 upwards	9	...	1	1	...	111,111	111,111	9	...	...	...	...	...	...					
All ages .	8,911	10	33	43	1,122	3,703	4,825	9,146	7	47	54	765	5,139	5,904					
JULY 1900										AUGUST 1900									
17-22 .	3,098	4	8	12	1,291	2,582	3,873	3,097	3	6	9	969	1,937	2,906					
23-27 .	3,465	2	14	16	577	4,040	4,617	3,455	2	9	11	579	2,605	3,184					
28-32 .	1,517	1	7	8	659	4,614	5,273	1,517	1	3	4	659	1,978	2,637					
33-37 .	866	3	...	3	3,464	...	3,464	868	1	...	1	1,152	...	1,152					
38-42 .	196	...	...	...	...	...	...	202	...	1	1	...	4,951	4,951					
43-47 .	63	1	...	1	15,873	...	15,873	63	...	...	...	...	...	...					
48 upwards .	9	...	...	...	...	...	...	9	...	...	...	...	...	...					
All Ages .	9,214	11	29	40	1,194	3,147	4,341	9,211	7	19	26	760	2,063	2,823					

TABLE VIII (continued).—Assured Yeomanry and Volunteers.

CALENDAR MONTHS.

OCTOBER 1900														
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000			Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 .	3,068	3	7	10	·978	2·282	3·260	3,049	2	7	9	·656	2·296	2·952
23-27 .	3,418	1	3	4	·293	·878	1·171	3,407	4	1	5	1·174	·294	1·468
28-32 .	1,499	...	2	2	...	1·334	1·334	1,489	...	3	3	...	2·015	2·015
33-37 .	859	1	1	2	1·164	1·164	2·328	851	4	1	5	4·700	1·175	5·875
38-42 .	200	...	...	...	...	...	...	199	...	...	...	...	...	...
43-47 .	63	...	...	...	...	...	...	62	...	...	...	...	...	...
48 upwards .	9	...	...	...	...	...	...	9	...	...	...	...	...	...
All Ages .	9,116	5	13	18	·548	1·426	1·974	9,066	10	12	22	1·103	1·324	2·427

NOVEMBER 1900														
17-22 .	3,002	1	7	8	·333	2·332	2·665	2,887	2	8	10	·693	2·771	3·464
23-27 .	3,368	...	6	6	...	1·781	1·781	3,250	2	16	18	·615	4·923	5·538
28-32 .	1,468	4	3	7	2·725	2·044	4·769	1,423	1	4	5	·703	2·811	3·514
33-37 .	838	2	...	2	2·387	...	2·387	809	1	4	5	1·236	4·944	6·180
38-42 .	195	...	1	1	...	5·128	5·128	191	...	1	1	...	5·236	5·236
43-47 .	62	...	...	...	...	...	...	62	...	...	...	...	...	...
48 upwards .	8	...	...	...	...	...	...	8	...	...	...	...	...	...
All Ages .	8,941	7	17	24	·783	1·901	2·684	8,630	6	33	39	·695	3·824	4·519



TABLE VIII (continued).—Assured Yeomanry and Volunteers.

CALENDAR MONTHS.

JANUARY 1901				FEBRUARY 1901			
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 .	2,831	2	14	16	706	4,945	5,651
23-27 .	3,184	2	13	15	628	4,083	4,711
28-32 .	1,398	...	3	3	...	2,146	2,146
33-37 .	790	...	1	1	...	1,266	1,266
38-42 .	187	...	...	...	...	...	...
43-47 .	60	...	...	...	...	...	...
48 upwards .	8	...	...	...	...	...	...
All Ages .	8,458	4	31	35	473	3,665	4,138
MARCH 1901							
17-22 .	5,401	1	14	15	185	2,592	2,777
23-27 .	5,061	3	4	7	593	790	1,383
28-32 .	2,031	1	4	5	492	1,969	2,461
33-37 .	1,134	2	2	4	1,764	1,764	3,528
38-42 .	219	...	1	1	...	4,566	4,566
43-47 .	64	...	...	...	...	...	...
48 upwards .	12	...	...	...	...	...	...
All Ages .	13,922	7	25	32	503	1,796	2,299
APRIL 1901							
17-22 .	5,664	3	7	10	530	1,236	1,766
23-27 .	5,109	...	17	17	...	3,327	3,327
28-32 .	1,983	...	4	4	...	2,017	2,017
33-37 .	1,139	...	3	3	...	2,634	2,634
38-42 .	207	...	1	1	...	4,831	4,831
43-47 .	61	...	...	...	...	...	...
48 upwards .	10	...	...	...	...	...	...
All Ages .	14,173	3	32	35	212	2,258	2,470

TABLE VIII (continued).—Assured Yeomanry and Volunteers.

CALENDAR MONTHS.

JUNE 1901									
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000	Months of Exposure to Risk (2)	DEATHS		
		Wounds (3)	Illness (4)	Total (5)			Wounds (3)	Illness (4)	Total (5)
17-22 .	5,565	4	20	24	·719	4,776	3	11	14
23-27 .	4,885	2	21	23	·409	4,044	3	10	13
28-32 .	1,881	2	3	5	1·063	1,573	...	6	6
33-37 .	1,112	...	2	2	...	903	...	2	2
38-42 .	192	...	1	1	...	173	...	...	...
43-47 .	57	...	...	...	...	56	2	...	2
48 upwards .	10	...	...	...	...	10	...	...	...
All Ages .	13,702	8	47	55	·584	11,535	8	29	37
MONTHLY DEATH-RATE PER 1,000									
		Wounds (6)	Illness (7)	Total (8)			Wounds (6)	Illness (7)	Total (8)
		·628	2·303	2·931			·628	2·303	2·931
		·742	2·473	3·215			·742	2·473	3·215
		...	3·814	3·814			...	3·814	3·814
		...	2·215	2·215			...	2·215	2·215
		...	...	...			...	...	...
		35·714	...	35·714			35·714	...	35·714
		...	...	...			...	...	...
		·691	2·514	3·208			·691	2·514	3·208
AUGUST 1901									
17-22 .	4,115	...	2	2	·486	4,115	...	·486	·486
23-27 .	3,313	2	6	8	1·811	3,313	·604	1·811	2·415
28-32 .	1,214	1	...	1	·824	1,214	·824	...	·824
33-37 .	694	...	...	...	...	694	...	...	...
38-42 .	115	...	1	1	8·696	115	...	8·696	8·696
43-47 .	39	...	...	...	...	39	...	...	...
48 upwards .	9	...	...	...	...	9	...	...	...
All ages .	9,499	3	9	12	·947	9,499	·316	·947	1·263

TABLE VIII (continued).—*Assured Yeomanry and Volunteers.*

CALENDAR MONTHS.

OCTOBER 1901									
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000	Months of Exposure to Risk (2)	DEATHS		
		Wounds (3)	Illness (4)	Total (5)			Wounds (3)	Illness (4)	Total (5)
17-22 .	4,022	3	3	6	746	3,980	5	...	5
23-27 .	3,196	5	4	9	1,564	3,125	6	...	6
28-32 .	1,168	...	1	1	...	1,141	2	1	3
33-37 .	679	...	...	...	...	665	2	2	4
38-42 .	107	1	...	1	9,346	104	...	...	...
43-47 .	37	...	...	...	...	36	1	...	1
48 upwards .	9	...	...	...	...	9	...	...	...
All ages .	9,218	9	8	17	976	9,060	16	3	19
NOVEMBER 1901									
17-22 .	3,905	4	11	15	1,024	3,868	4	21	25
23-27 .	3,057	...	...	...	...	3,017	5	10	15
28-32 .	1,118	1	3	4	894	1,095	2	4	6
33-37 .	641	...	...	...	...	622	...	7	7
38-42 .	98	...	...	...	...	95	1	...	1
43-47 .	32	...	...	...	...	32	1	...	1
48 upwards .	9	...	...	...	...	7	...	...	...
All ages .	8,860	5	14	19	564	8,736	13	42	55
DECEMBER 1901									
17-22 .	6,463	1,034	5,429	6,463	3,868	3,868	4	21	25
23-27 .	4,972	1,657	3,315	4,972	3,017	3,017	5	10	15
28-32 .	5,479	1,826	3,653	5,479	1,095	1,095	2	4	6
33-37 .	11,254	...	11,254	11,254	622	622	...	7	7
38-42 .	10,526	10,526	...	10,526	95	95	1	...	1
43-47 .	31,250	31,250	...	31,250	32	32	1	...	1
48 upwards .	...	...	...	...	7	7	...	...	...
All ages .	6,296	1,488	4,808	6,296	8,736	8,736	13	42	55



TABLE VIII (continued).—Assured Yeomanry and Volunteers.

CALENDAR MONTHS.

FEBRUARY 1902									
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000	Months of Exposure to Risk (2)	DEATHS		
		Wounds (3)	Illness (4)	Total (5)			Wounds (3)	Illness (4)	Total (5)
17-22 .	3,766	1	16	17	4.515	3,810	10	17	27
23-27 .	2,926	1	14	15	5.127	2,938	9	11	20
28-32 .	1,065	1	3	4	3.756	1,057	3	5	8
33-37 .	602	...	2	2	3.322	591	1	3	4
38-42 .	89	...	...	...	...	91	...	...	...
43-47 .	30	...	...	...	...	30	...	...	...
48 upwards .	7	...	...	...	...	7	...	...	...
All Ages .	8,485	3	35	38	4.479	8,524	23	36	59
							2.698	4.223	6.921
APRIL 1902									
Ages at Entry (1)	Months of Exposure to Risk (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000	Months of Exposure to Risk (2)	DEATHS		
		Wounds (3)	Illness (4)	Total (5)			Wounds (3)	Illness (4)	Total (5)
17-22 .	3,985	3	12	15	3.764	3,441	3	2	5
23-27 .	2,878	5	22	27	9.381	2,335	...	7	7
28-32 .	1,025	...	4	4	3.902	809	...	2	2
33-37 .	562	1	1	2	3.558	450	...	2	2
38-42 .	81	...	...	...	...	71	...	...	...
43-47 .	30	...	...	...	...	23	...	...	...
48 upwards .	7	...	...	...	...	7	...	...	...
All Ages .	8,568	9	39	48	4.552	7,136	3	13	16
							420	1.822	2.242

TABLE VIII (continued).  
*Assured Yeomanry and Volunteers.*

CALENDAR MONTHS.

MAY 1902							
Ages at Entry  (1)	Months of Exposure to Risk  (2)	DEATHS			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	3,122	...	2	2	...	·641	·641
23-27 . .	2,132	1	4	5	·469	1·876	2·345
28-32 . .	733	...	1	1	...	1·364	1·364
33-37 . .	420	...	...	...	...	...	...
38-42 . .	56	...	...	...	...	...	...
43-47 . .	19	...	...	...	...	...	...
48 upwards .	7	...	...	...	...	...	...
All Ages .	6,489	1	7	8	·154	1·079	1·233

In column 2 are entered the months of exposure, and in columns 3, 4, and 5, the corresponding deaths; these figures being simply copied from Table VI. Columns 6, 7, and 8 give the monthly death-rates per 1,000 for the particular group of ages from wounds, from other causes, and from all causes combined, respectively.

Table IX gives a summary of the results for all ages in calendar months of the war.

The total rates for all ages obtained by the calendar month method have been reproduced in the form of curves in Diagrams C, D and E. It will be seen on comparing the death-rates of the assured Yeomanry and Volunteers with those of the whole of the Yeomanry deduced from the official War Office returns, that the rates of the former are considerably the lower. There are two or three reasons for this, the chief being the different methods which we were compelled to adopt in obtaining the time of exposure to risk. As already explained, all the official War Office returns assumed the period of risk to commence upon landing in South Africa, and to end upon leaving that country. The assured lives on the other hand were assumed to commence their period of risk before sailing from the United Kingdom, and to end it on their return. Again, during the last eight or nine months of the war, a number of Yeomanry were kept at home

under orders, but did not actually sail for some months. In the assured experience they were treated as at risk for the whole period. As far as the calculation of war risk premiums is concerned, it would seem to be more correct to make the assumption used for assured lives, as an extra premium is usually paid before the policyholder leaves the United Kingdom.

TABLE IX.

*Assured Yeomanry and Volunteers.*

ALL AGES.

## SUMMARY IN CALENDAR MONTHS.

Month	Months of Exposure to Risk	DEATHS			MONTHLY DEATH-RATE PER 1,000		
		Wounds	Illness	Total	Wounds	Illness	Total
January 1900 .	100	...	...	...	...	...	...
February . . .	3,225	...	3	3	...	·930	·930
March . . . .	6,785	...	5	5	...	·737	·737
April . . . . .	8,053	1	11	12	·124	1·366	1·490
May . . . . .	8,911	10	33	43	1·122	3·703	4·825
June . . . . .	9,146	7	47	54	·765	5·139	5·904
July . . . . .	9,214	11	29	40	1·194	3·147	4·341
August . . . .	9,211	7	19	26	·760	2·063	2·823
September . .	9,116	5	13	18	·548	1·426	1·974
October . . . .	9,066	10	12	22	1·103	1·324	2·427
November . . .	8,941	7	17	24	·783	1·901	2·684
December . . .	8,630	6	33	39	·695	3·824	4·519
January 1901 .	8,458	4	31	35	·473	3·665	4·138
February . . .	9,570	7	29	36	·731	3·030	3·761
March . . . . .	13,922	7	25	32	·503	1·796	2·299
April . . . . .	14,173	3	32	35	·212	2·258	2·470
May . . . . .	13,702	8	47	55	·584	3·430	4·014
June . . . . .	11,535	8	29	37	·694	2·514	3·208
July . . . . .	10,117	7	17	24	·692	1·680	2·372
August . . . .	9,499	3	9	12	·316	·947	1·263
September . .	9,218	9	8	17	·976	·868	1·844
October . . . .	9,060	16	3	19	1·766	·331	2·097
November . . .	8,860	5	14	19	·564	1·580	2·144
December . . .	8,736	13	42	55	1·488	4·808	6·296
January 1902 .	8,485	3	35	38	·354	4·125	4·479
February . . .	8,524	23	36	59	2·698	4·223	6·921
March . . . . .	8,568	9	39	48	1·050	4·552	5·602
April . . . . .	7,136	3	13	16	·420	1·822	2·242
May . . . . .	6,489	1	7	8	·154	1·079	1·233
Total . . . .	256,450	193	638	831	·753	2·488	3·241

A third reason was the inclusion of Volunteers with the Yeomanry in the assured investigation. Although the larger number were Yeomanry, yet the inclusion of Volunteers



undoubtedly tended to lower the death-rate, for the latter were not, on the average, placed in positions of such danger as the former.

In Diagram E we see that the curve for the assured lives has very much the same conformation as that of the Yeomanry, being above it, however, at first, but later on falling appreciably below.

It will be remembered that the Yeomanry went out in three detachments, the first going out about February 1900, the second about a year later, whilst the third did not go out till very near the close of the war. It has been said that the second detachment were much inferior in general physique to the first, and possibly this may account for some of the divergences of the curves in the later stages of the war, as the proportion of assured in the second detachment was not so great as in the first. Another point may be suggested, though it would be unsafe to place any stress upon it from the diagram before us; namely, that the course of time enabled the Yeomanry of 1900 to become more acclimatized, or at all events more accustomed to the new life of hardship and fatigue. This may, however, better be discussed by the use of the "Policy Months" method.

Table X gives the original facts in months of exposure to risk for each month of assurance for each of the seven groups of ages, arranged in such a manner as to allow of the exposed to risk for yearly rates of mortality being obtained as well as for monthly, these two rates being convenient for the calculation of premiums.

As in the calendar month method, the discontinuances and the deaths were treated as being at risk until the end of the assurance month in which they occurred, the entrants being treated as coming in at the beginning of the month.

The exposed to risk for the monthly rate were thus obtained by a method similar to that adopted for the calendar month system.

For the yearly rate the exposed to risk were obtained by leaving in the deaths during intervening months, and subtracting them only at the close of the 12th, 24th, and 29th month, respectively.

TABLE X.

*Assured Yeomanry and Volunteers.*

SELECT.

ENTERED 7,826.

AGES 17—22.

Month (n)	Existing	Left Active Service or otherwise withdrew from observ- ation	DIED			Col. (2) + Col. (3)	Col. (6) + Col. (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	72	9	1	2	3	81	84	7,826	7,826
2	221	30	1	6	7	251	258	7,742	7,745
3	460	43	3	21	24	503	527	7,484	7,494
4	113	50	9	20	29	163	192	6,957	6,991
5	6	54	3	24	27	60	87	6,765	6,828
6	56	61	6	14	20	117	137	6,678	6,768
7	1	68	5	6	11	69	80	6,541	6,651
8	2	75	7	12	19	77	96	6,461	6,582
9	2	104	6	19	25	106	131	6,365	6,505
10	4	162	1	27	28	166	194	6,234	6,399
11	9	85	4	25	29	94	123	6,040	6,233
12	15	140	6	35	41	155	196	5,917	6,139
13	110	1,832	7	7	14	1,942	1,956	5,721	5,721
14	223	216	5	7	12	439	451	3,765	3,779
15	1,124	531	1	2	3	1,655	1,658	3,314	3,340
16	282	515	...	8	8	797	805	1,656	1,685
17	2	246	...	2	2	248	250	851	888
18	...	55	...	...	...	55	55	601	640
19	...	31	...	...	...	31	31	546	585
20	...	17	2	...	2	17	19	515	554
21	...	19	1	1	2	19	21	496	537
22	6	28	...	1	1	34	35	475	518
23	9	15	...	...	...	24	24	440	484
24	4	13	1	1	2	17	19	416	460
25	17	218	...	1	1	235	236	397	397
26	33	4	...	2	2	37	39	161	162
27	69	3	...	...	...	72	72	122	125
28	45	3	...	...	...	48	48	50	53
29	1	1	...	...	...	2	2	2	5
...	2,886	4,628	69	243	312	7,514	7,826	100,538	102,094

TABLE X (continued).

*Assured Yeomanry and Volunteers.*

SELECT.

AGES 23—27.

ENTERED 6,854.

Month (n)	Existing	Left Active Service or otherwise withdrew from observa- tion	DIED			Col (2)+ Col (3)	Col (6)+ Col (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	66	4	...	8	8	70	78	6,854	6,854
2	97	16	...	12	12	113	125	6,776	6,784
3	222	26	5	21	26	248	274	6,651	6,671
4	79	47	5	25	30	126	156	6,377	6,423
5	6	49	8	23	31	55	86	6,221	6,297
6	25	58	3	16	19	83	102	6,135	6,242
7	...	62	8	12	20	62	82	6,033	6,159
8	2	81	7	6	13	83	96	5,951	6,097
9	...	103	3	4	7	103	110	5,855	6,014
10	1	166	6	23	29	167	196	5,745	5,911
11	12	103	3	19	22	115	137	5,549	5,744
12	13	160	8	22	30	173	203	5,412	5,629
13	66	1,647	6	31	37	1,713	1,750	5,209	5,209
14	152	234	2	13	15	386	401	3,459	3,496
15	794	477	2	8	10	1,271	1,281	3,058	3,110
16	226	587	...	3	3	813	816	1,777	1,839
17	3	267	...	1	1	270	271	961	1,026
18	...	92	...	...	...	92	92	690	756
19	1	52	...	1	1	53	54	598	664
20	...	23	1	1	2	23	25	544	611
21	...	19	...	...	...	19	19	519	588
22	4	24	...	...	...	28	28	500	569
23	7	16	...	...	...	23	23	472	541
24	8	18	...	1	1	26	27	449	518
25	14	259	...	2	2	273	275	422	422
26	21	5	...	1	1	26	27	147	149
27	47	12	...	...	...	59	59	120	123
28	55	3	...	...	...	58	58	61	64
29	1	2	...	...	...	3	3	3	6
...	1,922	4,612	67	253	320	6,534	6,854	92,548	94,516



TABLE X (continued).

*Assured Yeomanry and Volunteers.*

SELECT.

AGES 28—32.

ENTERED 2,674.

Month (n)	Existing	Left Active Service or otherwise withdrew from observa- tion	DIED			Col. (2)+ Col. (3)	Col. (6)+ Col. (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	24	3	...	2	2	27	29	2,674	2,674
2	29	3	...	4	4	32	36	2,645	2,647
3	69	15	1	6	7	84	91	2,609	2,615
4	20	18	2	11	13	38	51	2,518	2,531
5	4	15	2	13	15	19	34	2,467	2,493
6	9	22	2	2	4	31	35	2,433	2,474
7	1	26	1	4	5	27	32	2,398	2,443
8	1	38	2	1	3	39	42	2,366	2,416
9	...	41	2	6	8	41	49	2,324	2,377
10	3	58	2	10	12	61	73	2,275	2,336
11	...	44	4	3	7	44	51	2,202	2,275
12	2	74	4	8	12	76	88	2,151	2,231
13	38	695	2	6	8	733	741	2,063	2,063
14	35	89	...	4	4	124	128	1,322	1,330
15	256	207	1	4	5	463	468	1,194	1,206
16	64	231	...	...	...	295	295	726	743
17	2	128	...	1	1	130	131	431	448
18	...	34	...	1	1	34	35	300	318
19	1	19	...	1	1	20	21	265	284
20	...	5	...	...	...	5	5	244	264
21	...	7	1	1	2	7	9	239	259
22	4	5	...	...	...	9	9	230	252
23	5	11	...	...	...	16	16	221	243
24	7	8	1	...	1	15	16	205	227
25	12	102	...	...	...	114	114	189	189
26	11	3	...	1	1	14	15	75	75
27	24	2	...	...	...	26	26	60	61
28	29	3	...	...	...	32	32	34	35
29	2	...	...	...	...	2	2	2	3
...	652	1,906	27	89	116	2,558	2,674	36,862	37,512

TABLE X (continued).

*Assured Yeomanry and Volunteers.*

SELECT.

ENTERED 1,530.

AGES 33—37.

Month (n)	Existing	Left Active Service or otherwise withdrew from observa- tion	DIED			Col. (2)+ Col. (3)	Col. (6)+ Col. (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	18	2	...	...	...	20	20	1,530	1,530
2	28	6	1	1	2	34	36	1,510	1,510
3	37	5	1	4	5	42	47	1,474	1,476
4	9	8	...	3	3	17	20	1,427	1,434
5	...	7	1	1	2	7	9	1,407	1,417
6	2	11	2	1	3	13	16	1,398	1,410
7	...	17	3	1	4	17	21	1,382	1,397
8	...	19	4	2	6	19	25	1,361	1,380
9	...	31	3	4	7	31	38	1,336	1,361
10	1	44	...	3	3	45	48	1,298	1,330
11	1	29	1	4	5	30	35	1,250	1,285
12	...	57	1	2	3	57	60	1,215	1,255
13	35	387	3	1	4	422	426	1,155	1,155
14	29	32	..	3	3	61	64	729	733
15	144	120	...	2	2	264	266	665	672
16	36	140	...	1	1	176	177	399	408
17	2	57	...	...	...	59	59	222	232
18	1	17	...	...	...	18	18	163	173
19	...	6	...	...	...	6	6	145	155
20	...	5	...	...	...	5	5	139	149
21	...	7	...	1	1	7	8	134	144
22	3	5	...	2	2	8	10	126	137
23	1	4	...	2	2	5	7	116	129
24	1	2	...	...	...	3	3	109	124
25	3	62	...	1	1	65	66	106	106
26	10	3	...	...	...	13	13	40	41
27	15	1	...	...	...	16	16	27	28
28	8	1	...	...	...	9	9	11	12
29	2	...	...	...	...	2	2	2	3
...	386	1,085	20	39	59	1,471	1,530	20,876	21,186

TABLE X (continued).  
*Assured Yeomanry and Volunteers.*

SELECT.

AGES 38—42.

ENTERED 281.

Month (n)	Existing	Left Active Service or otherwise withdrew from obser- vation	DIED			Col. (2)+ Col. (3)	Col. (6)+ Col. (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	2	1	...	2	2	3	5	281	281
2	2	...	...	1	1	2	3	276	278
3	4	...	1	2	3	4	7	273	276
4	3	1	...	...	...	4	4	266	272
5	...	1	...	1	1	1	2	262	268
6	1	2	...	...	...	3	3	260	267
7	...	2	1	...	1	2	3	257	264
8	...	3	...	...	...	3	3	254	262
9	...	10	...	...	...	10	10	251	259
10	1	1	1	1	2	2	4	241	249
11	...	4	...	1	1	4	5	237	247
12	1	6	...	...	...	7	7	232	243
13	6	78	...	2	2	84	86	225	225
14	...	11	...	...	...	11	11	139	141
15	17	15	...	...	...	32	32	128	130
16	5	28	...	...	...	33	33	96	98
17	...	12	...	...	...	12	12	63	65
18	...	4	...	1	1	4	5	51	53
19	...	2	...	...	...	2	2	46	49
20	...	1	...	...	...	1	1	44	47
21	...	4	...	...	...	4	4	43	46
22	...	...	...	...	...	...	...	39	42
23	...	3	...	...	...	3	3	39	42
24	2	3	...	...	...	5	5	36	39
25	...	21	...	...	...	21	21	31	31
26	2	...	...	...	...	2	2	10	10
27	4	2	...	...	...	6	6	8	8
28	2	...	...	...	...	2	2	2	2
29	...	...	...	...	...	...	...	...	...
...	52	215	3	11	14	267	281	4,090	4,194



TABLE X (continued).  
*Assured Yeomanry and Volunteers.*

SELECT.

AGES 43—47.

ENTERED 87.

Month (n)	Existing	Left Active Service or otherwise withdrew from observa- tion	DIED			Col. (2)+ Col. (3)	Col. (6)+ Col. (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	...	...	...	...	...	...	...	87	87
2	...	1	...	1	1	1	2	87	87
3	1	1	1	...	1	2	3	85	86
4	...	...	1	...	1	...	1	82	84
5	...	...	1	...	1	...	1	81	84
6	...	...	...	...	...	...	...	80	84
7	...	1	...	...	...	1	1	80	84
8	...	...	...	...	...	...	...	79	83
9	...	2	...	...	...	2	2	79	83
10	1	1	...	...	...	2	2	77	81
11	...	5	...	...	...	5	5	75	79
12	...	3	...	...	...	3	3	70	74
13	3	19	...	...	...	22	22	67	67
14	2	3	...	...	...	5	5	45	45
15	4	3	...	...	...	7	7	40	40
16	2	4	...	...	...	6	6	33	33
17	...	4	3	..	3	4	7	27	27
18	...	3	...	...	...	3	3	20	23
19	...	1	...	...	...	1	1	17	20
20	...	1	1	...	1	1	2	16	19
21	...	...	...	...	...	...	...	14	18
22	...	1	...	...	...	1	1	14	18
23	...	...	...	...	...	...	...	13	17
24	...	1	...	...	...	1	1	13	17
25	...	7	...	...	...	7	7	12	12
26	1	...	...	...	...	1	1	5	5
27	1	...	...	...	...	1	1	4	4
28	3	...	...	...	...	3	3	3	3
29	...	...	...	...	...	...	...	...	...
...	18	61	7	1	8	79	87	1,305	1,364

TABLE X (continued).  
*Assured Yeomanry and Volunteers.*

SELECT.

ENTERED 17.

AGES 48 AND UPWARDS.

Month ( <i>n</i> )	Existing	Left Active Service or otherwise withdrew from observa- tion	DIED			Col. (2)+ Col. (3)	Col. (6)+ Col. (7)	MONTHS OF EXPOSURE	
			Wounds	Illness	Total			For Monthly Rate	For Yearly Rate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	...	...	...	...	...	...	...	17	17
2	1	...	...	...	...	1	1	17	17
3	...	...	...	...	...	...	...	16	16
4	...	...	...	1	1	...	1	16	16
5	...	1	...	...	...	1	1	15	16
6	...	...	...	...	...	...	...	14	15
7	...	1	...	...	...	1	1	14	15
8	...	...	...	...	...	...	...	13	14
9	...	2	...	...	...	2	2	13	14
10	...	...	...	...	...	...	...	11	12
11	...	...	...	1	1	...	1	11	12
12	...	...	...	...	...	...	...	10	12
13	...	1	...	...	...	1	1	10	10
14	...	1	...	...	...	1	1	9	9
15	3	...	...	...	...	3	3	8	8
16	...	1	...	...	...	1	1	5	5
17	...	...	...	...	...	...	...	4	4
18	...	...	...	...	...	...	...	4	4
19	...	...	...	...	...	...	...	4	4
20	...	...	...	...	...	...	...	4	4
21	1	...	...	...	..	1	1	4	4
22	...	...	...	...	..	...	...	3	3
23	...	...	...	...	..	...	...	3	3
24	1	...	...	...	..	1	1	3	3
25	...	1	...	...	..	1	1	2	2
26	1	...	...	...	..	1	1	1	1
27	...	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...	...
...	7	8	..	2	2	15	17	231	240

Table XI gives the resulting monthly death-rates per 1,000 exposed to risk for each group of ages, distinguishing, as before, the rates from wounds, from other causes, and from all causes combined. Column 2 gives the months of exposure as obtained in Table X; columns 3, 4 and 5 show the corresponding deaths, whilst in columns 6, 7 and 8 are given the monthly death-rates.

TABLE XI.

*Assured Yeomanry and Volunteers.*

SELECT.

Ages (1)	1ST MONTH OF EXPOSURE						2ND MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	7,826	1	2	3	·128	·256	·383	7,742	1	6	7	·129	·775	·904
23-27 . .	6,854	...	8	8	...	1·167	1·167	6,776	...	12	12	...	1·771	1·771
28-32 . .	2,674	...	2	2	...	·748	·748	2,645	...	4	4	...	1·512	1·512
33-37 . .	1,530	...	...	...	...	...	...	1,510	1	1	2	·662	·662	1·325
38-42 . .	281	...	2	2	...	7·117	7·117	276	...	1	1	...	3·623	3·623
43-47 . .	87	...	...	...	...	...	...	87	...	1	1	...	11·494	11·494
48 upwards	17	...	...	...	...	...	...	17	...	...	...	...	...	...
...	19,269	1	14	15	·052	·727	·778	19,053	2	25	27	·105	1·312	1·417
3RD MONTH OF EXPOSURE														
17-22 . .	7,484	3	21	24	·401	2·806	3·207	6,957	9	20	29	1·294	2·875	4·168
23-27 . .	6,651	5	21	26	·752	3·157	3·909	6,377	5	25	30	·784	3·920	4·704
28-32 . .	2,609	1	6	7	·383	2·300	2·683	2,518	2	11	13	·794	4·369	5·163
33-37 . .	1,474	1	4	5	·678	2·714	3·392	1,427	...	3	3	...	2·102	2·102
38-42 . .	273	1	2	3	3·663	7·326	10·989	266	...	...	...	...	...	...
43-47 . .	85	1	...	1	11·765	...	11·765	82	1	...	1	12·195	...	12·195
48 upwards	16	...	...	...	...	...	...	16	...	1	1	...	62·500	62·500
...	18,592	12	54	66	·645	2·904	3·550	17,613	17	60	77	·964	3·401	4·364



TABLE XI (continued).  
Assured Yeomanry and Volunteers.

SELECT.

Ages	5TH MONTH OF EXPOSURE						6TH MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	6,765	3	24	27	·443	3·548	3·991	6,678	6	14	20	·898	2·096	2·995
23-27 . .	6,221	8	23	31	1·286	3·697	4·983	6,135	3	16	19	·489	2·608	3·097
28-32 . .	2,467	2	13	15	·811	5·270	6·080	2,433	2	2	4	·822	·822	1·644
33-37 . .	1,407	1	1	2	·711	·711	1·421	1,398	2	1	3	1·431	·715	2·146
38-42 . .	262	...	1	1	...	3·817	3·817	260	...	...	...	...	...	...
43-47 . .	81	1	...	1	12·346	...	12·346	80	...	...	...	...	...	...
48 upwards	15	...	...	...	...	...	...	14	...	...	...	...	...	...
...	17,218	15	62	77	·871	3·601	4·472	16,998	13	33	46	·765	1·941	2·706

Ages	7TH MONTH OF EXPOSURE						8TH MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	6,541	5	6	11	·764	·917	1·682	6,461	7	12	19	1·083	1·857	2·941
23-27 . .	6,033	8	12	20	1·326	1·989	3·315	5,951	7	6	13	1·176	1·008	2·185
28-32 . .	2,398	1	4	5	·417	1·668	2·085	2,366	2	1	3	·845	·423	1·268
33-37 . .	1,382	3	1	4	2·171	·724	2·894	1,361	4	2	6	2·939	1·470	4·409
38-42 . .	257	1	...	1	3·891	...	3·891	254	...	...	...	...	...	...
43-47 . .	80	...	...	...	...	...	...	79	...	...	...	...	...	...
48 upwards	14	...	...	...	...	...	...	13	...	...	...	...	...	...
...	16,705	18	23	41	1·078	1·377	2·454	16,485	20	21	41	1·213	1·274	2·487

TABLE XI (continued).  
*Assured Yeomanry and Volunteers.*

SELECT.

Ages	9TH MONTH OF EXPOSURE						10TH MONTH OF EXPOSURE							
	E	DIED			MONTHLY DEATH-RATE PER 1,000			E	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	6,365	6	19	25	·943	2·985	3·928	6,234	1	27	28	·160	4·331	4·492
23-27 . .	5,855	3	4	7	·512	·683	1·196	5,745	6	23	29	1·044	4·003	5·048
28-32 . .	2,324	2	6	8	·861	2·582	3·442	2,275	2	10	12	·879	4·396	5·275
33-37 . .	1,336	3	4	7	2·246	2·994	5·240	1,298	...	3	3	...	2·311	2·311
38-42 . .	251	...	...	...	...	...	...	241	1	1	2	4·149	4·149	8·299
43-47 . .	79	...	...	...	...	...	...	77	...	...	...	...	...	...
48 upwards	43	...	...	...	...	...	...	11	...	...	...	...	...	...
...	16,223	14	33	47	·863	2·034	2·897	15,881	10	64	74	·630	4·030	4·660

Ages	11TH MONTH OF EXPOSURE						12TH MONTH OF EXPOSURE							
	E	DIED			MONTHLY DEATH-RATE PER 1,000			E	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	6,040	4	25	29	·662	4·139	4·801	5,917	6	35	41	1·014	5·915	6·929
23-27 . .	5,549	3	19	22	·541	3·424	3·965	5,412	8	22	30	1·478	4·065	5·543
28-32 . .	2,202	4	3	7	1·817	1·362	3·179	2,151	4	8	12	1·860	3·719	5·579
33-37 . .	1,250	1	4	5	·800	3·200	4·000	1,215	1	2	3	·823	1·646	2·469
38-42 . .	237	...	1	1	...	4·219	4·219	232	...	...	...	...	...	...
43-47 . .	75	...	...	...	...	...	...	70	...	...	...	...	...	...
48 upwards	11	...	1	1	...	90·909	90·909	10	...	...	...	...	...	...
...	15,364	12	53	65	·781	3·450	4·231	15,007	19	67	86	1·266	4·465	5·731

TABLE XI (continued).  
Assured Yeomanry and Volunteers.

SELECT.

14TH MONTH OF EXPOSURE														
Ages (1)	13TH MONTH OF EXPOSURE						14TH MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	5,721	7	7	14	1·224	1·224	2·447	3,765	5	7	12	1·328	1·859	3·187
23-27 . .	5,209	6	31	37	1·152	5·951	7·103	3,459	2	13	15	·578	3·758	4·337
28-32 . .	2,063	2	6	8	·969	2·908	3·878	1,322	...	4	4	...	3·026	3·026
33-37 . .	1,155	3	1	4	2·597	·866	3·463	729	...	3	3	...	4·115	4·115
38-42 . .	225	...	2	2	...	8·889	8·889	139	...	...	...	...	...	...
43-47 . .	67	...	...	...	...	...	...	45	...	...	...	...	...	...
48 upwards	10	...	...	...	...	...	...	9	...	...	...	...	...	...
...	14,450	18	47	65	1·246	3·253	4·498	9,468	7	27	34	·739	2·852	3·591
15TH MONTH OF EXPOSURE														
Ages (1)	15TH MONTH OF EXPOSURE						16TH MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	3,314	1	2	3	·302	·604	·905	1,656	...	8	8	...	4·831	4·831
23-27 . .	3,058	2	8	10	·654	2·616	3·270	1,777	...	3	3	...	1·688	1·688
28-32 . .	1,194	1	4	5	·838	3·350	4·188	726	...	...	...	...	...	...
33-37 . .	665	...	2	2	...	3·008	3·008	399	...	1	1	...	2·506	2·506
38-42 . .	128	...	...	...	...	...	...	96	...	...	...	...	...	...
43-47 . .	40	...	...	...	...	...	...	33	...	...	...	...	...	...
48 upwards	8	...	...	...	...	...	...	5	...	...	...	...	...	...
...	8,407	4	16	20	·476	1·903	2·379	4,692	...	12	12	...	2·558	2·558



TABLE XI (continued).  
*Assured Yeomanry and Volunteers.*

SELECT.

17TH MONTH OF EXPOSURE													18TH MONTH OF EXPOSURE							
Ages (1)	E (2)	DIED					MONTHLY DEATH-RATE PER 1,000					E (2)	DIED				MONTHLY DEATH-RATE PER 1,000			
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)	Wounds (3)	Illness (4)	Total (5)	Wounds (6)		Illness (7)	Total (8)						
17-22 . .	851	...	2	2	...	2-350	601	...	...	...	...	...	...	...	...	...	...	...		
23-27 . .	961	...	1	1	...	1-041	690	...	...	...	...	...	...	...	...	...	...	...		
28-32 . .	431	...	1	1	...	2-320	300	...	1	1	...	...	...	...	3-333	...	...	3-333		
33-37 . .	222	...	...	...	...	...	163	...	...	...	...	...	...	...	...	...	...	...		
38-42 . .	63	...	...	...	...	...	51	...	1	1	...	...	...	...	19-608	...	...	19-608		
43-47 . .	27	3	...	3	111-111	...	20	...	...	...	...	...	...	...	...	...	...	...		
48 upwards	4	...	...	...	...	...	4	...	...	...	...	...	...	...	...	...	...	...		
...	2,559	3	4	7	1-172	1-563	1,829	...	2	2	...	...	...	...	1-093	...	...	1-093		
19TH MONTH OF EXPOSURE													20TH MONTH OF EXPOSURE							
Ages (1)	E (2)	DIED					MONTHLY DEATH-RATE PER 1,000					E (2)	DIED				MONTHLY DEATH-RATE PER 1,000			
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)	Wounds (3)	Illness (4)	Total (5)	Wounds (6)		Illness (7)	Total (8)						
17-22 . .	546	...	...	...	...	...	515	2	...	2	3-884	...	...	...	3-884	...	...	3-884		
23-27 . .	598	...	1	1	...	1-672	544	1	1	2	1-838	...	1	2	1-838	1-838	...	3-676		
28-32 . .	265	...	1	1	...	3-754	244	...	...	...	...	...	...	...	...	...	...	...		
33-37 . .	145	...	...	...	...	...	139	...	...	...	...	...	...	...	...	...	...	...		
38-42 . .	46	...	...	...	...	...	44	...	...	...	...	...	...	...	...	...	...	...		
43-47 . .	17	...	...	...	...	...	16	1	...	1	62-500	...	...	...	62-500	...	...	62-500		
48 upwards	4	...	...	...	...	...	4	...	...	...	...	...	...	...	...	...	...	...		
...	1,621	...	2	2	...	1-234	1,506	4	1	5	2-656	...	1	5	2-656	664	...	3-320		

TABLE XI (continued).  
Assured Yeomanry and Volunteers.

SELECT.

Ages	21st MONTH OF EXPOSURE						22nd MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	496	1	1	2	2·016	4·032	475	...	1	1	...	2·105	2·105	
23-27 . .	519	...	...	...	...	...	500	...	...	...	...	...	...	
28-32 . .	239	1	1	2	4·184	8·368	230	...	...	...	...	...	...	
33-37 . .	134	...	1	1	...	7·463	126	...	2	2	...	15·873	15·873	
38-42 . .	43	...	...	...	...	...	39	...	...	...	...	...	...	
43-47 . .	14	...	...	...	...	...	14	...	...	...	...	...	...	
48 upwards	4	...	...	...	...	...	3	...	...	...	...	...	...	
...	1,449	2	3	5	1·380	3·451	1,387	...	3	3	...	2·163	2·163	
Ages	23rd MONTH OF EXPOSURE						24th MONTH OF EXPOSURE							
	E (2)	DIED			MONTHLY DEATH-RATE PER 1,000			E (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 . .	440	...	...	...	...	...	416	1	1	2	2·404	2·404	4·808	
23-27 . .	472	...	...	...	...	...	449	...	1	1	...	2·227	2·227	
28-32 . .	221	...	...	...	...	...	205	1	...	1	4·878	...	4·878	
33-37 . .	116	...	2	2	...	17·241	109	...	...	...	...	...	...	
38-42 . .	39	...	...	...	...	...	36	...	...	...	...	...	...	
43-47 . .	13	...	...	...	...	...	13	...	...	...	...	...	...	
48 upwards	3	...	...	...	...	...	3	...	...	...	...	...	...	
...	1,304	...	2	2	...	1·534	1,231	2	2	4	1·625	1·625	3·249	

TABLE XI (continued).

*Assured Yeomanry and Volunteers.*

SELECT.

Ages	25TH MONTH OF EXPOSURE						26TH MONTH OF EXPOSURE					
	E	DIED			MONTHLY DEATH-RATE PER 1,000			E	DIED			MONTHLY DEATH-RATE PER 1,000
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	
(1)	(2)							(2)				
17-22 . .	397	...	1	1	...	2-519	2-519	161	...	2	2	12-422
23-27 . .	422	...	2	2	...	4-739	4-739	147	...	1	1	6-803
28-32 . .	189	...	...	...	...	...	...	75	...	1	1	13-333
33-37 . .	106	...	1	1	...	9-434	9-434	40	...	...	...	...
38-42 . .	31	...	...	...	...	...	...	10	...	...	...	...
43-47 . .	12	...	...	...	...	...	...	5	...	...	...	...
48 upwards	2	...	...	...	...	...	...	1	...	...	...	...
...	1,159	...	4	4	...	3-451	3-451	439	...	4	4	9-112
	27TH MONTH OF EXPOSURE						28TH MONTH OF EXPOSURE					
	E	DIED			MONTHLY DEATH-RATE PER 1,000			E	DIED			MONTHLY DEATH-RATE PER 1,000
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)		Wounds (3)	Illness (4)	Total (5)	
(1)	(2)							(2)				
17-22 . .	122	...	...	...	...	...	...	50	...	...	...	...
23-27 . .	120	...	...	...	...	...	...	61	...	...	...	...
28-32 . .	60	...	...	...	...	...	...	34	...	...	...	...
33-37 . .	27	...	...	...	...	...	...	11	...	...	...	...
38-42 . .	8	...	...	...	...	...	...	2	...	...	...	...
43-47 . .	4	...	...	...	...	...	...	3	...	...	...	...
48 upwards	...	...	...	...	...	...	...	...	...	...	...	...
...	341	...	...	...	...	...	...	161	...	...	...	...



TABLE XI (continued).

*Assured Yeomanry and Volunteers.*

SELECT.

Ages  (1)	29TH MONTH OF EXPOSURE						
	E  (2)	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds (3)	Illness (4)	Total (5)	Wounds (6)	Illness (7)	Total (8)
17-22 .	2	...	...	...	...	...	...
23-27 .	3	...	...	...	...	...	...
28-32 .	2	...	..	...	...	...	...
33-37 .	2	...	...	...	...	...	...
38-42 .	...	...	...	...	...	...	...
43-47 .	...	...	...	...	...	...	...
48 upwards	...	...	...	...	...	...	...
...	9	...	...	...	...	...	...

Tables XII and XIII summarize the facts given in Table XI, the former combining all months, and showing total results for each age group, and the latter combining all ages and showing total results for each policy month.

Referring to Table XII, and omitting the rates for ages beyond 42, where the facts are too few to enable us to draw any conclusions, we notice that the death-rate from wounds appears, as might be expected, on the whole to remain about the same for all ages, the tendency, perhaps, being slightly to increase with the age. Looking at column 10, giving the death-rates from "Other Causes", we see again that the age distribution has, on the whole, very little effect, except for the group 23 to 27, where a comparatively high rate is shown.

It will be remembered that about this age period most of the well-known tables of mortality show an apparently abnormal increase in the death-rates. It is possible that the same force which produced this result in other tables has operated to predispose those engaged in the war to fall more ready victims to disease, or to be less able to overcome it when attacked. In marked contrast to this there is a correspondingly low rate for ages 33 to 37, and it may be that the inference to be drawn here is that men of these ages have more power to resist the inroads of disease, or that they exercise greater discretion in their dietary.

TABLE XII.

*Assured Yeomanry and Volunteers.*

SUMMARY.

ENTERED (17-22)	.	7,826
(23-27)	.	6,854
(28-32)	.	2,674
(33-37)	.	1,530
(38-42)	.	281
(43-47)	.	87
(48 and upwards)	.	17
		<hr/>
		19,269
		<hr/>

SELECT.

ALL DURATIONS COMBINED.

Ages	Existing	Left Active Service or otherwise withdrew from observa- tion	DIED			Col. (2)+ Col. (3)	Months of Exposure	MONTHLY DEATH- RATE PER 1,000		
			Wounds	Illness	Total			Wounds	Illness	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
17-22 .	2,886	4,628	69	243	312	7,514	100,538	·686	2·417	3·103
23-27 .	1,922	4,612	67	253	320	6,534	92,548	·724	2·734	3·458
28-32 .	652	1,906	27	89	116	2,558	36,862	·732	2·414	3·146
33-37 .	386	1,085	20	39	59	1,471	20,876	·958	1·868	2·826
38-42 .	52	215	3	11	14	267	4,090	·733	2·689	3·422
43-47 .	18	61	7	1	8	79	1,305	5·364	·766	6·130
48 & upwards	7	8	...	2	2	15	231	...	8·658	8·658
	5,923	12,515	193	638	831	18,438	256,450	·753	2·488	3·241

TABLE XIII.

*Assured Yeomanry and Volunteers.*

## SUMMARY.

SELECT.

ALL AGES COMBINED.

Month of Exposure (n)	E	DIED			MONTHLY DEATH-RATE PER 1,000		
		Wounds	Illness	Total	Wounds	Illness	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	19,269	1	14	15	·052	·727	·779
2	19,053	2	25	27	·105	1·312	1·417
3	18,592	12	54	66	·645	2·904	3·549
4	17,643	17	60	77	·964	3·401	4·365
5	17,218	15	62	77	·871	3·601	4·472
6	16,998	13	33	46	·765	1·941	2·706
7	16,705	18	23	41	1·078	1·377	2·455
8	16,485	20	21	41	1·213	1·274	2·487
9	16,223	14	33	47	·863	2·034	2·897
10	15,881	10	64	74	·630	4·030	4·660
11	15,364	12	53	65	·781	3·450	4·231
12	15,007	19	67	86	1·266	4·465	5·731
13	14,450	18	47	65	1·246	3·253	4·499
14	9,468	7	27	34	·739	2·852	3·591
15	8,407	4	16	20	·476	1·903	2·379
16	4,692	...	12	12	...	2·558	2·558
17	2,559	3	4	7	1·172	1·563	2·735
18	1,829	...	2	2	...	1·093	1·093
19	1,621	...	2	2	...	1·234	1·234
20	1,506	4	1	5	2·656	·664	3·320
21	1,449	2	3	5	1·380	2·070	3·450
22	1,387	...	3	3	...	2·163	2·163
23	1,304	...	2	2	...	1·534	1·534
24	1,231	2	2	4	1·625	1·625	3·250
25	1,159	...	4	4	...	3·451	3·451
26	439	...	4	4	...	9·112	9·112
27	341	...	...	...	...	...	...
28	161	...	...	...	...	...	...
29	9	...	...	...	...	...	...
...	256,450	193	638	831	·753	2·488	3·241

The rates given in Table XIII for all classes combined are reproduced in the form of curves in Diagram F, which accordingly shows us the death-rates for all ages combined for each policy month, and enables us to trace more closely the effect of length of service. The exposures beyond the 24th month were so few that the corresponding rates have not been reproduced. Taking first the curves for "All Causes Combined", we see that for the first year of assurance the rate tended to increase, reaching a maximum at the end of that period. In the second year it declined, rising only to a much lower maximum towards the end.



In considering this curve it is to be noted that the average dates at which the assurances were effected were in the early part of calendar years ; so that the end of a policy year, that is the 12th and 24th month, would fall roughly about the time of the South African summer, when, as already stated, we might expect a heavy death-rate from disease. When this is borne in mind it makes the fall in the general death-rate as the duration increases more noticeable.

It will be observed, moreover, that this fall is occasioned entirely by the lower death-rates experienced from "Other Causes", the rates for wounds remaining fairly even up to the end of the 17th or 18th month, and afterwards rising only slightly.

From this we might fairly assume that the duration of the policy, or rather the length of the period for which the men were engaged upon active service, had an important bearing on the death-rate—a bearing more important than that of age. It may be that this arose from the men becoming more inured to hardships, and better able to withstand disease, or it may be that it was from the less hardy men being invalided home, and consequently being taken off the "exposed to risk." Probably both causes combined to produce the effect ; but at all events we may fairly argue from the diagram, that whilst the death-rates from wounds remained practically unaltered by lapse of time, those from disease diminished.

With the object of ascertaining, if possible, whether age had any appreciable effect on the death-rates from disease, a set of diagrams was prepared from Table XI, showing the curves for the respective age groups. These diagrams have not been reproduced in this paper, as the few facts brought out by them can be followed with little difficulty from a careful examination of Table XI, but we shall be pleased to show them to anyone interested in this part of the subject and desiring to pursue it further.

Dealing with death-rates from "Other Causes" the general results may be summarized as follows :

- (1) For the first six months of assurance, ages 23–27 and 38–42 show the highest death-rates.
- (2) From the 7th to the 12th month, ages 17–22 show particularly heavy death-rates.
- (3) From the 13th to the 15th month, ages 23–27 again take the first place.

- (4) In the 16th and 17th month, ages 17–22 return to the unenviable position occupied by them in the 7th–12th months; but it must be borne in mind that the number of deaths here referred to is very small.

Beyond this period, the rates vary so considerably that no general deductions can be drawn from a comparison of the various age groups.

Generally, as might have been expected, it must be concluded that the age distribution had very little effect upon the death-rates, the incidents of the campaign and the length of the period of exposure being much more potent factors.

Table XIV gives the various columns required to obtain the single premium for an assurance for the 29 months of the war, the radix of the  $l_x$  column being taken in each age group as 100,000. Thus, column 2 gives the monthly death-rate per 1,000 as obtained in Table X; by successive multiplications into these rates, the deaths have been obtained as shown in column 4, and by successive subtraction from the living at the beginning of the previous month, the living for the succeeding month have been obtained.

Column 5 gives the number of deaths discounted for the successive months. The sum of this column for all months gives the total value, at the commencement of the risk, of the payments to be made, and by dividing by 100,000, the assumed number of entrants, the "Single Premium" for an assurance of £1 for the whole duration of the war is obtained. The method adopted provides for payments to be made at the end of the policy month in which death occurs, and hence the resulting single premiums may be taken to include the adjustment for "Immediate Payment of Claims."

TABLE XIV.

*Assured Yeomanry and Volunteers.*

*Table for obtaining Single Premiums from the Monthly Death-rate.*

SELECT.

INTEREST 3 PER-CENT.

Month of Exposure ( <i>n</i> )	AGES 17—22				AGES 23—27			
	Monthly Death- rate per 1,000 from Table XI	$l_x$	$d_x$	$v^n d_x$	Monthly Death- rate per 1,000 from Table XI	$l_x$	$d_x$	$v^n d_x$
(1)	(2)	(3)	(4)	(5)	(2)	(3)	(4)	(5)
1	·383	100,000	38	37·9	1·167	100,000	117	116·7
2	·904	99,962	90	89·6	1·771	99,883	177	176·1
3	3·207	99,872	320	317·6	3·909	99,706	390	387·1
4	4·168	99,552	415	410·9	4·704	99,316	467	462·4
5	3·991	99,137	396	391·2	4·983	98,849	493	487·0
6	2·995	98,741	296	291·7	3·097	98,356	305	300·5
7	1·682	98,445	166	163·2	3·315	98,051	325	319·4
8	2·941	98,279	289	283·4	2·185	97,726	213	208·8
9	3·928	97,990	385	376·6	1·196	97,513	117	114·4
10	4·492	97,605	438	427·3	5·048	97,396	492	480·0
11	4·801	97,167	467	454·5	3·965	96,904	384	373·7
12	6·929	96,700	670	650·5	5·543	96,520	535	519·4
13	2·447	96,030	235	227·6	7·103	95,985	682	660·5
14	3·187	95,795	305	294·7	4·337	95,303	413	399·0
15	·905	95,490	86	82·9	3·270	94,890	310	298·8
16	4·831	95,404	461	443·2	1·688	94,580	160	153·8
17	2·350	94,943	223	213·9	1·041	94,420	98	94·0
18	...	94,720	...	...	...	94,322	...	...
19	...	94,720	...	...	1·672	94,322	158	150·8
20	3·884	94,720	368	350·3	3·676	94,164	346	329·4
21	4·032	94,352	380	360·8	...	93,818	...	...
22	2·105	93,972	198	187·6	...	93,818	...	...
23	...	93,774	...	...	...	93,818	...	...
24	4·808	93,774	451	425·1	2·227	93,818	209	197·0
25	2·519	93,323	235	221·0	4·739	93,609	444	417·5
26	12·422	93,088	1,156	1,084·3	6·803	93,165	634	594·7
27	...	91,932	...	...	...	92,531	...	...
28	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...
...	...	...	8,068	7,785·8	...	...	7,469	7,241·0



TABLE XIV (continued).

*Assured Yeomanry and Volunteers.**Table for obtaining Single Premiums from the Monthly Death-rate.*

SELECT.

INTEREST 3 PER-CENT.

Month of Exposure ( <i>n</i> )	AGES 28—32				AGES 33—37			
	Monthly Death- rate per 1,000 from Table XI	$l_x$	$d_x$	$v^{1\frac{1}{2}}d_x$	Monthly Death- rate per 1,000 from Table XI	$l_x$	$d_x$	$v^{1\frac{1}{2}}d_x$
(1)	(2)	(3)	(4)	(5)	(2)	(3)	(4)	(5)
1	·748	100,000	75	74·8	...	100,000	...	...
2	1·512	99,925	151	150·3	1·325	100,000	132	131·4
3	2·683	99,774	268	266·0	3·392	99,868	339	336·5
4	5·163	99,506	514	509·0	2·102	99,529	209	207·0
5	6·080	98,992	602	594·6	1·421	99,320	141	139·3
6	1·644	98,390	162	159·6	2·146	99,179	213	209·9
7	2·085	98,228	205	201·5	2·894	98,966	286	281·1
8	1·268	98,023	124	121·6	4·409	98,680	435	426·5
9	3·442	97,899	337	329·6	5·240	98,245	515	503·7
10	5·275	97,562	515	502·5	2·311	97,730	226	220·5
11	3·179	97,047	309	300·7	4·000	97,504	390	379·6
12	5·579	96,738	540	524·3	2·469	97,114	240	233·0
13	3·878	96,198	373	361·2	3·463	96,874	335	324·4
14	3·026	95,825	290	280·2	4·115	96,539	397	383·5
15	4·188	95,535	400	385·5	3·008	96,142	289	278·5
16	...	95,135	...	...	2·506	95,853	240	230·7
17	2·320	95,135	221	211·9	...	95,613	...	...
18	3·333	94,914	316	302·3	...	95,613	...	...
19	3·754	94,598	355	338·8	...	95,613	...	...
20	...	94,243	...	...	...	95,613	...	...
21	8·368	94,243	789	749·2	7·463	95,613	714	678·0
22	...	93,454	...	...	15·873	94,899	1,506	1,426·6
23	...	93,454	...	...	17·241	93,393	1,610	1,521·3
24	4·878	93,454	456	429·8	...	91,783	...	...
25	...	92,998	...	...	9·434	91,783	866	814·3
26	13·333	92,998	1,240	1,163·1	...	90,917	...	...
27	...	91,758	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...
...	...	...	8,242	7,956·5	...	...	9,083	8,725·8

TABLE XIV (continued).

*Assured Yeomanry and Volunteers.*

*Table for obtaining Single Premiums from the Monthly Death-rate.*

SELECT.

INTEREST 3 PER CENT.

Month of Exposure ( <i>n</i> )	AGES 38—42				AGES 43—47			
	Monthly Death- rate per 1,000 from Table XI	$l_x$	$d_x$	$v^{12}d_x$	Monthly Death- rate per 1,000 from Table XI	$l_x$	$d_x$	$v^{12}d_x$
(1)	(2)	(3)	(4)	(5)	(2)	(3)	(4)	(5)
1	7.117	100,000	712	710.2	...	100,000	...	...
2	3.623	99,288	360	358.2	11.494	100,000	1,149	1143.4
3	10.989	98,928	1,087	1079.0	11.765	98,851	1,163	1154.4
4	...	97,841	...	...	12.195	97,688	1,191	1179.3
5	3.817	97,841	373	368.4	12.346	96,497	1,191	1176.4
6	...	97,468	...	...	...	95,306	...	...
7	3.891	97,468	379	372.5	...	95,306	...	...
8	...	97,089	...	...	...	95,306	...	...
9	...	97,089	...	...	...	95,306	...	...
10	8.299	97,089	806	786.4	...	95,306	...	...
11	4.219	96,283	406	395.1	...	95,306	...	...
12	...	95,877	...	...	...	95,306	...	...
13	8.889	95,877	852	825.1	...	95,306	...	...
14	...	95,025	...	...	...	95,306	...	...
15	...	95,025	...	...	...	95,306	...	...
16	...	95,025	...	...	...	95,306	...	...
17	...	95,025	...	...	111.111	95,306	10,589	10154.7
18	19.608	95,025	1,863	1782.2	...	84,717	...	...
19	...	93,162	...	...	...	...	...	...
20	...	...	...	...	...	...	...	...
21	...	...	...	...	...	...	...	...
22	...	...	...	...	...	...	...	...
23	...	...	...	...	...	...	...	...
24	...	...	...	...	...	...	...	...
25	...	...	...	...	...	...	...	...
26	...	...	...	...	...	...	...	...
27	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...
...	...	...	6,838	6677.1	...	...	15,283	14808.2

TABLE XIV (continued).

*Assured Yeomanry and Volunteers.**Table for obtaining Single Premiums from the Monthly Death-rate.*

SELECT.

INTEREST 3 PER CENT.

Month of Exposure ( <i>n</i> )	AGES 48 AND UPWARDS			
	Monthly Death-rate per 1,000 from Table XI	$l_x$	$d_x$	$v^{12}d_x$
(1)	(2)	(3)	(4)	(5)
1	...	100,000	...	...
2	...	100,000	...	...
3	...	100,000	...	...
4	62·500	100,000	6,250	6188·7
5	...	93,750	...	...
6	...	93,750	...	...
7	...	93,750	...	...
8	...	93,750	...	...
9	...	93,750	...	...
10	...	93,750	...	...
11	90·909	93,750	8,523	8295·2
12	...	85,227	...	...
13	...	...	...	...
14	...	...	...	...
15	...	...	...	...
16	...	...	...	...
17	...	...	...	...
18	...	...	...	...
19	...	...	...	...
20	...	...	...	...
21	...	...	...	...
22	...	...	...	...
23	...	...	...	...
24	...	...	...	...
25	...	...	...	...
26	...	...	...	...
27	...	...	...	...
28	...	...	...	...
29	...	...	...	...
...	...	...	14,773	14483·9



Table XV has been prepared to enable us to pass from single premiums to annual. The usual relation between single and annual premiums does not help us here. We are dealing with temporary assurances, payable immediately upon the occurrence of death, whereas the premiums are paid yearly in advance. Thus it will be seen that the single premium and the annuity do not correspond, and, to obtain the latter, it is more convenient to employ yearly rates of mortality.

TABLE XV.

*Assured Yeomanry and Volunteers.*

SELECT.

YEARLY DEATH RATES.

Ages	MONTHS OF ASSURANCE 1 TO 12				MONTHS OF ASSURANCE 13 TO 24			
	Months of Exposure to Risk	Years of Exposure to Risk =Col.(2) ÷ 12	Deaths during Period	Yearly Death Rates	Months of Exposure to Risk	Years of Exposure to Risk =Col.(6) ÷ 12	Deaths during Period	Yearly Death Rates
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
17-22 .	82,161	6,847	263	·038411	19,191	1,599	46	·028768
23-27 .	74,825	6,235	247	·039615	18,927	1,577	70	·044388
28-32 .	29,512	2,459	92	·037414	7,637	637	23	·036107
33-37 .	16,785	1,399	43	·030736	4,211	351	15	·042735
38-42 .	3,166	264	11	·041667	977	81	3	·037037
43-47 .	996	83	4	·048193	344	29	4	·137931
48 & upwards	176	15	2	·133333	61	5	...	...

Column 2 gives the months of exposure as found in Table XV, in which the deaths have been left in amongst the "Exposed", and subtracted only at the end of the 12th and 24th months.

Column 3, by dividing by 12, reduces the months of exposure to years, and the corresponding yearly rate of mortality is given in column 5.

In Table XVI the annuity-value is deduced from this yearly rate of mortality.

Table XVII is, perhaps, the most interesting table to those who have to deal with war mortality amongst assured lives. In column 2 is the single premium obtained from Table XIV as already described. Column 3 gives the annuity due from Table XVI, and finally, in column 4 is the annual premium for the risk incurred during the war by the assured lives.

TABLE XVI.  
*Assured Yeomanry and Volunteers.*

ANNUITIES OBTAINED FROM YEARLY DEATH RATES

SELECT.

INTEREST 3 PER-CENT.

Ages (1)	MONTHS OF ASSURANCE, 1 TO 12			MONTHS OF ASSURANCE, 13 TO 24			Col. (4) × Col. (7) (8)	$\frac{1}{2} \times \text{Col. (8)}$ (9)	$a_{x:\overline{1\frac{1}{2}} }$ = Col. (4) + Col. (9) (10)
	Yearly Death Rate from Table XV = $q_x$ (2)	$p_x = 1 - q_x$ (3)	$vp_x$ (4)	Yearly Death Rate from Table XV = $q_x$ (5)	$p_x = 1 - q_x$ (6)	$vp_x$ (7)			
17-22	.038411	.961589	.93358	.028768	.971232	.94294	.88031	.36680	1.30038
23-27	.039615	.960385	.93241	.044388	.955612	.92778	.86507	.36045	1.29286
28-32	.037414	.962586	.93455	.036107	.963893	.93582	.87457	.36440	1.29895
33-37	.030736	.969264	.94103	.042735	.957265	.92938	.87457	.36440	1.30543
38-42	.041667	.958333	.93042	.037037	.962963	.93492	.86987	.36245	1.29287
43-47	.048193	.951807	.92408	.137931	.862069	.83696	.77342	.32226	1.24634
48 and upwards	.133333	.866667	.84142	...	1.000000	.97087	.81691	.34038	1.18180

TABLE XVII.

*Assured Yeomanry and Volunteers.*

SINGLE AND ANNUAL PREMIUMS.

SELECT.

INTEREST 3 PER-CENT

Ages (1)	Single Premium per £100 from Table XIV (2)	$1 + a_{x:1\frac{5}{12}}$ from Table XVI (3)	Annual Premium per £100 Col. (2) ÷ Col. (3) (4)
17-22. . .	7.7858	2.30038	3.38457
23-27. . .	7.2410	2.29286	3.15806
28-32. . .	7.9565	2.29895	3.46093
33-37. . .	8.7258	2.30543	3.78489
38-42. . .	6.6771	2.29287	2.91211
43-47. . .	14.8082	2.24634	6.59215
48 and upwards .	14.4839	2.18180	6.63851

With regard to this annual premium, it may be observed that it is obtained by the use of an annuity covering a period of  $2\frac{5}{12}$  years only. Strictly, therefore, the last premium charged should be only  $\frac{5}{12}$  of that given in column 4. In practice, however, such a procedure would be impossible, as five months before the cessation of hostilities there was no means of estimating how long the operations would last. By charging the full annual premium, and returning  $\frac{7}{12}$  (the proportion not required) on the declaration of peace, this difficulty is overcome. This was the method adopted in practice by many Offices charging extra premiums for war risk.

As has been previously remarked, the results for age 43 and upwards are deduced from so small an experience as to render them of little value, and although the premiums brought out vary considerably for the other age groups, it is not suggested for one moment that varying premiums ought to be charged to cover the risk of war.

In looking at these premiums it should also be remembered that although the experience includes a proportion of officers, the vast majority of the lives observed were non-commissioned officers and men. The rates for officers considered alone would no doubt be much higher.

Although the premiums are probably unsuitable as a basis for the calculation of any extra premium required for an officer in the Regulars, they are, we think, valuable should similar auxiliary forces be required in any future war—a not improbable contingency.



It may be thought by some that we have gone into unnecessary detail in this investigation; that for the purpose of arriving at approximate premiums we have taken undue trouble in verifying the facts upon which they are based. It has not been, however, the only object of this paper to arrive at monetary results, but as the title indicates, it is an attempt to investigate as accurately as possible the mortality experienced by the Imperial Forces during the late war, in the hope that some useful information may be derived from the discussion of the various points brought out, and if one of the results has been to confirm in a measure prevailing ideas amongst the actuarial profession on the subject of the incidence of war risks, the paper will have served a useful purpose.

During the progress of the investigation over 40,000 cards were written, involving very great labour. So far as we are aware, no other war statistics have ever been obtained in a similar manner, as it was impossible for previous investigators to enter into detail in the way in which we happily have been enabled to do. As explained in the early portion of the paper, we were able, by the courtesy of the War Office, to obtain the strength of the forces engaged at very frequent intervals, and every death recorded was closely examined; whilst the assured experience may be said to have been dealt with in much the same way as Tables of Mortality deduced from the general records of assured lives.

We have had to follow closely the progress of the war, and have been much impressed by many features which prove how difficult must have been the task before our generals and home authorities. In the first place, it will be noticed how the strength of the forces was maintained, necessitating a constant flow of reinforcements to replace the inevitable wastage of war, proving that those in authority never lost sight of the importance of this point, and never allowed it to get out of hand, although the base of operations was 6,000 miles away. In the next place, it has been strongly brought home to us that the care of the sick and wounded was exceptionally good. Although the medical staff were put to very severe tests on many occasions, they were always equal to any emergency, whether caused by disease or by the bullets of the enemy.

We cannot close without expressing our admiration for the endurance and courage exhibited by all arms throughout the war, and a feeling of thankfulness that a combat under circumstances of exceptional difficulty with so brave and stubborn a foe has been brought to a successful termination.

## DISCUSSION.

Mr. A. G. MACKENZIE felt certain that on no previous occasion had results from any campaign or war been treated in so scientific and thorough a manner, and he was sure that the paper would long have a high place in the estimation of all interested in mortality statistics. The manner in which the authors had treated the results in the form of diagrams was particularly informing, and showed very graphically the results obtained. It was interesting to learn from the paper that the mortality experience in the late war was at a lower rate than had been experienced in any of the great wars of the past century, and it was especially gratifying to learn that the mortality from disease was less than might have been anticipated. That, he thought, was owing to the great skill and organisation of the medical forces. One thing which had particularly struck him was that the premiums which were supposed to be adequate when fixed by the offices did not appear to have been sufficient. The mortality experienced by the officers of the regular forces during the first year of the war, appeared to have been about 8 per-cent., whereas the premium which was generally quoted for the extra risk was five guineas. Afterwards the rate of mortality was about 4 per-cent., and the premiums which were generally charged were, he believed, two guineas; so that it would appear as if the offices had charged just about 50 per-cent. of what was really required. Under those circumstances, it was somewhat curious to hear that there had been many applications made by officers returned from the front for some return of extra premiums. It was rather difficult for anyone who had not been trained as an actuary to understand that the matter must be looked at from the broad standpoint of average, and that it was impossible to go into particulars as regards individuals as to whether there had been gain or loss in any one transaction. Even if it had been proved that the amounts charged were more than sufficient to provide for the risk which really occurred, the question of the return of premiums would be one-sided, because the companies would have had no power to call for any additional premiums in the contrary event of the experience having been more unfavourable than was anticipated, and the question of the deterioration of health which might possibly affect the longevity of the assured, must also be taken into consideration. There had been a general feeling among many officers that companies, from patriotic motives, ought to waive all extra premiums. That, of course, would undoubtedly be unfair to the other policyholders in the companies, who would have to stand the brunt of the extra charge which had to be made. He thought one suggestion might be considered by the authorities, namely, whether it would not be practicable, considering the circumstances of officers, to provide in some degree for the extra premiums which they might have to pay on account of being called upon to take up military duties. He thought it would be well if offices were to impress upon officers generally, when they took out policies, the advisability of paying a small extra premium each year to cover the risk of war. That plan, he believed, was arranged on a very moderate scale at present by a large number of offices, the premiums varying from 10s. per-cent. upwards. If offices generally were to consider, in the face of the mortality experience and the results of the



last and of other campaigns, what extra premium might be supposed to be adequate, and were to come to some arrangement in regard to that matter, he thought it would be very valuable. It would also be of interest if the offices would in combination consider what had been the result to themselves of the extra war mortality, comparing and combining their experience as to the amount which was paid for the risk and the actual loss which they had suffered. He thought the Government might take an example from bankers, commercial establishments, and even some insurance companies, and consider whether, for policies for a certain amount, they could not see their way to paying the extra premium which was asked for the purpose. He did not think the taxpayers would object to that. The tax would fall properly upon the nation, and not upon the policyholders in particular offices; and he certainly hoped on the next occasion—which they all hoped would be far distant—when British soldiers and civilians had to take up arms for the defence, or in the interests, of their country, some provision of that kind would be made.

Surgeon-General W. F. STEVENSON, M.B., C.B., said he was afraid he was unable to add anything to this most interesting paper, which he was sure would be of very great value to everybody interested in the matter. Having, together with some other gentlemen who were present, had something to do with the statistics of the war, he knew the paper would be found of the greatest possible value to them. He desired to say how very much the service to which he had the honour to belong would appreciate one or two remarks made in the paper, especially towards the end of it. Such remarks would be greatly appreciated by his Department, because such tokens of appreciation had been wanting recently on many occasions, and he therefore desired, on the part of the officers of the Medical Service, to thank the authors for the remarks they had made in that respect.

Mr. W. A. WORKMAN thought the paper was so complete in every detail that it was almost above criticism, but he desired to offer a few remarks on some of the points which had been brought out. An interesting point was the difference between the proportion of the officers to the men amongst the Yeomanry and Regulars. It was stated on page 550 that in the Yeomanry and Colonial forces the proportion of officers to men was almost  $1\frac{1}{2}$  times as large as the proportion in the Regulars. On turning to Table III, and finding that the death-rate among the officers of the Yeomanry was very much greater than the death rate-among the officers in the Regulars, the question arose, did the difference in the distribution account for some of the difference in the mortality? The view that it did was rather strengthened by the fact that the death-rate from disease was practically the same among officers of the Yeomanry as among officers of the Regulars, and yet the death rate from wounds was very much higher in the one case than in the other. As an illustration, supposing there were two bodies of men, each numbering 1,000, where the proportion of officers in the one was very much larger than the proportion of officers in the other,—Did that difference in the distribution increase the risk officers ran? There was no doubt, in considering the death-rate among the general body of troops, it did do so, because where there was a larger proportion of men running a



bigger risk, there must necessarily be a larger death-rate among the whole body. The same argument of course applied to the Colonial forces, where the distribution between officers and men was much the same as it was in the Yeomanry, but seeing that the death-rate there was so much less than in the Regulars, it seemed to point to the fact that the immunity from death and disease, owing to acclimatisation and knowledge of the country, was even greater than that shown by the figures at first glance. In regard to the medical opinion given on page 565 to the effect that a cold had so much more serious effects in South Africa than in this country, that seemed to raise the question as to whether English officers were justified in charging ordinary home rates for civilians living in South Africa. Of course, as a set off to the increased risk named, officers had the great benefit derived from a climate in which people were so little likely to develop lung complaints, and he presumed if the one was set against the other there was a balance on the right side. One item of general interest to the public in the paper, was the proving of the idea that the Colonials being acclimatised and having a knowledge of the country, would undergo a much less death-rate than the ordinary troops. This was, he thought, established beyond doubt in the paper.

Lieut.-Col. E. M. WILSON, C.B., C.M.G., D.S.O., having, in the first place, thanked the authors and Mr. Mackenzie for the very kind words in which they had spoken of the medical department, referred to the part of the paper where the authors spoke of the pause at Bloemfontein being accompanied by an increased death-rate. That was not at all an uncommon thing. He thought it was very largely due to the inactivity which was imposed upon men who had lately gone through very severe experiences. It was very often found that the death-rate during the most severe portions of the campaign was not so heavy as that which followed a period of forced inactivity. The authors had stated, with reference to the mortality among officers and men, that it was possible the officers were better looked after when they were ill. He hoped that was not so, and believed it was not the case. The statement made, that troops which had been in the country longer suffered less was undoubtedly true. When Lord Kitchener went up the Nile during the first part of the war, there was one brigade of four infantry regiments employed, but before the advance to Khartoum was made, another brigade of four additional battalions was also employed, and when a comparison was made between the death-rate among those two brigades the difference was exceedingly marked. The first brigade—he would not say because it was largely composed of Highlanders—suffered very little; but the second brigade, which had only lately come from England, suffered very much more severely. With regard to enteric fever, that had been endemic in South Africa as long as he had known the country, and during the Zulu War it was very serious. If the authors could tabulate the returns of deaths from disease in the Zulu War of 1879, especially from enteric fever, and compare them with those in the late war, it would be seen that the medical profession had learned something during the last twenty years, because the death-rate of the two campaigns was very materially different.

Mr. S. G. WARNER considered that no war statistics had hitherto

been treated in so elaborately scientific a manner. As the authors had explained, a great deal of valuable information, which was essential, had been most courteously placed at their disposal by the War Office; but having obtained that material they had worked upon it with an ingenuity and diligence which had produced what would take rank for a long time as the most scientific treatment of such a subject on record. But, following on that, the chief impression left on his mind by such a paper was, how very much any such great historical event as the South African War must, for statistical purposes, be considered unique. It possessed features so individual and peculiar to itself that one doubted how far it would be of practical use in other similar events. The war had been distinguished by several outstanding features quite its own. The great mortality from disease, for instance, occurred to the mind at once. Such characteristics were not likely to occur in the same way in another war. It seemed to him that one of the leading lessons of the paper was that each such phenomenon would have to be analyzed by itself on its own merits, and it was very doubtful how far one would throw light upon the other. He was also impressed by the cognate and consequent fact of the arbitrary character of "war extras." Of course, that was inevitable; they had to be decided upon quickly, without much notice, and with a very slender basis of fact to go upon. It seemed to him that offices would always be, for the reasons he had just expressed, more or less in the dark in fixing extra rates for any individual war. Another aspect of the matter, which had been referred to by a previous speaker, was suggestive and interesting in the light of the fact that the late campaign was the first practical experience of war on a large scale since the growth of the practice of charging uniform extras, either during military service or during life. As a matter of fact that was now very freely done. Proposers who were officers in the Army or Navy had appreciated, and acted on, the principle of paying moderate fixed extra premiums throughout life, which should cover all risks of foreign service or war, at a somewhat higher rate fixed in the same way to endure as long as service lasted. If such a system became universal, it would give an entirely different aspect to statistical investigations of war experience, so far as extra premiums were concerned. Offices would no longer be interested in fixing the extra premium for each individual campaign, and he could not but think that that would be an advantage. Probably, having regard to the larger body of facts involved in the computation of extras on such a principle, experience would eventually lead to figures being arrived at which would approximately represent the real risk run. This could hardly be hoped for as regards the extras charged for individual wars, for there they were extremely unlikely to hit upon the right amount. On the whole, therefore, he hoped the practice of charging uniform extras would in time so prevail that offices would no longer be concerned on any great scale in fixing beforehand in a hurry, and with very imperfect data to guide them, what was likely to be an adequate extra premium for a particular campaign.

Mr. A. F. BURRIDGE thought the late war was unique in regard to the experience of life assurance offices in the respect that, for the



first time, offices did not know the full extent of the risks which were upon their books, for the reason that not only were the military officers of the regular army engaged in the campaign, but a great number of civilians, who held what were technically known as whole-world policies, volunteered in the capacity of Yeomanry or Volunteers. The consequence was that probably no office was able to this day to say exactly the amount of the sum assured which it had in South Africa exposed to the risks of war. At the commencement of the campaign, military officers were somewhat surprised, and even aggrieved, when they were informed of the extra premium they had to pay, and in some instances went to the length of expressing their feelings in the public press. The offices endeavoured to make the premiums as moderate as possible, and it was a very great satisfaction to them to find that they had been fixed below rather than above the rate sufficient to cover the risk. It must be remembered that, in the case of ordinary life offices, it was the risk of the officer rather than of the non-commissioned officer or private with which they were most concerned, and whom they had most largely represented on their books. It would be found very clearly shown in Diagram A that the mortality among the officers was considerably in excess of that of all other ranks. It came out at 5·3 per-cent. per annum, which, as a previous speaker had observed, showed that the extra premium of five guineas for the first year, which was reduced subsequently to two guineas, was not sufficient to meet the risks exposed. But if the figure of 5·3 per-cent. per annum was subdivided into the three years of the war, it would be found that in the first year the mortality was as high as 7·8, necessitating, as Mr. Mackenzie observed, an extra premium of £8. The two subsequent years dropped to half that amount, 4·1. Therefore offices were right, although he did not know they followed any scientific method, in charging a higher rate of extra for the first year, which carried the heaviest mortality as it turned out, and a lower rate for the subsequent years. It might also be interesting to supplement the large and exhaustive experience of the authors of 19,000 lives by a much smaller experience which had come under his notice. In this particular instance the lives of officers only were concerned. The risks known to the society were about £180,000. To those must be added, as he had already explained, all those policies on the lives of civilians who volunteered, and who were under no necessity of reporting themselves to their insurance offices because they held whole-world policies, so that upwards of £200,000 of insurance had to be covered by extra premiums. Although he was only speaking of an isolated case, it was interesting to know that the mortality experience among that class was equal to 6 per-cent. per annum, a figure somewhat higher than that shown by the authors. The rates charged were five guineas for the first year, and two guineas for the second, and therefore were much less than sufficient to meet the risks. It occurred to him that the beneficent climate of South Africa must be to a large extent credited for the lower mortality than would otherwise have been experienced, so that in that respect the experience of the paper must not be treated as a guide to future campaigns, which might take place in much more unwholesome climates, such as tropical climates,



and less beneficial ones than South Africa. The second point which he hoped officers would consider was the great advantage to themselves of always seeing that their policies carried a small extra premium from the outset, which would cover the whole of the risks of service, foreign residence, and active warfare.

Mr. H. W. MANLY thought he was expressing the general opinion of members that the authors had done a great service to the nation in the production of their paper. It had been accomplished through the confidence and the great public spirit shown by the War Office in placing all their returns at the service of the authors, and he hoped that they would feel that their confidence had not been misplaced. If other Government departments could be induced to show the same public spirit and confidence in actuaries, far better statistical returns and results might be attained than were at present produced.

Mr. ARCH. HEWAT said that Mr. Mackenzie had referred to a return of premiums. Some members who were engaged in "other places" had found there were no premiums to return. They did not know how much was exposed to risk in South Africa, but they knew the claims which had arisen. These had not only taken away all the reserve at the credit of each of the policies, but also all the extra premiums of everyone (including the survivors) who had paid the extras demanded; notwithstanding which there was a balance on the wrong side. So that when a refund was asked for they had a very good answer to give. As Mr. Manly had said, if other departments of the Government would put a little more confidence in actuaries, he thought they might be of still greater service to the nation than they had been in the past.

Mr. GEO. TODD said he had been asked to mention the figures of a small experience with which he had been connected. The experience included nearly 1,700 lives, insured for a total of close upon half a million. The lives were almost entirely of the better class of Yeomen and Volunteers; in fact, the greater portion of them were not only passed by the army surgeons, but were also subjected to medical examination on behalf of the office which he represented, and full papers were completed, with the result that in an appreciable percentage of cases the lives, although they had been passed by the army surgeons, were declined for insurance purposes. The consequence was that there had been excluded from the experience all weakly lives, and all lives whose inherited constitution was weak. Coming shortly to the result, which did not very much differ from that shown in the paper for the insured lives, his total loss amounted to 3.38 per-cent. in lives, and a rather smaller percentage in respect of amounts assured. There were two points upon which he would like the authors to give a little more information. With reference to the experience derived from the records of the War Office, supplemented as described in the paper, it was very clearly shown how the exposed to risk was arrived at, but he did not follow in the paper with quite so much clearness the statistics as to the deaths. It was known that a great many deaths took place upon lives invalided home, some of them on the voyage home, and some of them shortly after they arrived, and it did not appear clearly in the

paper whether those deaths were included. If they were included he might point out that they were compared with a record of exposed to risk, which only included those actually upon the soil of South Africa. As to the second point, on Schedule A, on page 548, it would be noticed how fast, in the earlier stages of the war, the exposed to risk were increasing, practically 20,000 a month for some time, and the deaths were compared with those monthly figures of exposed to risk. Now, a great number of the deaths which took place in April were due to wounds received or to illness acquired in the previous month, and those deaths should properly be compared with the exposed to risk in that previous month, the effect of which, in his opinion, would be to produce a higher rate of mortality during the earlier stages of the war than that actually deduced in the paper.

Major T. McCULLOCH, M.B., said it would be found in the last return that the number of those who had died after they were sent home was 508 all told. The number was actually very small.

Mr. T. G. ACKLAND said that, having had the unfortunate fate himself to grope—he used the word advisedly—amid statistics in connection with Military and Naval operations, and to try and deduce some intelligible results from them, it was exceedingly encouraging and satisfactory to find a contribution in which the data were so fully set forth, appeared to be so thoroughly reliable, were verified from different points of view, and in which every stage of the process was most clearly and lucidly stated, so that it was possible fully to understand the course of the processes which were followed, and to discuss them with a clear appreciation of what they really meant. He was glad reference had been made in the paper to the contribution of Mr. McLauchlan, included in the thirty-fourth volume of the *Journal*, because he thought it had some value as bearing upon the subject, not directly, perhaps, on war mortality, but as to the mortality of the Army in foreign parts in time of peace, which was an allied subject. The figures set out in the present paper in connection with past wars illustrated how very rough and unsatisfactory the data had been in connection with them, notably in the case of the Crimean War. The figures had been set out notably by Mr. Mackenzie, but he was sure that gentleman must have had an immense amount of trouble to get out the data and ratios in reference to the English and French armies engaged. With regard to the Franco-German War and the American War, the statistics were certainly better, and on the present occasion one rose to what one might say was the ideal condition of things in having figures placed before them in reference to the South African War, which gave in a full and satisfactory manner the information required. The notable point in connection with the figures tabulated in the latter part of the paper especially was that they had some relation to the age. It was true that the conclusion of the authors was on the whole that the rates from disease or from wounds did not materially differ with the age, but it was satisfactory to have an investigation made into that point, and to trace the curves through the different ages of lives, and it was the first time, he thought, that had been done in connection with any military operations. It was striking to compare the ratios of



death from disease and from battle, not only of the figures in the paper, but also in reference to previous campaigns. In the long struggle of the American Civil War from 1861 to 1865, the annual death-rate from disease was truly enormous, being no less, he thought it would be found, than four times that of the deaths arising on the field of battle. During the Franco-German War, which took place a few years later, the death-rate from disease was singularly low, but that was carried on in a European country, which was on the whole a healthy one. From the German statistics it would be found that the rate of death from disease was notably below the rate of death on the field of battle. It had been stated in the present discussion that, on the whole, the rate of death in the South African War was below that with which they were familiar in connection with large national wars, but he thought if the matter was looked at from another aspect that result would be somewhat modified. One might fairly consider from their recollection of the facts, and from the valuable epitome given in the paper of the events of the war, that the South African War proper lasted during the first twelve months, in the sense that that was really the great struggle, and that the rest was a sort of guerilla warfare. During the first twelve months' operations, the death-rate was exceedingly heavy, that of the officers being  $8\frac{3}{4}$  per-cent. per annum, and the N.C.O's and men practically 6 per-cent. per annum, those rates being considerably in excess of the corresponding rates per annum in the Franco-German War, which were  $7\frac{1}{2}$  per-cent. per annum, and  $4\frac{1}{2}$  per-cent. per annum, respectively. This indicated that the South African War was even a more serious event, judged by that standard, than the great national war between those two nations. The reference in the paper to deterioration of health, as a consequence of warlike operations, was a very interesting but a most difficult subject, and no writer had, he thought, dealt with it in an entirely satisfactory way. Mr. McLauchlan, in the paper to which he had already referred, had attempted to deal with it in a very interesting manner, upon the lines that the deterioration was represented by the difference between the  $H^M$  select mortality and the  $H^M$  normal mortality. That was confessedly an assumption, and as he did not know that it could be based either on data or on logical reasoning, one must take it for what it was worth. Turning now to the assured lives in the Yeomanry, it would be interesting if the authors could state the social position of the men, which might perhaps be roughly gauged by the average sum assured. He was not sure that the class of men was quite identical with those which they would have to deal with in most offices in insuring against military operations, and it would be interesting to hear whether that view was confirmed by the authors. Dealing with other minor matters, he would like to ask a question, on a matter which had been referred to by a previous speaker, as to the precise duration of the exposure in the case of the assured Yeomanry. In the paper it was stated: "It has been assumed that they remained exposed till the end of the month," but he fancied there must be something omitted there, because it was not quite clear what month was referred to. The difference of rate between the mortality of the assured Yeomanry and that of the Yeomanry as a whole, as deduced from the



War Office returns, was remarkable. In the assured class, the deaths from battle were 9 per 1,000, while in the Yeomanry, as a whole, they were 22 per 1,000, and the deaths from disease were 29 per 1,000 in the one class, against 33 per 1,000 in the other. The mortality was thus 38 per 1,000 as a whole for the assured class, and 55 per 1,000 for the Yeomanry generally. That seemed to be a very large difference in ratio, for which the reasons given by the authors no doubt would account in a measure; but it seemed to him to indicate that too much weight must not be given to the conclusions arrived at in respect of the assured class, and that in charging extra premiums they should have regard to the rate of 55 per 1,000 rather than to that of 38 per 1,000. He had also been a little puzzled to know why the particular process shown in Table XVI was adopted, of deducing the yearly extra premium from the monthly rates. He thought probably it must have been for some reason other than that stated in the paper; and it appeared to him that the yearly premiums could have been deduced directly from the monthly rates. He had worked out the annuity for seventeen months, payable monthly, and then, by the known relation between a term-annuity payable monthly and a term-annuity payable yearly, he arrived at a result which differed only in the third place of decimals from that deduced by the authors, and, he thought, with less work. He thought the paper deserved, and would no doubt receive, their further study, and that the discussion, lengthy and full as it had been, in no way represented the true value of this contribution to their proceedings.

The PRESIDENT (Mr. Hughes), before asking the authors to reply, congratulated them upon having presented the members with a paper of an interest surpassing that which usually formed the subject of their discussions. The interest of the paper was not merely professional and scientific, but it would be read by many people who never, under any circumstances, took any interest in matters forming the subject of most of the papers and discussions at the Institute. He thought it was no disparagement upon the authors of former investigations to say that the paper was in one respect, at all events, if not in others, superior to them all. The authors had had the advantage, with the kind assistance of the officials at the War Office, of getting their facts perfectly fresh and accurate, and having got them they had presented them in a form which was intelligible to all. He wished to emphasize Mr. Warner's remarks as to the danger of generalizing from the results of the paper, because, valuable as they were, they only represented the statistics of a particular war. Former wars were waged under different conditions, and it was to be expected that future wars would be conducted under yet other conditions of a totally different kind, so that the experience derived from the investigation must be used warily, and only as a matter of comparison, and with due regard to the circumstances to which they were applied in the future.

Mr. E. A. RUSHER, in reply, said Mr. Ackland had asked a question with reference to the social position of the men of the assured Yeomanry and Volunteers. They were taken for the most part in bulk, in battalions, and assured by public subscription for

sums generally of £250, the policies being held by trustees specially appointed for the purpose, so that there was nothing in the way of ordinary assurance selection either for or against the company. This circumstance, perhaps, might be a further reason, in addition to those given in the paper, why the rates of the Yeomanry assured lives were less than those of the Yeomanry as a whole. Especially towards the end of the war there were one or two battalions of Volunteers who suffered very heavily indeed, but they happened not to be those who were amongst the assured lives, and that in itself would have a great effect upon the resulting death-rate. Mr. Ackland had also asked how the date of return was dealt with. Probably he could explain that best by giving an example. From enquiries made it was found that one of the Yeomen returned, say, on the 15th June, 1901; that man would be taken off the risk on the 1st July, 1901, *i.e.*, he would be treated as having been at risk till the end of the month in which he was stated to have returned. The reasons for adopting that course were given in the paper. Mr. Ackland had referred to the question of deterioration, which, of course, was the great difficulty the authors had to face. They had no statistics whatever with which they could attempt to measure the effect of deterioration, although it had a very important bearing upon the premiums which should be charged for military risks. Mr. Todd had given a very interesting experience of some 1,700 lives, assured for half a million pounds, with a resulting death-rate of 3·38. That might be due to the fact that the Yeomen who were assured happened to be specially selected lives, having passed a medical examination at the office as well as the War Office ordeal. Mr. Todd had asked how the War Office deaths were recorded, and what had been done with those who died after their return to this country. The authors were supplied with manuscript volumes, which contained a record of every death that had occurred in connection with the South African War, including the 508 which took place after return to England. Those records contained the dates of death, which were very carefully noted on the cards, the deaths so obtained being set against the exposed to risk as already explained. The 508 deaths referred to were thus included in the experience, although the corresponding exposures refer only to troops actually engaged in South Africa. Mr. Mackenzie and several other speakers had referred to the difficulty of officers being called upon to pay extra premium when war broke out, and he wished to emphasize that point. It was a very great pleasure to find that assurance companies were more and more coming into line, and endeavouring to get officers to provide against the rainy day by giving them the option of paying a small extra premium throughout the whole duration of the policy instead of when the risk actually occurred. He hoped the officials from the War Office, who had honoured the Institute by their presence that evening, would do their best to impress upon officers that that was the best way of providing for the inevitable when it arrived. As to whether the Government would ever get to the happy stage, to which Mr. Mackenzie had referred, of paying the extra premium for the officers, one could only hope that that

consummation might come. He was very glad to hear Colonel Wilson strongly emphasize the effect of the length of service in hardening the soldiers, enabling them to endure the hardships of the campaign with much greater immunity from disease. He was sure all the members agreed with Mr. Manly that the kindness and courtesy experienced from the War Office, in so freely affording the authors opportunities for dealing with the matter in a scientific manner, in order that the results might be made public, was an example which they hoped would be followed by many of the other Government departments.

Mr. F. SCHOOLING, in reply, said the members would readily understand that in the preparation of the paper the authors must have had a large amount of skilled assistance, and he desired to heartily thank those gentlemen who so readily and willingly gave up their time and the benefit of their skill and brains in helping to make the paper complete. He was exceedingly pleased to hear the remarks of Surgeon-General Stevenson in regard to the concluding paragraphs of the paper. Originally it was written rather more strongly than it at present stood, because, having very closely followed the progress of the war, the authors were greatly impressed with the way in which those in authority, particularly those of the medical profession, had stuck to their work, and if the words in the paper had given any satisfaction, he was well repaid.

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101	XIX	3	169	XXX	5	206	XXXVI	6
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103	XIX	5	173	XXXI	3	208	XXXVII	2
104	XIX	6	174	XXXI	4	209	XXXVII	3
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107	XX	3	176	XXXI	6	211	XXXVII	5
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1874	†King, George, F.F.A., Mem. Act. Soc. Amer. (VICE-PRESIDENT AND EDITOR OF JOURNAL), <i>15 Walbrook, E.C.</i>	1864	McGedy, Frank, <i>14 Fitz-George-avenue, Kensington West.</i>
1887	†Kyd, Thomas, F.F.A., <i>Northern Assurance Company, 1 Union-terrace, Aberdeen.</i>	1883	†McGowan, James, B.A., <i>The Treasury, Cape Town.</i>
1882	Lancaster, William John, J.P., <i>South Lynn, Putney-hill, s.w.</i>	1885	Mackenzie, Alexander George, F.F.A., <i>47, York-terrace, Regent's-park, N.W.</i>
1894	†Laughton, Alexander Millar, F.F.A., <i>National Mutual Life Assoc. of Australasia, Limited, Corner of Collins and Queen-streets, Melbourne, Australia.</i>	1900	†Macnaghten, Stuart Edye, A.C.A., <i>Equity &amp; Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>
1887	†Lemon, William Kent, Barrister-at-Law, <i>5 Pump-court, E.C.</i>	1901	†Macphail, Donald, F.F.A., <i>Yorkshire Insurance Company, York.</i>
1896	†Levine, Abraham, M.A., <i>National Mutual Life Assur. Soc., 39 King-st., Cheapside, E.C.</i>	1870	†Manly, Henry William, Mem. Act. Soc. Amer. (PAST PRESIDENT, 1898-1900), <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>
1896	†Lewis, John Norman, F.F.A., <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1890	†Marks, Geoffrey (LIBRARIAN), <i>National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.</i>
1892	†Lidstone, George James, <i>Alliance Assurance Co., Limited, Bartholomew-lane, E.C.</i>	1900	†Marr, Vyvyan, F.F.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>
1901	†Little, James Fulton, <i>Mutual Life Association of Australasia, Sydney, Australia.</i>	1902	†May, Basil, <i>National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.</i>
1899	Low, George Macritchie, F.F.A., <i>Scottish Equitable Life Assur. Society, 28 St. Andrew-square, Edinburgh.</i>		



## FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1897	†May, George Ernest, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1887	†Nightingale, Harry Ethelston, <i>Royal Exchange Assurance Cor-</i> <i>poration, Royal Exchange, E.C.</i>
1851	†Meikle, James, F.F.A., Mem. Act. Soc. Amer., <i>Wardieburn-house, Edinburgh.</i>	1901	†Norton, William Ernest, <i>National Provident Institution,</i> <i>48 Gracechurch-street, E.C.</i>
1897	†Miller, Neville, <i>London Assurance Corporation,</i> <i>7 Royal Exchange, E.C.</i>	1899	†Parker, Robert Peter, <i>Sun Life Assurance Society,</i> <i>63 Threadneedle-street, E.C.</i>
1893	†Milner, John William, <i>North British &amp; Mercantile Insur.</i> <i>Co., 61 Threadneedle-street, E.C.</i>	1864	Pearson, Arthur, <i>Betchworth-house, The Bank,</i> <i>Highgate, N.</i>
1892	†Milton, Henry, M.A., <i>Law Debenture Corporation Ltd.,</i> <i>41 Threadneedle-street, E.C.</i>	1891	†Phelps, William Peyton, M.A., <i>Equity and Law Life Assur. Soc.,</i> <i>18 Lincoln's-inn-fields, W.C.</i>
1899	†Moir, Henry, F.F.A., <i>Provident Savings Life Assur.</i> <i>Soc., 346, Broadway, New York,</i> <i>U.S.A.</i>	Under the Charter.	Priestley, John George, <i>44 St. German's-road., Forest-</i> <i>hill, S.E.</i>
1890	†Molyneux, Arthur Ernest, <i>Provident Clerks' Mutual Life</i> <i>Assurance Association, 27 &amp; 29</i> <i>Moorgate-street, E.C.</i>	1891	†Pulley, William Pritchard, <i>Norwich Union Life Insur. Soc.,</i> <i>71 &amp; 72 King William-st., E.C.</i>
1901	†Moorhouse, Alfred, <i>Friends' Provident Institution,</i> <i>Bradford.</i>	1899	†Raisin, Arthur Herbert, <i>Pelican Life Ins.Co., 70 Lombard-</i> <i>street, E.C.</i>
1897	†Moors, Elphinstone MacMahon, M.A., <i>University of Sydney, Australia.</i>	1897	†Rees, Martin, <i>Law Reversionary Interest Soc.,</i> <i>Ltd., 24 Lincoln's-inn-fields, W.C.</i>
1896	†Moran, Joseph Flack, <i>Marine &amp; General Mutual Life</i> <i>Assurance Society, 14 Leaden-</i> <i>hall-street, E.C.</i>	1901	†Reeve, Charles Ernest, <i>Royal Exchange Assurance Cor-</i> <i>poration, Royal Exchange, E.C.</i>
1900	†Morgan, Benjamin Charles, M.A., <i>Commercial Union Assur. Co.,</i> <i>24, 25 &amp; 26 Cornhill, E.C.</i>	1902	†Richmond, George William, <i>Scottish Widows' Fund Life</i> <i>Assur. Society, 28 Cornhill, E.C.</i>
1895	†Muter, Percy, <i>New Zealand Government Life</i> <i>Insurance Department, Wel-</i> <i>lington, New Zealand.</i>	1898	†Robinson, George Frederick, <i>Legal and General Life Assur.</i> <i>Society, 10 Fleet-street, E.C.</i>
1888	†Nash, Willie Oscar, <i>Law Reversionary Interest Soc.,</i> <i>Ltd., 24 Lincoln's-inn-fields, W.C.</i>	1888	†Rusher, Edward Arthur, F.S.S., <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1883	Neison, Francis G. P., F.S.S., <i>19 Abingdon-st., Westminster, S.W.</i>	1882	†Ryan, Gerald Hemington, Mem. Act. Soc. Amer., <i>British Empire Mutual Life</i> <i>Assurance Co., 4 &amp; 5 King William-</i> <i>street, E.C.</i>
1888	†Newman, Philip Lewin, B.A., <i>Yorkshire Insurance Co., York.</i>	1898	†Salmon, Richard George, F.S.S., <i>Sun Life Assurance Society, 63</i> <i>Threadneedle-street, E.C.</i>
1865	Newton, Algernon, M.A., <i>c/o London &amp; Westminster Bank,</i> <i>94 &amp; 96 High-st., Kensington, W.</i>	1883	Saunders, Harris Charter Lindon, F.R.A.S., <i>"Marquise," Twickenham.</i>

## FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1886	†Schooling, Frederick (HON. SEC.), <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1892	†Straker, Edward Robert, <i>British Empire Mutual Life</i> <i>Assurance Co., 4 &amp; 5 King William-</i> <i>street, E.C.</i>
1901	†Searle, George Morley, <i>Sun Life Assurance Society, 63</i> <i>Threadneedle-street, E.C.</i>	1878	†Straker, Frank Arthur, <i>Legal and General Life Assur.</i> <i>Society, 10 Fleet-street, E.C.</i>
1901	†Sharman, William Charles, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1902	†Strong, William Richard, <i>London Guarantee &amp; Accident</i> <i>Co., 61 Moorgate-street, E.C.</i>
1896	†Sim, William Abernethy, F.F.A., <i>Scottish Union and National</i> <i>Insurance Co., 35 St. Andrew-</i> <i>square, Edinburgh.</i>	1884	†Stuart, John Moody, F.F.A., <i>The Leeds Permanent Building</i> <i>Society, Victoria-buildings, Park-</i> <i>lane, Leeds.</i>
1875	†Smither, Arthur, <i>National Provident Institution,</i> <i>48 Gracechurch-street, E.C.</i>	1900	†Sutherland, John, M.A., <i>Australasian Temperance and</i> <i>General Mutual Life Assurance</i> <i>Society, Swanston-street, Mel-</i> <i>bourne, Australia.</i>
1881	†Somerville, William Finlay, <i>Liverpool and London and Globe</i> <i>Insurance Co., 7 Cornhill, E.C.</i>	1889	†Tarn, Arthur Wyndham, <i>Westminster and General Life</i> <i>Assurance Association, 28 King-</i> <i>street, Covent-garden, W.C.</i>
1877	†Sorley, James, F.F.A., C.A., F.R.S.E., <i>Pelican Life Insurance Company,</i> <i>70 Lombard-street, E.C.</i>	1887	Teece, Richard, F.F.A., F.S.S., Mem. Act. Soc. Amer., <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>
1898	†Spencer, John, <i>English and Scottish Law Life</i> <i>Assurance Assoc., 12 Waterloo-</i> <i>place, S.W.</i>	1872	Templeton, Col. John M., C.M.G., <i>National Mutual Life Associa-</i> <i>tion of Australasia, Melbourne,</i> <i>Australia.</i>
1894	†Sprague, Alfred Ernest, B.Sc., M.A., F.F.A., <i>Edinburgh Life Assurance Co.,</i> <i>22 George-street, Edinburgh.</i>	1886	Tennant, John Bell, <i>Friends' Provident Institution,</i> <i>Bradford, Yorkshire.</i>
1857	Sprague, Thomas Bond, M.A., LL.D., F.F.A., F.S.S., F.R.S.E. (PAST PRESIDENT, 1882-86), <i>29 Buckingham-terrace, Edin-</i> <i>burgh.</i>	1864	†Terry, James, <i>Hernelee, Lyme Regis, Dorset.</i>
1896	†Stahlschmidt, Louis, <i>Alliance Assurance Co., Limited,</i> <i>Bartholomew-lane, E.C.</i>	1889	†Thiselton, Herbert Cecil, F.F.A., Mem. Act. Soc. Amer., <i>Hand-in-Hand Insurance Soc.,</i> <i>26 New Bridge-street, E.C.</i>
Under the Charter	Stevens, Charles, <i>Aberdeen Ho., Preston, Brighton.</i>	1901	†Thodey, Robert, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>
1888	Stewart, John, F.F.A., <i>City of Glasgow Life Assur. Co.,</i> <i>30 Renfield-street, Glasgow.</i>	1893	†Thomas, Ernest Charles, <i>Gresham Life Assurance Society,</i> <i>St. Mildred's-house, Poultry, E.C.</i>
1898	Stirling, Robert, F.F.A., <i>Law Union &amp; Crown Insurance</i> <i>Co., 126 Chancery-lane, W.C.</i>	1899	†Thomas, Robert Arthur Caradoc, <i>British Empire Mutual Life</i> <i>Assurance Co., 12 Dalhousie-</i> <i>square, Calcutta.</i>
1868	†Strachan, Thomas Young, <i>Jethart, High View-road, Sidcup.</i>	1895	†Thomson, Herbert Archer, B.A., <i>Parkwood-house, Whetstone, N.</i>

## FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1880	Thomson, Robert, <i>Colonial Mutual Life Assurance Society, Collins - street - west, Melbourne, Australia.</i>	1895	†Watson, James Douglas, <i>English &amp; Scottish Law Life Assr. Assoc., 12 Waterloo-place, S.W.</i>
1876	Thomson, Spencer Campbell, B.A., F.F.A., <i>Standard Life Assurance Co., 3 George-street, Edinburgh.</i>	1880	†Whittall, Wm. Joseph Hutchings, Mem. Act. Soc. Amer., <i>Clerical, Medical &amp; General Life Assur. Soc., 15 St. James's-sq., S.W.</i>
1893	†Thorne, Alfred Charles, <i>Equity &amp; Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>	1864	Wilson, Robert, 44 Talfourd-rd., Camberwell, S.E.
1891	†Tilt, Robert Ruthven, <i>General Reversionary &amp; Investment Co., Ltd., 26 Pall-mall, S.W.</i>	1888	†Wilson, Robert, Jr., <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1902	†Tinner, Thomas, <i>Comptroller's Depart., London County Council, Spring-gardens, S.W.</i>	Under the Charter.	Winser, Thomas Boorman, 81 Shooter's-hill-road, Blackheath, S.E.
1881	†Todd, George, M.A. (HON. SEC.), <i>Economic Life Assurance Society, 6 New Bridge-street, E.C.</i>	1899	†Winter, Arthur Thomas, <i>British Empire Mutual Life Assurance Company, 4 and 5 King William-street, E.C.</i>
1894	†Todhunter, Ralph, M.A., <i>University Life Assur. Soc., 25 Pall-mall, S.W.</i>	1897	†Wintle, Lancelot Andrewes, <i>Economic Life Assurance Soc., 6 New Bridge-street, E.C.</i>
1899	†Trouncer, Harold Moltke, B.A., <i>London Life Association Ltd., 81 King William-street, E.C.</i>	1884	†Woods, Ernest, Mem. Act. Soc. Amer., <i>Westminster and General Life Assur. Assoc., 28 King-street, Covent-garden, W.C.</i>
1878	Turnbull, Andrew Hugh, F.F.A., F.R.S.E., <i>Scottish Widows' Fund Life Assur. Soc., 9 St. Andrew-square, Edinburgh.</i>	1902	†Woolmer, Alfred Henry, <i>Star Life Assurance Society, 32 Moorgate-street, E.C.</i>
Under the Charter.	Tyndall, William Henry, F.S.S., F.R.Met.S., <i>Morlands, Oxford-road, Redhill, Surrey.</i>	1902	†Workman, William Arthur, <i>Equitable Life Assur. Society, Mansion-house-street, E.C.</i>
1889	Wallace, Thomas, F.F.A., <i>North British &amp; Mercantile Insurance Co., Edinburgh.</i>	1902	†Worthington, William, <i>Royal Insurance Co., Liverpool.</i>
1888	†Warner, Samuel George, <i>Law Union &amp; Crown Insur. Co., 126 Chancery-lane, W.C.</i>	1875	†Wyatt, Frank Bertrand, Mem. Act. Soc. Amer., <i>Clergy Mutual Assurance Soc., 2 &amp; 3 The Sanctuary, S.W.</i>
1893	†Watson, Alfred William, <i>Manchester Unity Friendly Soc., Nottingham.</i>	1874	Young, Thomas Emley, B.A., F.R.A.S. (PAST-PRESIDENT, 1896-8), Mem. Act. Soc. Amer., 108 Evering-road, Stoke Newington, N.



## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1900	<sup>2</sup> Adams, Cecil Francis, <i>New Zealand Accident Insurance Company, Wellington, New Zealand.</i>	1896	Brown, George Andrew, <i>Clerical, Medical &amp; General Life Assurance Society, Mansion-house-buildings, E.C.</i>
1869	<sup>2</sup> Adey, Theodore Henry, <i>Scottish Provident Institution, 17 King William-street, E.C.</i>	1899	<sup>2</sup> Brown, Harold, <i>Scottish Union and National Insurance Co., 3 King William-street, E.C.</i>
1899	<sup>3</sup> Adlard, Stanley, A.K.C., <i>London Life Association Ltd., 81 King William-street, E.C.</i>	1900	<sup>2</sup> Brown, Henry, B.A., <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>
1899	<sup>2</sup> Ansell, George Frederic, <i>National Debt Office, Finsbury Pavement House, E.C.</i>	1886	Buckley, Thomas John Wesley, <i>9 St. Andrew-street, Holborn-circus, E.C.</i>
1883	<sup>2</sup> Ashley, John Geo., M.A., <i>War Office, S.W.</i>	1882	Burke, David, F.S.S., <i>Royal Victoria Life Insur. Co., Montreal, Canada.</i>
1901	<sup>2</sup> Ashton, William Richard, <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>	1900	<sup>2</sup> Burnley, Isaac, <i>Australian Mutual Prov. Society, Sydney, Australia.</i>
1881	<sup>2</sup> Ayling, Charles Stephen, <i>Commercial Union Assur. Co., 20 New Bridge-street, E.C.</i>	1895	<sup>3</sup> Butterfield, William Thos., A.C.A., <i>9 Market-street, Bradford.</i>
1902	<sup>3</sup> Bacon, James, <i>c/o T. G. Ackland, Esq., 5 &amp; 6 Clements-inn, Strand, W.C.</i>	1876	<sup>1</sup> Carter, Eric Mackay, <i>33 Waterloo-street, Birmingham.</i>
1885	Barton, Arthur, <i>United Kent Insurance Institution, Maidstone.</i>	1899	<sup>2</sup> Catchlove, Chas. Hamilton Leyland, <i>Australian Mutual Provident Society, Adelaide, S. Australia.</i>
1894	<sup>3</sup> Barton, Robert Whitchurch, <i>Clerical, Medical &amp; General Life Assurance Society, 15 St. James's-square, S.W.</i>	1900	<sup>2</sup> Chandler, Thomas Richard, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1901	<sup>2</sup> Benjamin, Stanley O., <i>Australian Mutual Provident Society, Sydney.</i>	1898	<sup>2</sup> Coates, Thomas Linnaeus, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1881	Birks, Edmund Alfred, <i>Yorkshire Insurance Co., York.</i>	1871	Cook, Arthur James, M.J.I., <i>Victoria Mutual Assur. Society, Farringdon-street, E.C.</i>
1873	<sup>2</sup> Block, Robert John, <i>Essex - villa, Chelsham - road, Clapham, S.W.</i>	1899	<sup>3</sup> Cook, William Playfair, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>
1898	Blount, Edward Thos. J., F.F.A., F.S.S., <i>Standard Life Assurance Co., Shanghai, China.</i>	1897	<sup>2</sup> Coop, Charles Rowland, <i>United Kingdom Temperance and General Provident Institution, 5 Bennett's-hill, Birmingham.</i>
1873	<sup>2</sup> Boon, Gerald Inglis, <i>United Legal Indemnity Insur. Soc., Limited, 222 Strand, W.C.</i>	1891	<sup>2</sup> Coote, Ernest Charles, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1889	Bremner, Thomas William, F.F.A., <i>Mutual Life Insurance Co. of New York, Sydney, Australia.</i>		

## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1900	<sup>2</sup> Corbett, Edwin Somerville, <i>Australasian Temperance and General Mutual Life Assurance Soc., Swanston-street, Melbourne, Australia.</i>	1881	Donaldson, John, <i>Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.</i>
1871	Countts, Edwin Arthur, <i>North British and Mercantile Insurance Company, Victoria-street, Nottingham.</i>	1899	<sup>2</sup> Dougharty, Harold, F.S.S., F.I.S., <i>London and Lancashire Life Assurance Company, 66 &amp; 67 Cornhill, E.C.</i>
1900	<sup>2</sup> Covington, Oliver Henry, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902	<sup>2</sup> Doust-Smith, Ernest Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1884	Craig, Robert Alexander, <i>Abstainers' and General Assur. Co., City Buildings, Birmingham.</i>	1881	Dovey, William Roadly, F.F.A., Mem. Act. Soc. Amer., 26 Crouch Hall Road, Crouch End, N.
1901	<sup>2</sup> Cross, Howard Turner, <i>Economic Life Assurance Soc., 6 New Bridge-street, E.C.</i>	1870	<sup>1</sup> Dowson, John, <i>Royal Insur. Company, Liverpool.</i>
1901	<sup>2</sup> Culley, Alfred Benjamin, <i>Star Life Assurance Society, 32 Moorgate-street, E.C.</i>	1898	<sup>2</sup> Doyle, Arthur James, 54 Bourke-st., Sydney, Australia.
1901	<sup>2</sup> Curjel, H. W., M.A., <i>Royal Insurance Co., Liverpool.</i>	1901	<sup>2</sup> Earle, Arthur Percival, <i>North American Life Assur. Co., North American Life Building, 112-118 King-st.-west, Toronto, Canada.</i>
1900	<sup>2</sup> Curtis, William Allen, <i>Clerical, Medical &amp; General Life Assurance Society, 15 St. James's-square, S.W.</i>	1868	<sup>1</sup> Eaton, Henry William, <i>Liverpool and London and Globe Insurance Company, William-street, New York, U.S.A.</i>
1902	<sup>2</sup> Dawson, Miles Menander, F.S.S., 11 Broadway, New York, U.S.A.	1872	<sup>2</sup> Evans, William, F.F.A., F.R.S.E., 38 Morningside-park, Edinburgh.
1901	Denham, Walter, F.F.A., <i>City of Glasgow Life Assurance Co., 30 Renfield-street, Glasgow.</i>	1902	<sup>3</sup> Falconer, Herbert Francis, F.F.A., <i>Scottish Life Assurance Co., 19 St. Andrew-sq., Edinburgh.</i>
1902	<sup>2</sup> Denmead, John Charles, M.A., <i>Estate Duty Office, Somerset House, W.C.</i>	1896	<sup>2</sup> Featherstonelough, William Irwin, <i>Commercial Union Assurance Co., 24, 25 &amp; 26 Cornhill, E.C.</i>
1901	<sup>2</sup> Diamond, George Frederick, <i>City Mutual Life Assurance Society, Hunter-street, Sydney, Australia.</i>	1897	<sup>2</sup> Findlay, Alexander Wynaud, LL.B., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1900	<sup>2</sup> Diver, Oswald Francis, M.A., <i>Clerical, Medical &amp; General Life Assur. Soc., 15 St. James'-sq., S.W.</i>	1902	<sup>2</sup> FitzGerald, Charles R., <i>Home Life Association of Canada, Toronto, Canada.</i>
1855	Dix, James, <i>Hurstdale, Wood-la., Highgate, N.</i>	1901	<sup>2</sup> FitzGerald, William George, 260 McLaren-street, Ottawa, Canada.
1901	Donald, Alexander Graham, M.A., F.F.A., <i>Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.</i>	1890	Fox, Charles Edward, F.F.A., <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>

## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.
1886	Fox, Morris, Mem. Act. Soc. Amer., <i>New Zealand Government Life Insur. Dept., Wellington, New Zealand.</i>	1902 <sup>2</sup> Grant, Milton Daniel, B.A., <i>Government Insurance Department, Ottawa, Canada.</i>
1894 <sup>2</sup>	Fraser, Thomas John, <i>Australian Alliance Assurance Company, Melbourne, Australia.</i>	1888 Gray, John, <i>Scottish Widows' Fund Life Assur. Society, 28 Baldwin-st., Bristol.</i>
1901	Gaff, William Robertson, C.A., F.F.A., <i>3 Crown-court, Old Broad-st., E.C.</i>	1902 <sup>2</sup> Gray, Robert Alexander, B.A., <i>324 Markham-street, Toronto, Canada.</i>
1873 <sup>2</sup>	Gage, Uriah Woodard, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>	1898 <sup>2</sup> Green, George, B.A., <i>Union Assurance Society, 81 Cornhill, E.C.</i>
1902 <sup>2</sup>	Gaines, John M., <i>New York Life Insurance Co., 346 &amp; 348 Broadway, New York, U.S.A.</i>	1868 <sup>1</sup> Greig, John Andrew, <i>Sun Life Assurance Society, 60 Charing-cross, S.W.</i>
1897 <sup>3</sup>	Galer, Frederic Bertram, B.A., <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>	1869 Griffith, E. Clifton, <i>4 Carlton-chambers, S.W.</i>
1895 <sup>2</sup>	Galwey, Charles Edmund, <i>New Zealand Government Life Insur. Dept., Wellington, New Zealand.</i>	1893 <sup>2</sup> Hall, John Francis Edmund, <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>
1893 <sup>2</sup>	Gardiner, Robert Edward, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1869 Hann, Robert George, Mem. Act. Soc. Amer., <i>Equitable Life Assur. Soc. of the United States, 120 Broadway, New York.</i>
1885 <sup>2</sup>	Gayford, Herbert Stannard, <i>Northern Assurance Co., 1 Moor-gate-street, E.C.</i>	1894 <sup>2</sup> Harcastle, Edwd. Edgington, M.A., <i>Union Central Life Office, Cincinnati, Ohio, U.S.A.</i>
1899 <sup>3</sup>	Gibb, James Burnett, F.F.A., <i>Penn Mutual Life Insc. Co. of Philadelphia, 923 Chestnut-st., Philadelphia, U.S.A.</i>	1900 <sup>2</sup> Harding, Harry Burnard, <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>
1871 <sup>2</sup>	Glennie, William Gordon, <i>Scottish Union &amp; National Insur. Co., 3 King William-street, E.C.</i>	1896 <sup>3</sup> Harris, Frederick Joseph, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1895 <sup>3</sup>	Glover, Henry Walter, <i>Pullinger House, Beaconsfield, Cape Colony.</i>	1897 <sup>2</sup> Hayeraft, William Melhuish, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1897 <sup>2</sup>	Goggs, Frank Sidney, <i>Scottish Metropolitan Life Assur. Co., Ltd., 25 St. Andrew-sq., Edinburgh.</i>	1897 <sup>2</sup> Hazell, James Stanley, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
1882	Goldman, Leopold, <i>North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.</i>	1895 <sup>2</sup> Heness, Leonard Thomas, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1897 <sup>2</sup>	Goodwyn, John, Jr., <i>41 St. Giles'-street, Norwich.</i>	1878 Henry, Alfred, F.C.A., <i>Throgmorton-house, Copthall-avenue, E.C.</i>
		1900 <sup>2</sup> Hicks, Arthur Joseph, <i>Reversionary &amp; General Securities Company, Ltd., Craven House, Northumberland-avenue, W.C.</i>



## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1884	Higham, William Samuel, <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>	1894	<sup>2</sup> Johnston, Frederick H., <i>Prudential Life Insurance Co. of America, Newark, N.J., U.S.A.</i>
1894	<sup>2</sup> Hollingworth, Albert Chas., <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1898	<sup>2</sup> Kaufman, Henry N., Assoc. Act. Soc. Amer., <i>Phoenix Mutual Life Insurance Co., Hartford, Connecticut, U.S.A.</i>
1883	Holt, Edward Hallett, <i>Law Life Assurance Society, 187 Fleet-street, E.C.</i>	1876	Kearry, Joseph, <i>44 Charlwood-street, Belgrave- road, s.w.</i>
1894	<sup>2</sup> Home, Noel Charles Minchin, LL.B., F.S.S., <i>Westcott, Dorking, Surrey.</i>	1899	<sup>2</sup> Kelly, John Joseph, <i>Citizens' Life Assurance Co., Sydney, Australia.</i>
1898	<sup>2</sup> Howell, Chas. Edward, B.A., LL.D., <i>Standard Life Assurance Compy., 66 Upper Sackville-st., Dublin.</i>	1897	<sup>2</sup> Kemp, Julian Ernest Sandford, <i>Eagle Insurance Company, 79 Pall-mall, s.w.</i>
1899	<sup>2</sup> Hudson, Alfred James, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>	1858	Kilford, George William, <i>Rue de Grétry, Paris.</i>
1875	Hunt, Richard Aldington, F.S.S., <i>Wesleyan &amp; General Assur. Soc., Corporation-street, Birmingham.</i>	1902	<sup>2</sup> Kilgour, David Errett, <i>14 Grenville-street, Toronto, Canada.</i>
1893	Hunter, Arthur, F.F.A., <i>New York Life Insurance Co., 346 &amp; 348 Broadway, New York.</i>	1874	<sup>1</sup> King, Arthur Thomas, <i>National Debt Office, Finsbury Pavement House, E.C.</i>
1902	<sup>2</sup> Hunter, Robertson G., <i>New York Life Insurance Co., 346 &amp; 348 Broadway, New York.</i>	1882	<sup>2</sup> King, William Alfred, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1887	<sup>2</sup> Hunter, Samuel, <i>Patriotic Assurance Company, 9 College-green, Dublin.</i>	1902	<sup>2</sup> Kitchin, Frederick Hareourt, B.A., <i>Broad Clyst, Teddington.</i>
1889	Jacobs, Frederick Job, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1861	Knowles, Richard, <i>35 Tilson-road, Tottenham, N.</i>
1876	<sup>2</sup> James, George Trevelyan, <i>12 Waterloo-place, s.w.</i>	1893	<sup>2</sup> Laing, William Claud, <i>North British and Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>
1871	Jellicoe, George Rogers, <i>Eagle Insurance Company, 79 Pall-mall, s.w.</i>	1897	<sup>2</sup> Lane, Arthur Vere, B.A., <i>City of Glasgow Life Assurance Company, 38 Baldwin-street, Bristol.</i>
1883	Jerman, Richard, <i>Commercial Union Assurance Company, Exeter.</i>	1899	<sup>2</sup> Lawton, George Herbert, <i>Clerical, Medical &amp; General Life Assurance Society, 15 St. James's- square, s.w.</i>
1896	<sup>2</sup> Jobson, Alexander, <i>Australian Mutual Provident Society, Melbourne.</i>	1885	Ledward, Archibald Prentice, B.Sc., <i>29 Langland-gardens, N.W.</i>
1894	<sup>2</sup> Johannessen, Nikolai Mikal, <i>Hygea Life Assurance Company, Bergen, Norway.</i>		

# ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1879	Leitch, Alexander, <i>Scottish Provident Institution,</i> 17 King William-street, E.C.	1883	Mannering, George Willsher, <i>London and Lancashire Life</i> <i>Assur. Co.,</i> 66 & 67 Cornhill, E.C.
1897	<sup>2</sup> Le Maitre, Frank William, <i>Sun Life Assurance Society,</i> 63 <i>Threadneedle-street,</i> E.C.	1880	<sup>1</sup> Manwaring, Henry, <i>National Debt Office,</i> Finsbury- <i>pavement-house,</i> E.C.
1885	Leveaux, Arthur Michael, F.S.S. (AUDITOR), <i>Registry of Friendly Societies,</i> <i>Central Office,</i> 28 Abingdon- <i>street,</i> Westminster, S.W.	1878	Marshall, William, <i>South African Mutual Life Assur.</i> <i>Soc.,</i> Cape Town, South Africa.
1885	<sup>2</sup> Lidbury, Isaac Stephen, <i>Prudential Assurance Company,</i> <i>Holborn-bars,</i> E.C.	1896	<sup>2</sup> Martin, Sidney George, <i>National Mutual Life Assoc. of</i> <i>Australasia, Ltd.,</i> 150 Queen- <i>street,</i> Brisbane, Australia.
1868	<sup>1</sup> Litchfield, Edward, 92 St. Vincent-street, Glasgow.	1897	<sup>2</sup> Mascall, Alfred John, <i>Standard Life Assurance Co.,</i> 3 Pall-mall East, S.W.
1876	<sup>2</sup> Lucey, Herbert, <i>General Assurance Company,</i> 103 Cannon-street, E.C.	1900	<sup>2</sup> Maunder, George Harvard, <i>Liverpool and London and Globe</i> <i>Insurance Co.,</i> 7 Cornhill, E.C.
1890	Lugton, Hugh, F.F.A., <i>North British and Mercantile</i> <i>Insurance Co.,</i> 61 Threadneedle- <i>street,</i> E.C.	1902	Maxwell, Benjamin Bell, F.F.A., <i>Scottish Equitable Life Assur.</i> <i>Society,</i> 28 St. Andrew-square, Edinburgh.
1900	<sup>3</sup> McArthur, Harry de C., <i>Economic Life Assur. Society,</i> 6 New Bridge-street, E.C.	1899	<sup>2</sup> Meade, Gerald Willoughby, <i>North British &amp; Mercantile In-</i> <i>surance Company,</i> 61 Thread- <i>needle-street,</i> E.C.
1867	<sup>1</sup> Macdonald, William Rae, F.F.A., <i>Scottish Metropolitan Life Assur.</i> <i>Co., Limited,</i> 25 St. Andrew- <i>square,</i> Edinburgh.	1896	<sup>2</sup> Merfield, Percy Henry, <i>Law Life Assurance Society,</i> 187 Fleet-street, E.C.
1882	<sup>3</sup> McDougald, Alfred, <i>British Empire Mutual Life</i> <i>Assur. Co.,</i> Montreal, Canada.	1874	Miller, John W., F.S.S., <i>Scottish Widows' Fund Life</i> <i>Assur. Soc.,</i> 28 Cornhill, E.C.
1884	Mackay, Alexander, <i>Law Union &amp; Crown Insur. Co.,</i> 126 Chancery-lane, W.C.	1902	<sup>2</sup> Milligan, Charles Livingstone, <i>Provident Life Office,</i> 50 Regent- <i>street,</i> W.
1901	<sup>2</sup> Mackenzie, Michael Alexander, <i>Trinity College,</i> Toronto, Canada.	1884	Mills, Daniel Yarnton, <i>Scottish Equitable Life Assur.</i> <i>Society,</i> 28 St. Andrew-square, Edinburgh.
1896	<sup>2</sup> Macmillan, John Campbell, <i>Royal Insurance Co.,</i> Apartado <i>Postal No. 657,</i> Mexico.	1879	<sup>1</sup> Monilaws, William Macgeorge, <i>Scottish Provident Institution,</i> 17 King William-street, E.C.
1867	Macpherson, Ronald, <i>Law Union &amp; Crown Insurance</i> <i>Co.,</i> 126 Chancery-lane, W.C.	1877	Moon, James, <i>Prudential Assurance Company,</i> 30 Dale-street, Liverpool.
1883	<sup>2</sup> Makeham, William Reed, <i>Alliance Assurance Co., Ltd.</i> <i>(Imperial Life Assurance Fund),</i> 47 Chancery-lane, W.C.		

## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1877	<sup>2</sup> Moon, John, <i>Prudential Assurance Company,</i> <i>76 King-street, Manchester.</i>	1895	<sup>2</sup> Paradise, William Henry, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1879	<sup>1</sup> Moon, Sidney Norman Laming, <i>74, 76 &amp; 78 William-street,</i> <i>New York, U.S.A.</i>	1869	<sup>1</sup> Park, David Francis, C.A., F.F.A., <i>Crédit Foncier of Mauritius (Limited), 39 Lombard-st., E.C.</i>
1898	<sup>2</sup> Moore, Joseph Patrick, <i>Citizens' Life Assurance Co.,</i> <i>Sydney, Australia.</i>	1884	Park, Leslie John, <i>Colonial Mutual Life Assurance Society, Melbourne, Australia.</i>
1871	<sup>2</sup> Moore, Roderick Mackenzie, <i>United Kingdom Temperance and General Provident Institution,</i> <i>1 Adelaide-place, London-bridge,</i> <i>E.C.</i>	1882	<sup>2</sup> Paterson, William Brockie, F.F.A., <i>Mem. Act. Soc. Amer.,</i> <i>Norwich Union Life Insurance Society, Norwich.</i>
1893	<sup>2</sup> Munro, Donald Alexander, <i>Brook House, 10 Walbrook, E.C.</i>	1898	Pearce, Henry John, F.F.A., <i>Edinburgh Life Assurance Co.,</i> <i>122 St. Vincent-street, Glasgow.</i>
1900	<sup>2</sup> Nash, Alfred Charles, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1899	<sup>2</sup> Peele, Thomas, <i>Refuge Assurance Company,</i> <i>Oxford-street, Manchester.</i>
1897	<sup>2</sup> Newling, Sidney Wallis, B.A., <i>Woodleigh, South Woodford,</i> <i>Essex.</i>	1901	<sup>2</sup> Penman, William, Jr., <i>Northern Assurance Company,</i> <i>1 Moorgate-street, E.C.</i>
1884	Nicoll, John, F.F.A., <i>Life Association of Scotland,</i> <i>82 Princes-street, Edinburgh.</i>	1900	<sup>2</sup> Peters, Charles Furness, <i>L'pool. Victoria Legal Friendly Society, 18 St. Andrew-street, E.C.</i>
1897	<sup>3</sup> Norris, Charles Arthur, <i>National Mutual Life Assoc. of Australasia, Ltd., Melbourne, Australia.</i>	1895	Pierson, Israel Coriell, Mem. Act. Soc. Amer., <i>141 Broadway, New York, U.S.A.</i>
1900	<sup>2</sup> Oakley, Henry John Percy, <i>North British and Mercantile Insurance Company, 61 Thread-needle-street, E.C.</i>	1902	<sup>2</sup> Pigrome, George Davey, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1883	Orr, Lewis P., F.F.A., <i>Scottish Life Assur. Co., Ltd.,</i> <i>19 St. Andrew-sq., Edinburgh.</i>	1899	<sup>2</sup> Pipe, Sidney Herbert, <i>Independent Order of Foresters,</i> <i>Temple-bldgs., Toronto, Canada.</i>
1886	Owen, Evan Frederick, F.S.S., <i>Actuary for Friendly Societies,</i> <i>Melbourne, Australia.</i>	1883	Pitts, Thomas, <i>Commercial Union Assurance Company, Exeter.</i>
1895	<sup>2</sup> Pagden, Lionel King, <i>Union Assurance Society, 81 Cornhill, E.C.</i>	1876	<sup>1</sup> Pound, Thomas James, <i>Clerical, Medical &amp; General Life Assurance Soc., 15 St. James's-square, S.W.</i>
1864	Panton, Edward Henry, <i>50 Wood-vale, Forest Hill, S.E.</i>	1890	<sup>2</sup> Powell, Alfred, <i>Alliance Assurance Company Limited, Bartholomew-lane, E.C.</i>
1901	<sup>2</sup> Papps, Percy Charles Herbert, <i>Canada Life Assurance Company,</i> <i>Toronto, Canada.</i>	1881	<sup>1</sup> Price, William John, <i>Life Association of Scotland,</i> <i>5 Lombard-street, E.C.</i>



## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1869	<sup>1</sup> Pringle, James, C.A., F.F.A., 42 Drumsheugh-gardens, Edinburgh.	1897	<sup>2</sup> Ryley, Edmund, Prudential Assurance Company, Holborn-bars, E.C.
1884	Pullar, James, F.F.A., Colonial Mutual Life Assurance Society, Melbourne, Australia.	1896	<sup>2</sup> Sanderson, Frank, M.A., F.F.A., F.S.S., Mem. Act. Soc. Amer., Canada Life Assurance Company, Toronto, Canada.
1881	Purves, Thomas Peter, New York Life Insurance Company, Sydney, Australia.	1884	Schooling, John Holt, Fotheringay-house, Montpelier-row, Twickenham.
1899	<sup>3</sup> Rae, Joseph, Finance Department, Town-hall, Upper-street, N.	1899	<sup>2</sup> Schouten, Pieter, Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.
1867	Rattray, Patrick, C.A., Gresham-house, 45 West Nile-street, Glasgow.	1873	Scott, Ernest Willem, Mem. Act. Soc. Amer., Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.
1874	<sup>2</sup> Ray, Charles Richard, Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.	1861	<sup>2</sup> Searle, Thomas John, Mansion-house-chambers, Bucklersbury, E.C.
1885	<sup>1</sup> Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., 3 & 4 Clement's-inn, W.C.	1900	<sup>2</sup> Searls, Edwin Richard, Northern Assurance Company, 1 Moorgate-street, E.C.
1898	<sup>2</sup> Reid, Edward E., B.A., London Life Insurance Co., London, Ontario.	1900	<sup>2</sup> Sharpe, Edgar Cecil Engledue, London Life Association, Ltd., 81 King William-street, E.C.
1901	<sup>2</sup> Rhodes, Francis, B.A., Royal Insurance Company, Liverpool.	1894	<sup>3</sup> Sheppard, Herbert Norman, B.A., Mem. Act. Soc. Amer., Union Central Life Office, Cincinnati, Ohio, U.S.A.
1887	Richardson, Josephus Hargreaves, F.F.A., Mem. Act. Soc. Amer., New Zealand Government Life Insurance Department, Wellington, New Zealand.	1899	<sup>2</sup> Sherriff, Francis Henry, Provident Clerks' Mutual Life Assurance Association, 27 & 29 Moorgate-street, E.C.
1879	Roberts, Thomas B., Australian Alliance Assurance Company, Collins-street, Melbourne, Australia.	1897	<sup>2</sup> Shimmell, James Edward, Scottish Imperial Insurance Co., 183 West George-st., Glasgow.
1878	Robertson, William, F.F.A., 54 Queen-street, Edinburgh.	1896	<sup>2</sup> Shlager, Joseph, 1, Lionel Villa, Gladstone-street, Cape Town, South Africa.
1876	<sup>1</sup> Robinson, Andrew, Sunningdale-park, Sunningdale, Berks.	1897	<sup>2</sup> Slade, Henry, Balgownie, Warwick-rd., Acocks-green, near Birmingham.
1902	<sup>2</sup> Robinson, Hugh Thomas Kay, Clergy Mutual Assurance Society, 2 & 3 The Sanctuary, Westminster, S.W.		
1885	Ronald, Thomas Robert, Law Guarantee and Trust Soc., Ltd., 49 Chancery-lane, W.C.		

## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1864	<sup>1</sup> Smith, Howard Samuel, F.F.A., F.S.S., F.C.A., <i>Bank-chambers, 11 Waterloo-street, Birmingham.</i>	1875	Taylor, J. Wilford, <i>North British and Mercantile Insur. Co., 61 Threadneedle-st., E.C.</i>
1898	<sup>2</sup> Smith, Robert Parker, <i>Royal Insurance Company, Liverpool.</i>	1898	<sup>2</sup> Thompson, Thomas Percy, B.A., <i>British Empire Mutual Life Assurance Co., 4 &amp; 5 King William-street, E.C.</i>
1884	Smithett, Edward Henry, <i>"Hillside," Fitzroy-park, High-gate, N.</i>	1883	<sup>2</sup> Titmuss, Walter George, <i>Provident Life Office, 50 Regent-street, W.</i>
1871	Spencer, Robert James, F.S.S., <i>75 King's-road, Southsea.</i>	1902	<sup>2</sup> Traversi, Antonio Thomas, <i>Government Life Insurance Department, Wellington, New Zealand.</i>
1868	Spens, William George, <i>Scottish Amicable Life Assur. Soc., 35 St. Vincent-pl., Glasgow.</i>	1883	<sup>1</sup> Tregaskis, George Alfred, <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>
1902	<sup>2</sup> Spurgeon, Ernest Frank, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1894	<sup>2</sup> Trenerry, Charles Farley, B.A., F.S.S., <i>3 North-road, Clapham-park, S.W.</i>
1860	<sup>1</sup> Stark, James, <i>Reversionary Interest Society, 30 Coleman-street, E.C.</i>	1869	<sup>2</sup> Trew, Edward Bellingham, <i>Law Life Assurance Society, 187 Fleet-street, E.C.</i>
1866	Stark, William Emery, F.S.S., <i>Chapel-walks, Manchester.</i>	1891	<sup>2</sup> Turnbull, A. D. Lindsay, C.A., F.F.A., <i>Commercial Union Assurance Co., 24, 25 &amp; 26 Cornhill, E.C.</i>
1878	Stevenson, Charles, <i>9 Albert-square, Manchester.</i>	1877	<sup>2</sup> Turpin, William Gibbs, <i>National Debt Office, Finsbury Pavement House, E.C.</i>
1880	Stock, Edward James, <i>National Mutual Life Assoc. of Australasia, Melbourne, Australia.</i>	1884	Vian, William Collett, <i>Railway Passengers' Assurance Company, 64 Cornhill, E.C.</i>
1896	<sup>2</sup> Stuckey, Jos. James, M.A., <i>Salisbury Chambers, 49a King William-street, Adelaide, South Australia.</i>	1884	Vineent, Frederiek James, F.S.S. <i>London, Edinburgh &amp; Glasgow Assurance Co., Ltd., Insurance-buildings, Farringdon-street, E.C.</i>
1869	Surenne, David John, F.F.A., <i>29 Inverleith-road, Edinburgh.</i>	1899	<sup>2</sup> Vokins, George Alfred, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1899	<sup>2</sup> Symmons, Frank Percy, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1883	<sup>2</sup> Walker, Davidson, F.F.A., <i>Norwich Union Life Assurance Society, Norwich.</i>
1882	Tarn, Walter George, <i>Reversionary Interest Society, 30 Coleman Street, E.C.</i>	1879	<sup>1</sup> Wall, Walter George, <i>3 Shrewsbury-road, Birkenhead.</i>
1893	<sup>2</sup> Taylor, Arthur, <i>Westminster and General Life Assurance Assoc., 28 King-street, Covent-garden, W.C.</i>		

## ASSOCIATES.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming an Associate.		Date of becoming an Associate.	
1878	Walton, William Gandy, F.F.A., <i>Scottish Provident Institution,</i> <i>6 St. Andrew-square, Edinburgh.</i>	1870	<sup>2</sup> Wilson, Henry Edward (AUDITOR), <i>Northern Ass. Co., 1 Moorgate-street, E.C.</i>
1902	<sup>2</sup> Wandless, John Robert, <i>Northumbria, Hockley, Essex.</i>	1901	<sup>2</sup> Wilson, John Sydney, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1902	<sup>2</sup> Wares, Harold Wallace, <i>Yorkshire Insurance Company, York.</i>	1873	<sup>2</sup> Windett, Charles, <i>Legal &amp; General Life Assur. Soc., 10 Fleet-street, E.C.</i>
1862	<sup>1</sup> Waterhouse, Edwin, M.A., F.C.A., F.S.S., <i>3 Frederick's-place, Old Jewry, E.C.</i>	1898	<sup>3</sup> Wood, Arthur Barton, B.A., Asso. Act. Soc. Amer., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1883	<sup>2</sup> Watson, John Robertson, <i>British Law Fire Insurance Co., 176 West George-st., Glasgow.</i>	1883	Woodhouse, Lister, A.C.A., F.S.S., <i>City Comptroller, City-hall, Westminster, S.W.</i>
1894	<sup>2</sup> Watt, George, <i>Royal Insurance Co., Liverpool.</i>	1877	<sup>2</sup> Woods, Arthur Biddle, <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>
1900	Watt, James, F.F.A., <i>28 Charlotte-square, Edinburgh.</i>	1866	Woods, Bernard (AUDITOR), <i>Metropolitan Life Assur. Soc., 13 Moorgate-street, E.C.</i>
1883	<sup>1</sup> Weall, Bertram, <i>16 Waldegrave-park, Twickenham.</i>	1875	Woods, Edward, <i>Victoria Life and General Insur. Co., Market-street, Collins-street-west, Melbourne, Australia.</i>
1902	<sup>2</sup> Weatherill, Charles, <i>The Scottish Office, Whitehall, S.W.</i>	1897	<sup>2</sup> Woolfe, Archibald William, B.A., <i>42 Church-crescent, Muswell-hill, N.</i>
1899	<sup>3</sup> Weatherill, Henry, <i>National Debt Office, Finsbury Pavement House, E.C.</i>	1879	<sup>1</sup> Wornum, Thornton Selden, <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>
1894	Weeks, Rufus Wells, Mem. Act. Soc. Amer., <i>New York Life Insurance Co., 346 &amp; 348 Broadway, New York.</i>	1893	<sup>2</sup> Wright, Robert Young Murray M.A., <i>Royal Insurance Co., 45 Dame-street, Dublin.</i>
1898	<sup>3</sup> Whigham, Charles Frederick, F.F.A., <i>Messrs. Moncrieff &amp; Horsburgh, 46 Castle-street, Edinburgh.</i>	1871	Yardley, John, <i>Prudential Assurance Company Holborn-bars, E.C.</i>
1884	Whyte, Alexander, <i>c/o Messrs. Lever Bros., Ltd., Port Sunlight, Cheshire.</i>	1873	Young, Alexander Hunter, <i>60 Market-street, Melbourne, Australia.</i>
1897	<sup>2</sup> Wickens, Charles H., <i>Registrar-General's Office, Perth, W. Australia.</i>	1900	<sup>3</sup> Young, Arthur Stanley, <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1896	<sup>2</sup> Wilkinson, Edward Berkeley, <i>12 Highlever-road, N. Kensington, W.</i>		
1900	<sup>2</sup> Wilson, George, <i>Standard Life Assurance Company, Edinburgh.</i>		



## STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming a Student.		Date of becoming a Student.	
1892	<sup>1</sup> Aaron, David Hyam, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1896	<sup>1</sup> Barry, David, <i>Royal Commission on the University of Melbourne, Supreme Court Library, Melbourne, Australia.</i>
1894	<sup>1</sup> Anderson, Adam Thomson, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1900	<sup>1</sup> Baxter, Edwin Herbert, <i>Scottish Provident Institution, 17 King William-street, E.C.</i>
1886	Arnold, Thomas, Jr., <i>British Equitable Life Assurance Company, Queen-street-place, E.C.</i>	1901	<sup>1</sup> Beddall, H. Muir, <i>Ocean Accident and Guarantee Corporation, 11 Pall Mall, S.W.</i>
1896	<sup>1</sup> Ashley, Charles Henry, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1900	<sup>1</sup> Bell, Henry Soady, <i>"Netherhall," The Drive, Sidcup, Kent.</i>
1902	<sup>1</sup> Askwith, Thomas Nowell, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1898	<sup>1</sup> Bennell, Samuel Thomas, <i>20 Narford-road, Brooke-road, Clapton, N.E.</i>
1901	<sup>1</sup> Atkins, Leonard George, <i>Law Union &amp; Crown Insurance Co., 126 Chancery-lane, W.C.</i>	1898	<sup>1</sup> Bennett, Samuel, <i>National Deposit Friendly Soc., 11 Red Lion-square, Holborn, W.C.</i>
1899	<sup>1</sup> Baber, Walter Crosbie, <i>Royal Victoria Life Insurance Co. of Canada, Montreal.</i>	1902	<sup>1</sup> Beresford, Cecil, <i>Prudential Assurance Company, Holborn Bars, E.C.</i>
1897	<sup>1</sup> Backett, William Albert, <i>London and Lancashire Fire Insurance Company, 73-76 King William-street, E.C.</i>	1902	<sup>1</sup> Biden, Norman Fredk., <i>Standard Life Association, Sydney, Australia.</i>
1902	Baguley, John Minty, B.A., <i>Rose Lyne, Fulham-palace-road, S.W.</i>	1895	<sup>1</sup> Bigby, Robert Frederick Mitchell, <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1896	<sup>2</sup> Ball, Sidney Robertson, <i>English and Scottish Law Life Assurance Association, 12 Waterloo-place, S.W.</i>	1900	<sup>1</sup> Bingeman, Milton H., <i>Great West Life Assurance Company, Winnipeg, Manitoba, Canada.</i>
1897	Barfield, Edmund John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1891	<sup>1</sup> Bird, Edward William, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1899	Barnett, Isaac, <i>North British and Mercantile Insee. Co., 61 Threadneedle-street, E.C.</i>	1898	<sup>1</sup> Bishop, Harold Garfield, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1899	Barrett, William Goodsman, <i>United Kingdom Temperance and General Provident Institution, 1 Adelaide-place, London-bridge, E.C.</i>	1898	<sup>1</sup> Blake, Frederick Edward, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
		1901	<sup>1</sup> Blake, Francis Seymour, <i>62 Oakhurst-grove, East Dulwich, S.E.</i>

## STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming a Student		Date of becoming a Student	
1898	<sup>1</sup> Blake, Henry Prince, <i>Union Assurance Society, 81 Cornhill, E.C.</i>	1894	<sup>1</sup> Brough, Frank, <i>Federal Life Assurance Company, Hamilton, Ontario.</i>
1895	Blanch, Frederick William, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1891	<sup>1</sup> Brown, William Heron, <i>Gresham Life Assurance Society, Limited, St. Mildred's-house, Poultry, E.C.</i>
1902	Blanchard, Norman, B.A., <i>Equity &amp; Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>	1889	Buckle, Frederick, <i>Aberfoyle, Valkyrie-road, West-cliff-on-Sea, Essex.</i>
1901	<sup>1</sup> Blehl, Ernest M., A.M., <i>Penn Mutual Life Insurance Co., 923 Chestnut-street, Philadelphia, U.S.A.</i>	1901	<sup>1</sup> Caldwell, Richard H., <i>North British &amp; Mercantile Insurance Co., Birmingham.</i>
1887	Blossom, James, <i>186 South-view-road, Sheffield.</i>	1902	<sup>1</sup> Capon, Geoffrey William, <i>Norwich Union Life Insurance Society, Norwich.</i>
1892	<sup>1</sup> Boddy, Henry Mitchell, <i>Manufacturers' Life Insurance Co., Toronto, Canada.</i>	1901	<sup>2</sup> Carter, George Stanley, <i>Life Association of Scotland, 5 Lombard-street, E.C.</i>
1897	Bond, Frederic D., <i>122 South 39th Street, Philadelphia, U.S.A.</i>	1899	<sup>2</sup> Carter, Norman John, <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>
1900	<sup>1</sup> Borrajo, Edward Joseph William, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1900	<sup>1</sup> Chambers, John Joseph, <i>100 Linaker-street, Southport.</i>
1902	<sup>1</sup> Bowerman, Judah Philip, <i>Provident Life and Trust Co. of Philadelphia, 149 Broadway, New York, U.S.A.</i>	1902	<sup>1</sup> Chandler, Frederick Joseph, <i>Eagle Insurance Co., 79 Pall-mall, S.W.</i>
1897	<sup>1</sup> Bowles, Francis Marsh, <i>Pearl Life Assurance Company, London-bridge, E.C.</i>	1901	<sup>1</sup> Chubb, William, <i>Sun Life Assurance Company of Canada, Montréal, Canada.</i>
1891	<sup>1</sup> Boyd, Henry Norris, <i>City of Glasgow Life Assurance Co., 21 St. Andrew-square, Edinburgh.</i>	1901	<sup>1</sup> Clarke, Eustace Edgar, <i>British Empire Mutual Life Assurance Co., 4 &amp; 5 King William-street, E.C.</i>
1899	<sup>1</sup> Brady, John Francis, <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1897	<sup>1</sup> Clinton, George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1897	<sup>1</sup> Brierley, William Ernest, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1902	<sup>1</sup> Clinton, Louis Ernest, <i>15 Devereux-road, New Wandsworth, S.W.</i>
1893	<sup>1</sup> Briggs, Frederick William, <i>Carlton-villa, Wood-green, N.</i>	1902	<sup>1</sup> Coates, Frederick George, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>
1902	<sup>1</sup> Brook, Frank A., <i>Refuge Assurance Co., Oxford-street, Manchester.</i>	1901	<sup>1</sup> Cockerton, John Leonard, <i>Pioneer Life Assurance Co., Ltd., 11 Dale-street, Liverpool.</i>

## STUDENTS.

*Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.*

Date of becoming a Student.		Date of becoming a Student.	
1895	Cogar, William Edward, <i>New York Life Insurance Co., Trafalgar-square, W.C.</i>	1887	<sup>1</sup> Cross, Henry John, <i>3 Park-rd., Wandsworth-common, S.W.</i>
1898	<sup>1</sup> Collier, Charles Aubrey, <i>46 Crockerton-road, Tooting, S.W.</i>	1897	<sup>1</sup> Crump, Percy C., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1895	<sup>3</sup> Collins, Frank Lakeman, <i>Clerical, Medical &amp; General Life Assurance Soc., 15 St. James's- square, S.W.</i>	1897	<sup>1</sup> Dalton, John, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>
1899	<sup>1</sup> Collins, Patrick A., <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1902	Daman, Gerald William, B.A., <i>Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.</i>
1902	<sup>1</sup> Collins, William Ernest, <i>61 Osmond-terrace, Norwood, South Australia.</i>	1891	Daniell, Ferrers Aitken, <i>Royal Exchange Assurance Cor- poration, Royal Exchange, E.C.</i>
1896	<sup>1</sup> Cook, Henry Milton, <i>Standard Life Assurance Com- pany, Dalhousie-square, Calcutta, India.</i>	1889	<sup>1</sup> Davies, Hugh Myddelton, <i>Royal Insurance Co., Liverpool.</i>
1900	<sup>1</sup> Cooper, Bernard Hugh, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	Davies, T. Ridler, B.A., <i>1143 Dorchester-street, Montreal, Canada.</i>
1902	<sup>1</sup> Corbett, Archibald Gladstone, <i>Australian Mutual Provident Society, Collins-st., Melbourne, Australia.</i>	1900	<sup>1</sup> Davies, William Allison, <i>33 Newland Drive, Wallasey- road, Liscard, Cheshire.</i>
1899	<sup>1</sup> Cotterill, William Ernest, <i>Mutual Life Assoc. of Austra- lasia, Ltd., Sydney, Australia.</i>	1899	<sup>1</sup> Davison, Horace Williams, <i>17 Torrington-square, W.C.</i>
1897	<sup>2</sup> Court, Alexander George Dacus, <i>Coningsby, Hyde-vale, Green- wich, S.E.</i>	1891	<sup>1</sup> Dawson, Frank Aubrey, <i>Ecclesiastical Insurance Office, Limited, 11 Norfolk-street, Strand, W.C.</i>
1901	<sup>1</sup> Coventry, Cameron H., <i>Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.</i>	1902	<sup>1</sup> Deck, James Gilbert, <i>National Provident Institution 48 Gracechurch-street, E.C.</i>
1894	Cox, Edward William, <i>Canada Life Assurance Co., Toronto, Canada.</i>	1902	<sup>1</sup> Denmark, Robert John, <i>Norwich Union Life Insurance Society, Norwich.</i>
1894	Cox, Herbert Coplin, <i>Canada Life Assurance Co., Toronto, Canada.</i>	1901	<sup>1</sup> Dent, Ernest Edward, <i>London and Lancashire Life Assurance Company, 66 &amp; 67 Cornhill, E.C.</i>
		1896	<sup>1</sup> de Ville, Francis, <i>Clergy Pensions Institution, 11 Norfolk-street, Strand, W.C.</i>



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Date of becoming a Student.		Date of becoming a Student.	
1897	<sup>1</sup> Dick, William Thos., B.A., M.L.A., <i>Newcastle, N.S.W.</i>	1892	<sup>1</sup> Farrell, John, <i>Citizens' Life Assur. Co., 210 Queen-st., Brisbane, Australia.</i>
1890	<sup>1</sup> Doeker, Leslie, <i>North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>	1902	<sup>1</sup> Farrow, Alfred Ellis, <i>Yorkshire Insurance Co., York.</i>
1897	<sup>1</sup> Dorrian, John Christopher, <i>Citizens' Life Assurance Com- pany, Sydney, Australia.</i>	1901	<sup>1</sup> Ferguson, Colin C., B.A., <i>Canada Life Assurance Co. Toronto, Canada.</i>
1899	<sup>1</sup> Douglas, J. Joseph, <i>Victoria-place, Blackrock, Co. Dublin.</i>	1901	<sup>1</sup> Fielder, William Crowhurst, <i>National Mutual Life Assurance Society, 39 King-street, Cheap- side, E.C.</i>
1900	<sup>1</sup> Doust, William Frederick, <i>National Debt Office, Finsbury- pavement-house, E.C.</i>	1887	Fisher, Hugh Strettell, <i>1 Avoca-terrace, Blackrock, co. Dublin.</i>
1901	<sup>2</sup> Downes, Sidney Cecil, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	<sup>1</sup> Fisher, John William, B.A., <i>Crown Life Insurance Co., Toronto, Canada.</i>
1897	<sup>1</sup> Eeroyd, Cuthbert W., <i>Friends' Provident Institution, Bradford.</i>	1896	<sup>1</sup> Fisk, George William Victor, F.S.S., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1892	<sup>1</sup> Edwards, Edward Samuel, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1901	<sup>1</sup> Franklin, Herbert Dare, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1902	<sup>1</sup> Edwards, Thomas Baker, <i>Comptroller's Dept., London County Council, Spring-gardens, S.W.</i>	1902	Frater, George Douglas, B.A., B.Sc., <i>22 Canonbury-villas, N.</i>
1892	<sup>1</sup> Eedy, Arthur Malcolm, <i>Citizens' Life Assurance Com- pany, Sydney, Australia.</i>	1890	Gamman, Robert Ebenezer, <i>London Joint Stock Bank, Princes-street, E.C.</i>
1901	<sup>1</sup> Eggleton, Harold Edward, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1886	Garcke, Emile, F.S.S., M.I.E.E., <i>Witton house, near Maidenhead.</i>
1900	<sup>1</sup> Elderton, Robert Lapidge, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1900	<sup>1</sup> Garner, James, <i>9 Arlington-gardens, Chiswick, W.</i>
1902	<sup>1</sup> Ellis, Reginald George Gregson, <i>12 Manson-pl., Queen's-gate, W.</i>	1901	<sup>1</sup> Gemmill, William, <i>7 Royal-terrace, Queen's-park, Glasgow.</i>
1893	<sup>1</sup> Emery, John M., <i>American Union Life Insurance Co., Bowling Green-building, Broadway, New York.</i>	1901	Gerrish, Frank Wilfred, <i>158 Brecknock-road, Camden- road, N.</i>
		1899	<sup>1</sup> Giles, Hylton Lloyd, <i>British Empire Mutual Life Assurance Co., 4 &amp; 5 King William-street, E.C.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1895	<sup>1</sup> Gill, James Stewart, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>	1897	<sup>2</sup> Gosset, Thorold, <i>21 Old-bldgs., Lincoln's-inn, W.C.</i>
1900	<sup>1</sup> Gillespie, Joseph Hugh Ross, M.A., <i>Manufacturers' &amp; Temperance &amp; General Life Assurance Co., Toronto, Canada.</i>	1902	<sup>1</sup> Gould, W. H., M.A., <i>Canada Life Assurance Co., Toronto, Canada.</i>
1901	<sup>1</sup> Glassford, David Murray, <i>Mutual Life Association of Australasia, Sydney, Australia.</i>	1886	Gover, Frederick Field, F.S.S., <i>10 Lee-park, Blackheath, S.E.</i>
1893	Glasson, George Cornish, <i>Economic Life Assurance Soc., 4 St. Stephen's-chbrs., Baldwin-street, Bristol.</i>	1895	<sup>1</sup> Grant, Kenneth Stuart, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1902	<sup>1</sup> Gleave, Charles Sheldon, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>	1900	<sup>1</sup> Green, James Proctor, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1893	<sup>1</sup> Gledstone, W. L., <i>Royal Exchange Assur. Corporation, Royal Exchange, E.C.</i>	1886	Greening, Herbert Joseph, <i>Abstainers &amp; General Insur. Co., City-buildings, Birmingham.</i>
1902	<sup>1</sup> Godsill, Richard Collis, <i>Liverpool Victoria Legal Friendly Soc., 18 St. Andrew-street, E.C.</i>	1899	<sup>1</sup> Grigg, Benjamin, <i>Sun Life Assur. Co. of Canada, Montreal, Canada.</i>
1894	<sup>1</sup> Golding, Arthur, <i>41 Digby-rd., Finsbury-park, N.</i>	1902	<sup>1</sup> Gysin, Howard, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1888	<sup>1</sup> Gooding, Harold John, <i>Law Guarantee and Trust Soc., Ltd., 56 Moorgate-street, E.C.</i>	1901	<sup>1</sup> Hall, Arthur F., <i>North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.</i>
1900	<sup>1</sup> Goodman, Gilbert, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1900	<sup>1</sup> Hall, John Bertram, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>
1892	Gordon, Alexander, <i>168 Islington, Liverpool.</i>	1902	Hallett, William Sebastian, B.A., <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>
1896	<sup>1</sup> Gordon, Harry Duncan Lockhart, <i>221 George-st., Toronto, Canada.</i>	1896	<sup>1</sup> Hallman, M. S., <i>The Mutual Life Assurance Co. of Canada, Waterloo, Ontario.</i>
1902	<sup>1</sup> Gordon, Walter Hamilton, <i>45, Braydon-road, Stamford Hill, N.</i>	1899	<sup>1</sup> Halloran, George Henry, <i>20 Castlereagh-street, Sydney, Australia.</i>
1902	<sup>1</sup> Gorham, Edwin Arthur, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1901	<sup>1</sup> Hamilton, George Powell, <i>North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.</i>

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1902	<sup>1</sup> Hammant, Francis Clive, <i>Prudential Assurance Company, Holborn Bars, E.C.</i>	1901	<sup>1</sup> Hilary, Henry Jephson, <i>Dryhill-park, Tonbridge.</i>
1900	<sup>1</sup> Hammond, Reginald, <i>British Equitable Life Assur. Co., Queen-street-place, E.C.</i>	1896	<sup>2</sup> Hines, Walter Robert, <i>Norwich Union Life Office, Norwich.</i>
1892	Hancock, Arthur Tom, <i>Clerical, Medical &amp; General Life Assurance Society, 15 St. James's- square, S.W.</i>	1897	Hitchins, William Richmond, B.A., <i>Manufacturers' Life Insurance Company, Toronto.</i>
1902	<sup>1</sup> Hardy, Reginald Herbert, <i>32 Highfield-street, Leicester.</i>	1902	<sup>1</sup> Hodge, Cecil Wilfred, <i>Star Life Assurance Society, 32, Moorgate-street, E.C.</i>
1901	<sup>1</sup> Harpell, James John, B.A., <i>North American Life Assurance Co., North American Life Building, 112-118 King-street West, Toronto, Canada.</i>	1896	<sup>1</sup> Hogg, Charles, <i>10 Whitehall-place, S.W.</i>
1901	<sup>1</sup> Harper, Henry, <i>96 Byron-road, Birmingham.</i>	1894	Holdsworth, David Arundell, <i>Star Life Assurance Society, 22 Eldon-square, Newcastle-on- Tyne.</i>
1895	<sup>1</sup> Harper, Sidney, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1898	<sup>2</sup> Hooper, George Duncan, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1889	<sup>1</sup> Harris, Henry, <i>Friends' Provident Institution, Bradford.</i>	1895	<sup>2</sup> Horn, Ernest Frederick, <i>Clerical, Medical and General Life Assurance Society, Mansion- house-buildings, E.C.</i>
1897	<sup>1</sup> Harriss, Walter James, <i>Life Association of Scotland, 5 Lombard-street, E.C.</i>	1902	<sup>1</sup> Houston, Charles Cornelius, <i>Metropolitan Asylums Board, Victoria-embankment, E.C.</i>
1896	Haskins, George Frederick, A.C.A., <i>18 Walbrook, E.C.</i>	1901	<sup>1</sup> Howell, Archibald Rennie, B.A., <i>Royal Insur. Co. of England, Montreal, Canada.</i>
1894	<sup>1</sup> Hatten, David Leslie, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>	1898	Hughes, Arthur J., <i>Crown Life Insurance Co., Toronto, Canada.</i>
1897	<sup>1</sup> Hay, John Dalziel, <i>Crown Lands Office, Wellington, New Zealand.</i>	1900	<sup>1</sup> Hughes, Arthur Sidney, <i>6 Telford-avenue, Streatham- hill, S.W.</i>
1892	Hellyer, Arthur Lee, <i>Shannon-court, Bristol.</i>	1902	<sup>1</sup> Hughes, Charles, <i>Insurance Department of the State of Connecticut, Hartford, Conn., U.S.A.</i>
1897	<sup>1</sup> Hepburn, Charles James, <i>Clerical, Medical and General Life Assurance Society, Mansion House-buildings, E.C.</i>	1902	<sup>1</sup> Hugill, Herbert, <i>19 Barlow-terrace, Keighley.</i>
1891	Higinbotham, Harry Newburgh, <i>Royal Exchange Assur. Corpora- tion, Royal Exchange, E.C.</i>	1897	<sup>1</sup> Humphrey, Bernard, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>



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Date of becoming a Student.		Date of becoming a Student.	
1902	<sup>1</sup> Humphreys, Charles Alwyn, <i>Century Insurance Co., Limited,</i> 35 Moorgate-street, E.C.	1899	<sup>1</sup> Jones, Leonard Alexander Mouat, <i>Hand-in-Hand Insur. Society,</i> 26 New-bridge-street, E.C.
1901	<sup>1</sup> Humphreys, Henry Thompson, <i>Sun Life Assurance Society,</i> 63 Threadneedle-street, E.C.	1896	<sup>1</sup> Jones, Richard Foxley, <i>Refuge Assurance Co., Oxford-</i> <i>street, Manchester.</i>
1902	<sup>1</sup> Humphreys, John A., 9 Court-rd., West Norwood, S.E.	1896	<sup>1</sup> Jones, Wallace Mouat, <i>General Reversionary &amp; Invest-</i> <i>ment Company, Limited, 26 Pall-</i> <i>mall, S.W.</i>
1902	<sup>1</sup> Humphry, Edmund William, <i>Life Association of Scotland,</i> 5 Lombard-street, E.C.	1893	<sup>2</sup> Kelham, Cyril Stephen, <i>Prudential Assurance Company</i> <i>Holborn-bars, E.C.</i>
1891	Hunt, Arthur Leonard, <i>Wesleyan and General Assur.</i> <i>Society, 101 Finsbury-pavement,</i> E.C.	1902	<sup>1</sup> Kemper, J. M. de Bosch, <i>Sun Life Assur. Co. of Canada,</i> <i>Montreal, Canada.</i>
1899	<sup>1</sup> Jackaman, Arthur Fredk. Samuel, <i>National Mutual Life Assur.</i> <i>Soc., 39 King-street, Cheapside,</i> E.C.	1898	Kidson, Leonard Douglas, "Oaklands," 1 Morley-road, Southport.
1902	Jackson, Charles William, M.A., 267 University-street, Montreal, Canada.	1894	<sup>2</sup> Kingsbury, James William, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>
1902	<sup>1</sup> Jackson, Herbert Moore, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>	1900	<sup>1</sup> Kirkham, Alfred, 229 Chapel-street, Prahran, Victoria, Australia.
1890	<sup>2</sup> Jackson, Samuel, <i>Scottish Widows' Fund Life</i> <i>Assurance Society, Liverpool.</i>	1899	<sup>1</sup> Kissan, Edgar Duguid, <i>Atlas Assurance Company, 92</i> <i>Cheapside, E.C.</i>
1900	<sup>2</sup> Jarman, William Rees, B.A., <i>National Debt Office, Finsbury</i> <i>Pavement House, E.C.</i>	1895	<sup>1</sup> Knight, Alfred Murray, <i>Bank-house, Chapel-st., Devon-</i> <i>port.</i>
1895	<sup>1</sup> Jenkyn, John, <i>Squirrel's-heath, Romford,</i> Essex.	1895	Laing, Oswald George, <i>North British and Mercantile</i> <i>Insurance Co., Park-row, Leeds.</i>
1896	<sup>1</sup> Jepps, John Blacklee, <i>English and Scottish Law Life</i> <i>Assurance Assoc., 12 Waterloo-</i> <i>place, S.W.</i>	1902	<sup>1</sup> Lancaster, Cyril Henry, 154 Westbourne-terrace, Hyde- park, W.
1898	<sup>1</sup> Johnston, Arthur Edward, 3 Cumnor-road, Sutton.	1902	<sup>1</sup> Lang, Frederiek John, <i>Royal London Friendly Society,</i> 6 Paul-street, Finsbury, E.C.
1902	<sup>1</sup> Jones, Edward Furnival, A.S.A.A., <i>Hearts of Oak Benefit Society,</i> 17 Charlotte-street, W.		

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Date of becoming a Student		Date of becoming a Student.	
1902	<sup>1</sup> Langstaff, James Miles, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1902	<sup>1</sup> McKellar, John A., <i>c/o Miles M. Dawson, Esq., 11 Broadway, New York, U.S.A.</i>
1901	<sup>1</sup> Latham, Bertrand, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1895	McLeod, James Stirling, <i>c/o Messrs. Dalgety &amp; Co., Ltd., Napier, New Zealand.</i>
1891	Layzell, Phillip Cuddington, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	<sup>1</sup> Macmillan, Alexander, <i>Westfield-house, Westgate-hill, Bradford.</i>
1901	<sup>1</sup> Leigh, Samuel George, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>	1902	<sup>1</sup> Macorquodale, F. D., <i>61, Shannon-street, Toronto, Canada.</i>
1894	Leonard, Maurice, <i>14 Sotheby-road, Highbury, N.</i>	1897	<sup>1</sup> McPhail, Frederick Charles, <i>Colonial Mutual Life Assurance Society, Limited, Melbourne, Australia.</i>
1896	<sup>1</sup> Ley, James, <i>Office of the Actuary for Friendly Societies, Melbourne, Australia.</i>	1893	<sup>1</sup> Martin, William Anderson, M.A., <i>Scottish Provident Institution, Dublin.</i>
1889	<sup>1</sup> Lighton, Harold John, <i>Law Union &amp; Crown Insurance Co., 126 Chancery-lane, W.C.</i>	1902	<sup>1</sup> Maudling, Reginald G., <i>London and Lancashire Life Insur. Co., 66 &amp; 67 Cornhill, E.C.</i>
1895	<sup>1</sup> Littell, Lewis Lloyd, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>	1900	<sup>1</sup> May, Walter Thomas, <i>Scottish Amicable Life Assurance Society, 1 Threadneedle-st., E.C.</i>
1890	Love, Robert, <i>Pelican Life Insurance Company, 70 Lombard-street, E.C.</i>	1895	<sup>1</sup> Mayhew, Percy Craske, <i>Westminster and General Life Assurance Assoc., 28 King-st., Covent-garden, W.C.</i>
1894	Lucey, Frederick Samuel, F.C.A., <i>15 George-street, Mansion-house, E.C.</i>	1890	<sup>1</sup> Meikle, Henry George Watson, F.F.A., <i>Oriental Government Security Life Assurance Co., Limited, Bombay.</i>
1888	<sup>1</sup> McConway, James Robert, <i>Royal Insurance Company, Liverpool.</i>	1897	<sup>1</sup> Melville, Charles Edward, <i>Australasian Temperance and General Mutual Life Assurance Society, 62 Lydiard-street, Ballarat, Victoria, Australia.</i>
1902	<sup>1</sup> Macfarlane, James Allan, <i>North American Life Assurance Co., North American Life Building, 112-118 King-street West, Toronto, Canada.</i>	1901	<sup>1</sup> Melville, Henry Edward, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>
1902	<sup>1</sup> McGee, Cyril H. <i>Box 981, St. Thomas, Ontario, Canada.</i>		

## STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of  
becoming  
a Student.

- 1892 <sup>1</sup> Meyers, Henry Wilson,  
*National Mutual Life Association of Australasia, Melbourne, Australia.*
- 1894 <sup>1</sup> Mills, Thomas Percy,  
*Mutual Life Association of Australasia, Wellington, New Zealand.*
- 1899 <sup>2</sup> Minns, Ernest Edwin,  
*Norwich Union Life Insurance Society, Norwich.*
- 1897 <sup>1</sup> Mirams, Arthur Greyford,  
*Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.*
- 1898 <sup>1</sup> Moore, George Cecil,  
*Imperial Life Insurance Co. of Canada, Toronto, Canada.*
- 1900 <sup>1</sup> Moore, George Edward,  
*Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*
- 1895 <sup>1</sup> Moore, Gerald Leslie, A.C.A.,  
1 *Rosebery-gardens, Muswell-hill, N.*
- 1902 <sup>1</sup> Moore, Hubert Fred,  
*London Assurance Corporation, 7 Royal Exchange, E.C.*
- 1898 <sup>1</sup> Moore, Stanley,  
*Prudential Assurance Company, Holborn-bars, E.C.*
- 1895 <sup>1</sup> Morgan, George Frederick Hughes,  
66 *Grafton-road, Acton, W.*
- 1902 <sup>1</sup> Morton, Francis,  
*Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.*
- 1902 <sup>1</sup> Muckle, Charles Park,  
*Excelsior Life Insurance Co., Toronto, Canada.*
- 1902 <sup>1</sup> Mullin, Alexander, B.A.,  
76 *Major-st., Toronto, Canada.*

Date of  
becoming  
a Student.

- 1896 <sup>1</sup> Neale, Maurice Baldwin,  
*Alliance Assurance Company, Ltd., 61 New-street, Birmingham*
- 1900 <sup>2</sup> Neill, Samuel Bennett,  
*London, Edinburgh & Glasgow Assurance Co., Ltd., Insurance-buildings, Farringdon-st., E.C.*
- 1895 <sup>1</sup> Newnham, Ernest Whiffin,  
*Prudential Assurance Company, Holborn-bars, E.C.*
- 1902 <sup>1</sup> Nicholls, Arthur William,  
*Australian Mutual Provident Society, Brisbane, Australia.*
- 1900 <sup>1</sup> Norsworthy, Edward C.,  
*The Dominion Securities Corporation, Limited, Canada Life Building, Montreal, Canada.*
- 1900 <sup>1</sup> Norsworthy, Stanley Counter,  
*Bank of Montreal, St. John's, Newfoundland.*
- 1901 <sup>1</sup> Nugent, James,  
*Cornwall, Ontario, Canada.*
- 1902 <sup>1</sup> O'Connor, William, M.A., M.D.,  
*Toronto Technical School, Toronto, Canada.*
- 1892 <sup>1</sup> O'Reilly, Anthony James,  
*Government Insurance Department, Ottawa, Canada.*
- 1897 <sup>1</sup> Osborn, Nathaniel Banner Francis,  
11 *Bruce-grove, Tottenham, N.*
- 1893 <sup>1</sup> Owen, Edgar Theodore, F.S.S.,  
*Registrar of Friendly Societies, Perth, Western Australia.*
- 1901 <sup>1</sup> Papworth, Frederick William,  
*The Admiralty, S.W.*
- 1895 <sup>1</sup> Pascoe, William Yeoman Bennett,  
*Prudential Assurance Company, Holborn-bars, E.C.*
- 1901 <sup>1</sup> Paton, Albert George,  
115 *Grosvenor-road, Highbury-new-park, N.*



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Date of becoming a Student.		Date of becoming a Student.	
1897	<sup>1</sup> Paton, Harry Arthur, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>	1894	<sup>2</sup> Rietschel, Hermann Julius, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1896	<sup>2</sup> Penny, Charles Augustus, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902	<sup>1</sup> Robertson, Aubrey Charles, <i>London Assurance Corporation, 7, Royal Exchange, E.C.</i>
1901	<sup>1</sup> Petter, Herbert, <i>British Workman's and General Assurance Co., Broad-street-corner, Birmingham.</i>	1901	<sup>1</sup> Robertson, A. W. L., <i>Guardian Assurance Co., 11 Lombard-street, E.C.</i>
1902	<sup>1</sup> Phillips, Tom Herbert Bartholomew, 2 Edith-road, W.	1898	Robertson, Douglas Gordon, <i>Essex-lodge, Muswell-hill, N.</i>
1898	Poort, Willem Anthonie, Phil. Nat. Doct., <i>Algemeene Friesche Levens-verzekerings Maatschappij Leeuwarden, Middleburg, Holland.</i>	1896	<sup>1</sup> Robinson, Frederick Charles, <i>Royal Exchange Assur. Corporation, Royal Exchange, E.C.</i>
1893	<sup>1</sup> Pownall, Herbert Wilfred, <i>Australian Mutual Provident Society, Adelaide, Australia.</i>	1893	<sup>1</sup> Roll, Frederick James, <i>Pearl Life Assurance Company, London-bridge, E.C.</i>
1897	Proctor, William, Jr., <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1893	<sup>1</sup> Roodenburch, Bartholomeus Adrianus, <i>Verzekeringsbank Victoria, 689, Prinsengracht, Amsterdam.</i>
1902	<sup>1</sup> Proud, Charles Winton, <i>c/o Messrs. Whiteheads &amp; Coles, 39 Throgmorton-street, E.C.</i>	1895	<sup>1</sup> Ross, Christopher Watson, <i>c/o Messrs. M. Moss &amp; Co., Flinder's-lane, Melbourne, Australia.</i>
1886	<sup>1</sup> Quick, John Richard, <i>Equity &amp; Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>	1901	<sup>1</sup> Rountree, Arthur FitzGerald, <i>The Vicarage, Werneth, Oldham.</i>
1901	<sup>1</sup> Ramsay, Cecil Byron, <i>Mutual Life Insurance Co. of New York, 16, 17 &amp; 18 Cornhill, E.C.</i>	1895	Rowley, James Edward, A.C.A., 7 Waterloo-street, Birmingham.
1898	<sup>1</sup> Reynell, Guy Courtenay, <i>National Mutual Life Assurance Society, 39 King-st., Cheapside, E.C.</i>	1895	<sup>1</sup> Rudd, Alfred James, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1894	<sup>1</sup> Richards, Gilbert P. A., <i>Oak Cottage, Bulwer-road, New Barnet.</i>	1899	<sup>1</sup> Rutter, Edward Valentine, 129 Tredegar-road, Bow, E.
		1894	Salter, George Ferry, Mem. Aet. Soc. Amer., 123 N. 16th-street, E. Orange, N.J., U.S.A.
		1902	<sup>1</sup> Sare, Thomas Henry, <i>Commercial Union Assur. Co., 24, 25 &amp; 26 Cornhill, E.C.</i>

## STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming a Student.		Date of becoming a Student.	
1894	<sup>1</sup> Saunders, Herbert Stewart, M.A., 3 Bolton-gardens, s.w.	1888	<sup>2</sup> Slimon, William James, F.F.A., 2 James'-place, Leith.
1892	<sup>1</sup> Savery, Robert S. B., <i>Gresham Life Assurance Society,</i> <i>Giselastrasse, No. 1, Vienna.</i>	1895	<sup>1</sup> Smeaton, John Richard, <i>Alliance Assurance Co., Ltd.,</i> <i>61 New-street, Birmingham.</i>
1897	<sup>1</sup> Scott, Alexander Lewis, <i>Australian Mutual Provident Society, Melbourne.</i>	1902	<sup>1</sup> Smith, Septimus Wontner, <i>Equitable Life Assurance Soc.,</i> <i>Mansion-house-street, E.C.</i>
1900	<sup>1</sup> Searle, Arthur Joseph, <i>English &amp; Scottish Law Life Assurance Association, Limited,</i> <i>12 Waterloo-place, s.w.</i>	1902	<sup>1</sup> Smither, Herbert Buxton, <i>University Life Assurance Soc.,</i> <i>25 Pall-mall, s.w.</i>
1888	Sewell, Richard, C.A., F.F.A., 63 Threadneedle-street, E.C.	1900	<sup>1</sup> Somerville, Walter Harold, <i>Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.</i>
1886	<sup>1</sup> Sharp, Joseph Benjamin, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, s.w.</i>	1897	<sup>2</sup> Stamp, Horatio E., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1896	Shawyer, John William, <i>Law Union &amp; Crown Insurance Co., 126 Chancery-lane, w.c.</i>	1901	<sup>1</sup> Steffensen, Johan F., "Nordisk" Re-insurance Co., 3 Romersgade, Copenhagen.
1900	<sup>1</sup> Shovelton, Sydney Taverner, B.A., 532 Eccles New-road, Manchester.	1898	<sup>2</sup> Stewart, Lionel William, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1902	<sup>1</sup> Shrubsole, Stanley Smith, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1886	<sup>2</sup> Stirling, James, <i>Scottish Imperial Insurance Co., 183 West George-st., Glasgow.</i>
1896	<sup>1</sup> Shute, Oxenham Bent, <i>National Provincial Bank of England, 53 Baker-street, w.</i>	1888	<sup>1</sup> Stott, Walter, <i>Royal Insurance Co., Liverpool.</i>
1895	Simmons, Lancelot, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1893	<sup>1</sup> Streeter, Theodore Edward, <i>Hampden House, Phoenix-street, King's-cross, N.W.</i>
1892	<sup>1</sup> Simpson, William Murray, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>	1902	<sup>1</sup> Strong, William Boughton, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1891	<sup>1</sup> Sindall, Alfred John, <i>London and Lancashire Life Assurance Co., 66 &amp; 67 Cornhill, E.C.</i>	1899	<sup>1</sup> Stuckey, Reginald Robert, <i>Australian Mutual Provident Society, Adelaide, S. Australia.</i>
1899	<sup>1</sup> Skelton, Reginald Albert, "Sunnymead," Elstree, Herts.	1902	<sup>1</sup> Sturt, Herbert Rothsay, c/o C. H. E. Rea, Esq., 3 & 4 Clement's-inn, Strand, w.c.

## STUDENTS.

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1902	<sup>1</sup> Sudell, Alfred Henry, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1901	<sup>1</sup> Tregaskis, George, <i>Sun Insur. Office, 40 Chancery-</i> <i>lane, W.C.</i>
1902	<sup>1</sup> Sugars, Robert Morrison, B.A., <i>223 Ashley-gardens, Westminster,</i> <i>S.W.</i>	1902	<sup>1</sup> Tully, Arthur Patrick Thomas, <i>The Mill-house, Wimbledon-</i> <i>common, S.W.</i>
1901	<sup>1</sup> Sutcliffe, Charles Ernest, <i>Hand-in-Hand Insurance Soc.,</i> <i>Manchester.</i>	1891	Tyler, Edgar Alfred, F.S.S., <i>9, Old Jewry-chambers, Bank,</i> <i>E.C.</i>
1895	<sup>1</sup> Sutton, Cecil Norman Stafford, <i>13 The Avenue, Barnes, Surrey.</i>	1902	Watherston, Charles F., B.A., <i>War Office, S.W.</i>
1895	Taylor, Leopold Victor, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1900	<sup>1</sup> Watt, Arthur W., <i>Sun Life Assur. Co. of Canada,</i> <i>Montreal, Canada.</i>
1901	<sup>1</sup> Taylor, L. W., <i>Collegiate Institute, Lindsay,</i> <i>Ontario, Canada.</i>	1898	<sup>1</sup> Webb, Lloyd, <i>Hand-in-Hand Insurance Soc.,</i> <i>26 New-bridge-street, E.C.</i>
1895	<sup>1</sup> Thistlethwaite, William, <i>4 Warren-terrace, Wakefield.</i>	1902	<sup>1</sup> Wellisch, Frederick, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>
1900	<sup>1</sup> Thomson, Frederick Robert T., <i>Kent house, Church-end, Finch-</i> <i>ley, N.</i>	1893	<sup>1</sup> Welman, Arthur Joseph, <i>Legal &amp; General Life Assurance</i> <i>Society, 10 Fleet-street, E.C.</i>
1902	<sup>1</sup> Thwaites, Frederick George, <i>Norwich Union Life Insurance</i> <i>Society, Norwich.</i>	1902	<sup>1</sup> White, Wilfred Clare, <i>Trinity College, Toronto, Canada.</i>
1897	<sup>1</sup> Tipping, Oswald, <i>Trustees', Executors', and Agency</i> <i>Co., Limited, 412 Collins-street,</i> <i>Melbourne, Victoria, Australia.</i>	1897	<sup>1</sup> Wigner, John Gurney, <i>92 Tyrwhitt-road, St. John's,</i> <i>S.E.</i>
1901	<sup>1</sup> Todhunter, Joseph, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1900	<sup>1</sup> Wilkinson, William Magnay, Jr., <i>Citizens' Life Assurance Co.,</i> <i>Sydney, Australia.</i>
1902	<sup>1</sup> Tope, Maurice William, <i>National Mutual Life Assurance</i> <i>Society, 39 King-street, Cheap-</i> <i>side, E.C.</i>	1886	<sup>1</sup> Williams, David, <i>181 Queen Victoria-street, E.C.</i>
1897	<sup>1</sup> Touzel, Philip Duncan, <i>Australian Mutual Provident</i> <i>Society, Melbourne, Australia.</i>	1894	<sup>1</sup> Williams, Frederick Alfred, <i>Hurstpierpoint, Hornchurch,</i> <i>Essex.</i>
1897	<sup>1</sup> Townshend, Edward Villiers, <i>North British and Mercantile</i> <i>Insurance Co., 7 Tithebarn-street,</i> <i>Liverpool.</i>	1895	<sup>1</sup> Williams, Henry Samuel Walter, <i>The Imperial Insur. Co., Ltd.,</i> <i>Wellington, New Zealand.</i>



## STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations for the Class of Fellow.

Date of becoming a Student.		Date of becoming a Student.	
1900	Williams, Lewis, B.A., <i>Hand-in-Hand Insurance Soc.,</i> 26 New-bridge-street, E.C.	1896	<sup>1</sup> Woodhouse, Hubert Allen, <i>Union Assurance Society,</i> 81 Cornhill, E.C.
1901	<sup>1</sup> Wilton, Herbert George, <i>Norwich Union Life Office,</i> Norwich.	1900	<sup>1</sup> Woolston, Paul Livingston, B.S., <i>New York Life Insurance Co.,</i> 100 Broadway, New York.
1894	<sup>1</sup> Windett, Sydney V., <i>Eagle Insurance Company,</i> 79 Pall-mall, S.W.	1900	<sup>1</sup> Worth, Bertram Oliver, <i>Clerical, Medical &amp; General</i> <i>Life Assurance Society,</i> 15 St. James's-square, S.W.
1899	<sup>1</sup> Winstanley, Charles William, <i>North British &amp; Mercantile</i> <i>Insurance Co.,</i> 8 Waterloo- place, S.W.	1894	<sup>1</sup> Wyatt, George Matthew, <i>Law Guarantee &amp; Trust Society,</i> 49 Chancery-lane, W.C.
1895	<sup>1</sup> Wood, David James, <i>Commercial Union Assurance</i> <i>Co.,</i> 24, 25 & 26 Cornhill, E.C.	1894	<sup>1</sup> Wylie, Samuel Brown, A.M., 112 N. Broad-st., Philadelphia, U.S.A.
1901	<sup>1</sup> Wood, Roland Stuart, <i>Liverpool and London and Globe</i> <i>Insurance Co.,</i> 7 Cornhill, E.C.	1886	Yeatman, Alexander Alfred, 2 Gresham-buildings, E.C.
1900	<sup>1</sup> Wood, William Archibald Porter, B.A., <i>Canada Life Assurance Co.,</i> Toronto, Canada.	1895	<sup>1</sup> Yeldham, William James, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
1902	<sup>1</sup> Woodhouse, David Alfred, <i>Refuge Assurance Co.,</i> Oxford- street, Manchester.	1897	<sup>1</sup> Younger, R. Hugh, <i>Hand-in-Hand Insurance Soc.,</i> 26 New-bridge-street, E.C.

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\* \* \* It is requested that any inaccuracy in the foregoing list may be pointed out to the ASSISTANT SECRETARY.

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Lijfrente, (Damrak, 74); Actuary  
of the "Nederlandsch Werklieden-  
denfonds," the Hague; Membre  
Correspondant de l'Association  
des Actuaires Belges; Membre  
Correspondant de l'Institut des  
Actuaires Français; de Genestet-  
straat, 17.*

## Hungary.

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*Manager and Actuary of the  
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viii. Baross-utca, 10.*

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*Actuary of "La Fondiaria" Life  
 Assurance Company.*

**Russia.****ST. PETERSBURG.**

M. Serge de Savitch,  
*Member of the Assurance Committee  
 of the Ministry of the Interior,  
 Nikolaevskaia, 35.*

**Spain.****MADRID.**

José Maluquer y Salvador,  
*Insurance Counsellor (Actuary) at  
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 pomanes, 10.*

**Sweden.****STOCKHOLM.**

M. Anders Lindstedt, Dr. Phil.,  
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 Director of the Polytechnicum;  
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 Co.; Member of the Royal Swedish  
 Academy of Science, etc.*

**Switzerland.****ZURICH.**

Herr Dr. Gottfried Schaertlin,  
*Direktor der Schweizerischen Lebens-  
 versicherungs-und Rentenanstalt.*

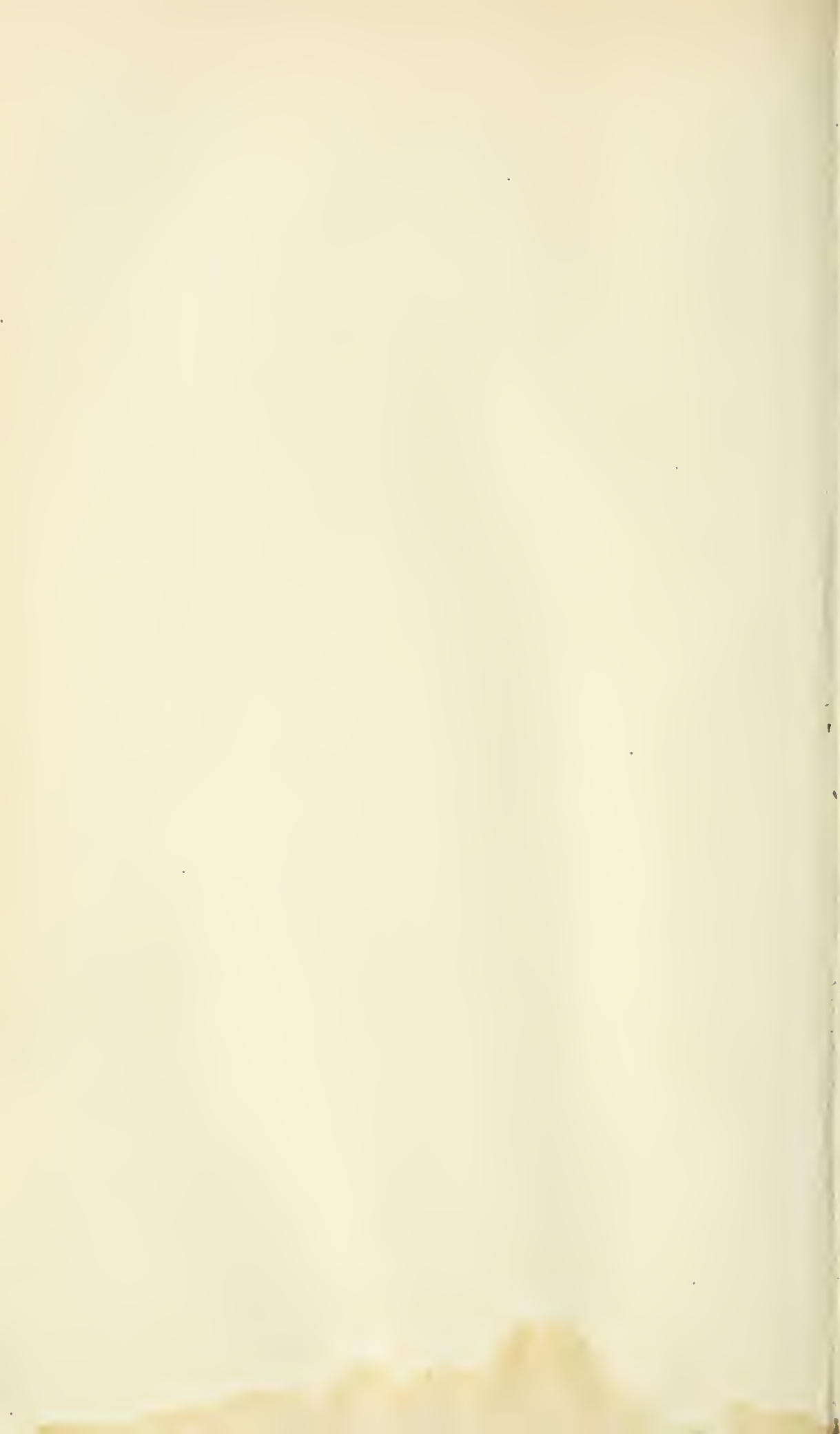
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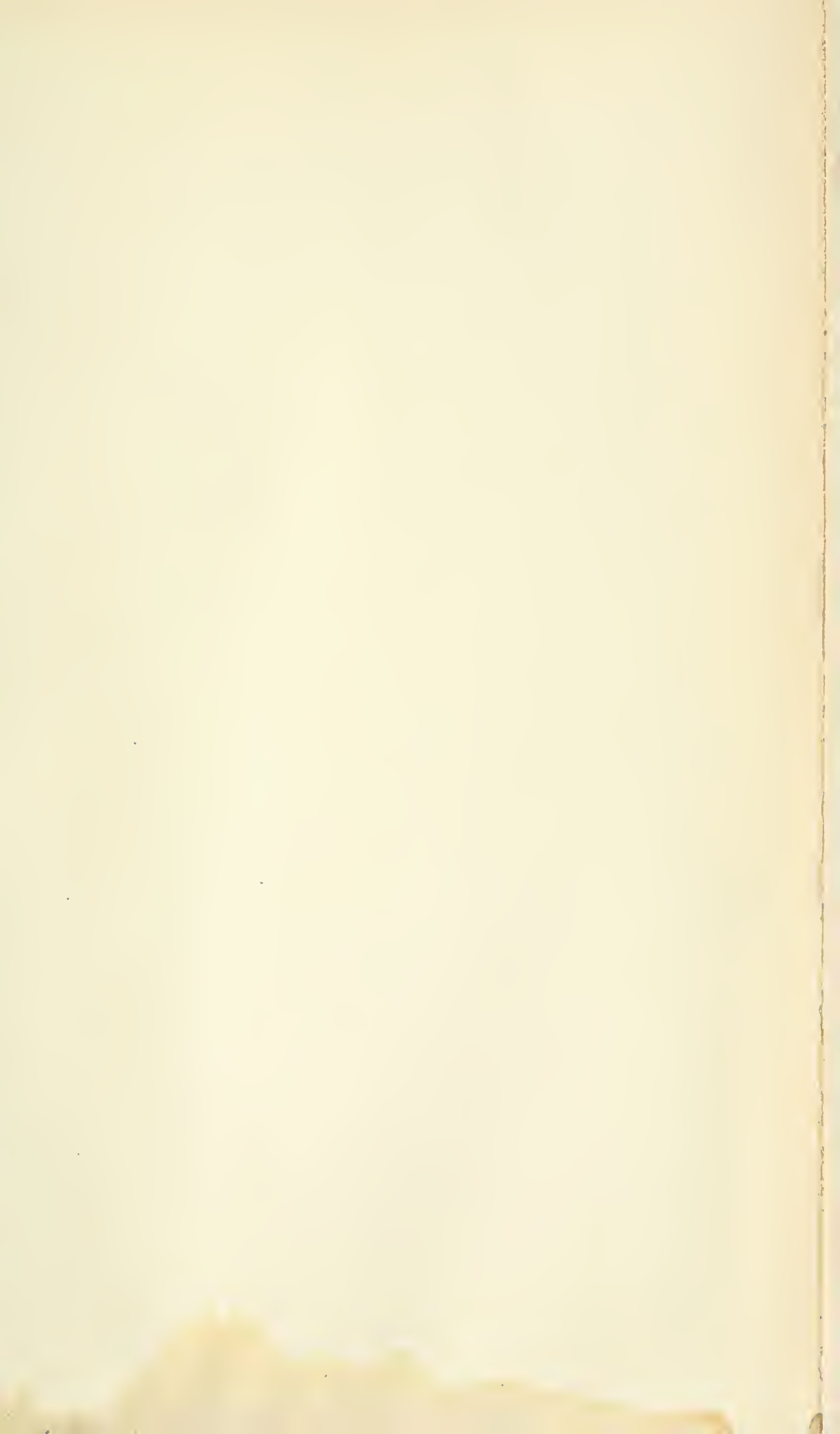
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